Bogoliubov Laboratory of Theoretical Physics

Search for new symmetries of hadron production in high energy collisions of protons and nuclei

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- Conservation law of fractal cumulativity
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Principles & Symmetries

"Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter and define the fundamental forces in nature."

Leon M. Lederman

"…for every conservation law there must be a continuous symmetry...."

Emmy Nöether

Discrete (C,P,T,..) and continuous symmetries correspond to fundamental principles (gauge, special, general and scale relativity, …) and conservation laws (charge,….) and vice versa.

- Principles are reflected as regularities in measurable observables and can be usually expressed as scaling in a suitable representation of data.
- z-Scaling of differential cross sections of inclusive particle production in p+p, p+A and A+A is used as a tool to search for and study of principles and symmetries that reflect properties of hadron interactions at constituent level.
- z-Scaling is based on the principles of *self-similarity*, *fractality*, *and locality*.

There exists a symmetry inherent to them:

Symmetry with respect to structural degrees of freedom - structural relativity.

Motivation & Goals

Search for new symmetries in Nature

Systematic analysis of inclusive cross sections of particle production in p+p, p+A and A+A collisions to search for general features of hadron and nucleus structure, constituent interaction and fragmentation process over a wide scale range

z-Scaling is a tool in high energy physics

Development of z-scaling approach for description of processes with unpolarized and polarized particle production in inclusive reactions and verification of fundamental physical principles of self-similarity, locality, fractality, maximal entropy, etc.

The suggested approach can be used to study

- Symmetry of constituent interactions at small scales
- Origin of flavor - u,d,s,c,b,t
- Origin of mass, spin, charge,…, fractal topology of space-time,…
- New phenomena in A+A in comparison with p+p

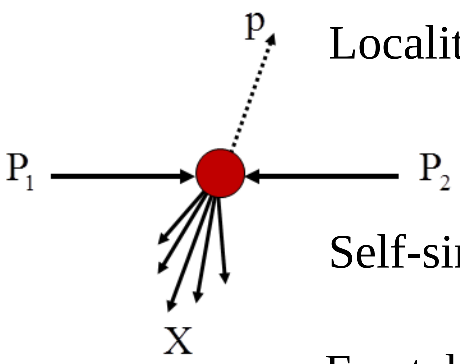
z-Scaling:
hypothesis, ideas, definitions, …

Basic principles:
locality, self-similarity, fractality,…

Int.J.Mod.Phys. A 27 (2012) 1250115
J.Mod.Phys. 3 (2012) 815
Int.J.Mod.Phys. A 32 (2017) 750029
Phys. Part. Nucl. 51 (2020) 141
Nucl.Phys. A 993 (2020) 121646
Nucl.Phys. A 1025 (2022) 122492

z-Scaling

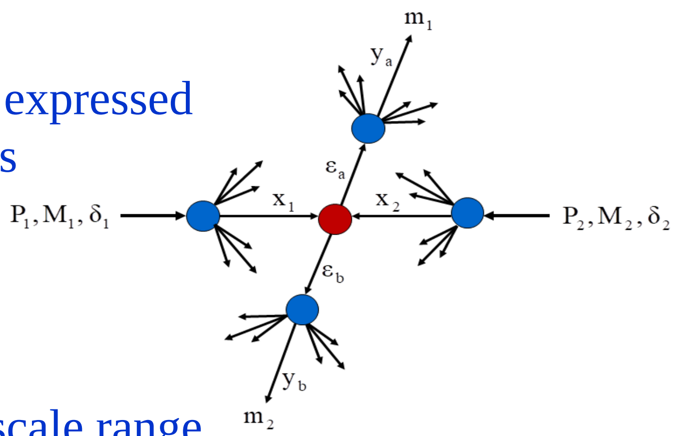
Principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).

Self-similarity: interactions of constituents are mutually similar.

Fractality: self-similarity is valid over a wide scale range.



Hypothesis of z-scaling :

$s^{1/2}$, p_T, θ_{cms}

Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

x_1, x_2, y_a, y_b

$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b, c$

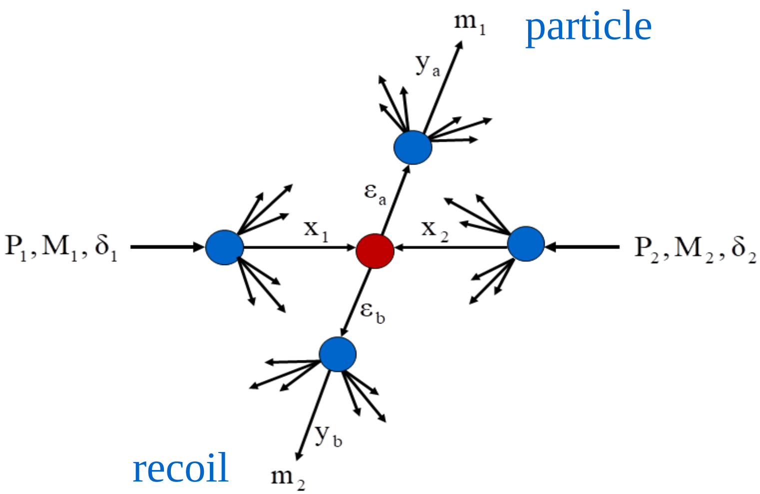
$Ed^3\sigma/dp^3$

Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable z.

$\Psi(z)$

Locality

Collisions of colliding objects are expressed via interactions of their constituents



P_1, P_2, p – momenta of colliding and produced particles

M_1, M_2, m_1 – masses of colliding and produced particles

x_1, x_2 – momentum fractions of colliding particles carried by constituents

y_a, y_b – momentum fractions of scattered constituents carried by inclusive particle and its recoil

δ_1, δ_2 – fractal dimensions of colliding particles

ε_a, ε_b – fractal dimensions of scattered constituents (fragmentation dimensions)

m_2 – mass of recoil particle

Elementary sub-process:

$$(x_1M_1) + (x_2M_2) \rightarrow (m_1/y_a) + (x_1M_1 + x_2M_2 + m_2/y_b)$$

Momentum conservation law for sub-process

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = M_X^2$$

Mass of recoil system

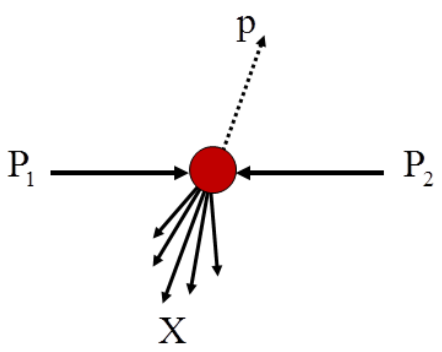
$$M_X = x_1M_1 + x_2M_2 + m_2/y_b$$

M.T., I.Zborovský
Yu.Panebratsev, G.Skoro
Phys.Rev.D54 5548 (1996)
Int.J.Mod.Phys.A16 1281 (2001)

Self-similarity

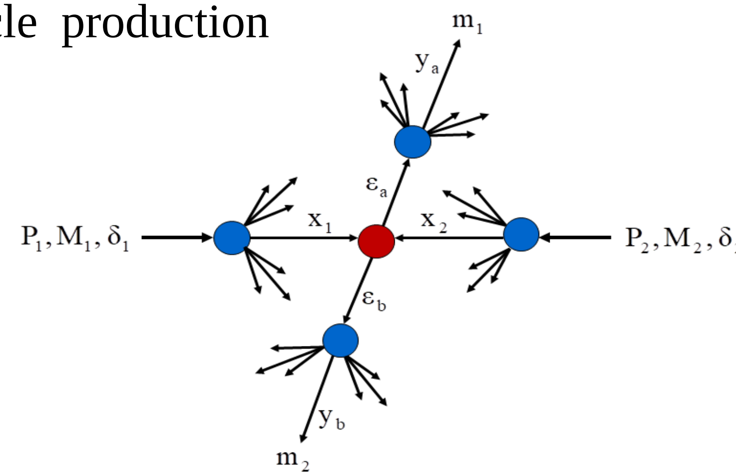
Interactions of constituents are mutually similar

The self-similarity parameter z is a dimensionless variable, expressed through the dimensional quantities $P_1, P_2, p, M_1, M_2, m_1, m_2$, characterizing the process of inclusive particle production



$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{ch}/d\eta|_0)^c m_N}$$



- Ω^{-1} – minimal resolution at which a constituent sub-process can be singled out of the inclusive reaction
- $\sqrt{s_\perp}$ – the transverse kinetic energy of the sub-process consumed on production of m_1 & m_2
- $dN_{ch}/d\eta|_0$ – multiplicity density of charged particles at $\eta = 0$
- c – parameter interpreted as a “specific heat” of created medium
- m_N – arbitrary constant (fixed at the value of nucleon mass)

Fractality

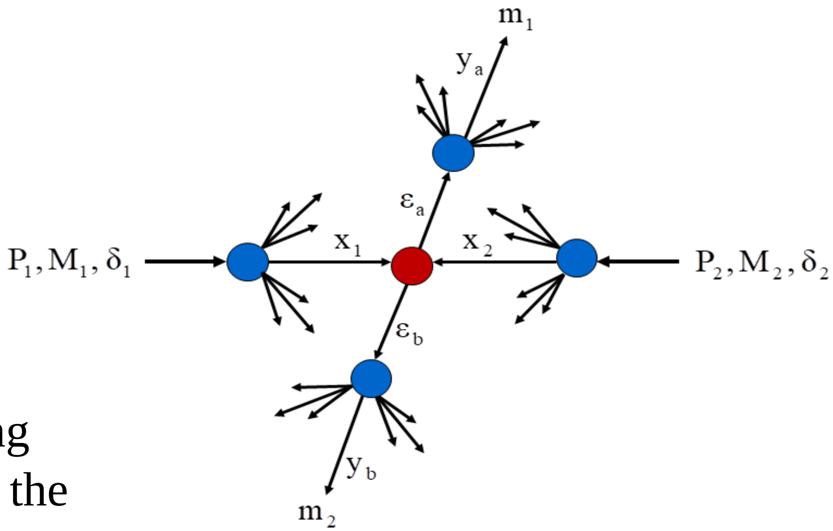
Self-similarity over a wide scale range

Fractal measure

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

$$0 < x_1, x_2 < 1 \qquad 0 < y_a, y_b < 1$$



Ω relative number of configurations containing a sub-process with fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

δ_1, δ_2, ε_a, ε_b parameters characterizing structure of the colliding objects and fragmentation process, respectively

$\Omega^{-1}(x_1, x_2, y_a, y_b)$ characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

The fractal measure z diverges as the resolution Ω^{-1} increases.

$$z(\Omega)|_{\Omega^{-1} \to \infty} \to \infty$$

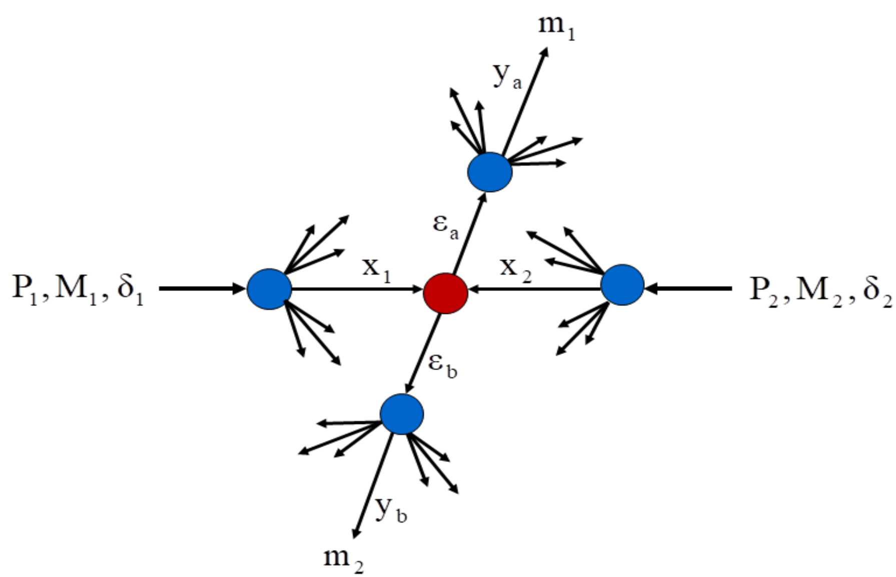
Minimal resolution of sub-process

Principle of minimal resolution: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to minimize the resolution Ω^{-1} of the fractal measure z with respect to all constituent sub-processes taking into account 4-momentum conservation law.

Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

$$\begin{cases} \partial\Omega/\partial x_1 \left.\right|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial x_2 \left.\right|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial y_b \left.\right|_{y_a=y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



Resolution of sub-process

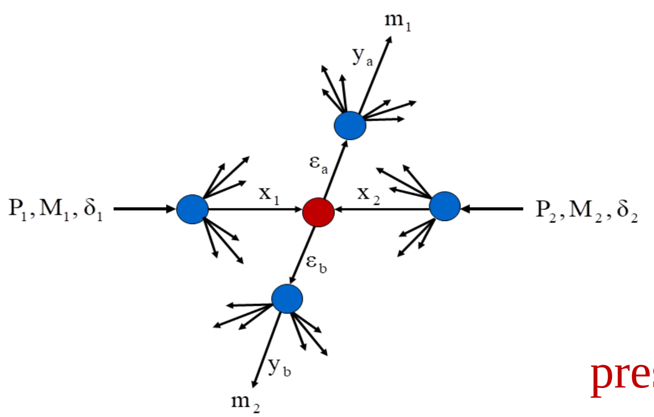
$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

Fractions x_1, x_2, y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.

Mass of the recoil system

$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

Scaling function $\Psi(z)$



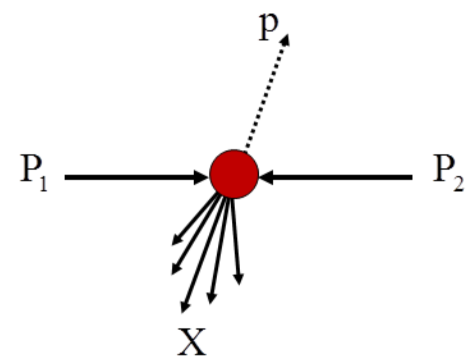
Normalization condition

$$\int_0^\infty \Psi(z)dz = 1$$

Scale transformation

$$z \to \alpha_F \cdot z \qquad \Psi \to \alpha_F^{-1} \cdot \Psi$$

preserves the normalization condition



$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3} \quad \Longleftrightarrow \quad \int E\frac{d^3\sigma}{dp^3} dy d^2p_\perp = \sigma_{inel} \cdot \langle N \rangle$$

- σ_{inel} - inelastic cross section
- $\langle N \rangle$ - average multiplicity
- $dN/d\eta$ - multiplicity density
- $J(z,\eta;p_T^2,y)$ - Jacobian
- $Ed^3\sigma/dp^3$ - inclusive cross section

The scaling function $\Psi(z)$ is a probability density to produce the inclusive particle with the corresponding value of self-similarity variable z.

Properties of $\Psi(z)$ in p+p collisions

- Energy independence of $\Psi(z)$ ($s^{1/2} > 20$ GeV)
- Angular independence of $\Psi(z)$ ($\theta_{cms}=3^0$-90^0)
- Multiplicity independence of $\Psi(z)$ ($dN_{ch}/d\eta=1.5$-26)
- Saturation of $\Psi(z)$ at low z ($z < 0.1$)
- Power law, $\Psi(z) \sim z^{-\beta}$, at high z ($z > 4$)
- Flavor independence of $\Psi(z)$ (π,K,φ,Λ,..,D,J/ψ,B,ϒ,…, top)

These properties reflect self-similarity, locality, and fractality of hadron interactions at a constituent level.

It concerns the structure of the colliding objects, constituent interactions and fragmentation process.

Some results of data analysis for unpolarized p+p collisions in z-scaling approach

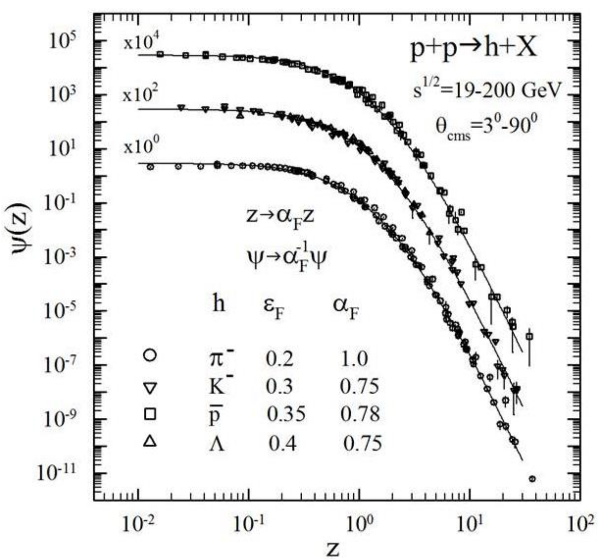
Self-similarity at RHIC

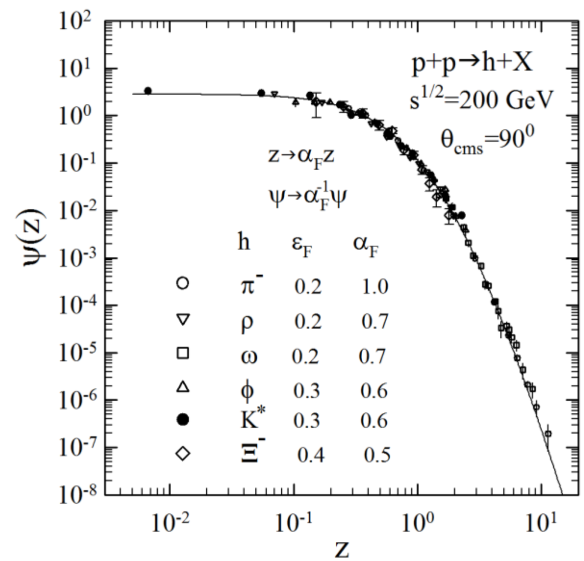
Flavor independence of scaling function

M.T. & I.Zborovský
Int.J.Mod.Phys.
A24,1417(2009)

π^-, ρ, ω, ϕ, K^*, Λ, Ξ, J/ψ

"Collapse" of data points onto a single curve





STAR:
PRL 92 (2004) 092301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901

PHENIX:
PRC 75 (2007) 051902

- Energy independence
- Angular independence
- Flavor independence
- Saturation for $z < 0.1$

- Power law $\Psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

Self-similarity of particle production with various flavor content.

Self-similarity of strangeness production in p+p

Universality: flavor independence of the scaling function

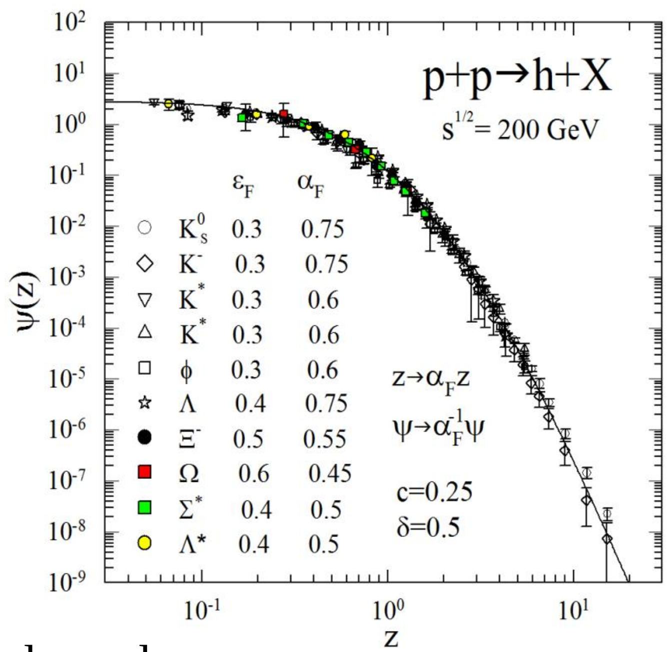
K_S^0, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

"Collapse" of data points onto a single curve

M.T.& I.Zborovský
Int.J.Mod.Phys.
A24,1417(2009)

Solid line for π^- meson is a reference frame

$\varepsilon_\pi = 0.2, \quad \alpha_\pi = 1$



STAR:
PRL 92 (2004) 092301
PRL 97 (2006) 132301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901
PRL 108 (2012) 072302

PHENIX:
PRC 75 (2007) 051902
PRD 83 (2011) 052004
PRC 90 (2014) 054905

- Energy independence
- Angular independence
- Flavor independence
- Saturation for $z < 0.1$

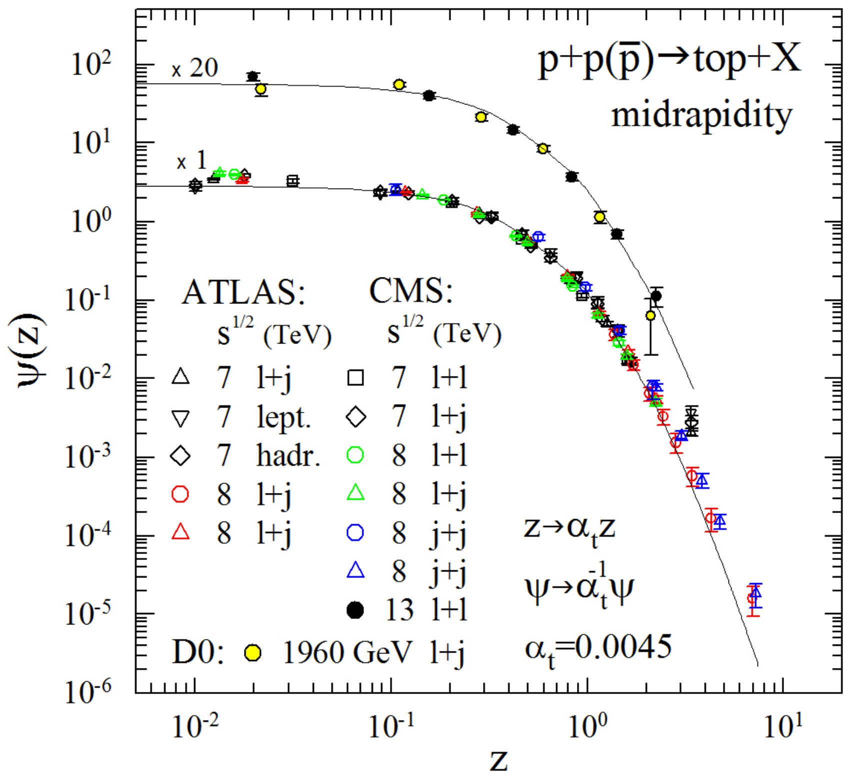
- Power law $\Psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

Self-similarity of top quark production at LHC & Tevatron

$s^{1/2}$ (GeV)

- ● 1960 PRD90(2014)092006
- △ 7000 PRD90(2014)072004
- ▽ 7000 JHEP06(2015)100
- ◇ 7000 JHEP06(2015)100
- □ 7000 EPJC73(2013)2339
- ◇ 7000 EPJC73(2013)2339
- ○ 8000 PRD93(2016)032009
- △ 8000 arXiv:1511.04716
- ○ 8000 EPJC75(2015)542
- △ 8000 EPJC75(2015)542
- ○ 8000 EPJC76(2016)128
- △ 8000 arXiv:1605.00116
- ◆ 13000 CMS TOP 16-011

"Collapse" of data points onto a single curve



- ➢ Energy independence of $\Psi(z)$
- ➢ Flavor independence of $\Psi(z)$
- ➢ Saturation of $\Psi(z)$ for or $z < 0.1$
- ➢ Fractal dimensions $\delta = 0.5$, $\varepsilon_{top} = 0$
- ➢ "Specific heat" $c = 0.25$

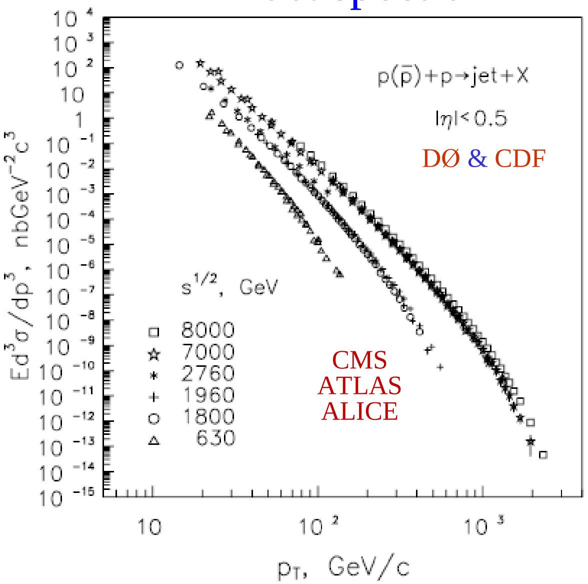
J. Mod. Phys. 32, 815 (2012)
ISMD'16, Jeju Island, South Korea, 2016

LHC & Tevatron data
confirm self-similarity
of top quark production in pp & p$\bar{p}$

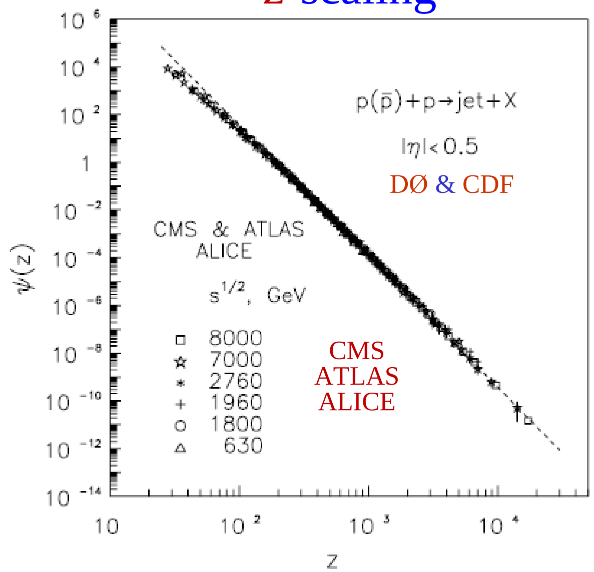
Self-similarity of jet production at Tevatron and LHC

Energy $\sqrt{s} = 8$ TeV up to the momentum $p_T \approx 2.4$ TeV/c and scale ~ $8 \cdot 10^{-5}$ fm

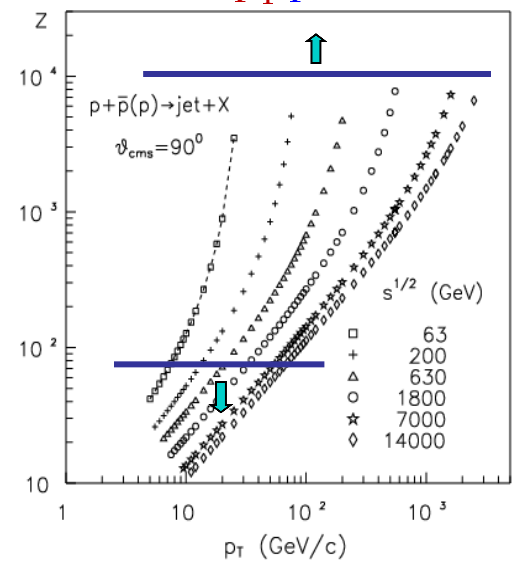
Jet spectra



z-scaling



z-p_T plot



Test of z-scaling at LHC

M.Tokarev, T.Dedovich, I.Z.
Int.J.Mod.Phys.A15 (2000) 3495
Int.J.Mod.Phys.A27 (2012)1250115

Structural phenomena ⟺ constituent substructure,…
Self-similarity at small scales ⟺ fractal topology of momentum space,…

Phys. Part. Nucl. 51 (2020) 141

What about z-scaling hypothesis for processes with polarized protons ?

F.Lehar

Some results of data analysis for polarized p+p collisions in z-scaling approach

Phys. Part. Nucl. Lett., 12 (2015) 81
Phys. Part. Nucl. Lett., 12 (2015) 214

Workshop "Physics programme for the first stage of the NICA SPD experiment",
JINR, Dubna, Russia, October 5-6, 2020

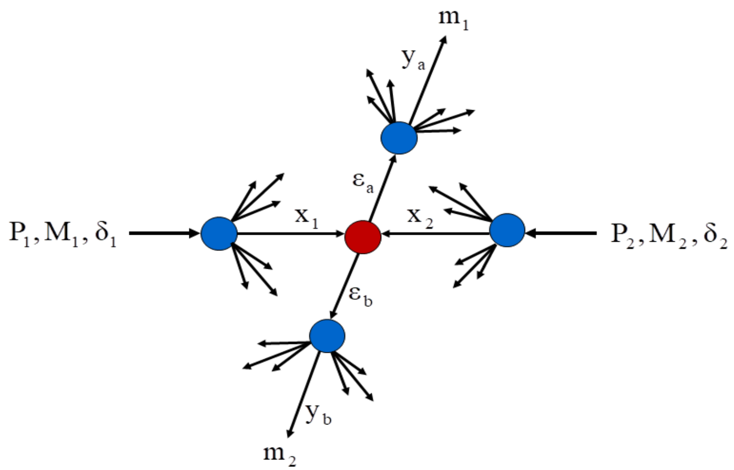
Spin-dependent version of z-scaling

Self-similarity of spin structure

z-scaling for processes with polarized particles

Inclusive spin-dependent particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

$s^{1/2}$, p_T, θ_{cms}
spin



spin-dependent fractions
x_1, x_2, y_a, y_b

spin-dependent dimensions
$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$

spin-dependent cross section
$Ed^3\sigma/dp^3$

Scaled spin-dependent inclusive cross section of particle production depends in a self-similar way on a single spin-dependent scaling variable z.

spin-dependent
z & Ψ

Universality of the shapes of $\Psi(z)$ for spin-dependent processes

Self-similarity in processes with polarized protons

New hypothesis:

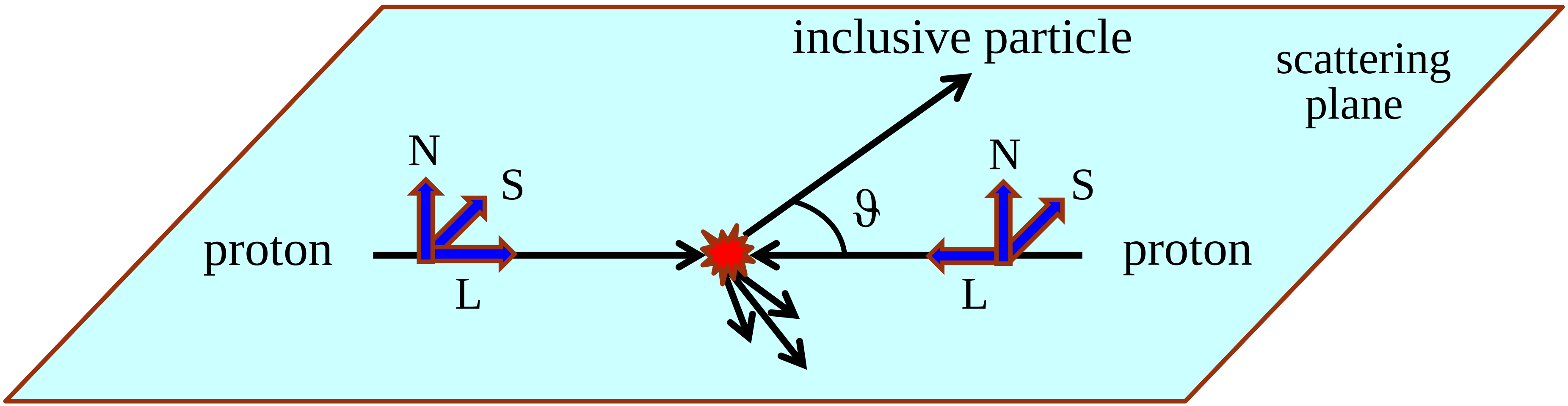
- Self-similarity of spin structure
- Fractality of proton spin
- Spin-dependent fractal dimensions

L, N, S represent the unit vectors along spin directions of initial particles

L is along the incident momentum

N is along the normal to the scattering plane

S is along N×L



Double spin asymmetry

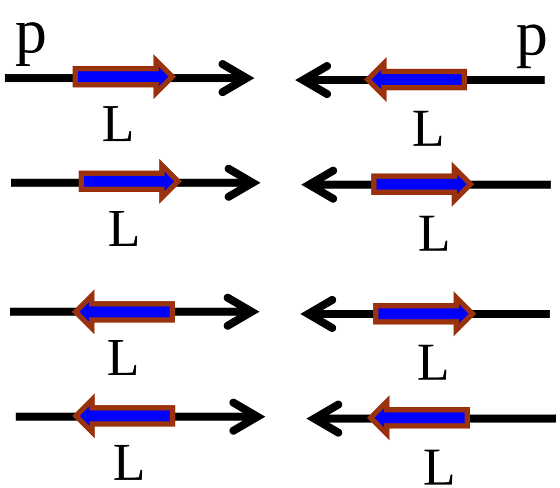
$$p^{\uparrow} + p^{\downarrow} \to h + X \qquad p^{\rightarrow} + p^{\leftarrow} \to h + X \qquad p^{\rightarrow} + p^{\uparrow} \to h + X$$

$$A_{NN} \qquad A_{LL} \qquad A_{LN}$$

Double spin asymmetry of pion production in pp

$$\vec{p} + \vec{p} \to \pi + X$$

$$A_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}$$



$\equiv \sigma_{++}$

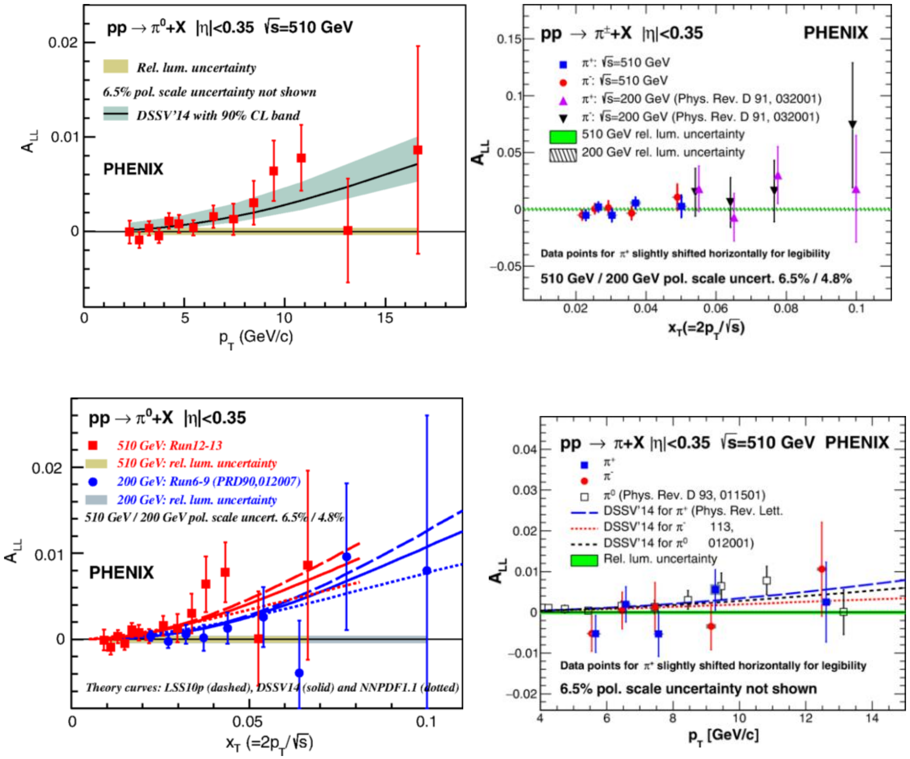
$\equiv \sigma_{+-}$

$\equiv \sigma_{--}$

$\equiv \sigma_{-+}$

$$\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+} = 4\sigma_{00}$$

STAR & PHENIX at RHIC



PHENIX Collaborartion
Adare A. et al. Phys. Rev. D 90 (2014) 012007
Adare A. et al. Phys. Rev. D 93 (2016) 011501
Acharya U. et al. Phys. Rev. D 102 (2020) 032001
RHIC SPIN Collaboration
Arschenauer E.C. et al. nucl-ex:1304.0079

Self-similarity of spin-dependent processes

Hypothesis of spin independence:

$$\Psi_{++} = \Psi_{+-} = \Psi_{00}$$

$$\vec{p} + \vec{p} \to \pi + X$$

$$\Psi_{++} \stackrel{\text{def}}{=} \Psi(z_{++}),\ \Psi_{+-} \stackrel{\text{def}}{=} \Psi(z_{+-}),\ \Psi_{00} \stackrel{\text{def}}{=} \Psi(z_{00})$$

$$z_{++} = z_0 \cdot \Omega^{-1}_{++00}\ ,\ldots$$

$$\Omega = (1\text{-}x_1)^{\delta_1}(1\text{-}x_2)^{\delta_2}(1\text{-}y_a)^{\varepsilon_F}(1\text{-}y_b)^{\varepsilon_F}$$

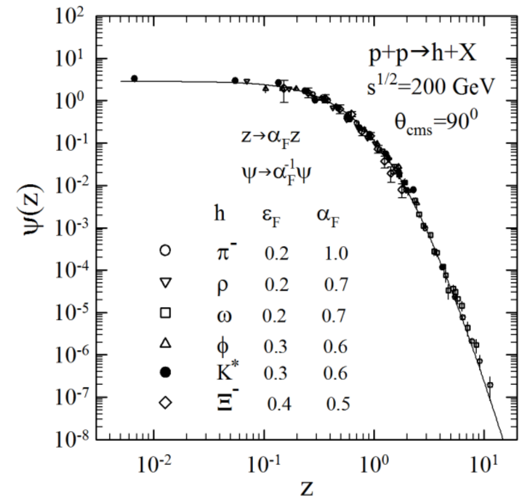
$$\Omega_{0000} =: \{\delta, \delta, \varepsilon_F, \varepsilon_F\}$$

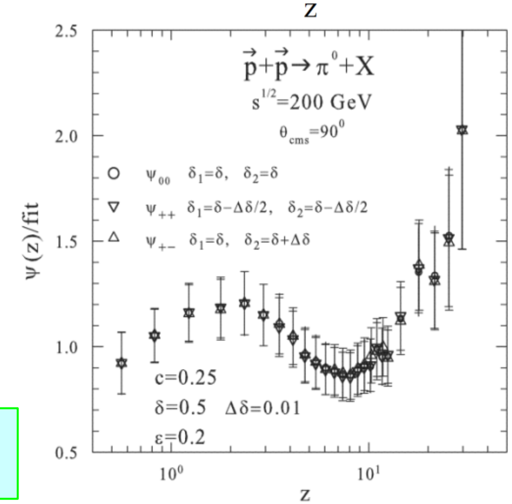
$$\Omega_{++00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\}$$

$$\Omega_{--00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\}$$

$$\Omega_{-+00} =: \{\delta + \Delta\delta, \delta, \varepsilon_F, \varepsilon_F\}$$

$$\Omega_{+-00} =: \{\delta, \delta + \Delta\delta, \varepsilon_F, \varepsilon_F\}$$





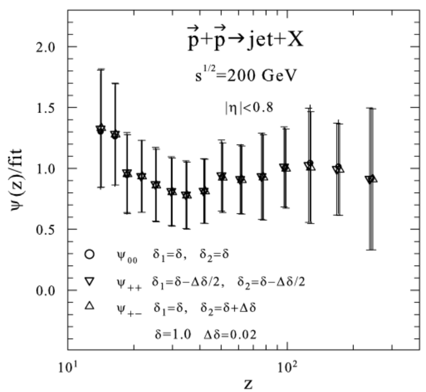
Spin correction to fractal dimension: δ, Δδ

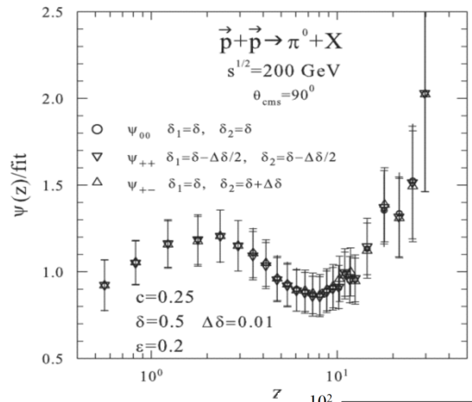
Self-similarity of spin-dependent processes

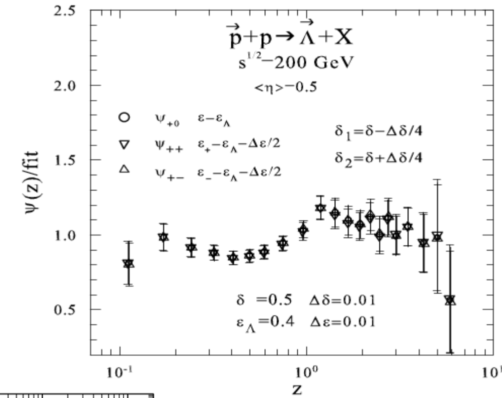
$\vec{p} + \vec{p} \rightarrow jet + X$

$\vec{p} + \vec{p} \rightarrow \pi + X$

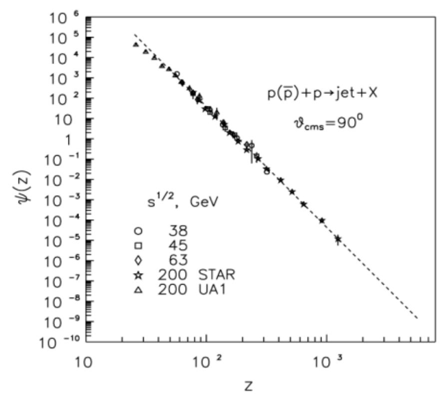
$\vec{p} + p \rightarrow \vec{\Lambda} + X$

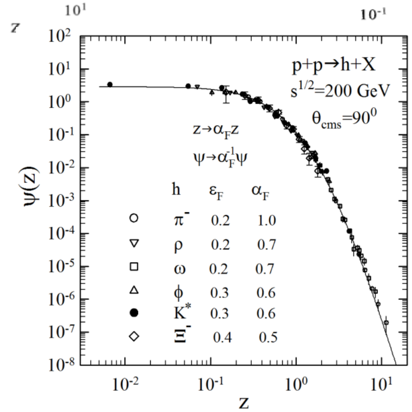






Phys. Part. Nucl. Lett., 12 (2015) 81
Phys. Part. Nucl. Lett., 12 (2015) 214





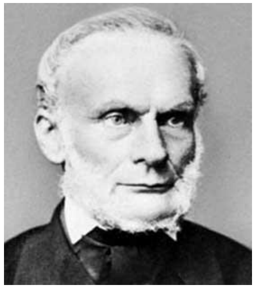
- Self-similarity and fractality of proton spin
- New insight into understanding of space-time origin of spin

Fractal entropy of nuclear system
&
self-similarity variable z

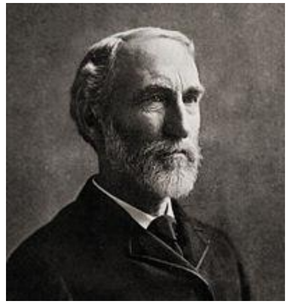
Physics 5 (2023) 537
Phys. Part. Nucl. 54 (2023) 640
Nucl. Phys. A1025 (2022) 122492
Nucl. Phys. A993 (2020) 121646

Entropy – basis notion of thermodynamics and stat. physics

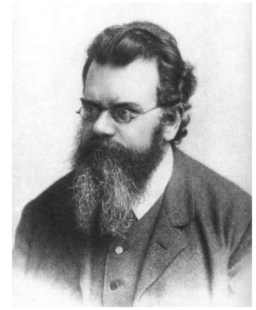
Thermodynamics ἐντροπία Statistical physics



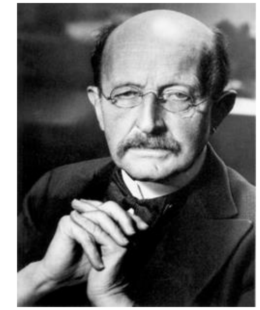
Rudolf Julius Emanuel Clausius



Josiah Willard Gibbs



Ludwig Eduard Boltzmann



Max Karl Ernst Ludwig Planck

Entropy is a function of state

$$dS = dU/T + pdV/T - \mu dN/T$$

$$S = S(U,V,N), \quad T=T(U,V,N)$$

Thermodynamic quantities and potentials are expressed via entropy

$$U=TS-pV+\mu N \qquad 1/T = \partial S/\partial U|_{V,N}$$

$$H=U+pV \qquad p/T = \partial S/\partial V|_{U,N}$$

$$F=U-TS \qquad c_V = T\partial S/\partial T|_V$$

$$G=U+pV-TS \qquad \partial p/\partial T|_{V,N} = \partial S/\partial V|_{T,N}$$

$$\Omega=U-TS-\mu N \qquad \partial V/\partial T|_{p,N} = - \partial S/\partial p|_{T,N}$$

$$S = k \cdot \ln W$$

k - Boltzmann constant

W - number of microstate

Various forms of entropy:

Clausius	(1865)	S_C
Gibbs	(1876)	S_G
Boltzmann	(1872)	S_B
Sharma, Mittal	(1975)	S_{SM}
Tsallis	(1988)	S_q
Kaniadakis	(2001)	S_k
………		

von Neumann	(1932)	
Shannon	(1948)	S_S
Kolmogorov	(1954)	
Khinchin	(1957)	
Rényi	(1961)	S_R
……….		

Properties of entropy

- Entropy is function of state of a (thermodynamic) system.
- Entropy is smooth function of thermodynamic parameters.
- For reversible processes $\oint dS = 0$.
- Basic concept in 2nd and 3d laws of thermodynamics.
- Entropy of phase transition.
- Entropy of extensive and non-extensive systems.
- Fractal entropy - entropy of systems with fractal objects.
- Entropy in quantum statistical mechanics.
- Entropy of Entanglement, Black Hole, Big Band, Universe, …
- Entropy and information content of the human genome, …
- ……...

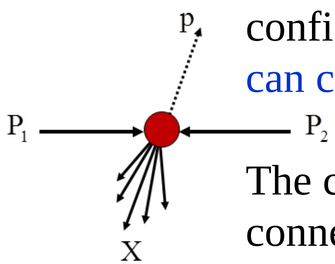
Entropy of nuclear system produced in p+p & A+A collisions

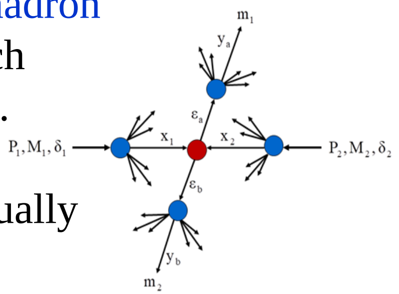
According to statistical physics, entropy of a system is given by a number Ws of its statistical states:

$$S = \ln W_S$$

The most likely configuration of the system is given by the maximal value of S.

For inclusive reactions, the quantity Ws is a number of all parton and hadron configurations in the initial and final states of the colliding system which can contribute to the production of inclusive particle with momentum p.





The configurations comprise all constituent configurations that are mutually connected by independent binary subprocesses:

$$(x_1M_1)+(x_2M_2) \to (m_a/y_a)+(x_1M_1+x_2M_2+m_b/y_b)$$

The subprocesses corresponding to the production of the inclusive particle with the 4-momentum p are subject to the momentum conservation law:

$$(x_1P_1+x_2P_2-p/y_a)^2=(x_1M_1+x_2M_2+m_b/y_b)^2$$

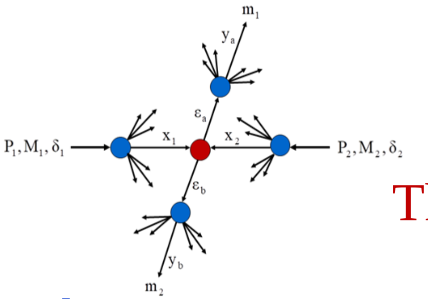
The underlying subprocess, which defines the variable z, is singled out from the corresponding subprocesses by the principle of maximal entropy S.

Self-similarity variable z & Fractal entropy $S_{\delta,\varepsilon}$

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{ch}/d\eta|_0)^c m_N}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$



The quantity W_S is a number of all parton and hadron configurations in the initial and final states of the colliding system which can contribute to the production of inclusive particle with momentum p

$$z = \frac{\sqrt{s_\perp}}{W}$$

$$W_S = W \cdot W_0 = (dN_{ch}/d\eta|_0)^c \cdot \Omega \cdot W_0$$

Statistical entropy

$$S = \ln W_S$$

Thermodynamical entropy for ideal gas

$$S = c_V \ln T + R \ln V + S_0$$

Fractal entropy for independent processes

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln(V_{\delta,\varepsilon}) + \ln W_0$$

Entropy $S_{\delta,\varepsilon}$ for systems with structural constituents

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + \ln W_0$$

- $dN_{ch}/d\eta|_0$ characterizes "temperature" of the colliding system.
- c has meaning of a "specific heat" of the produced medium.
- Fractional exponents $\delta_1,\delta_2,\varepsilon_a,\varepsilon_b$ are fractal dimensions in the space of $\{x_1,x_2,y_a,y_b\}$.
- $V_{\delta,\varepsilon} = \Omega$ is fractal volume in the space of momentum fraction.

Maximum entropy principle & New conservation law

Principle of maximal entropy:

The momentum fractions x_1, x_2, y_a, y_b are determined in a way to maximize the entropy $S_{\delta,\varepsilon}$ with a kinematic constraint (momentum conservation law).

Maximum of $S_{\delta,\varepsilon}$

$$\begin{cases} \partial\Omega/\partial x_1 = 0 & \partial\Omega/\partial y_a = 0 \\ \partial\Omega/\partial x_2 = 0 & \partial\Omega/\partial y_b = 0 \end{cases}$$

Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

Mass of recoil system

$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

Equivalence of minimal resolution and maximal entropy principle

Conservation law

$$\delta_1\frac{x_1}{1-x_1}+\delta_2\frac{x_2}{1-x_2}=\varepsilon_a\frac{y_a}{1-y_a}+\varepsilon_b\frac{y_b}{1-y_b}$$

for arbitrary $P_1, P_2, p, \delta_1, \delta_2, \varepsilon_a, \varepsilon_b$!!!

The conservation law corresponds to maximum of fractal entropy $S_{\delta,\varepsilon}$

Resolution wrt. to constituent sub-processes

$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

I.Zborovsky & MT
Int. J. Mod. Phys. A 33, 1850057 (2018)
ICHEP 2020, Prague, July 28 –August 6

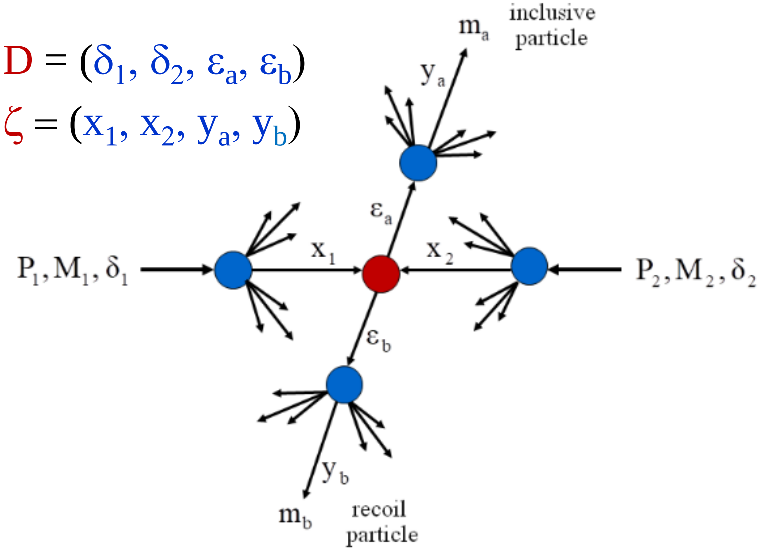
Conservation law for fractal cumulativity $C(D, \zeta)$

"Fractal cumulativity"

$$C(D,\zeta) = D \cdot \frac{\zeta}{1-\zeta}$$

$D = (\delta_1, \delta_2, \varepsilon_a, \varepsilon_b)$

$\zeta = (x_1, x_2, y_a, y_b)$



The fractal cumulativity before a constituent interaction is equal to the fractal cumulativity after a constituent interaction for any binary constituent sub-process

$$\sum_{i}^{in} C(D_i, \zeta_i) = \sum_{j}^{out} C(D_j, \zeta_j)$$

We assume that

- every physical particle is a structural one
- particle's constituents possess a fractal-like structure
- fragmentation is a fractal-like process
- compactness of the fractal structures is governed by the Heisenberg uncertainty principle

Fractal cumulativity $C(D, \zeta)$ is a property of a fractal-like object (or fractal-like process) with fractal dimension D to form a local compact "structural aggregate" - a FRACTALON, which carries the fraction ζ of momentum of its parent fractal.

Fractal entropy $S_{\delta,\varepsilon}$ near a fractal limit $\Omega^{-1} \to \infty$

Entropy decomposition near a fractal limit

$$S_{\delta,\varepsilon} = S_{\Upsilon} - S_{\Gamma} + S_0$$

S_{Υ} depends on momenta and masses of the colliding and inclusive particles

S_0 is a constant guaranteeing positivity of $S_{\delta,\varepsilon}$

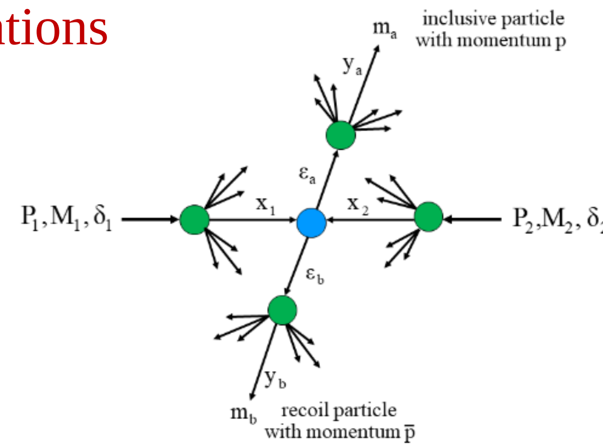
S_{Γ} depends *solely* on fractal dimensions

$$S_{\Gamma} = (\delta_1+\delta_2+\varepsilon_a+\varepsilon_b)\ln(\delta_1+\delta_2+\varepsilon_a+\varepsilon_b) - \delta_1\ln\delta_1 - \delta_2\ln\delta_2 - \varepsilon_a\ln\varepsilon_a - \varepsilon_b\ln\varepsilon_b$$

S_{Γ} enters with minus sign in the decomposion and diminishes the fractal entropy $S_{\delta,\varepsilon}$

Entropy S_Γ of a statistical ensemble

Statistical ensemble of interacting fractal configurations



Large collection of the interacting fractals

– with random configurations $\{x_1, x_2, y_a, y_b, \ldots\}$

– with the same fractal dimensions $\{\delta_1, \delta_2, \varepsilon_a, \varepsilon_b\}$

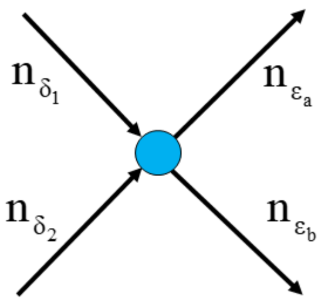
Number of configurations

n_{δ_1} – internal structure of M_1

n_{δ_2} – internal structure of M_2

n_{ε_a} – fragmentation process to m_a

n_{ε_b} – fragmentation process to m_b



Statistical weight

$$\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} \equiv \frac{\left(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!\,n_{\delta_2}!\,n_{\varepsilon_a}!\,n_{\varepsilon_b}!}$$

The statistical ensemble is considered as a collection of n_{δ_1} fractals with random configurations but with the same fractal dimension δ_1, together with an analogous set of n_{δ_2} interacting fractals with the fractal dimension δ_2, which recombined via binary sub-processes with the collection of n_{ε_a} fractals with random configurations but with the same fractal dimension ε_a in the final state, and the corresponding set of n_{ε_b} fractals with the fractal dimension ε_b.

Entropy of the whole statistical ensemble

$$S_\Gamma = d \cdot \ln\left(\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b}\right)$$

Statistical interpretation of entropy S_Γ

The entropy S_Γ can be presented as logarithm of the number of different ways to share identical dimensional quanta d among fractal dimensions of the interacting fractal structures.

$$S_\Gamma = d \cdot \ln\left(\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b}\right)$$

$$\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} \equiv \frac{\left(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!n_{\delta_2}!n_{\varepsilon_a}!n_{\varepsilon_b}!} = \Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b} \cdot \Gamma_{\delta_1,\delta_2} \cdot \Gamma_{\varepsilon_a,\varepsilon_b}$$

$$\Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b} = \frac{(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b})!}{(n_{\delta_1}+n_{\delta_2})!(n_{\varepsilon_a}+n_{\varepsilon_b})!} \qquad \Gamma_{\delta_1,\delta_2} = \frac{\left(n_{\delta_1}+n_{\delta_2}\right)!}{n_{\delta_2}!\ n_{\delta_3}!} \qquad \Gamma_{\varepsilon_a,\varepsilon_b} = \frac{\left(n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\varepsilon_a}!\ n_{\varepsilon_b}!}$$

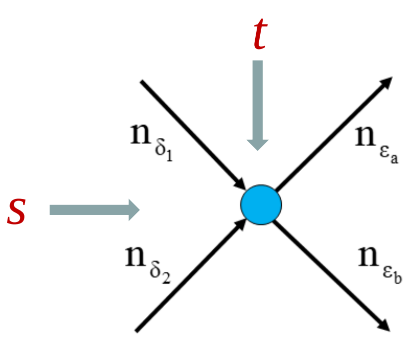
Such interpretation of the entropy S_Γ within statistical ensemble of fractal configurations of the internal structures of the colliding hadrons (or nuclei) and fractal configurations corresponding to the fragmentation processes in the final state is only possible if quantization of fractal dimensions takes place:

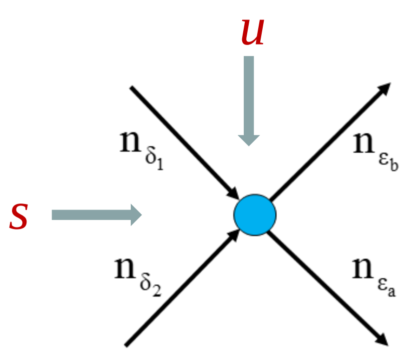
$$\delta_1 = d \cdot n_{\delta_1}\ ,\ \ \delta_2 = d \cdot n_{\delta_2}\ ,\ \ \varepsilon_a = d \cdot n_{\varepsilon_a}\ ,\ \ \varepsilon_b = d \cdot n_{\varepsilon_b}$$

d – quant of fractal dimension

$n_{\delta_1}, n_{\delta_2}, n_{\varepsilon_a}, n_{\varepsilon_b}$ – quantum numbers of fractal dimension

Crossing symmetry for entropy S_Γ





Statistical weight

$$S_\Gamma = d\cdot\ln\left(\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b}\right) \qquad \Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} \equiv \frac{\left(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!n_{\delta_2}!n_{\varepsilon_a}!n_{\varepsilon_b}!}$$

s - channel

$$\boxed{\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} = \Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b}\cdot\Gamma_{\delta_1,\delta_2}\cdot\Gamma_{\varepsilon_a,\varepsilon_b}}$$

$$\Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b} = \frac{(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b})!}{(n_{\delta_1}+n_{\delta_2})!(n_{\varepsilon_a}+n_{\varepsilon_b})!}$$

$$\Gamma_{\delta_1,\delta_2} = \frac{\left(n_{\delta_1}+n_{\delta_2}\right)!}{n_{\delta_1}!\;n_{\delta_2}!}$$

$$\Gamma_{\varepsilon_a,\varepsilon_b} = \frac{\left(n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\varepsilon_a}!\;n_{\varepsilon_b}!}$$

t - channel

$$\boxed{\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} = \Gamma_{\delta_1,\varepsilon_a;\delta_2,\varepsilon_b}\cdot\Gamma_{\delta_1,\varepsilon_a}\cdot\Gamma_{\delta_2,\varepsilon_b}}$$

$$\Gamma_{\delta_1,\varepsilon_a;\delta_2,\varepsilon_b} = \frac{(n_{\delta_1}+n_{\varepsilon_a}+n_{\delta_2}+n_{\varepsilon_b})!}{(n_{\delta_1}+n_{\varepsilon_a})!(n_{\delta_2}+n_{\varepsilon_b})!}$$

$$\Gamma_{\delta_1,\varepsilon_a} = \frac{\left(n_{\delta_1}+n_{\varepsilon_a}\right)!}{n_{\delta_1}!\;n_{\varepsilon_a}!}$$

$$\Gamma_{\delta_2,\varepsilon_b} = \frac{\left(n_{\delta_2}+n_{\varepsilon_b}\right)!}{n_{\delta_2}!\;n_{\varepsilon_b}!}$$

u - channel

$$\boxed{\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} = \Gamma_{\delta_1,\varepsilon_b;\delta_2,\varepsilon_a}\cdot\Gamma_{\delta_1,\varepsilon_b}\cdot\Gamma_{\delta_2,\varepsilon_a}}$$

$$\Gamma_{\delta_1,\varepsilon_b;\delta_2,\varepsilon_a} = \frac{(n_{\delta_1}+n_{\varepsilon_b}+n_{\delta_2}+n_{\varepsilon_a})!}{(n_{\delta_1}+n_{\varepsilon_b})!(n_{\delta_2}+n_{\varepsilon_a})!}$$

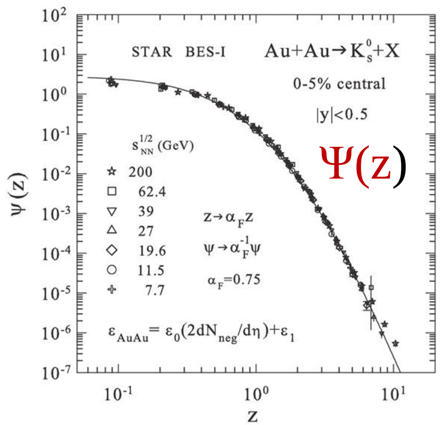
$$\Gamma_{\delta_1,\varepsilon_b} = \frac{\left(n_{\delta_1}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!\;n_{\varepsilon_b}!}$$

$$\Gamma_{\delta_2,\varepsilon_a} = \frac{\left(n_{\delta_2}+n_{\varepsilon_a}\right)!}{n_{\delta_2}!\;n_{\varepsilon_a}!}$$

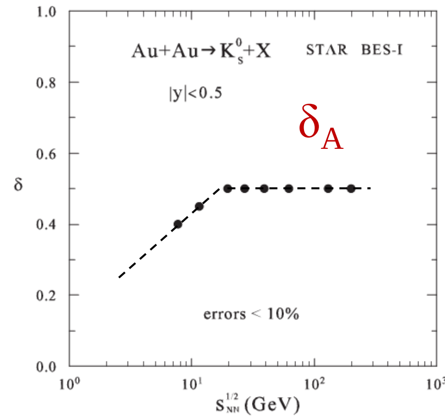
Crossing symmetry for entropy S_Γ in terms of quantum numbers n_i

Features of K^0_S meson production in Au+Au at RHIC

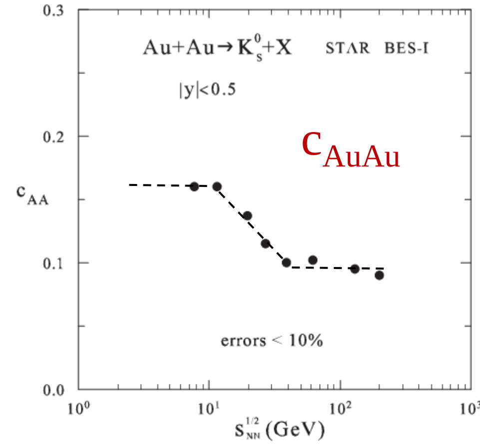
"Collapse" of data points onto a single curve



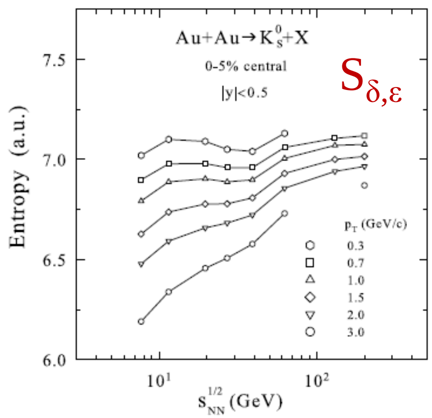
Additivity of nucleus fractal dimension



Anomalous behavior of specific heat



Anomaly of fractal entropy in central Au+Au collsions

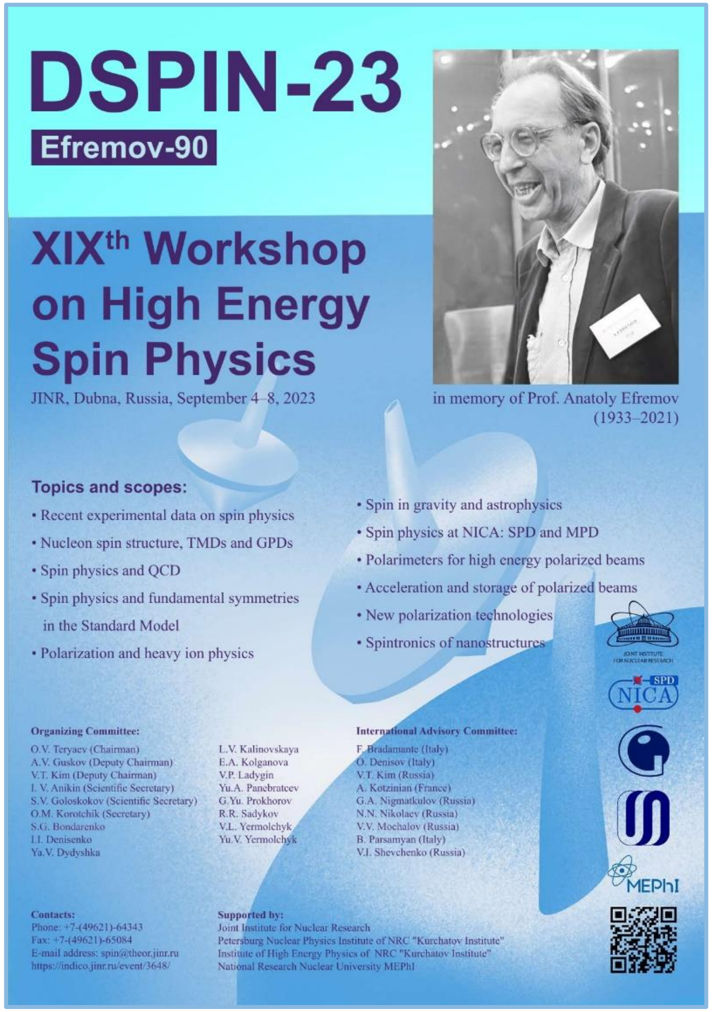


Results of analysis:

- Self-similarity and fractality over a wide range of energy, centrality, p_T were established.
- Fractal entropy vs. energy, centrality, p_T was studied.
- Anomalous behavior of $S_{\delta,\varepsilon}$ in central Au+Au collisions at small p_T was found.
- Constancy of fractal dimension δ_A at high energy was found.
- Abrupt decrease of specific heat c_{AuAu} in the range $\sqrt{s_{NN}}$ = 11.5-39 GeV was observed.

Summary

- Brief review of z-scaling was given.
- Principles of self-similarity, locality, and fractality were discussed.
- Correspondence of the principles of minimal resolution and maximal entropy was shown.
- Fractal entropy introduced in z-scaling approach was discussed.
- Conservation law of fractal cumulativity was formulated.
- Quantization of fractal dimensions was shown.
- A hypothesis of self-similarity of proton spin was formulated.
- Method of data analysis based on z-scaling was justified for description of processes with polarized protons.



Thank You for Your Attention !