Simulation of spin effects with ultracold gases in optical traps

Vladimir S. Melezhik

Bogoliubov Laboratory of Theoretical Physics JINR, Dubna

XIXth Workshop on High Energy Spin Physics
(dedicated to 90th anniversary of A.V. Efremev birth)
Dubna, 4-8 September 2023

Outline

- Quantum simulations: why cold atoms ?
- Solid state physics: modeling matter phase-transitions
- Simulations with degenerate quantum gases
- Dipole confinement-induced resonances in traps

- Outlook, goals and opportunities

Quantum simulations: why cold atoms ?

R.Feynman's vision: a quantum simulator
to study the quantum dynamics of another system

R. Feynman, Int. J. Theor. Phys. 21, 467 (1982)

Y. Manin, Computable and Uncomputable (Sovetskoye Radio Press, Moscow) (in Russian) 1980.

development of physics of ultracold atoms has opened unique possibility for realisation of R. Feynman's idea:

to use simple quantum systems with desiered properties
(amenable quantitative description and modeling)
to describe more complex systems and phenomena

Quantum simulations: why cold atoms ?

Quantum simulation with fully controlled systems

control over: particle number

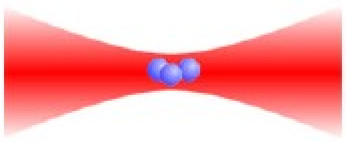
quantum state

interaction

Quantum simulations: why cold atoms ?

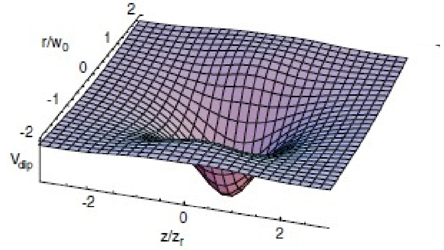
control over: particle number

the focus of a laser beam

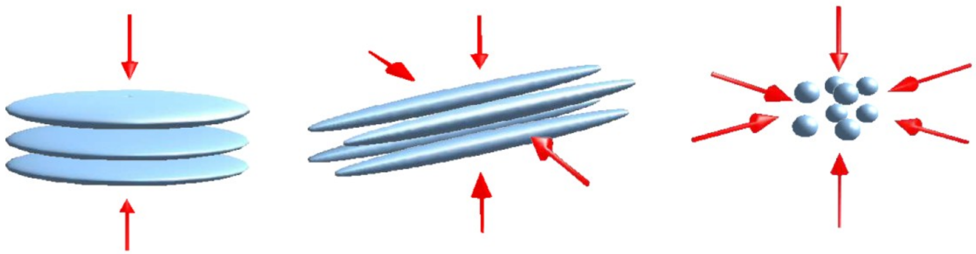


$$d = \alpha(\omega)E.$$

Optical dipole trap



$$V_{dip} = -\frac{1}{2}\langle \mathrm{dE} \rangle = -\frac{1}{2\epsilon_0 c}\mathrm{Re}(\alpha)I$$

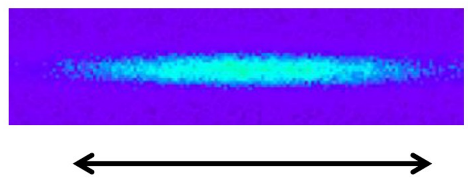


Lattices formed by applying orthogonal standing waves in one, two, and three directions.

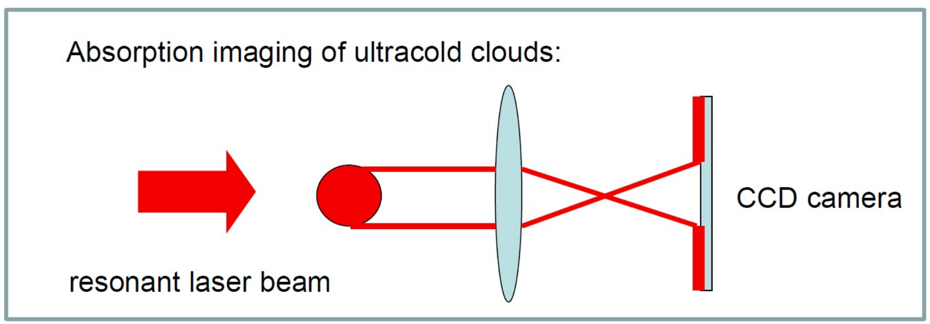
Quantum simulations: why cold atoms ?

control over: particle number

About 50000 atoms @ 250nK, T_F~1µK

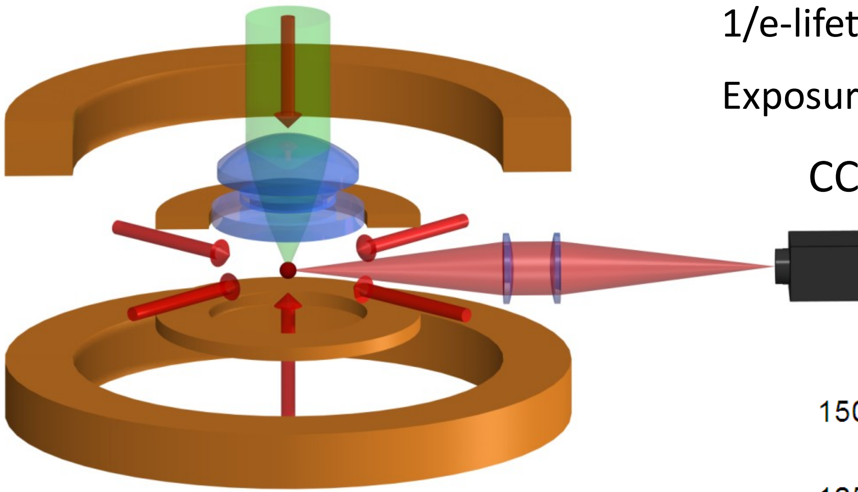


~100µm



Quantum simulations: why cold atoms ?

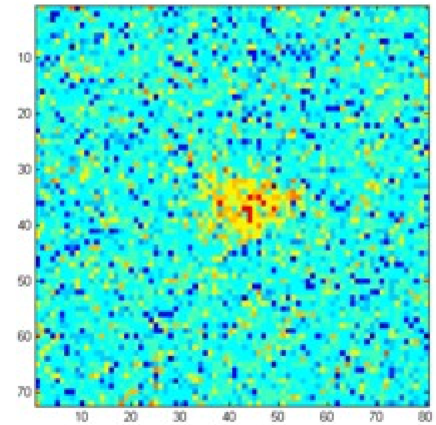
control over: particle number



1/e-lifetime: 250s

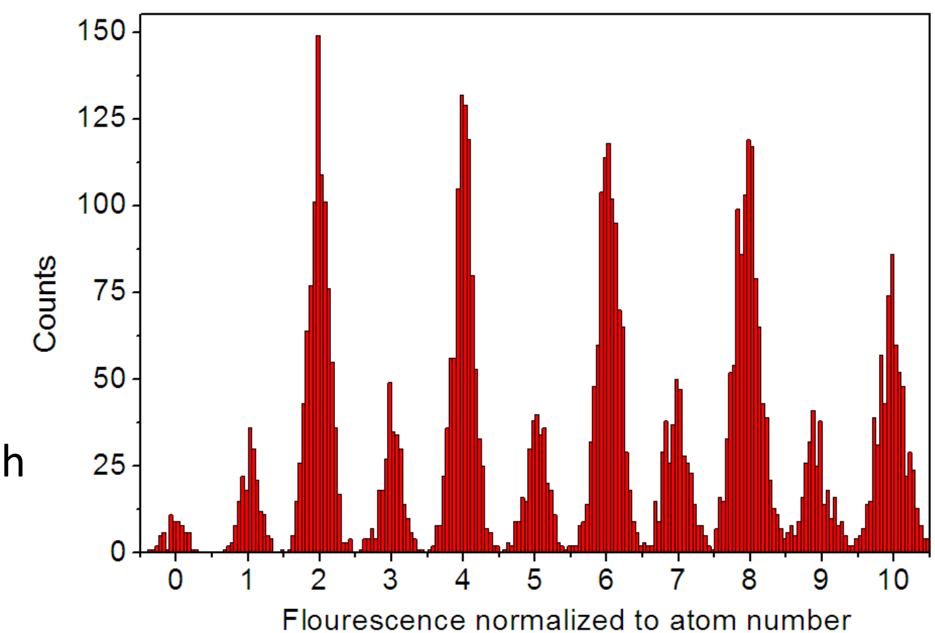
Exposure time 0.5s

CCD



distance between 2 neighbouring atom peaks: ~ 6σ

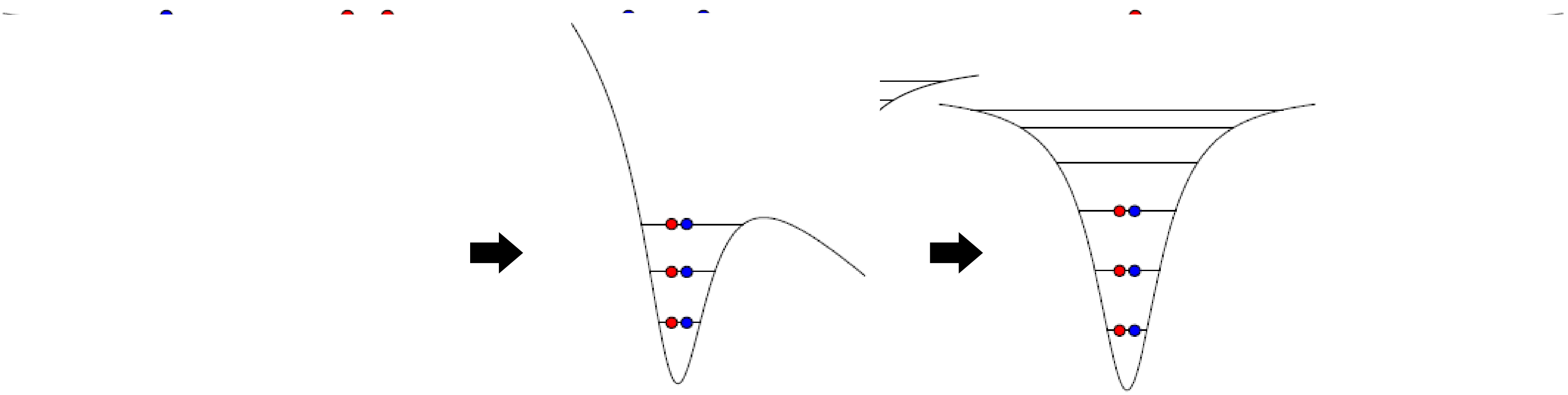
1-10 atoms can be distiguished with high fidelity > 99%



Quantum simulations: why cold atoms ?

control over: particle number

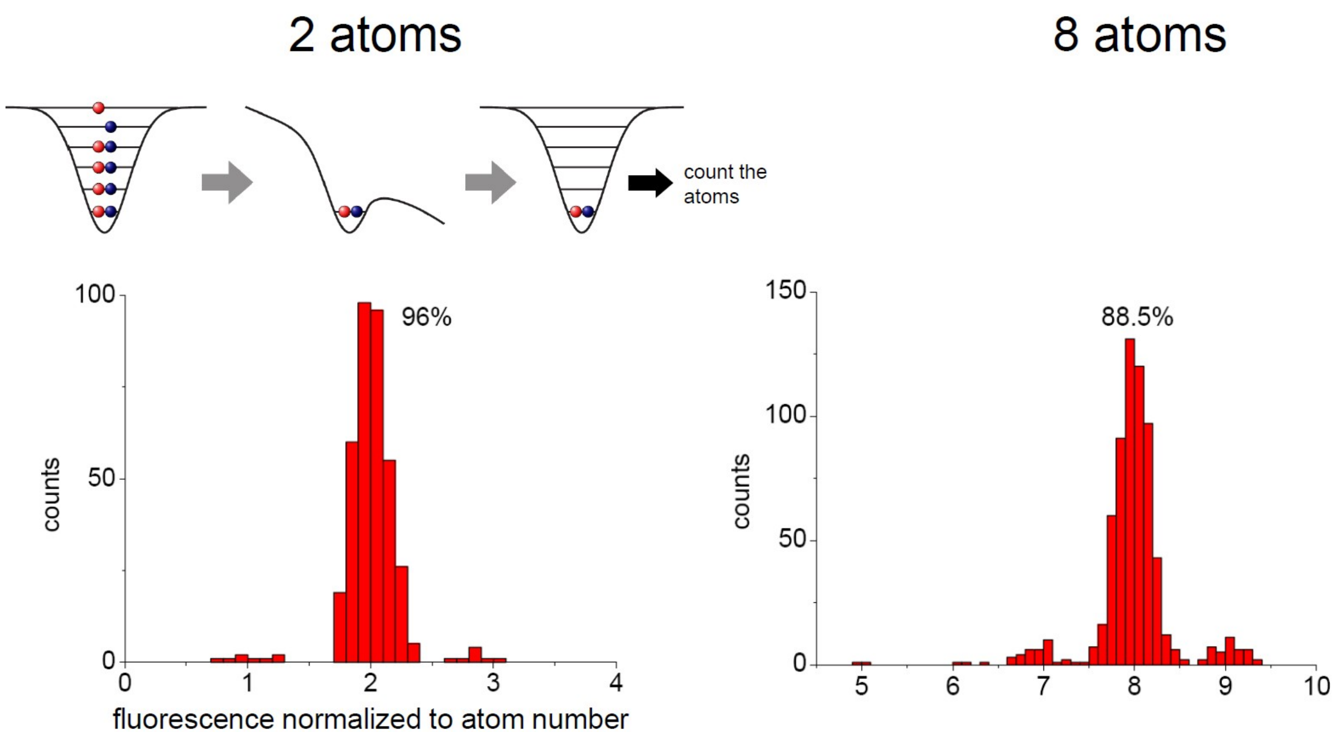
- 2-component mixture in reservoir T=250nK
- superimpose microtrap
- switch off reservoir



+ magnetic field gradient in axial direction

Quantum simulations: why cold atoms ?

control over: particle number with high fidelity



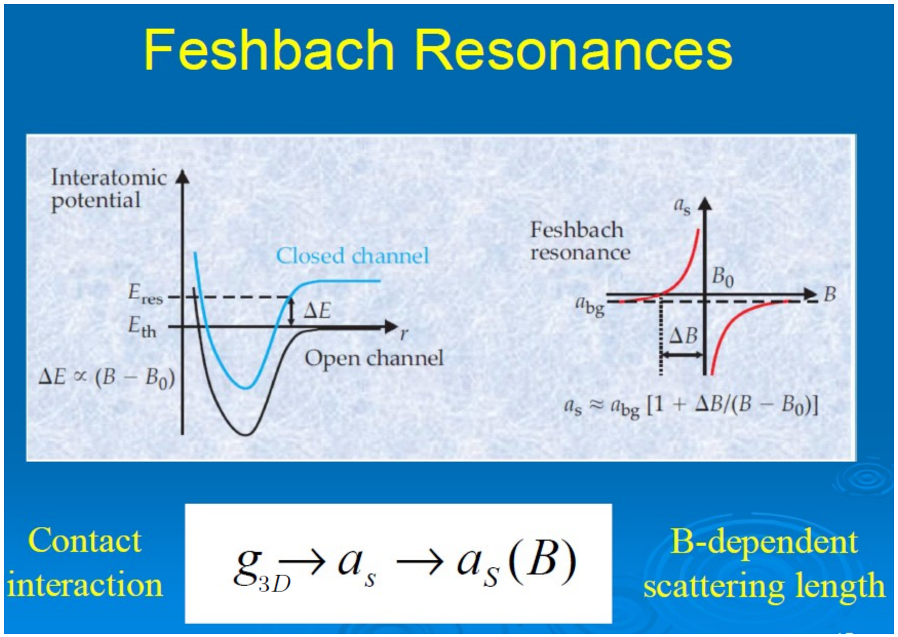
lifetime in ground state ~ 60s

F. Serwane et al., Science **332**, 336 (2011)

Quantum simulations: why cold atoms ?

control over: interaction

$$V(r)$$



$$V(r) = g_{3D}\delta(r)$$

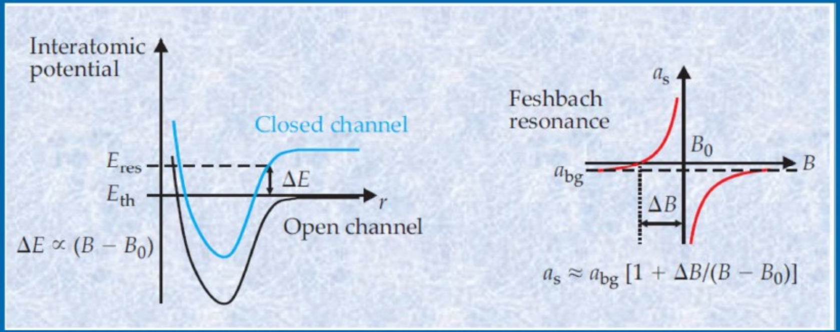
$$g_{3D} = \frac{2\pi\hbar^2}{\mu} a_S(B)$$

Quantum simulations: why cold atoms ?

control over: interaction

$$V(r)$$

Feshbach Resonances



Contact interaction

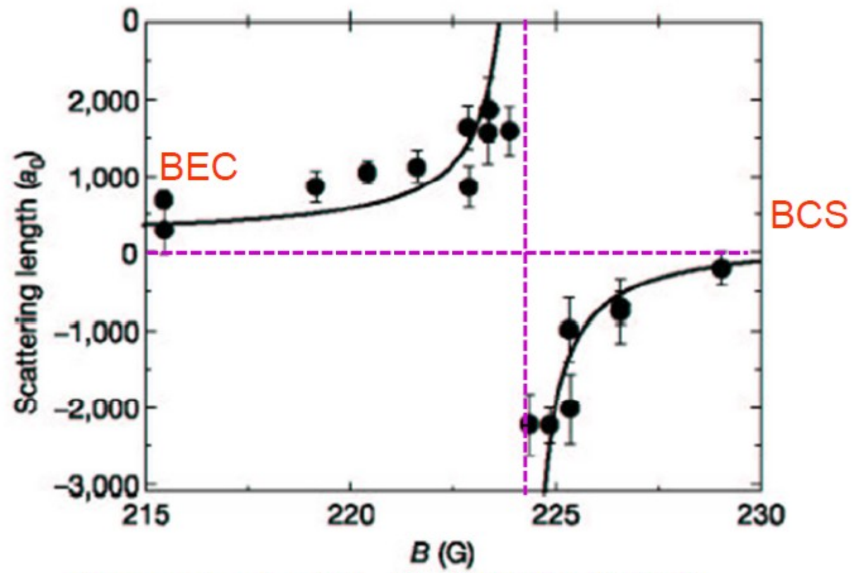
$$g_{3D} \rightarrow a_s \rightarrow a_S(B)$$

B-dependent scattering length

$$V(r) = g_{3D}\delta(r)$$

$$g_{3D} = \frac{2\pi\hbar^2}{\mu} a_S(B)$$

S-wave scattering length (^{40}K)

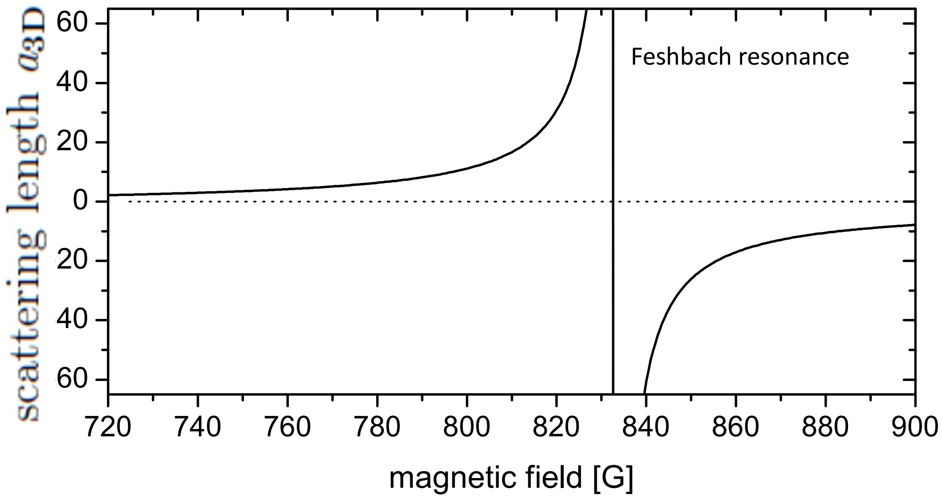


Regal et. al., PRL 90, 230404 (2003)

Quantum simulations: why cold atoms ?

control over: interaction

3D



single-channel pseudopotential

$$V(r) = g_{3D}\delta(r)$$

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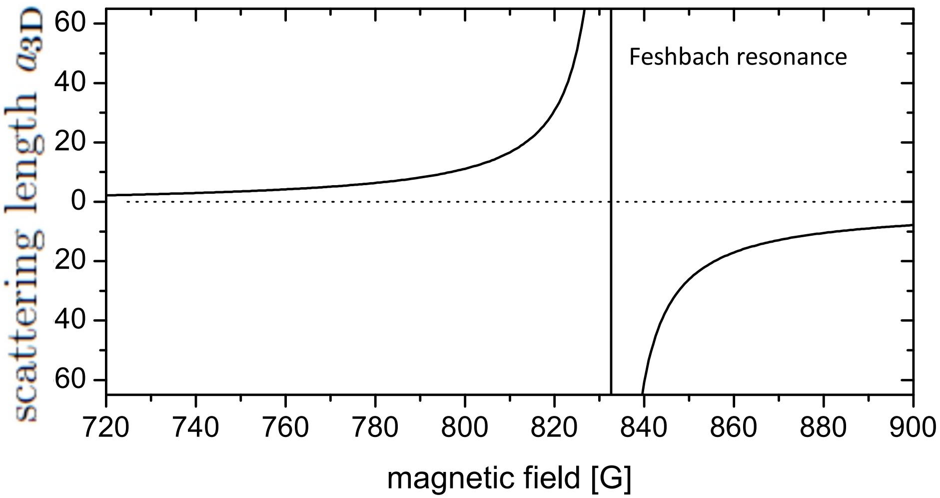
Quantum simulations: why cold atoms ?

control over: interaction

3D



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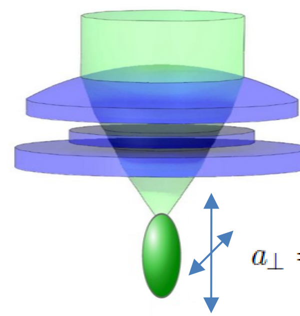
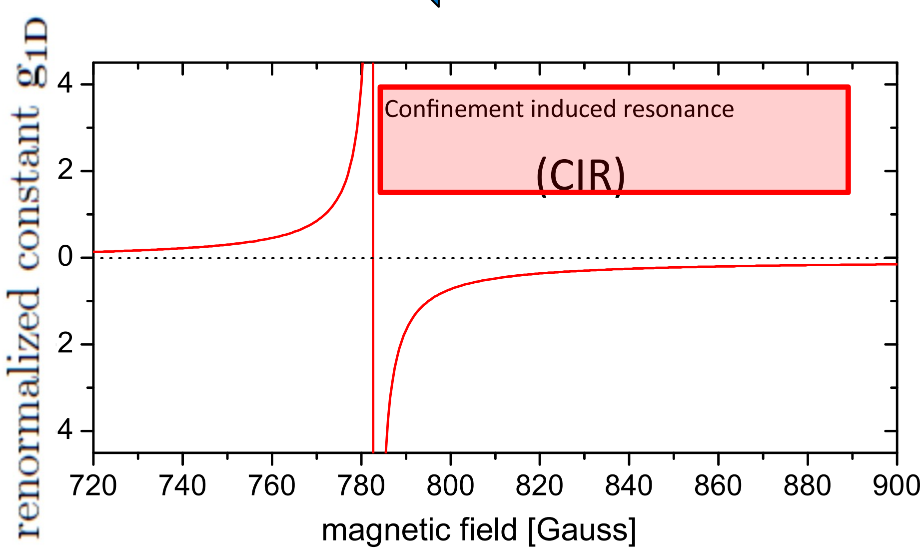
strong confinement



$$a_{\perp} = \sqrt{\hbar/(\mu\omega)}$$

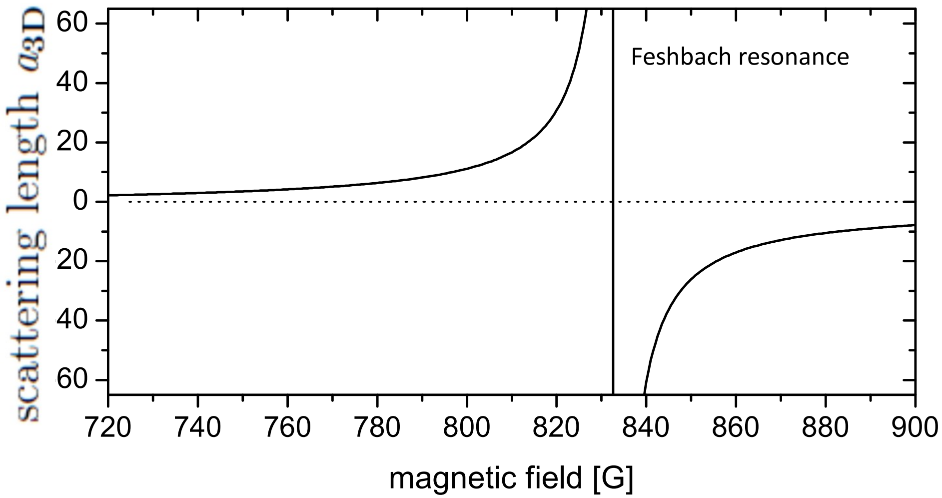
1D



Quantum simulations: why cold atoms ?

control over: interaction

3D



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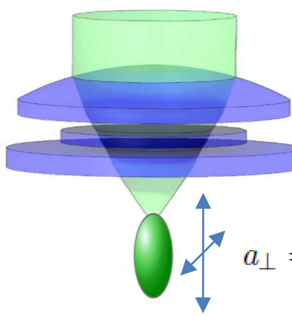

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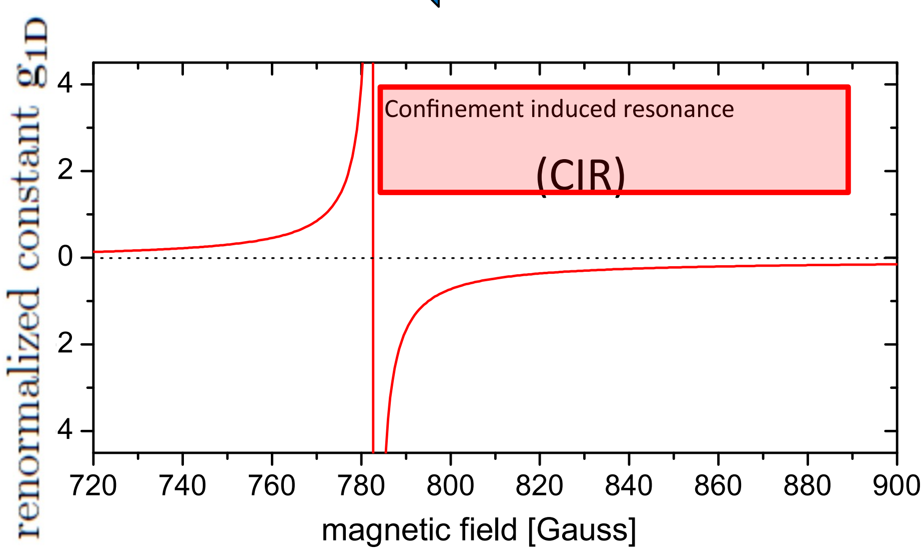


single-channel pseudopotential with renormalized interaction constant

$$g_{1D} = \frac{2\hbar^2 a_{3D}(B)}{\mu a_\perp} \frac{1}{(a_\perp - C a_{3D}(B))}$$

M. Olshanii, PRL 81, 938 (1998).

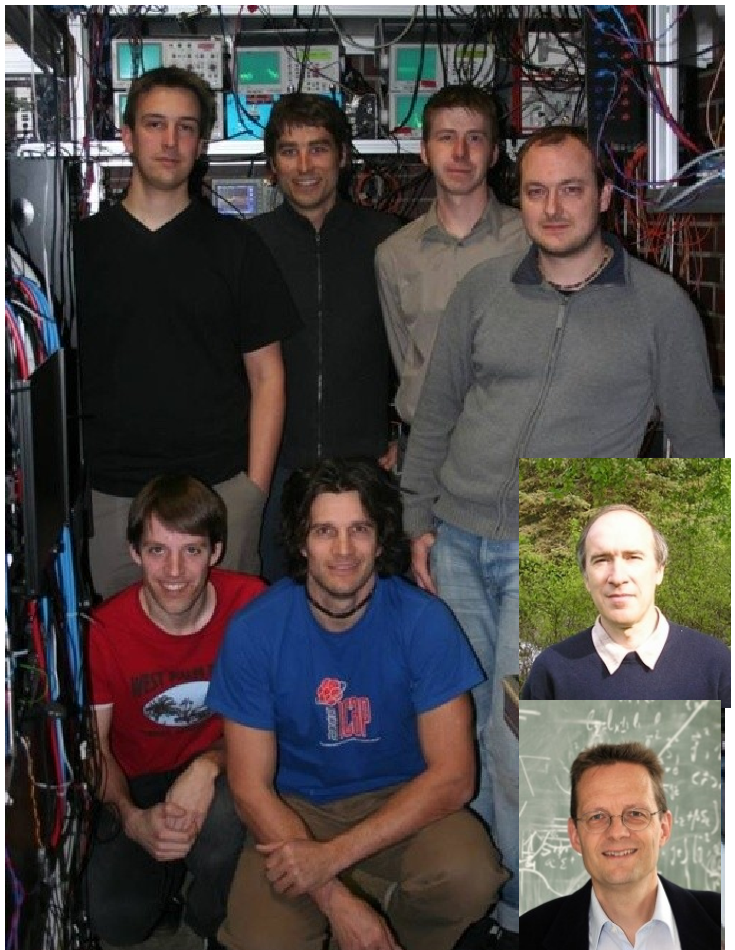
PRL **104**, 153203 (2010) PHYSICAL REVIEW LETTERS week ending 16 APRIL 2010

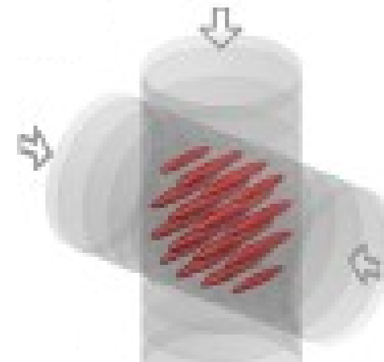
Confinement-Induced Resonances in Low-Dimensional Quantum Systems

Elmar Haller,[1] Manfred J. Mark,[1] Russell Hart,[1] Johann G. Danzl,[1] Lukas Reichsöllner,[1] Vladimir Melezhik,[2] Peter Schmelcher,[3] and Hanns-Christoph Nägerl[1]

[1]*Institut für Experimentalphysik and Zentrum für Quantenphysik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria*
[2]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, 141980 Dubna, Russia*
[3]*Zentrum für Optische Quantentechnologien, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany*
(Received 19 February 2010; published 14 April 2010)

Elmar Haller –>
Outstanding Doctoral Thesis in AMO Physics Recipients for 2011

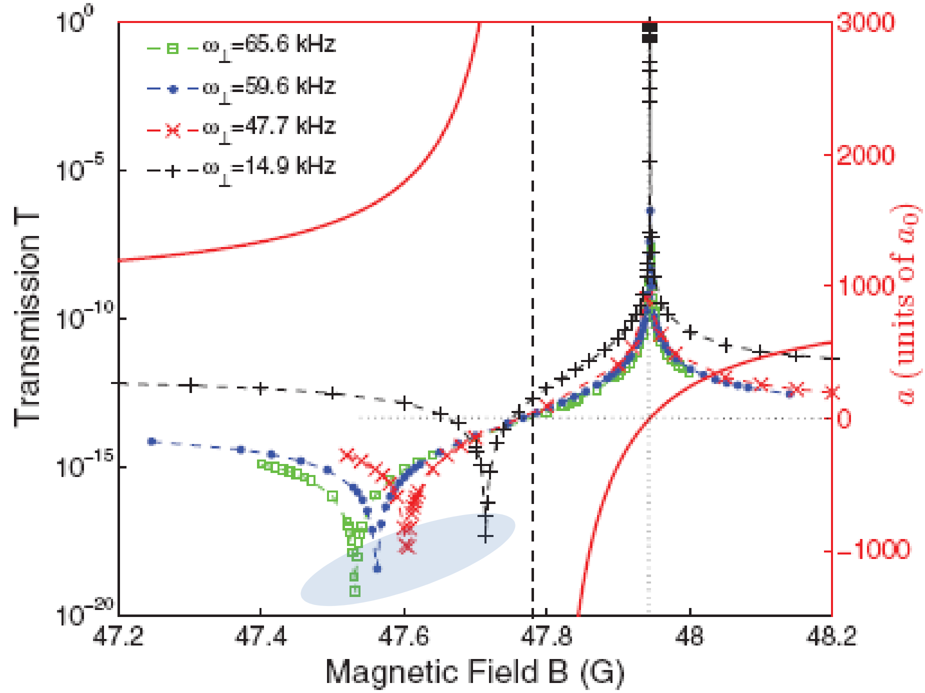




Shifts and widths of Feshbach resonances in atomic waveguides

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)

$$a_\perp = \sqrt{\hbar/(m\omega_\perp)}$$



d-wave FR at 47.8G develops in waveguide as depending on $\omega_\perp$minimums and stable maximum of transmission coefficient T

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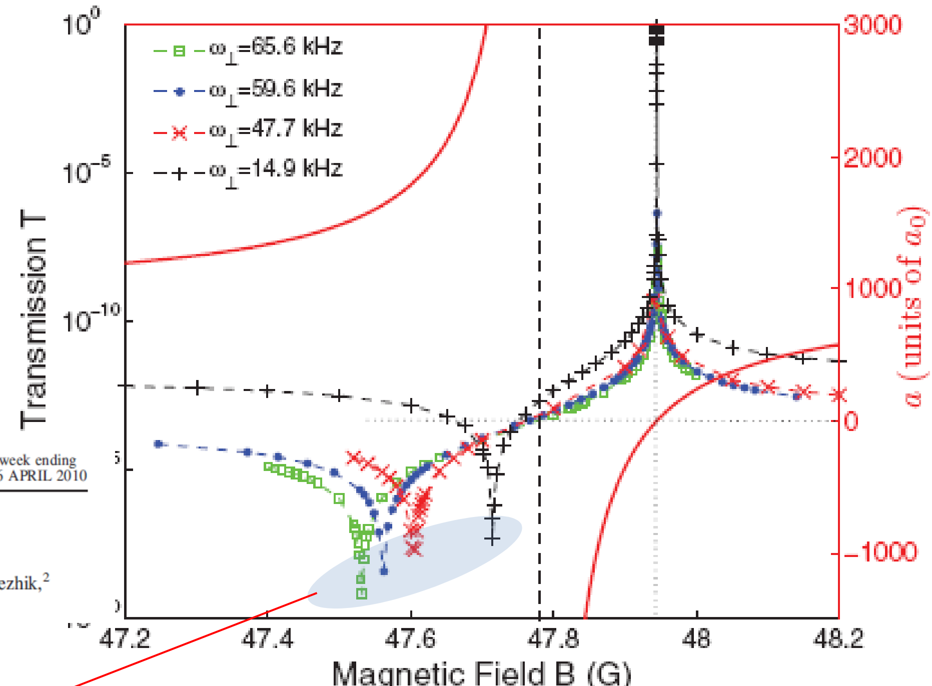

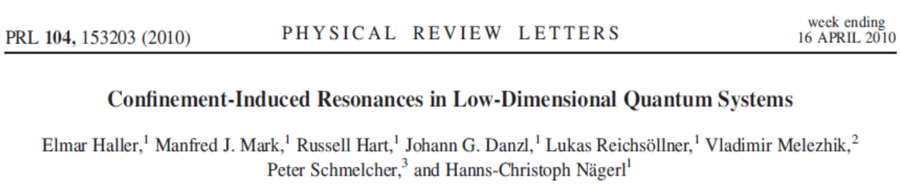
PRL 104, 153203 (2010) PHYSICAL REVIEW LETTERS week ending 16 APRIL 2010

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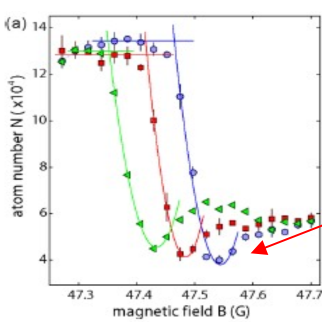

experiment

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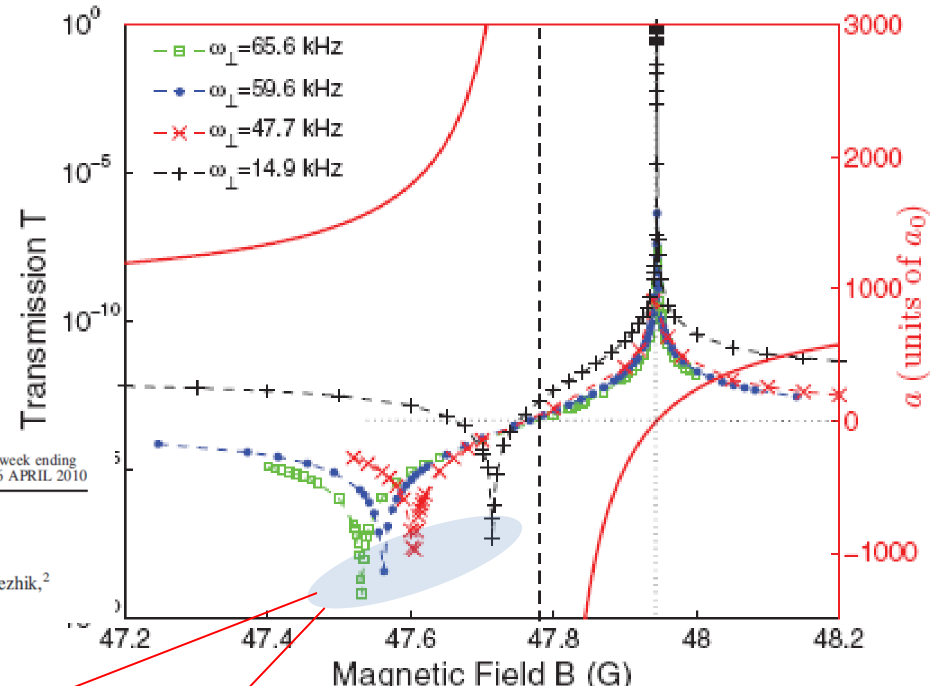

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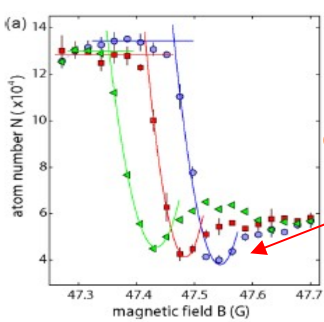
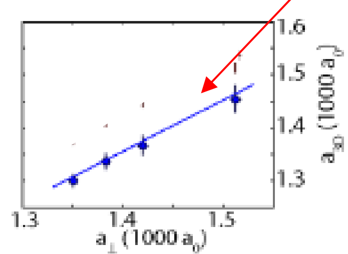


experiment

theory



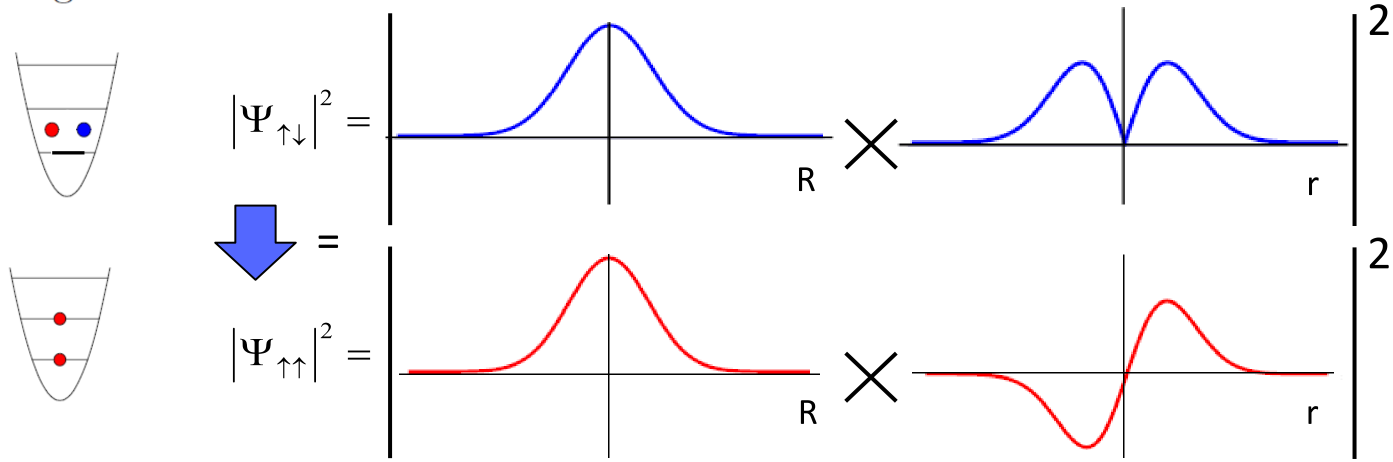
Olshanii formula works for s,d,and g FRs

$$a = 0.68a_\perp$$

Quantum simulations: why cold atoms ?

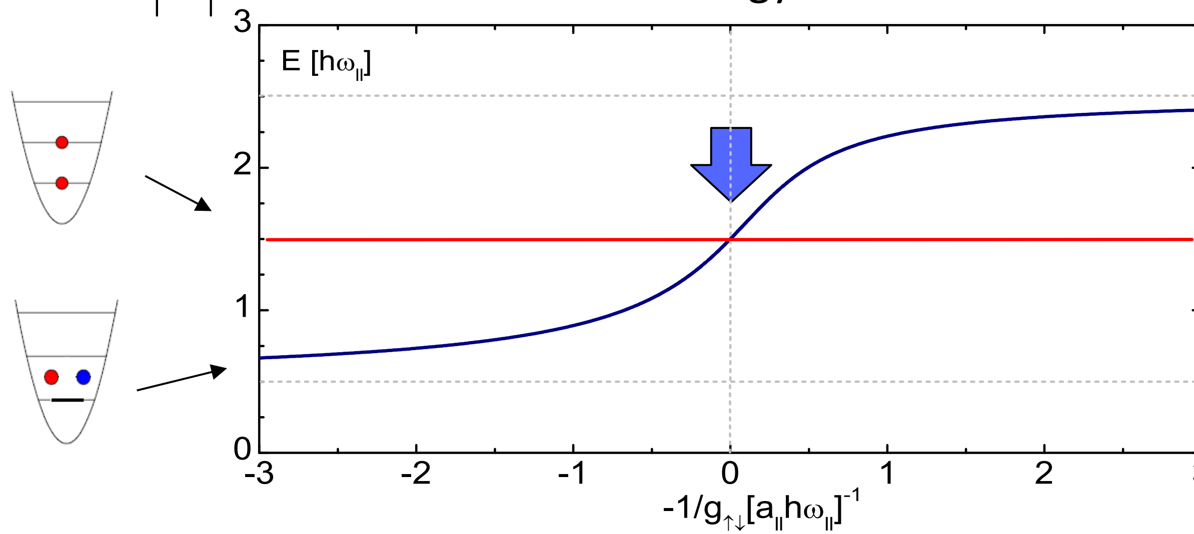
control over: quantum state

distinguishable fermions behave as identical ones at $g_{1D} \to \pm\infty$



M.D. Girardeau, PRA 82, 011607(R) (2010)

in 1D: $|\Psi|^2$ same ⟷ same energy



analytic solution for energy:

T. Busch et al., Found Phys Vol.28, No.4 549-559 (1998)

$g_{1D} > 0$ $g_{1D} \to \pm\infty$ $g_{1D} < 0$

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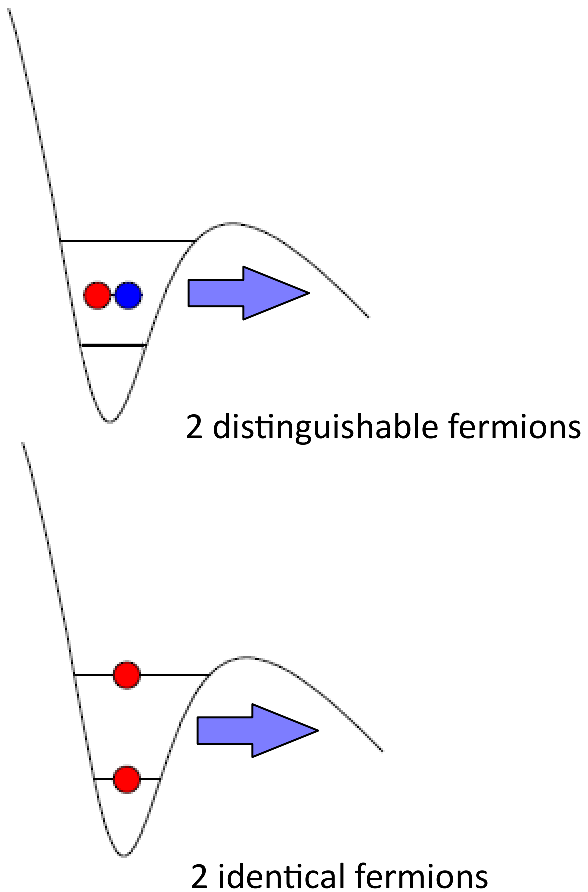
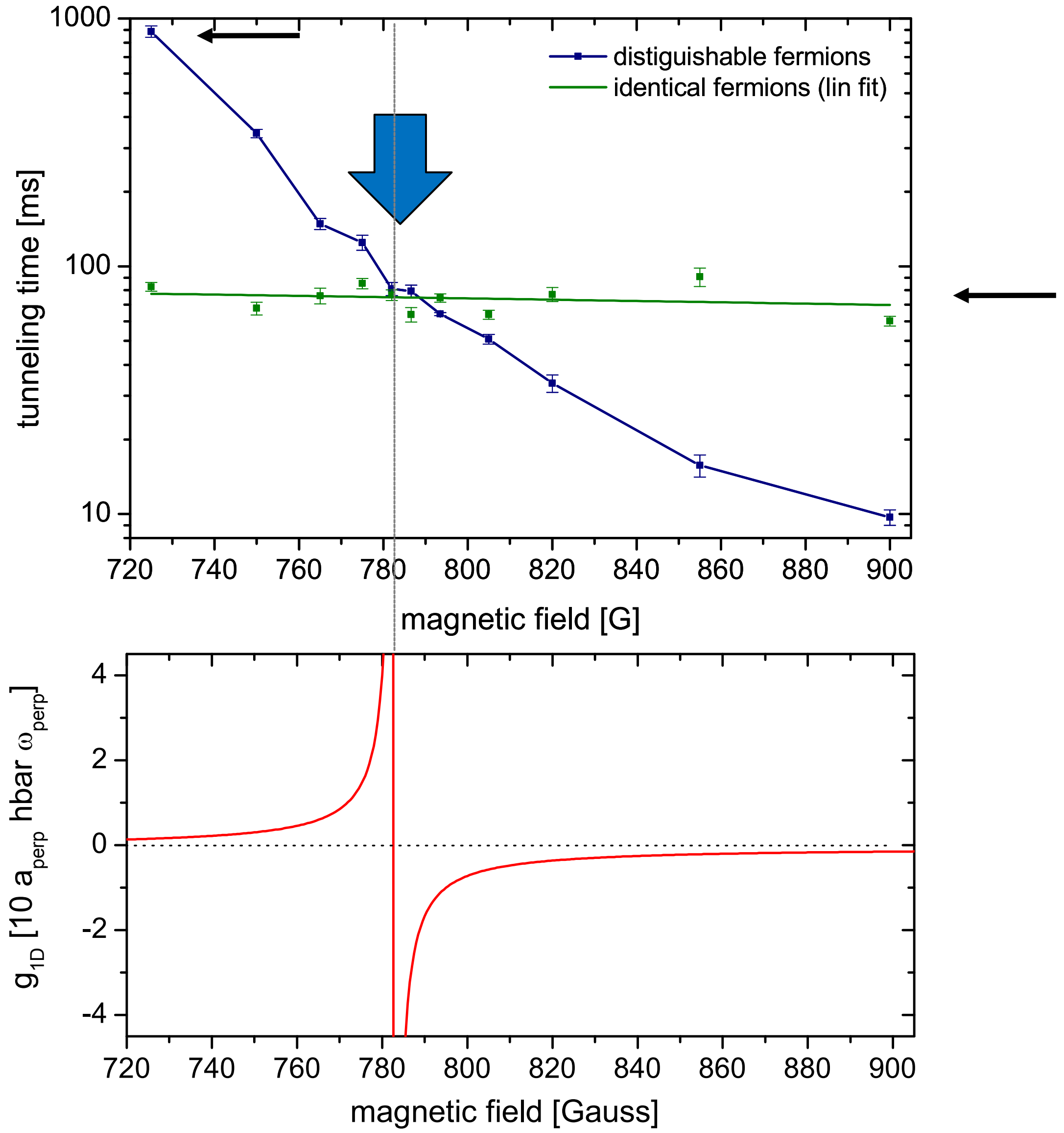
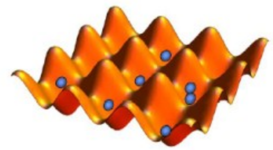




F.Serwane et al. Science 332(2011)336

Solid state physics: modeling phase-transitions

The ultra-cold atom simulator



Atoms ↔ Electrons

Optical lattice ↔ Ionic Crystal

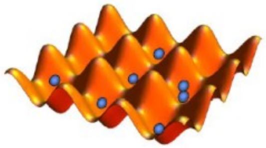
Optical Lattices	Solid state crystals
• Fully controllable, no defects, no vibrations	• Very complex condensed matter environment
• Lattice spacing micrometers	• Lattice spacing Angstroms
• Trapped atom mass ~ 10-100 amu	• Electron mass 1/1900 amu
• Temperature : T~1 nK	• Temperature : T~ 100 K

Solid state physics: modeling phase-transitions

Bose-Hubbard Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^{\dagger} \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

The ultra-cold atom simulator



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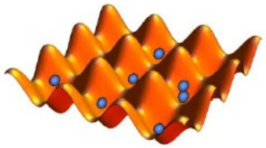
Tunnelmatrix element/Hopping element

$$J = -\int d^3x\, w(\mathbf{x} - \mathbf{x}_i) \left(-\frac{\hbar^2}{2m} \Delta + V_{lat}(\mathbf{x}) \right) w(\mathbf{x} - \mathbf{x}_j)$$

Onsite interaction matrix element

$$U = \frac{4\pi\hbar^2 a}{m} \int d^3x |w(\mathbf{x})|^4$$

The ultra-cold atom simulator



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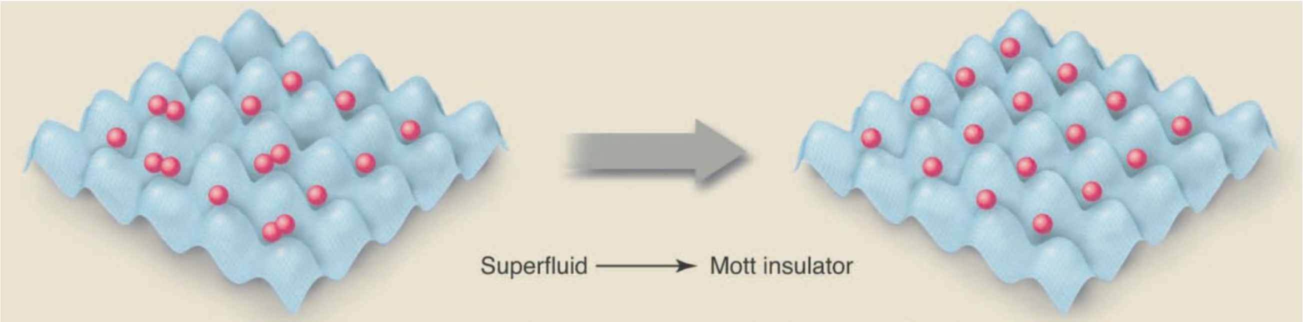
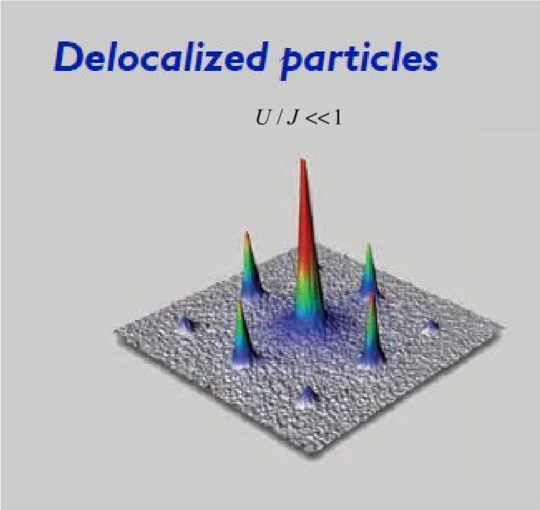
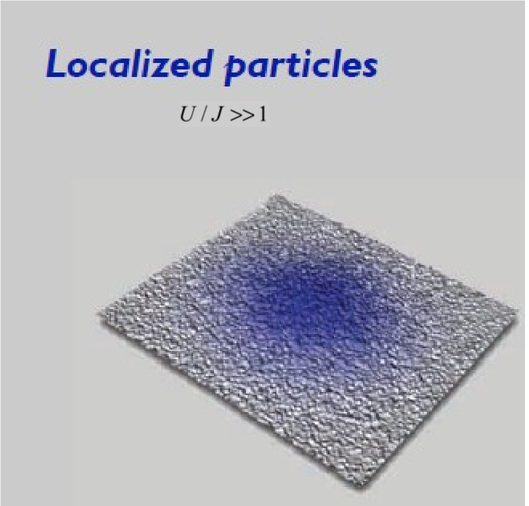


Phase coherence





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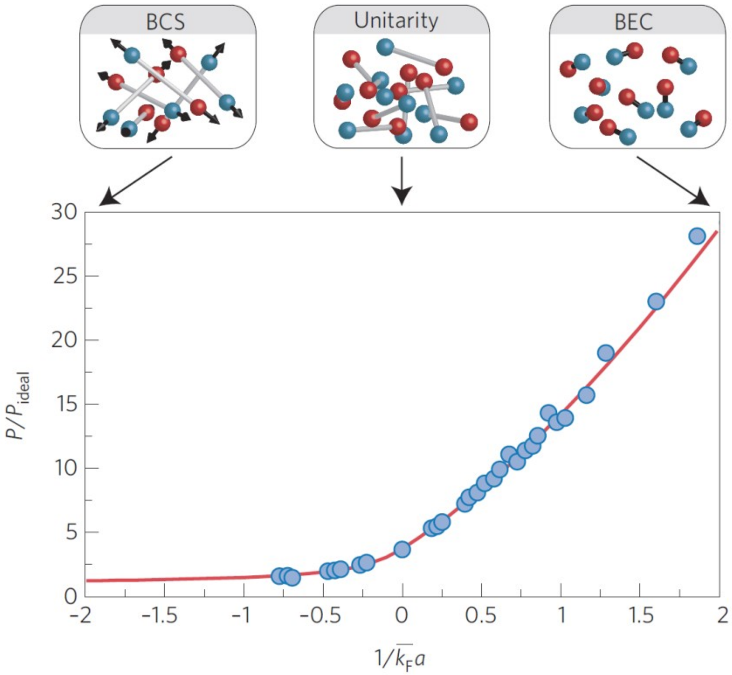
Greiner, M., O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, 2002, Nature (London) **415**, 39.

BCS-BEC crossover in ultracold Fermi gas



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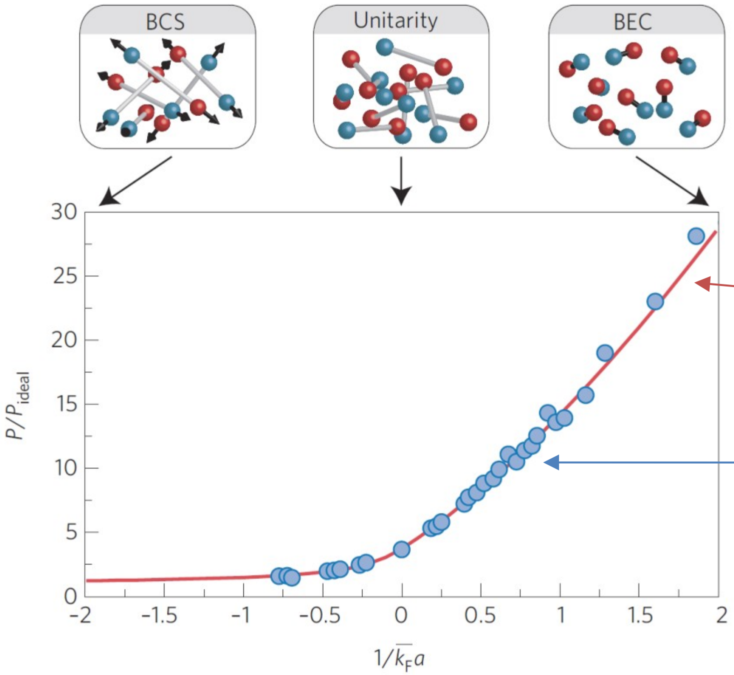

diffusion Monte-Carlo simulation
S.Pilati & S.Giorgini PRL 100 (2008) 030401

experiment ^{40}K , ^{6}Li
N.Navon et al Science 328 (2010) 729

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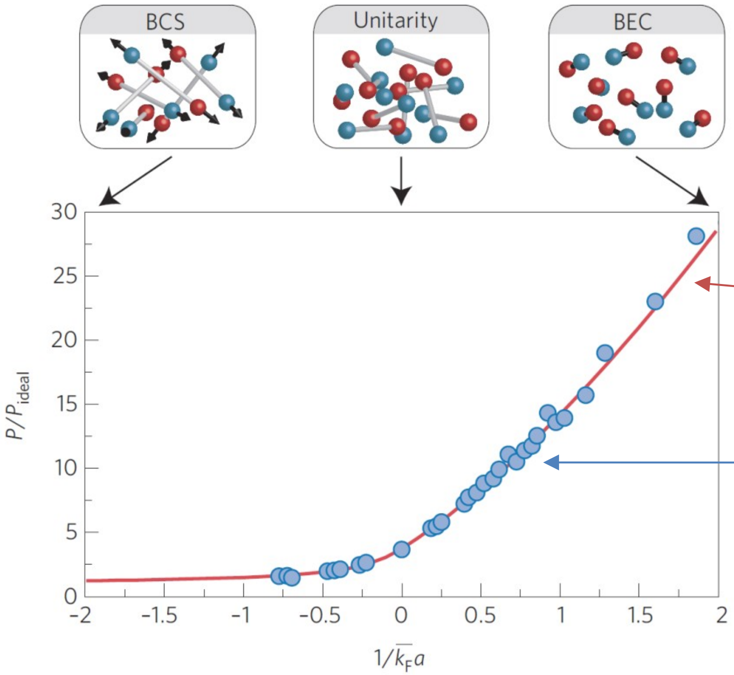

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unitary limit $1/(k_F a) \to \pm 0$ $(a \to \pm\infty)$

equation of state $\mu = \xi E_F$

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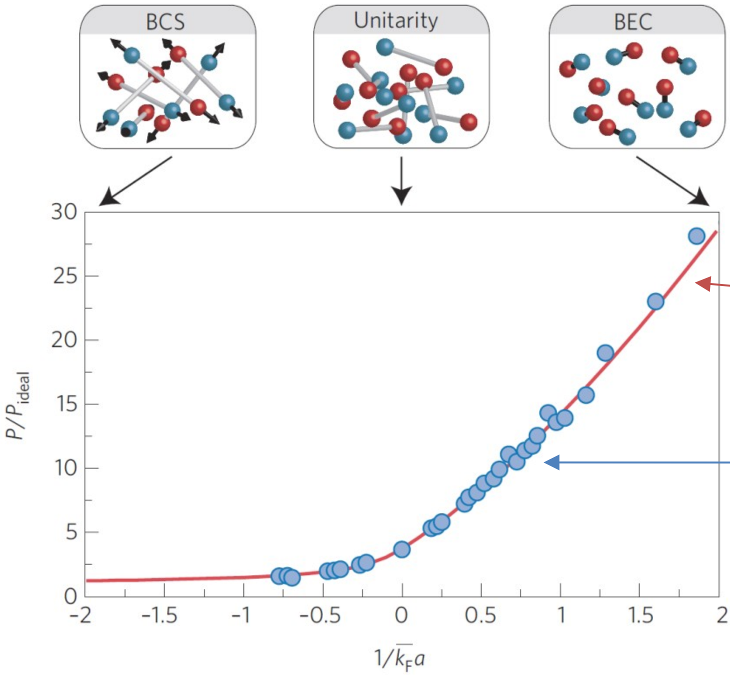

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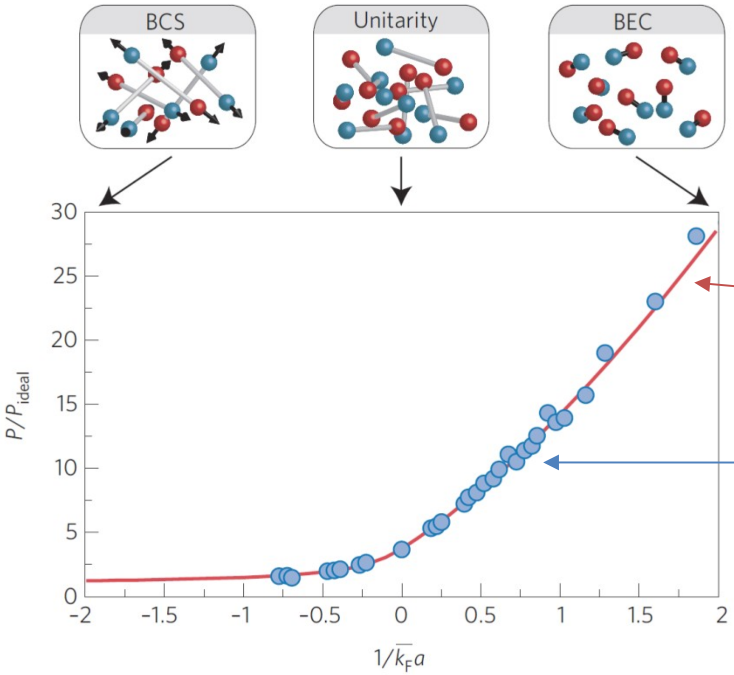
experiment $\xi = 0.4$

BCS-BEC crossover in ultracold Fermi gas



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experiment ^{40}K , ^{6}Li
N.Navon et al Science 328 (2010) 729

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equation of state $\mu = \xi E_F$

experiment $\xi = 0.4$

simple mean-field theories, variational Monte Carlo methods

Dipolar confinement-induced resonances in waveguides

P.Giannakeas, V. Melezhik & P.Schmelcher, PRL,111(2013)

$\vec{d} \quad \vec{d}$

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \underbrace{\frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}}_{V_{sr}} + \underbrace{\frac{d^2}{r^3}[1 - 3(\hat{z}\cdot\hat{r})]}_{V_{dd}}$$

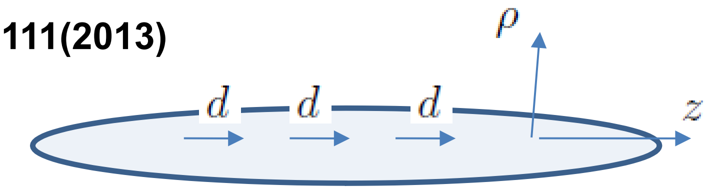
$$\underline{K}^{3\mathrm{D}} = \begin{pmatrix} K_{ss} & K_{sd} & 0 \\ K_{ds} & K_{dd} & K_{dg} \\ 0 & K_{gd} & K_{gg} \end{pmatrix}$$

$$a_{ll'} = -\frac{K_{ll'}}{k}$$

$$l_d = \frac{\mu d^2}{\hbar^2}$$

Dipolar confinement-induced resonances in waveguides

P.Giannakeas, V. Melezhik & P.Schmelcher, PRL,111(2013)



$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{\mu}{2}\omega_\perp^2\rho^2 + \underbrace{\frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}}_{V_{sr}} + \underbrace{\frac{d^2}{r^3}[1 - 3(\hat{z}\cdot\hat{r})]}_{V_{dd}}$$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$

$$\underline{\tilde{K}}_{oo}^{1D} = \underline{K}_{oo}^{1D} + i\underline{K}_{oc}^{1D}(\mathcal{I} - i\underline{K}_{cc}^{1D})^{-1}\underline{K}_{co}^{1D}$$

$$\underline{K}^{3D} = \begin{pmatrix} K_{ss} & K_{sd} & 0 \\ K_{ds} & K_{dd} & K_{dg} \\ 0 & K_{gd} & K_{gg} \end{pmatrix}$$

$$\det(\mathcal{I} - i\underline{K}_{cc}^{1D}) = 0$$

$$a_{ll'} = -\frac{K_{ll'}}{k} \qquad \bar{a}_{ll'} = \frac{a_{ll'}}{a_\perp}$$

$$l_d = \frac{\mu d^2}{\hbar^2} \qquad \bar{l}_d = \frac{l_d}{a_\perp}$$

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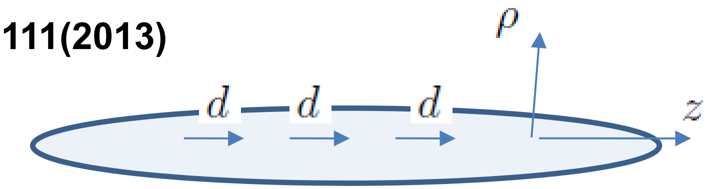

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$$\det(\mathcal{I} - i\underline{K}_{cc}^{1D}) = 0$$

$$a_{ll'} = -\frac{K_{ll'}}{k} \qquad \bar{a}_{ll'} = \frac{a_{ll'}}{a_\perp}$$

we obtained resonance condition:

$$\bar{a}_{ss}(ka_\perp, d) = \mathcal{F}(\{\bar{a}_{\ell\ell'}(ka_\perp, d)\})$$

$$l_d = \frac{\mu d^2}{\hbar^2} \qquad \bar{l}_d = \frac{l_d}{a_\perp}$$

$$\mathcal{F}_{\mathrm{BA}} = -\frac{1 + \eta_1\bar{l}_d + \eta_2\bar{l}_d^2 + \eta_3\bar{l}_d^3}{\sigma_0 + \sigma_1\bar{l}_d + \sigma_2\bar{l}_d^2}$$

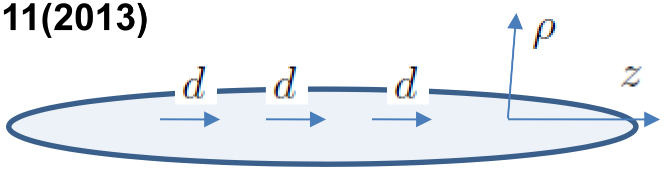
For $l_d = 0$, the resonance condition $\bar{a}_{ss} = \mathcal{F}_{\mathrm{BA}}$

reduces to $\bar{a}_s = -1/\sigma_0 = 0.68$

$$a_s = 0.68a_\perp$$

Dipolar confinement-induced resonances in waveguides

P.Giannakeas, V. Melezhik & P.Schmelcher, PRL,111(2013)



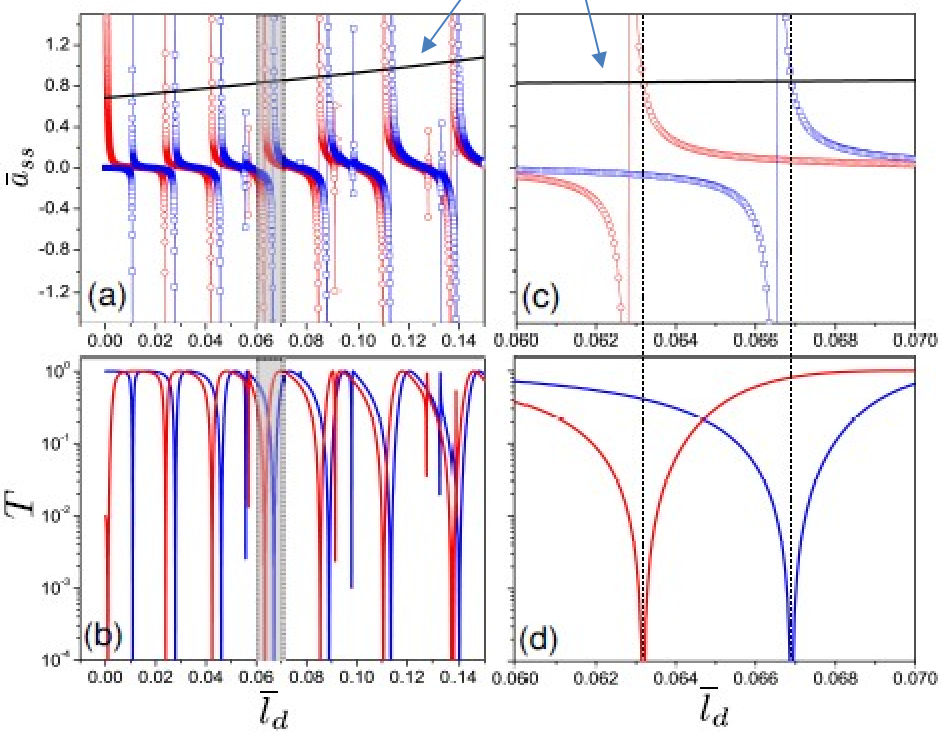
$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{\mu}{2}\omega_\perp^2\rho^2 + \underbrace{\frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}}_{V_{sr}} + \underbrace{\frac{d^2}{r^3}[1-3(\hat{z}\cdot\hat{r})]}_{V_{dd}}$$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$

$a_s \gg l_{\rm vdW}$ (−○−)

$a_s \ll l_{\rm vdW}$ (−□−)

$$\mathcal{F}_{\rm BA} = -\frac{1+\eta_1\bar{l}_d+\eta_2\bar{l}_d^2+\eta_3\bar{l}_d^3}{\sigma_0+\sigma_1\bar{l}_d+\sigma_2\bar{l}_d^2}$$



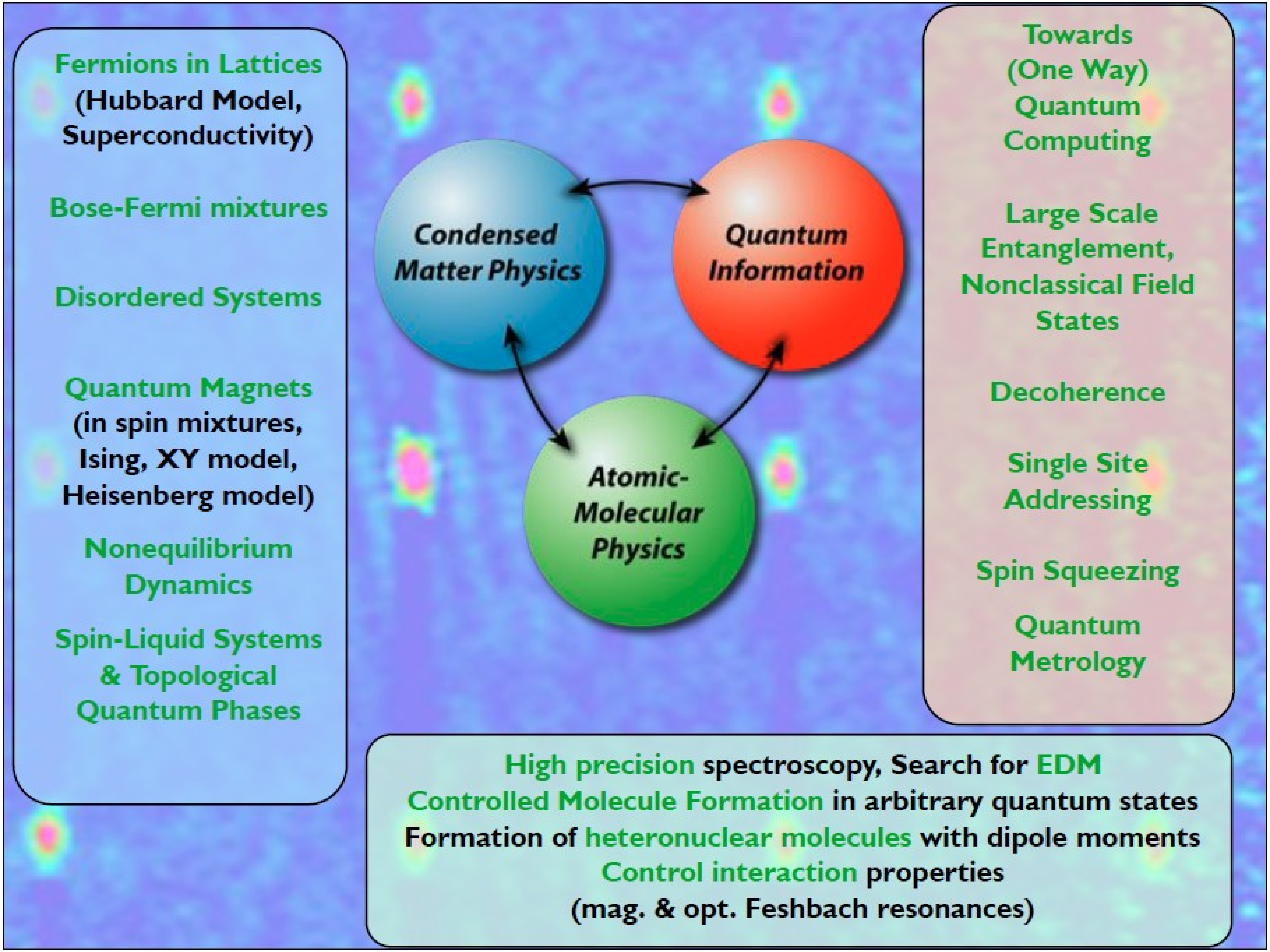
$$\bar{a}_{ll'} = \frac{a_{ll'}}{a_\perp} \qquad a_{ll'} = -\frac{K_{ll'}}{k}$$

$$\bar{l}_d = \frac{l_d}{a_\perp} \qquad l_d = \frac{\mu d^2}{\hbar^2}$$

For $l_d = 0$, the resonance condition $\bar{a}_{ss} = \mathcal{F}_{\rm BA}$ reduces to $\bar{a}_s = -1/\sigma_0 = 0.68$

$$a_s = 0.68 a_\perp$$

analytically derived resonant condition $\bar{a}_{ss} = \mathcal{F}_{\rm BA}$ predict position of dipolar confinement-induced resonances


Fermions in Lattices
(Hubbard Model,
Superconductivity)
Bose-Fermi mixtures
Disordered Systems
Quantum Magnets
(in spin mixtures,
Ising, XY model,
Heisenberg model)
Nonequilibrium
Dynamics
Spin-Liquid Systems
& Topological
Quantum Phases
Condensed
Matter Physics
Quantum
Information
Atomic-
Molecular
Physics
Towards
(One Way)
Quantum
Computing
Large Scale
Entanglement,
Nonclassical Field
States
Decoherence
Single Site
Addressing
Spin Squeezing
Quantum
Metrology
High precision spectroscopy, Search for EDM
Controlled Molecule Formation in arbitrary quantum states
Formation of heteronuclear molecules with dipole moments
Control interaction properties
(mag. & opt. Feshbach resonances)

Outlook, goals and opportunities

Quantum simulation with fully controlled systems

control over: particle number, quantum states, interaction

Fast-growing field, promising applications in study of many problems

I.M.Georgescu et al. Quantum simulations, Rev.Mod.Phys. 86 (2014) 153
J.I. Cirac and P.Zoller, Goals and opportunities in quantum simulation, Nature Phys. 8 (2012) 264
M.Dalmonte and S.Montangero, Lattice gauge theories simulations in the quantum Information era, Contem. Phys. 57 (2016) 388

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~ few tens experimental groups worldwide

Rb,Cs,K,Sr,Li … Rb_2 , Cs_2 , RbK …   1D, 2D, 3D recently -> hybrid “atom-ion” systems Li-Yb+, Rb-Ba+ …

M. Tomza, K. Jachymski, R. Gerritsma, A. Negretti, T. Calarco, Z. Idziaszek, and P.S. Julienne; Cold hybrid ion-atom systems; Rev. Mod. Phys. 91 (2019) 035001

L. Chomaz, I.Ferrier-Barbut, F.Ferlaino, B. Laburthe-Tolra, B. L. Lev and T. Pfau;
Dipolar physics: a review of experiments with magnetic quantum gases
Rep. Prog. Phys. 86 (2023) 026401

S.I. Mistakidis, A.G. Volosniev, R.E. Barfknecht, T. Fogarty, Th. Busch, A. Foerster, P. Schmelcher, N.T. Zinner
Cold atoms in low dimensions - a laboratory for quantum dynamics
arXiv:2202.1107

in proposed quantum simulators improved controllability and scalability are required.