A.V.Kotikov, JINR, Dubna

(in collab. with I.R. Gabdrakhmanov, N.A Gramotkov, D.A. Volkova
and I.A. Zemlyakov, JINR, Dubna).

based on 2307.16225

The XIX International Workshop High Energy Spin Physics (DSPIN-23) dedicated to the 90-th anniversary of prof. A.V. Efremov September 4-8, 2023, Dubna.

Bjorken sum rule with analytic coupling at low Q^2 values

OUTLINE

1. Introduction
2. Results
3. Conclusions

Abstract

The experimental data obtained for the polarized Bjorken sum rule $\Gamma_1^{p-n}(Q^2)$ for small values of Q^2 are approximated by the predictions obtained in the framework of analytic QCD up to the 5th order perturbation theory, whose coupling constant does not contain the Landau pole. We found an excellent agreement between the experimental data and the predictions of analytic QCD, as well as a strong difference between these data and the results obtained in the framework of standard QCD.

1. Introduction

Polarized Bjorken sum rule (BSR) $\Gamma_1^{p-n}(Q^2)$ (Bjorken: 1966), i.e. the difference between the first moments of the spin-dependent structure functions (SFs) of a proton and neutron, is a very important space-like QCD observable (Deur, Brodsky, De Téramond: 2018), (Kuhn, Chen, Leader: 2009).

Its isovector nature facilitates its theoretical description in perturbative QCD (pQCD) in terms of the operator product expansion (OPE), compared to the corresponding SF integrals for each nucleon.

Experimental results for this quantity obtained in polarized deep inelastic scattering (DIS) are currently available in a wide range of the spacelike squared momenta Q^2: 0.021 GeV$^2 \leq Q^2 < 5$ GeV^2 (see, e.g., (Deur et al.: 2022) and references therein).

Theoretically, pQCD (with OPE) in the $\overline{MS}$-scheme was the usual approach to describing such quantities. This approach, however, has the theoretical disadvantage that the running coupling constant (*couplant*) $\alpha_s(Q^2)$ has the Landau singularities for small Q^2 values: $Q^2 \leq 0.1$ GeV^2, which makes it inconvenient for estimating spacelike observables at small Q^2, such as BSR.

In the recent years, the extension of pQCD couplings for low Q^2 without Landau singularities called (fractional) analytic perturbation theory [(F)APT)] (Shirkov, Solovtsov: 1996,1997), (Milton, Solovtsov, Solovtsova: 1997), (Shirkov: 2001) and (Bakulev, Mikhailov, Stefanis: 2007,2007,2010) (or the minimal analytic (MA) theory (Cvetic, Valenzuela: 2008,2011)), were applied to match the theoretical OPE expression with the experimental BSR data (Pasechnik et al.: 2008,2010,2012), (Ayala et al.: 2017,2018).

Following (Cvetic, Valenzuela: 2006), we introduce here the derivatives (in the k-order of perturbation theory (PT))

$$\tilde{a}_{n+1}^{(k)}(Q^2) = \frac{(-1)^n}{n!} \frac{d^n a_s^{(k)}(Q^2)}{(dL)^n}, a_s^{(k)}(Q^2) = \frac{\beta_0 \alpha_s^{(k)}(Q^2)}{4\pi} = \beta_0 \, \overline{a}_s^{(k)}(Q^2),$$

which are very convenient in the case of analytic QCD. β_0 is the first coefficient of the QCD β-function:

$$\beta(\overline{a}_s^{(k)}) = -\left(\overline{a}_s^{(k)}\right)^2 \left(\beta_0 + \sum_{i=1}^{k} \beta_i \left(\overline{a}_s^{(k)}\right)^i\right),$$

where β_i are known up to $k = 4$ (Baikov, Chetyrkin, Kuhn: 2008).

The series of derivatives $\tilde{a}_n(Q^2)$ can successfully replace the corresponding series of a_s-powers (see, e.g. (Kotikov, Zemlyakov: 2022)). Indeed, each derivative reduces the a_s power but is accompanied by an additional β-function $\sim a_s^2$. Thus, each application of a derivative yields an additional a_s, and thus it is indeed possible to use a series of derivatives instead of a series of a_s-powers.

In LO, the series of derivatives $\tilde{a}_n(Q^2)$ are exactly the same as a_s^n. Beyond LO, the relationship between $\tilde{a}_n(Q^2)$ and a_s^n was established in (Cvetic, Valenzuela: 2006), (Cvetic, Kogerler, Valenzuela: 2010) and extended to the fractional case, where $n \to$ is a non-integer ν, in (Cvetic, Kotikov: 2012)

In this short paper, we apply the inverse logarithmic expansion of the MA couplants, recently obtained in (Kotikov, Zemlyakov: 2023) for any PT order. This approach is very convenient: for LO the MA couplants have simple representations (see (Bakulev, Mikhailov, Stefanis: 2007,2007,2010)), while beyond LO the MA couplants are very close to LO ones, especially for $Q^2 \to \infty$ and $Q^2 \to 0$, where the differences between MA couplants of various PT orders become insignificant. Moreover, for $Q^2 \to \infty$ and $Q^2 \to 0$ the (fractional) derivatives of the MA couplants with $n \geq 2$ tend to zero, and therefore only the first term in perturbative expansions makes a valuable contribution.

2. Bjorken sum rule

The polarized (nonsinglet) BSR is defined as the difference between the proton and neutron polarized SFs, integrated over the entire interval x

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 dx\,[g_1^p(x,Q^2) - g_1^n(x,Q^2)].$$

Theoretically, the quantity can be written in the OPE form (Shuryak, Vainshtein: 1982), (Balitsky, Braun, Kolesnichenko: 1990)

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\,(1 - D_{\rm BS}(Q^2)) + \sum_{i=2}^{\infty} \frac{\mu_{2i}(Q^2)}{Q^{2i-2}}\,,$$

where g_A=1.2762 $\pm$ 0.0005 (PDG: 2020) is the nucleon axial charge, $(1-D_{BS}(Q^2))$ is the leading-twist contribution, and μ_{2i}/Q^{2i-2} $(i \geq 1)$ are the higher-twist (HT) contributions.

Since we include very small Q^2 values here, this representation of the HT contributions is inconvenient. It is much better to use the so-called "massive" representation for the HT part (introduced in (Teryaev: 2013), (Khandramai, Teryaev, Gabdrakhmanov: 2016)):

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\left(1 - D_{\rm BS}(Q^2)\right) + \frac{\hat{\mu}_4 M^2}{Q^2 + M^2},$$

where the values of $\hat{\mu}_4$ and M^2 have been fitted in (Ayala et al.: 2018) in the different analytic QCD models.

In the case of MA QCD, from (Ayala et al.: 2018)

$$M^2 = 0.439 \pm 0.012 \pm 0.463, \quad \hat{\mu}_{\rm MA,4} = -0.173 \pm 0.002 \pm 0.666,$$

where the statistical (small) and systematic (large) uncertainties are presented.

Another form, which is correct at very small Q^2 values, has been proposed in (Gabdrakhmanov, Teryaev, Khandramai: 2017))

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}(1 - D_{\rm BS}(Q^2)) + \frac{\hat{\mu}_4 M^2(Q^2+M^2)}{(Q^2+M^2)^2+M^2\sigma^2},$$

where small value $\sigma \equiv \sigma_\rho = 145$ MeV (the ρ-meson decay width) has been used.

Up to the k-th PT order, the perturbative part has the form

$$D_{\rm BS}^{(1)}(Q^2) = \frac{4}{\beta_0} a_s^{(1)}, D_{\rm BS}^{(k\geq 2)}(Q^2) = \frac{4}{\beta_0} a_s^{(k)} \left(1 + \sum_{m=1}^{k-1} d_m (a_s^{(k)})^m\right),$$

where d_1, d_2 and d_3 are known from exact calculations. The exact d_4 value is not known, but it was recently estimated in (Ayala, Pineda: 2022))

Converting the powers of couplant into its derivatives, we have

$$D^{(1)}_{\rm BS}(Q^2) = \frac{4}{\beta_0}\,\tilde{a}^{(1)}_1, D^{(k\geq 2)}_{\rm BS}(Q^2) = \frac{4}{\beta_0}\,\left(\tilde{a}^{(k)}_1 + \sum_{m=2}^{k} \tilde{d}_{m-1}\tilde{a}^{(k)}_m\right),$$

where $b_i = \beta_i/\beta_0^{i+1}$ and

$$\tilde{d}_1 = d_1, \quad \tilde{d}_2 = d_2 - b_1 d_1, \quad \tilde{d}_3 = d_3 - \frac{5}{2}b_1 d_2 - (b_2 - \frac{5}{2}b_1^2)\,d_1,$$
$$\tilde{d}_4 = d_4 - \frac{13}{3}b_1 d_3 - (3b_2 - \frac{28}{3}b_1^2)\,d_2 - (b_3 - \frac{22}{3}b_1 b_2 + \frac{28}{3}b_1^3)\,d_1\,.$$

For the case of 3 active quark flavors ($f=3$), we have

$$d_1 = 1.59, \quad d_2 = 3.99, \quad d_3 = 15.42 \quad d_4 = 63.76,$$
$$\tilde{d}_1 = 1.59, \quad \tilde{d}_2 = 2.73, \quad \tilde{d}_3 = 8.61, \quad \tilde{d}_4 = 21.52\,,$$

i.e., the coefficients in the series of derivatives are slightly smaller.

In MA QCD, the results for BSR become as follows

$$\Gamma^{p-n}_{\mathrm{MA},1}(Q^2) = \frac{g_A}{6}\,(1 - D_{\mathrm{MA,BS}}(Q^2)) + \frac{\hat{\mu}_{\mathrm{MA},4} M^2 (Q^2 + M^2)}{(Q^2 + M^2)^2 + M^2\sigma^2},$$

where the perturbative part $D_{\mathrm{BS,MA}}(Q^2)$ takes the form

$$D^{(1)}_{\mathrm{MA,BS}}(Q^2) = \frac{4}{\beta_0}\, A^{(1)}_{\mathrm{MA}},$$

$$D^{k\geq 2}_{\mathrm{MA,BS}}(Q^2) = \frac{4}{\beta_0}\,(A^{(k)}_{\mathrm{MA}} + \sum_{m=2}^{k} \tilde{d}_{m-1}\, \tilde{A}^{(k)}_{\mathrm{MA},\nu=\mathrm{m}})\,.$$

	M^2 for $\sigma = \sigma_\rho$ (for $\sigma = 0$)	$\hat{\mu}_{\mathrm{MA},4}$ for $\sigma = \sigma_\rho$ (for $\sigma = 0$)	$\chi^2/(\mathrm{d.o.f.})$ for $\sigma = \sigma_\rho$ (for $\sigma = 0$)
LO	1.592 ± 0.300 (1.631 ± 0.301)	-0.168 ± 0.002 (-0.166 ± 0.001)	0.788 (0.789)
NLO	1.505 ± 0.286 (1.545 ± 0.287)	-0.157 ± 0.002 (-0.155 ± 0.001)	0.755 (0.757)
N^2LO	1.378 ± 0.242 (1.417 ± 0.241)	-0.159 ± 0.002 (-0.156 ± 0.002)	0.728 (0.728)
N^3LO	1.389 ± 0.247 (1.429 ± 0.248)	-0.159 ± 0.002 (-0.157 ± 0.002)	0.747 (0.747)
N^4LO	1.422 ± 0.259 (1.462 ± 0.259)	-0.159 ± 0.002 (-0.157 ± 0.001)	0.754 (0.754)

Table 1: The values of the fit parameters with $\sigma = \sigma_\rho$ ($\sigma = 0$).

3. Results

The results of calculations are presented in Table 1 and in Fig. 5. Here we use the Q^2-independent M and $\hat{\mu}_4$ values and the twist-two parts for the cases of usual PT and APT, respectively.

In the case of using MA couplants, we see in Table 1 that the cases $\sigma = 0$ and $\sigma = \sigma_\rho$ lead to very similar values for the fitting parameters and χ^2-factor. So, in Fig. 5 we show only the case with $\sigma = \sigma_\rho$. The quality of the fits is very good, as evidenced quantitatively by the values of $\chi^2/(\mathrm{d.o.f.})$. Moreover, our results obtained for different PT orders are very similar to each others: the corresponding curves in Fig. 5 are indistinguishable. One can also see the important role of the twist-four term. Without it, the value of $\Gamma_1^{p-n}(Q^2)$ is about 0.16, which is very far from the experimental data.

At $Q^2 \leq 0.3$ GeV^2 we also see good agreement with the phenomenological models: Burkert-Ioffe one (Burkert, Ioffe: 1992,1994) and especially LFHQCD one (Brodsky, de Teramond, Dosch, Erlich: 2015). For larger Q^2 values our results are below the results of the phenomenological models and at $Q^2 \geq 0.5$ GeV^2 are below the experimental data. We hope to improve agreement with using "massive" forms of HT higher twist contributions h_{2i} with $i \geq 3$. This is a subject of future investigations.

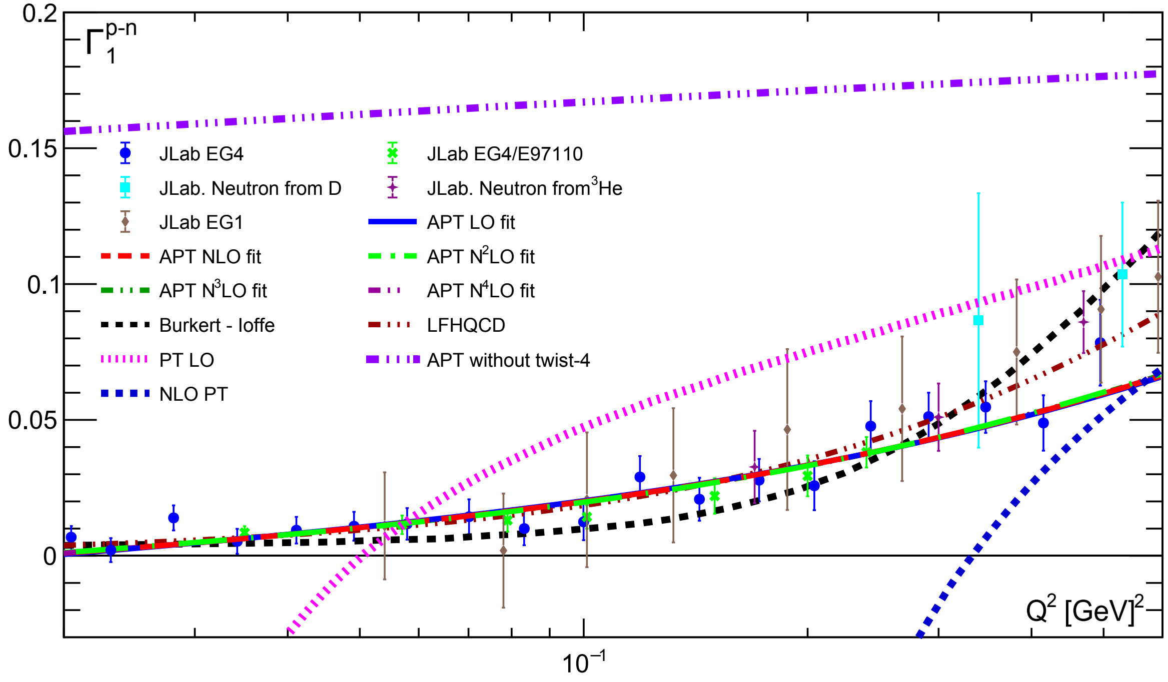

Figure 1: The results for $\Gamma_1^{p-n}(Q^2)$ in the first four orders of APT with $\sigma = \sigma_\rho$.

As seen in Fig. 5, the results obtained using conventional couplants are not good and worse for the NLO case to compare to the LO one. Indeed, the deterioration increases with the PT order in this case (see (Pasechnik et al.: 2008,2010,2012), (Ayala et al.: 2017,2018), (Kotikov, Zemlyakov: 2023). Thus, the use of the "massive" twist-four form does not improve these results, since at $Q^2 \to \Lambda_i^2$ conventional couplants become to be singular, that leads to large and negative results for the twist-two part. As the PT order increases, usual couplants become singular for ever larger Q^2 values, while BSR tends to negative values for ever larger Q^2 values. (see also Fig. 15 in (Kotikov, Zemlyakov: 2023)). Thus, the discrepancy between theory and experiment increases with the PT order.

6. Conclusions

We have considered the Bjorken sum rule in the framework of MA and conventional QCD and obtained results similar to those obtained in previous studies (Pasechnik et al.: 2008,2010,2012), (Ayala et al.: 2017,2018), (Kotikov, Zemlyakov: 2023) for the first 4 orders of PT.

The results based on the conventional PT do not agree with the experimental data. For some Q^2 values, the PT results become negative, since the high-order corrections are large and enter the twist-two term with a minus sign.

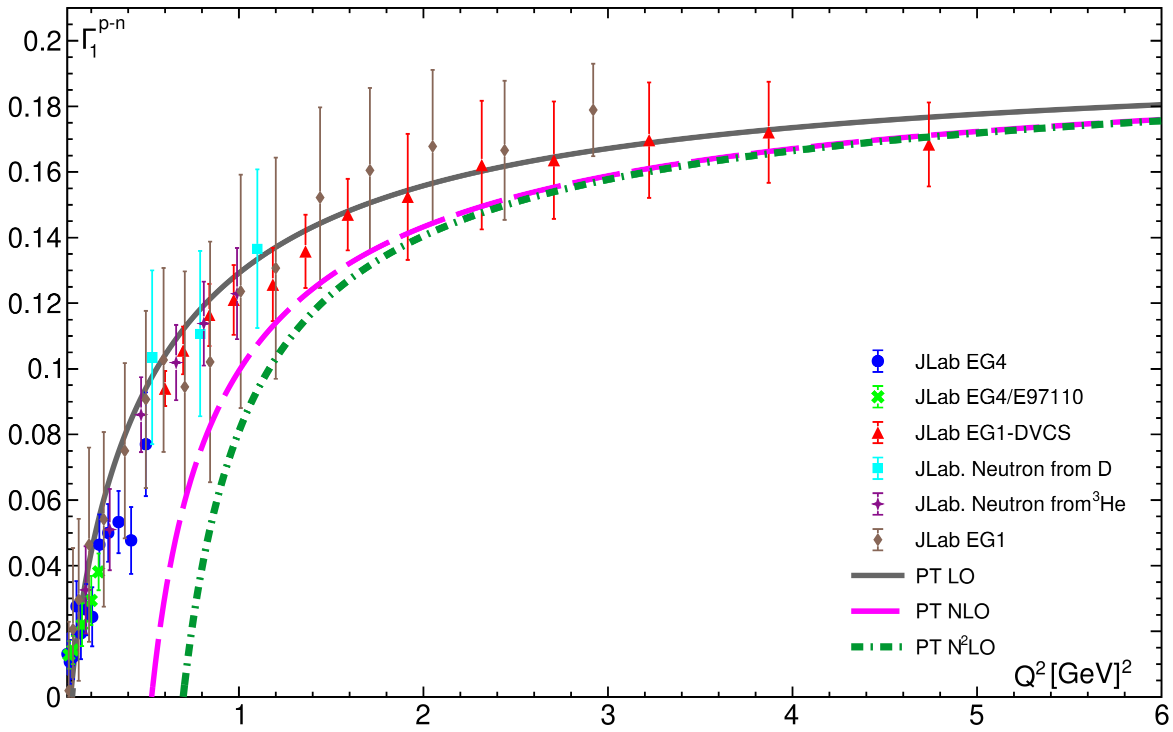

Figure 2: The results for $\Gamma_1^{p-n}(Q^2)$ in the first three orders of PT.

APT in the minimal version leads to a good agreement with experimental data when we used the "massive" version for the twist-four contributions.

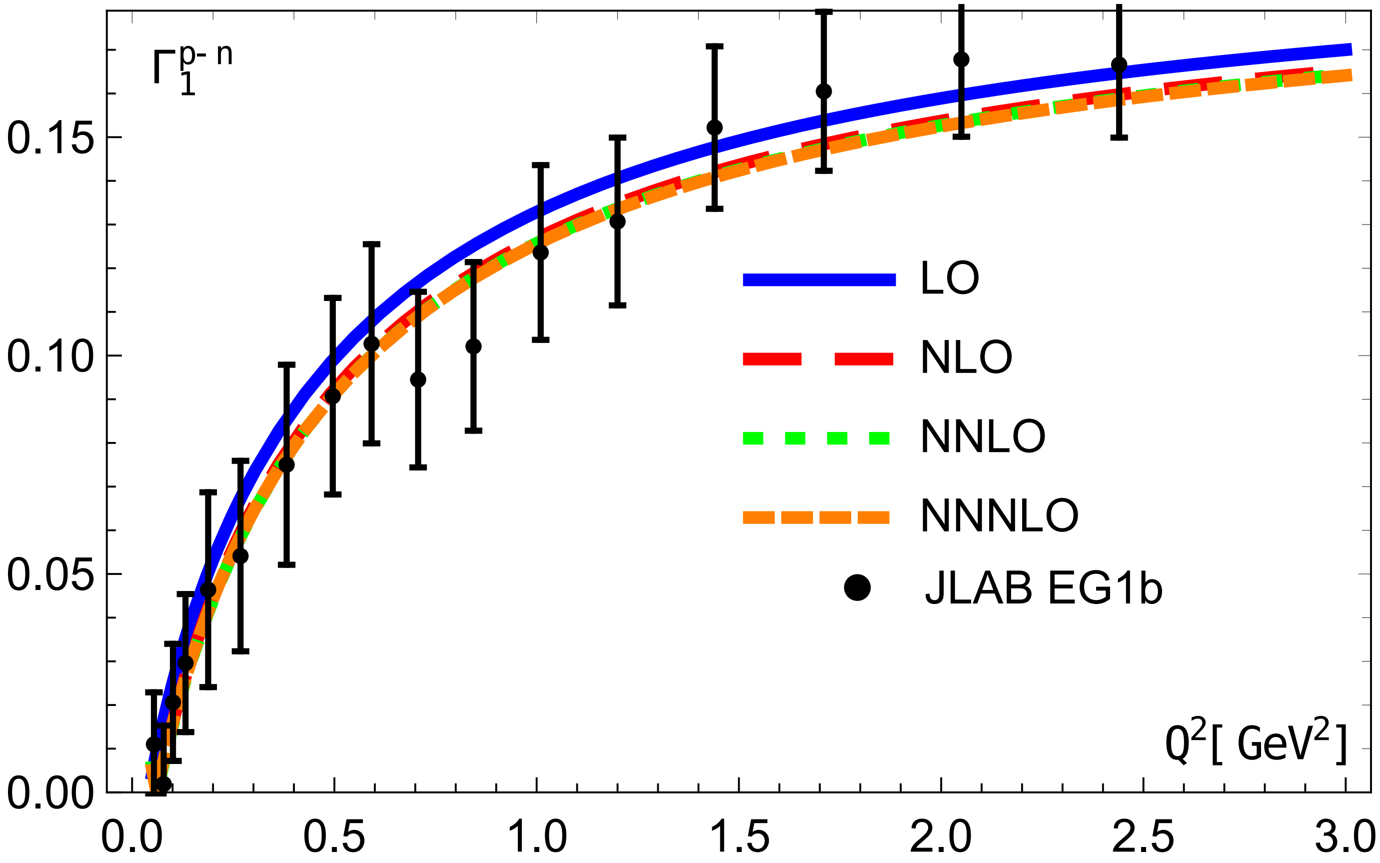

Figure 3: The results for $\Gamma_1^{p-n}(Q^2)$ in the first three orders of APT with $\sigma = 0$.

Consider lower Q^2 values

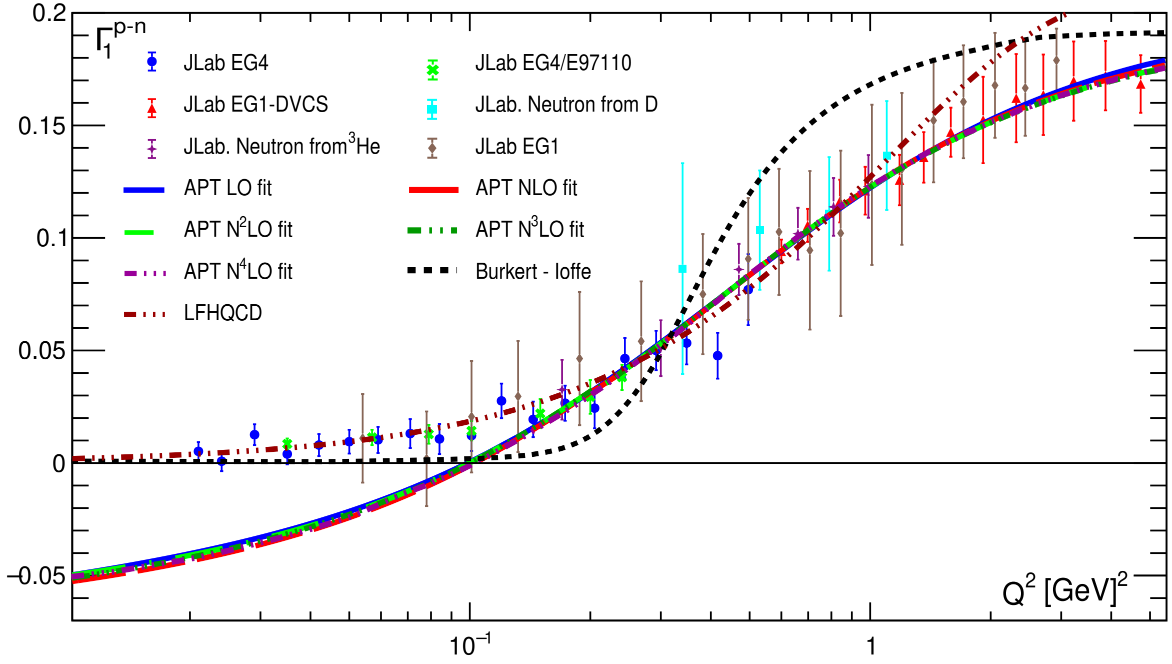

Figure 4: The results for $\Gamma_1^{p-n}(Q^2)$ in the first four orders of APT with $\sigma = 0$.

Consider fits of experimental data at $Q^2 \leq 0.5$ values: with a purpose to decreare $\hat{\mu}_{\mathrm{MA},4}$. It is a subject of the study..

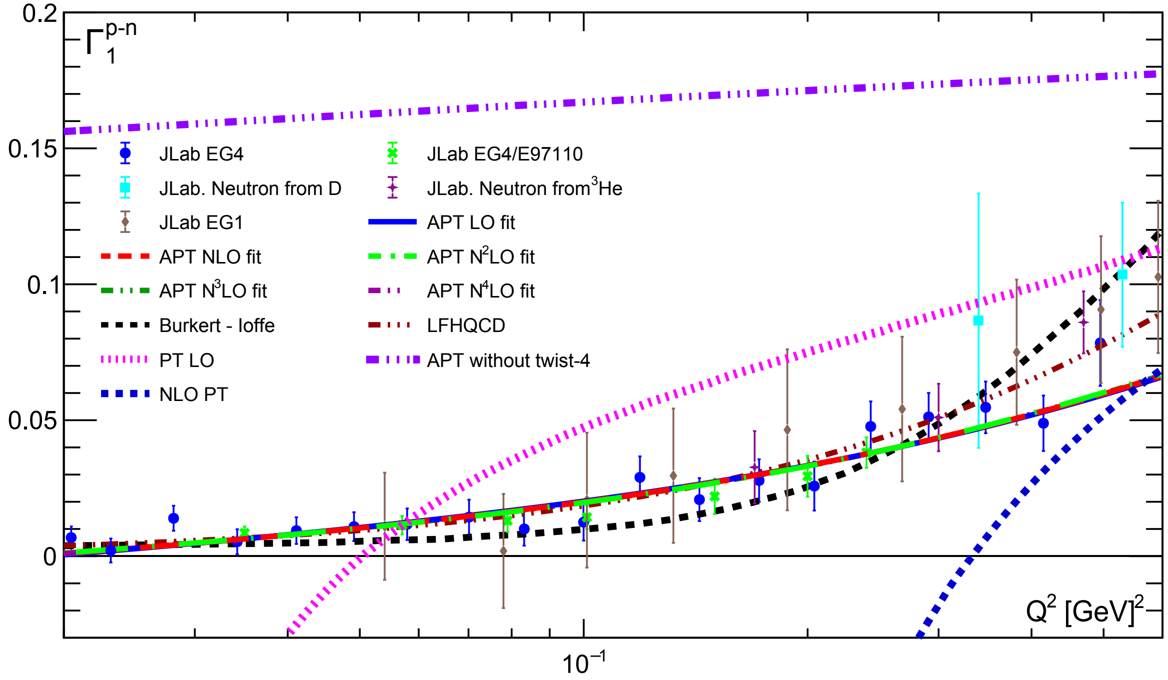

Figure 5: The results for $\Gamma_1^{p-n}(Q^2)$ in the first four orders of APT with $\sigma = \sigma_\rho$.

Newertheless, there is a problem at very low Q^2 values.

Now we would like to discuss the photoproduction (PhP) case, i.e. the $Q^2 \to 0$ limit. In MA QCD,

$A_{\rm MA}(Q^2=0)=1$ and $\tilde{A}_{\rm MA,m}=0$ for $m>1$

and we have

$$D_{\rm MA,BS}(Q^2=0)=\frac{4}{\beta_0} \quad \text{and}$$

$$\Gamma^{p-n}_{\rm MA,1}(Q^2=0,\sigma=0)=\frac{g_A}{6}(1-\frac{4}{\beta_0})+\hat{\mu}_{\rm MA,4}\,.$$

The finitness of cross-section in the real photon limit leads (Teryaev: 2013), (Khandramai, Teryaev, Gabdrakhmanov: 2016))

$$\Gamma^{p-n}_{\rm MA,1}(Q^2=0)=0 \quad \text{and, hence,} \quad \hat{\mu}^{\rm php}_{\rm MA,4}=-\frac{g_A}{6}(1-\frac{4}{\beta_0}).$$

In the case of 3 active quarks, i.e. $f=3$, we have

$$\hat{\mu}^{\rm php}_{\rm MA,4}=-0.118 \quad \text{and, hence,} \quad |\hat{\mu}^{\rm php}_{\rm MA,4}|<|\hat{\mu}_{\rm MA,4}|,$$

shown in Table I.

This is a common situation that appears as a consequence of the use of analytic versions of QCD for the Bjorken sum rule (see, e.g., Ref. (Ayala et al.: 2018)). Note that our results for $\hat{\mu}_4$ shown in Table I are smaller than in (Ayala et al.: 2018).

So, in our fits the finitness of cross-section in the real photon limit is violated. We note that the results for $\hat{\mu}_{\mathrm{MA},4}$ (shown in Table I) obtained only with the statistical uncertanties for experimental data. If we take into account also the systematic uncertanties, which are very large, the results for $\hat{\mu}^{\mathrm{php}}_{\mathrm{MA},4}$ and $\hat{\mu}_{\mathrm{MA},4}$ are in full agreement.

In future investigations we plan to improve this analysis by taking several "massive" twists by analogy with twist-four one. We hope that this will lead to better agreement with the real photon limit.