

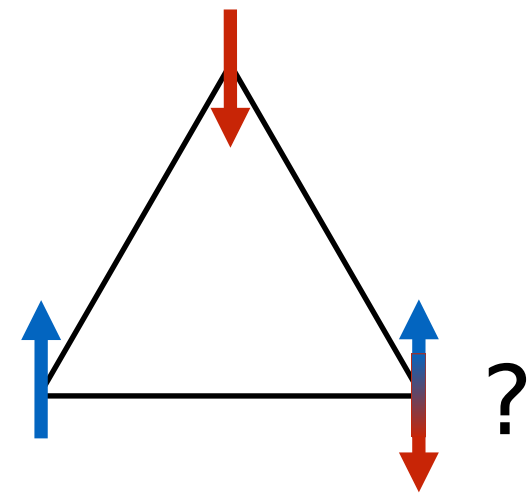
# Frustrated magnetism and quantum computing

P.A. Maksimov

JINR, Dubna

DSPIN-23

# Frustrated magnets



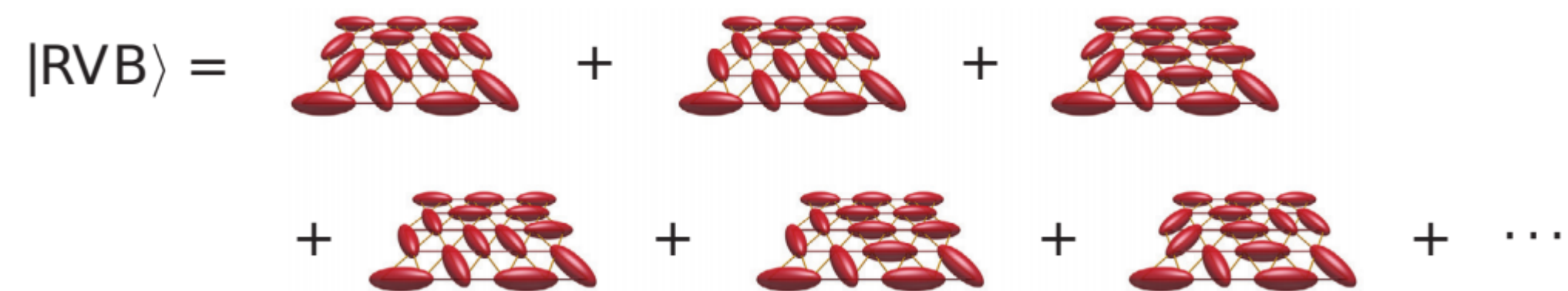
G. H. Wannier, *Physical Review* 79, 357 (1950)

Classical degeneracy  $\rightarrow$  Finite entropy at  $T=0$

Resonating valence bond (RVB)

P. W. Anderson, *Materials Research Bulletin* 8, 153 (1973)

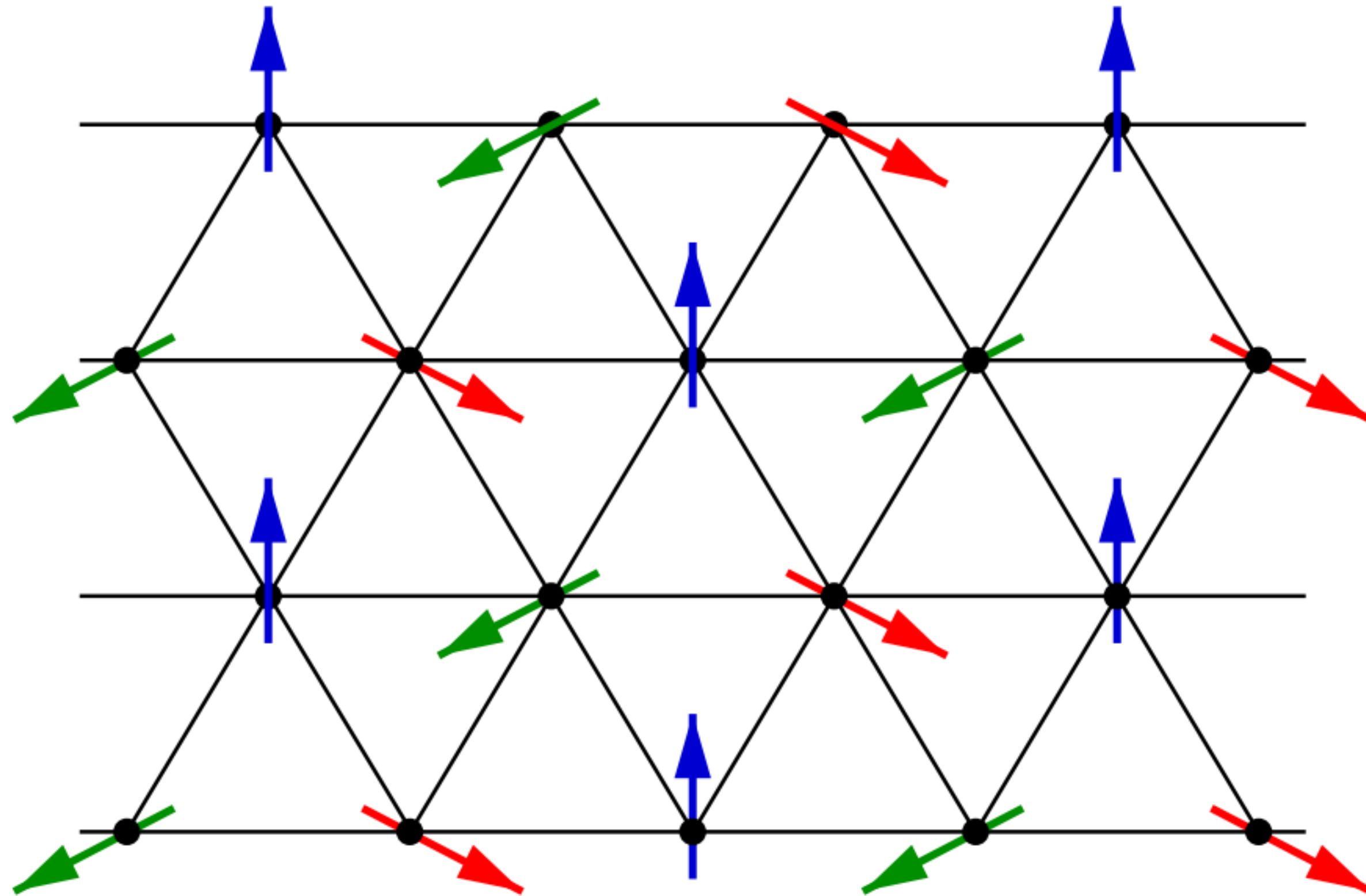
P. Fazekas and P. W. Anderson, *Philosophical Magazine* 30, 423 (1974)



Spin liquids: fractionalized excitations, topological properties

L. Savary and L. Balents, *Reports on Progress in Physics* 80, 016502 (2017)

# Suppressed long-range order

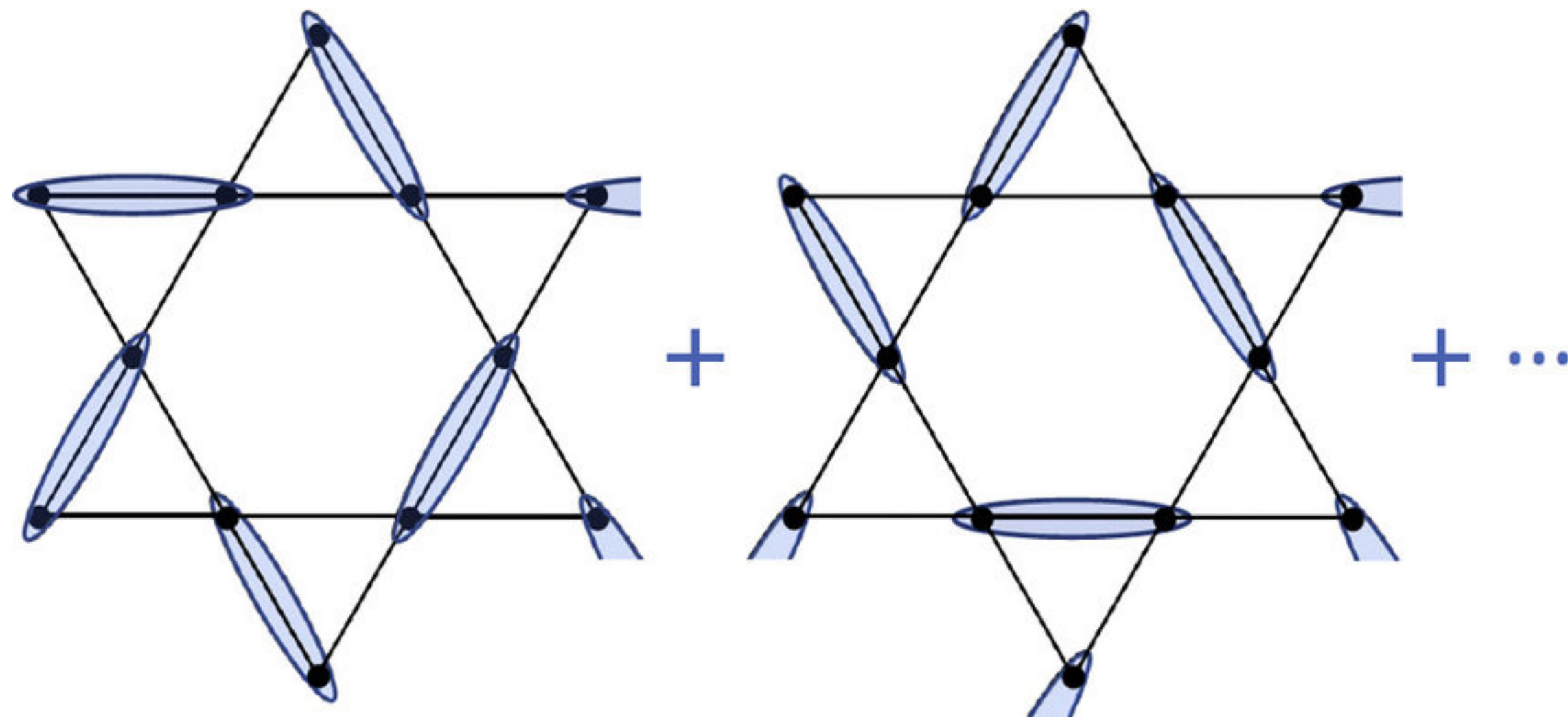


Strong magnetic moment  
renormalization because of  
frustration

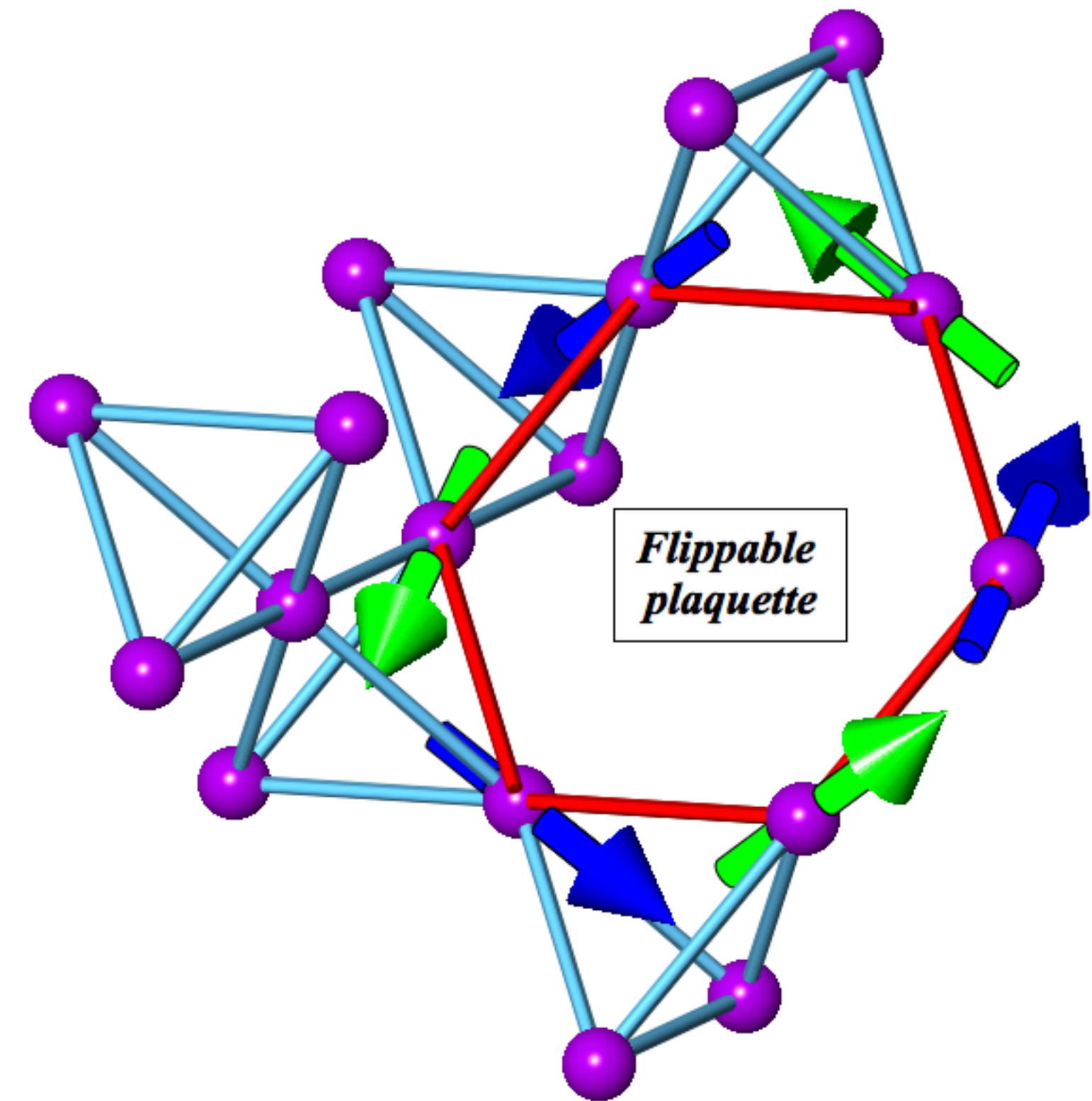
$$\langle S^z \rangle = S - 0.261$$

Th. Jolicoeur and J.C. Le Guillou, Phys. Rev. B 40, 2727 (1989)

# Frustration from geometry: kagome and pyrochlore lattices



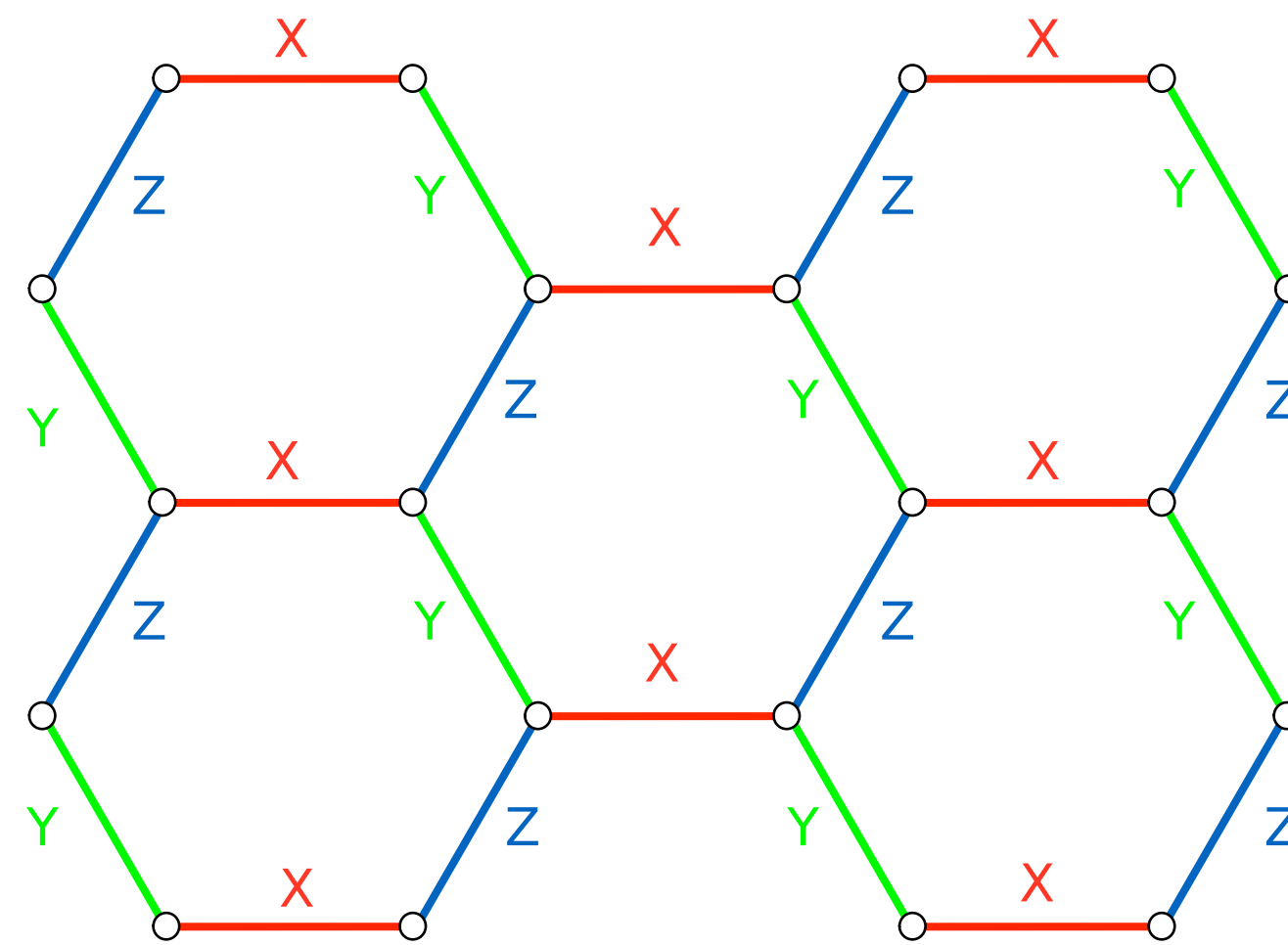
P. Mendels and F.Bert, Comptes Rendus Physique 17, 455 (2016)



M.J.P. Gingras and P.A. McClarty, Rep. Prog. Phys. 77, 056501 (2014)

# Frustration from anisotropic interactions: Kitaev model

$$\mathcal{H} = K \sum_{\langle ij \rangle^\gamma} S_i^\gamma S_j^\gamma = K \sum_{\langle ij \rangle^x} S_i^x S_j^x + K \sum_{\langle ij \rangle^y} S_i^y S_j^y + K \sum_{\langle ij \rangle^z} S_i^z S_j^z$$

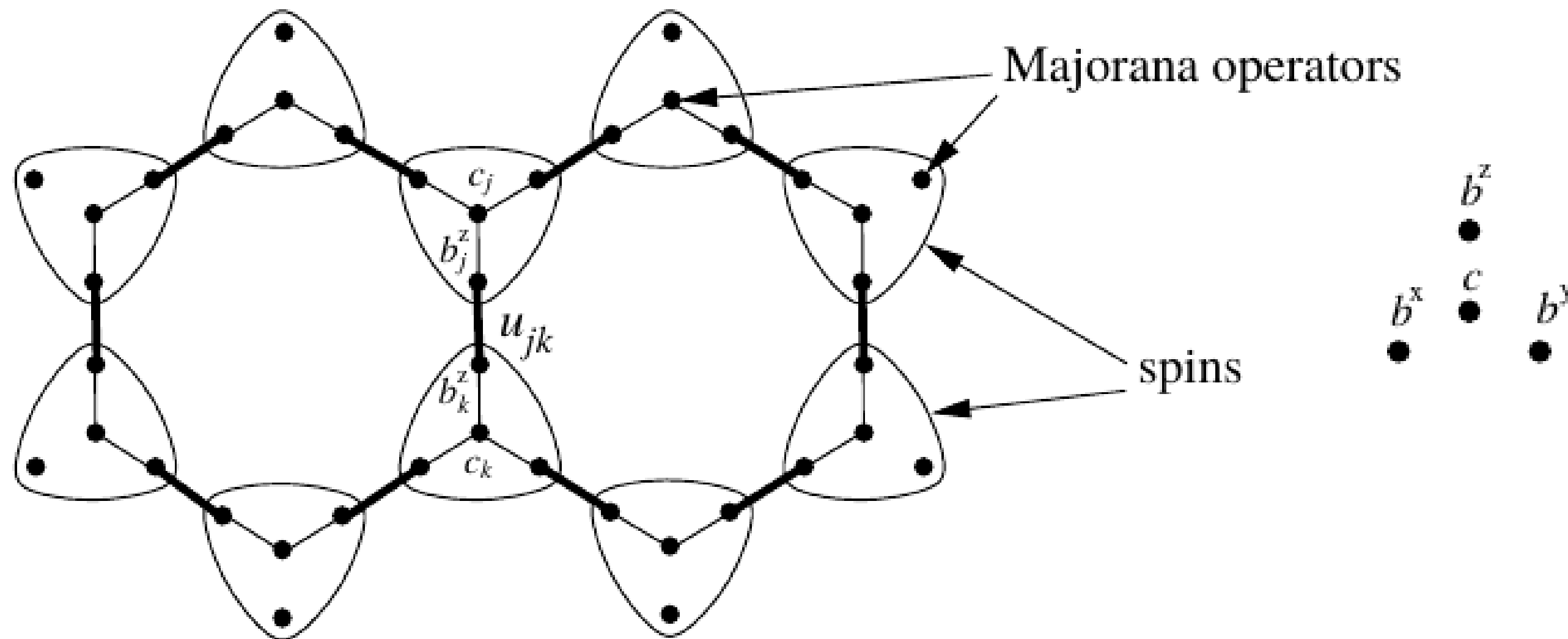


A. Kitaev, *Annals of Physics* 321, 2 (2006),  
January Special Issue

Exactly solvable for  $S=1/2$ , massive classical degeneracy

# Kitaev model: Majorana fermions

$$\mathcal{H} = K \sum_{\langle ij \rangle^\gamma} S_i^\gamma S_j^\gamma = K \sum_{\langle ij \rangle^x} S_i^x S_j^x + K \sum_{\langle ij \rangle^y} S_i^y S_j^y + K \sum_{\langle ij \rangle^z} S_i^z S_j^z$$



Free fermion solution!

$$S^x = ib^x c, \quad S^y = ib^y c, \quad S^z = ib^z c$$

Majorana fermions:

$$c_j^2 = 1, \quad c_i c_j = -c_j c_i, \quad i \neq j$$

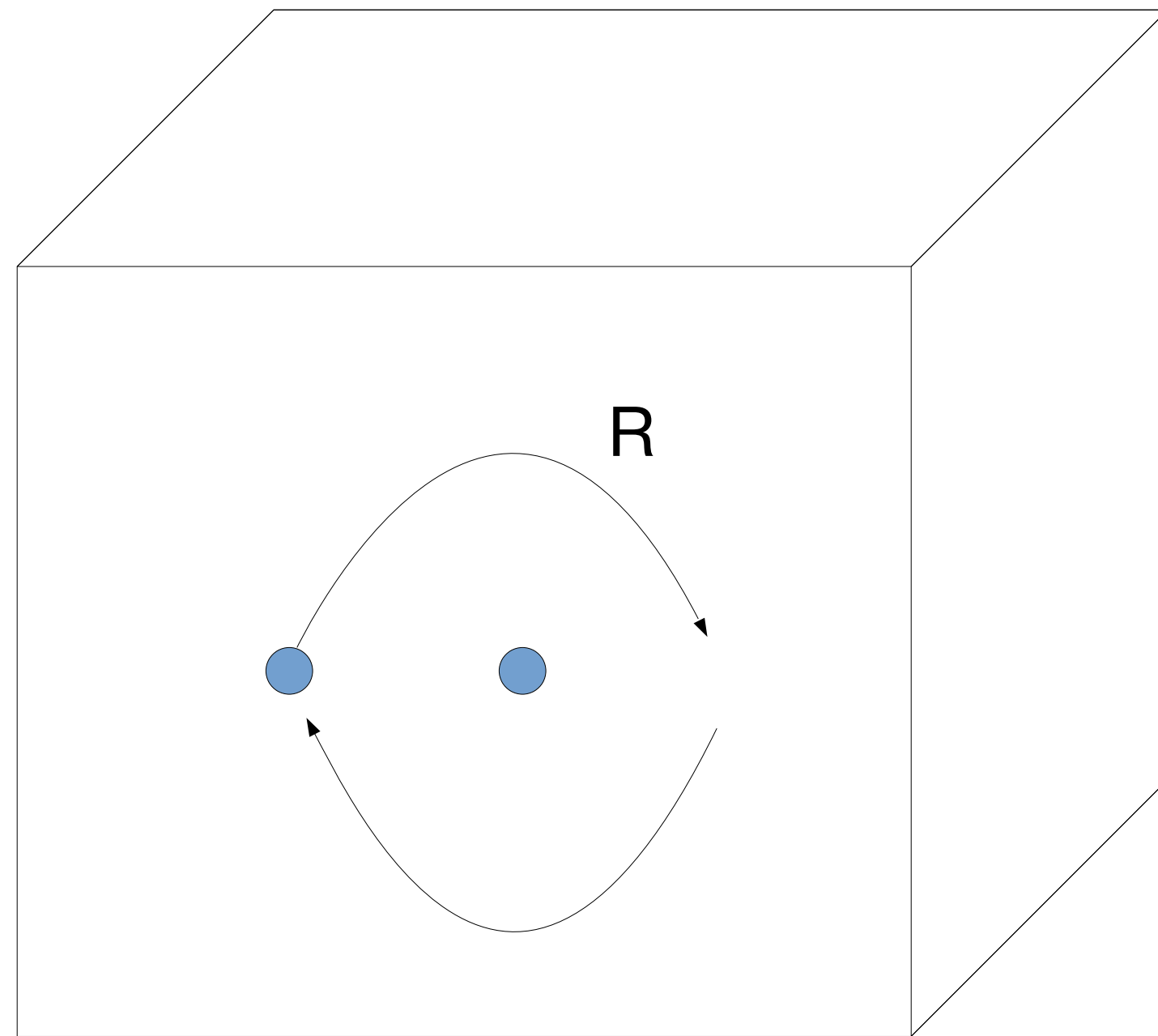
Non-abelian statistics in magnetic field: path to topological quantum computing

А. Китаев, А. Шень, М. Вялый

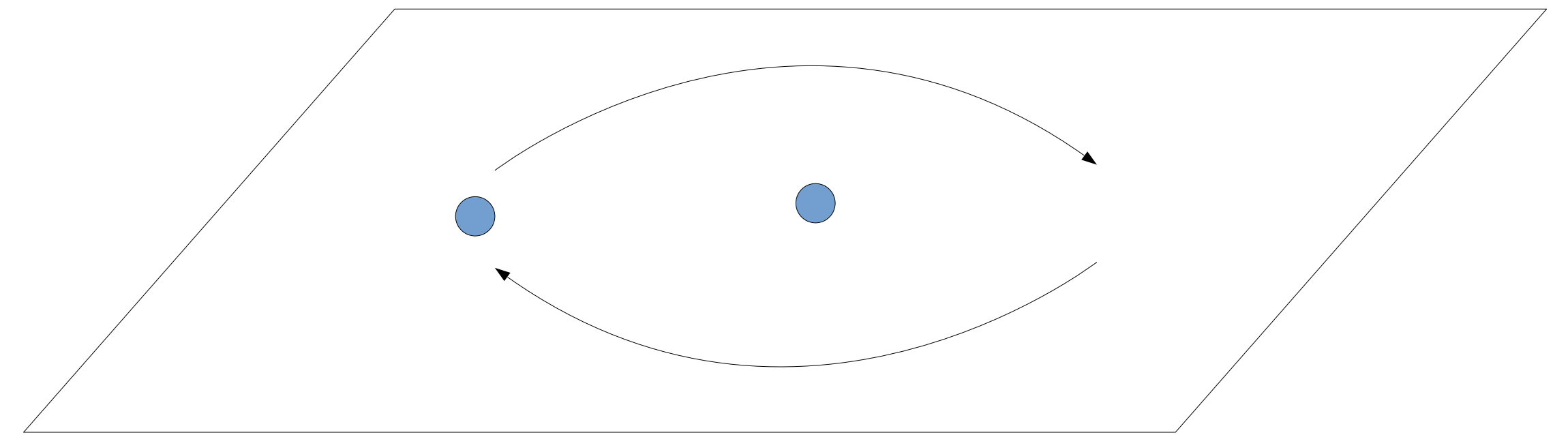
КЛАССИЧЕСКИЕ И КВАНТОВЫЕ ВЫЧИСЛЕНИЯ

4. Анионы. Анионы — это особые возбуждения в двумерных квантовых системах, в частности, в двумерной электронной жидкости в магнитном поле. Один из авторов (А.К.) считает этот подход наиболее интересным (поскольку он же его и придумал [32]), поэтому опишем его более подробно.

# Topological quantum computing

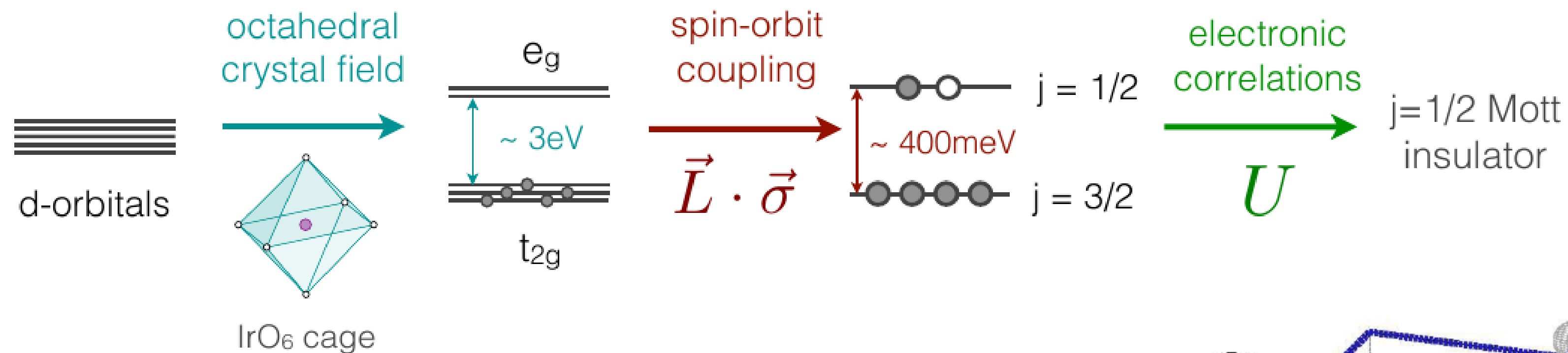


$$R^2 = 1$$
$$R = \pm 1$$



$$R = e^{i\theta}$$

# Kitaev materials: $d^5$ ions



$$|j_{1/2}\rangle = \begin{cases} \frac{1}{\sqrt{3}}(-|xy, \uparrow\rangle - i|xz, \downarrow\rangle - |yz, \downarrow\rangle) & (m_j = +\frac{1}{2}) \\ \frac{1}{\sqrt{3}}(|xy, \downarrow\rangle + i|xz, \uparrow\rangle - |yz, \uparrow\rangle) & (m_j = -\frac{1}{2}) \end{cases}$$

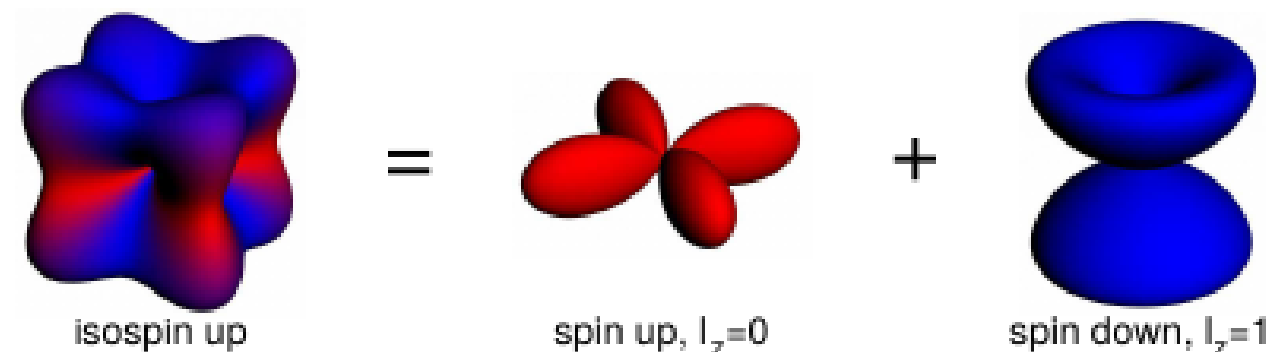
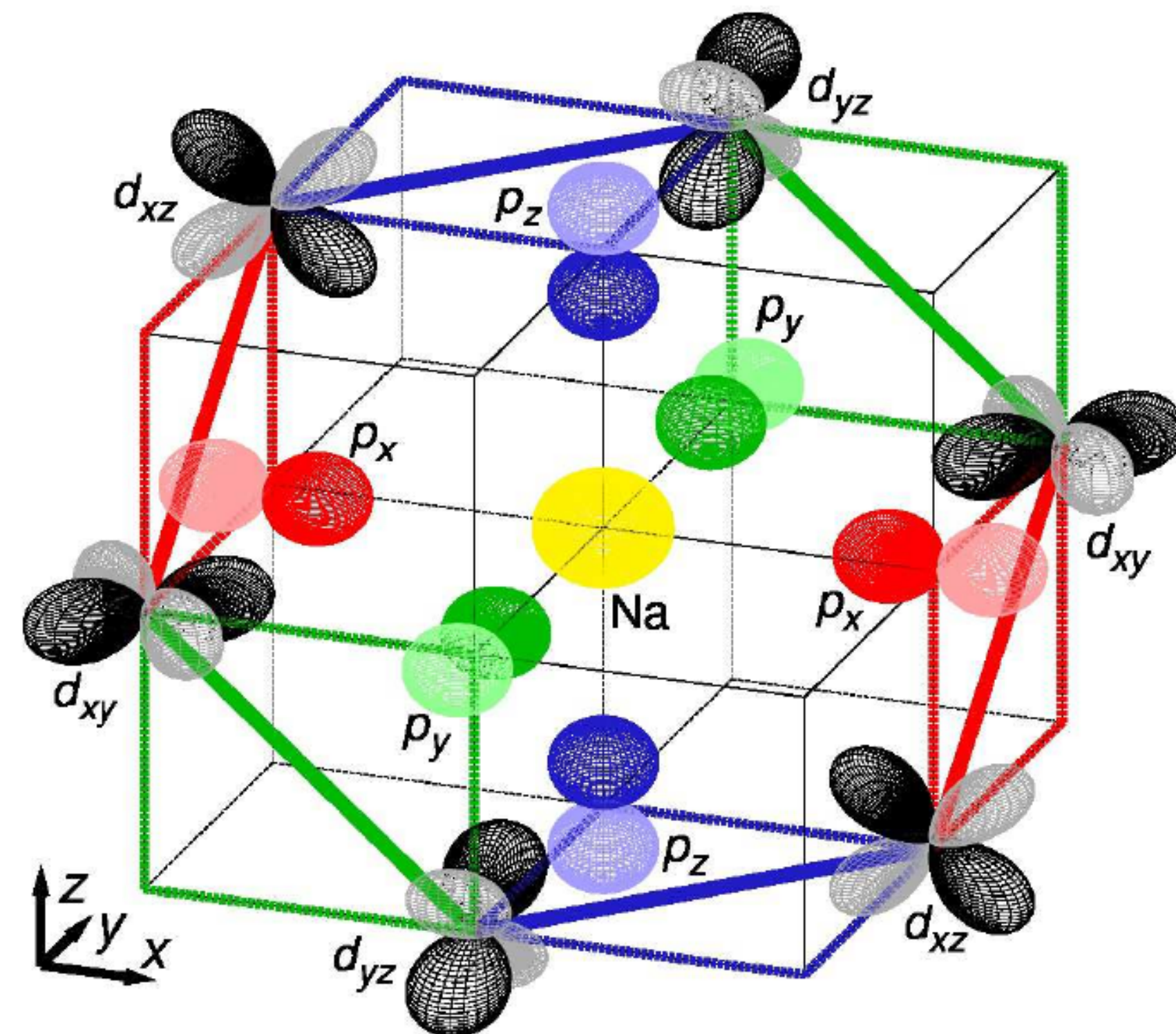


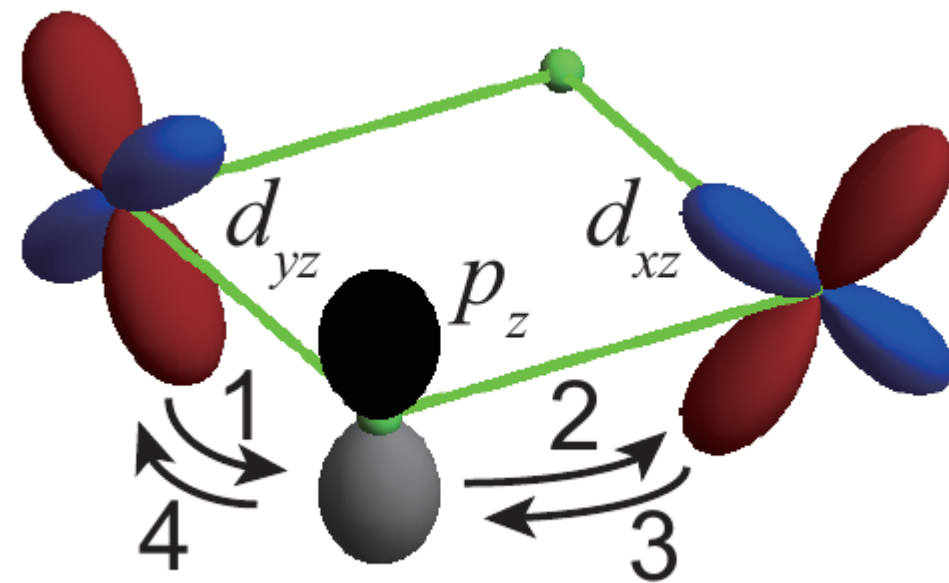
FIG. 1: (Color online) Density profile of a hole in the isospin up state (without tetragonal distortion). It is a superposition of a spin up hole density in  $|xy\rangle$ -orbital,  $l_z = 0$ , (middle) and spin down one in  $(|yz\rangle + i|xz\rangle)$  state,  $l_z = 1$ , (right).



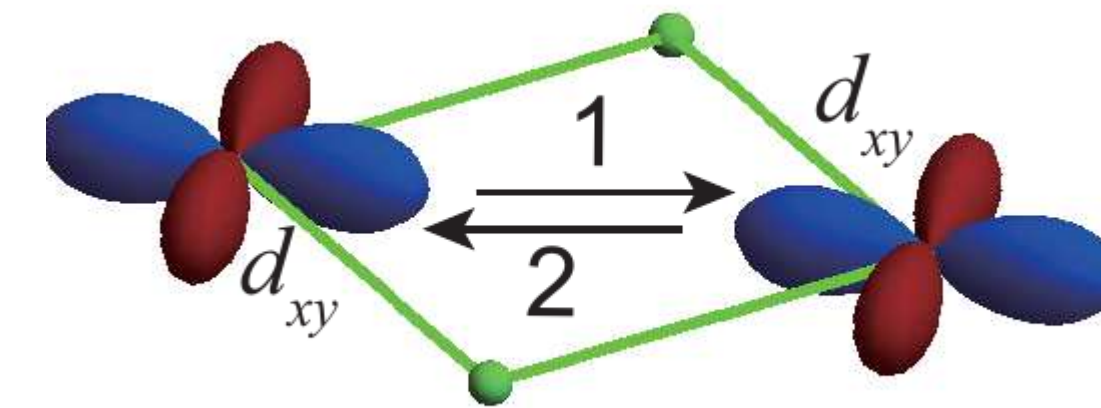


# Kitaev-Heisenberg model

Kitaev bond-dependent  
exchange  $K$

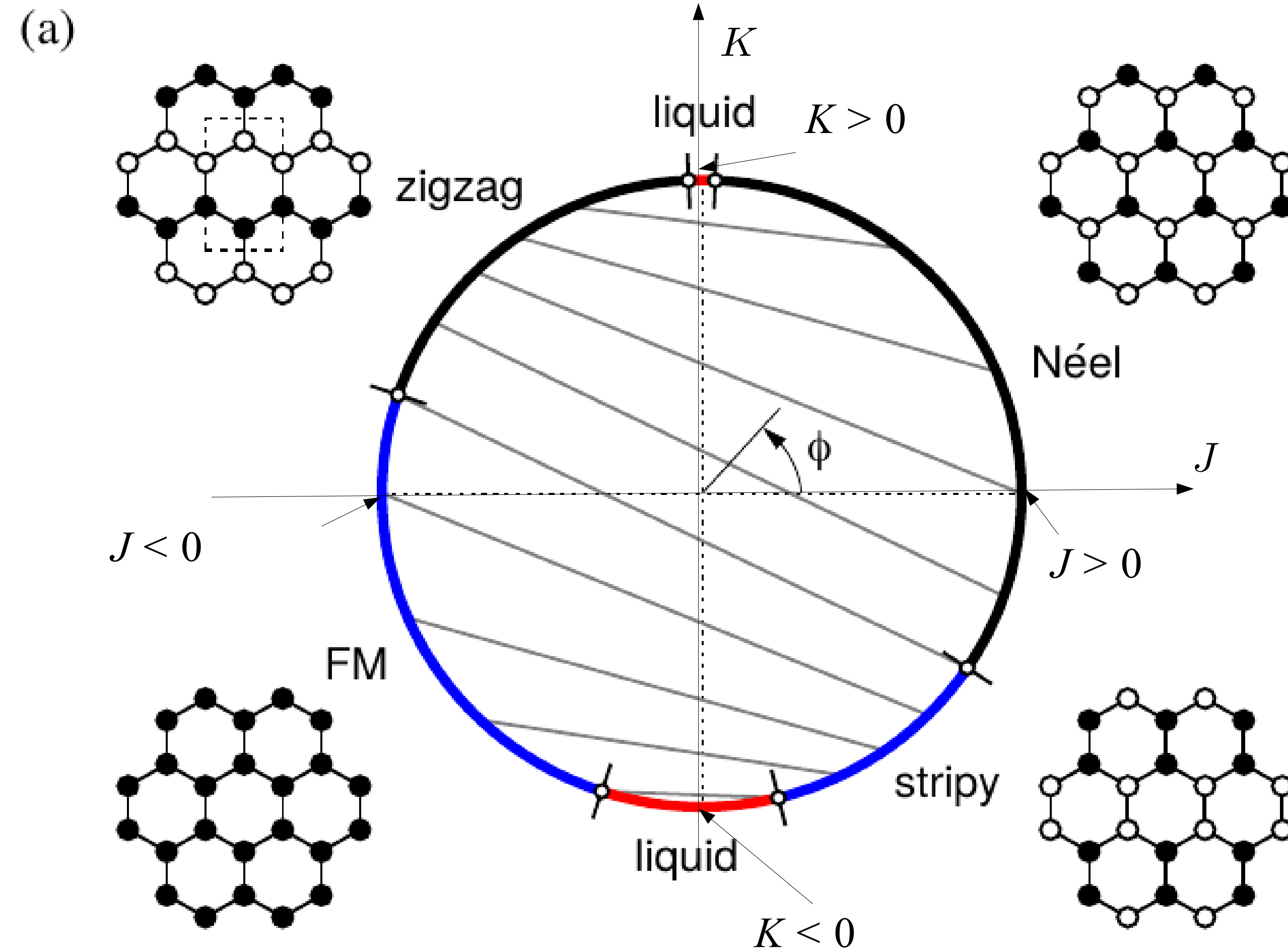


Heisenberg bond-independent  
exchange  $J$



$$\mathcal{H} = \sum_{\langle ij \rangle^\gamma} J \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma$$

# Kitaev-Heisenberg model phase diagram



$$\mathcal{H} = \sum_{\langle ij \rangle^\gamma} J \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma$$

$$J = \cos \phi, \quad K = \sin \phi$$

Mostly — ordered states



# Kitaev materials: Na<sub>2</sub>IrO<sub>3</sub>

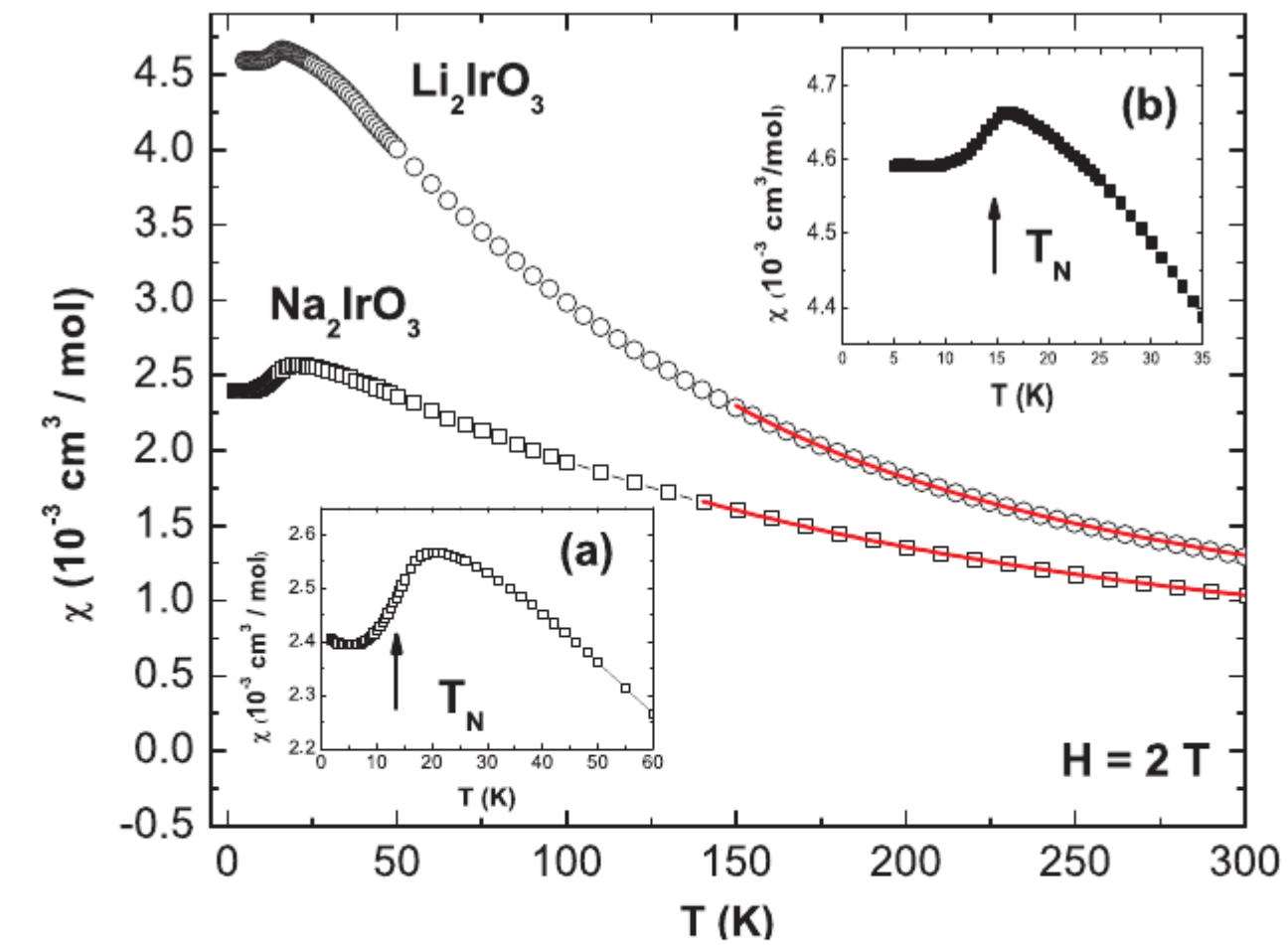
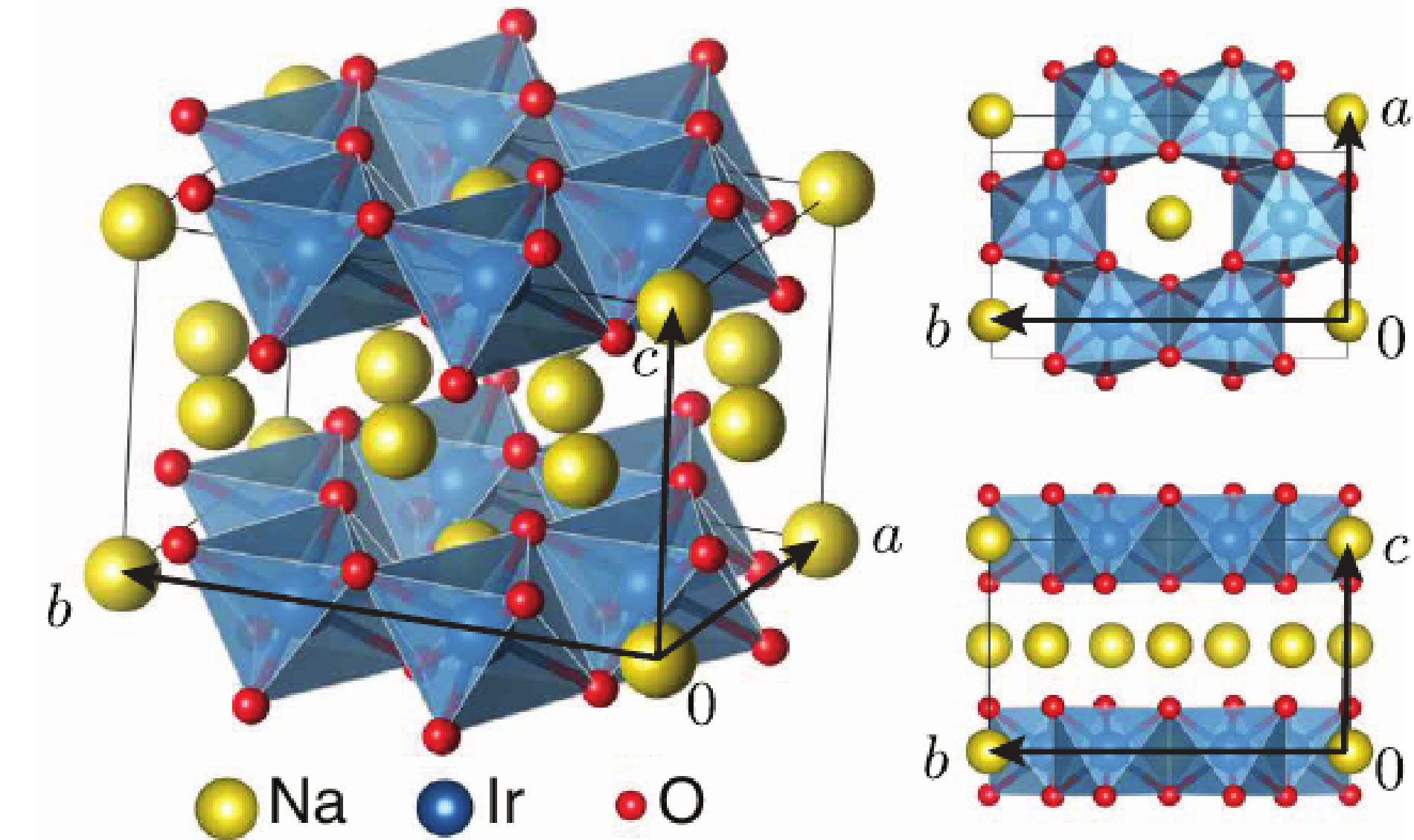


FIG. 1: (Color online) Magnetic susceptibility  $\chi$  versus temperature  $T$  for  $A_2\text{IrO}_3$  ( $A = \text{Na}, \text{Li}$ ). The fit by the Curie-Weiss (CW) expression  $\chi = \chi_0 + C/(T - \theta)$  is shown as the curve through the data. The insets (a) and (b) shows the anomaly at the antiferromagnetic ordering for the Na and Li systems, respectively.

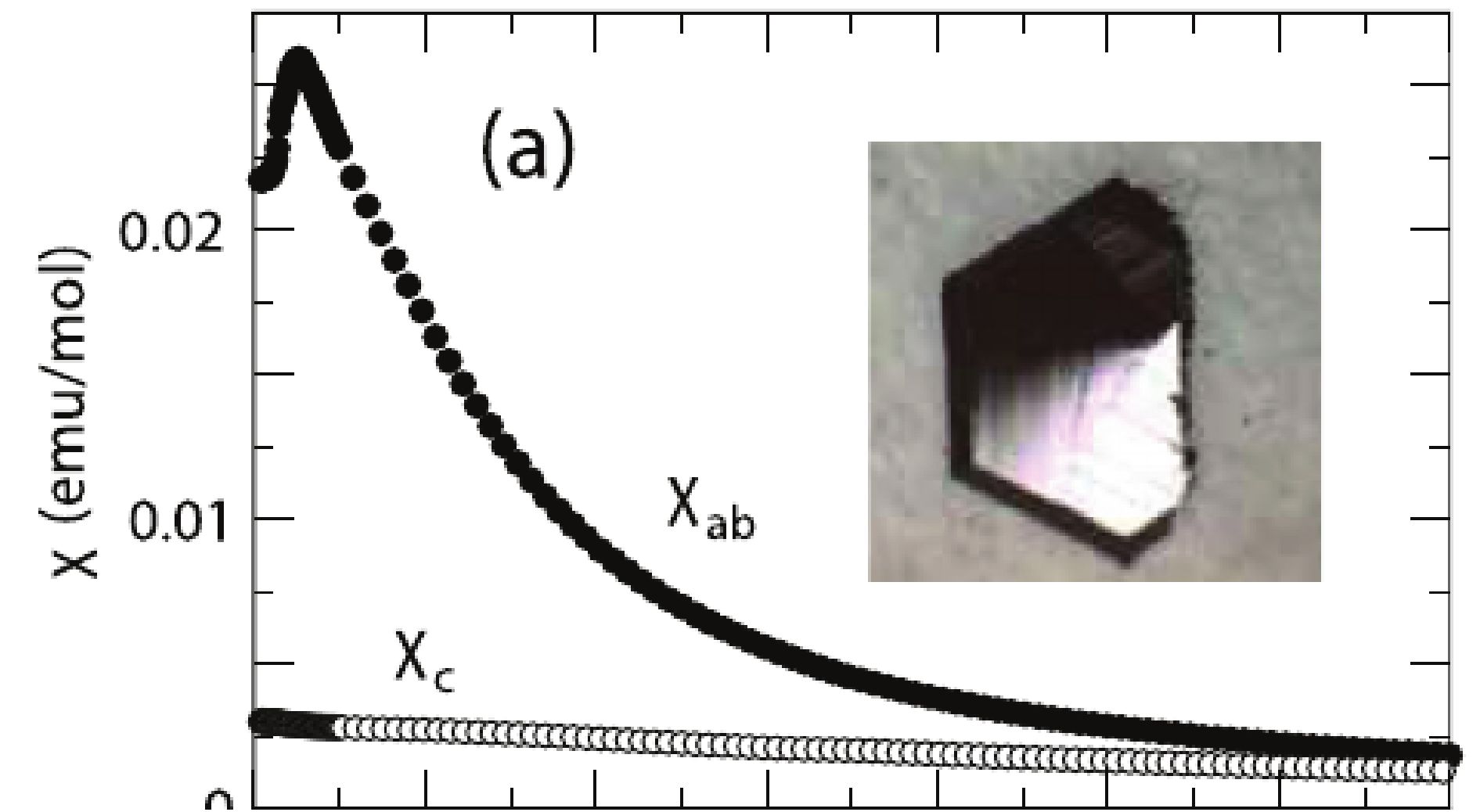
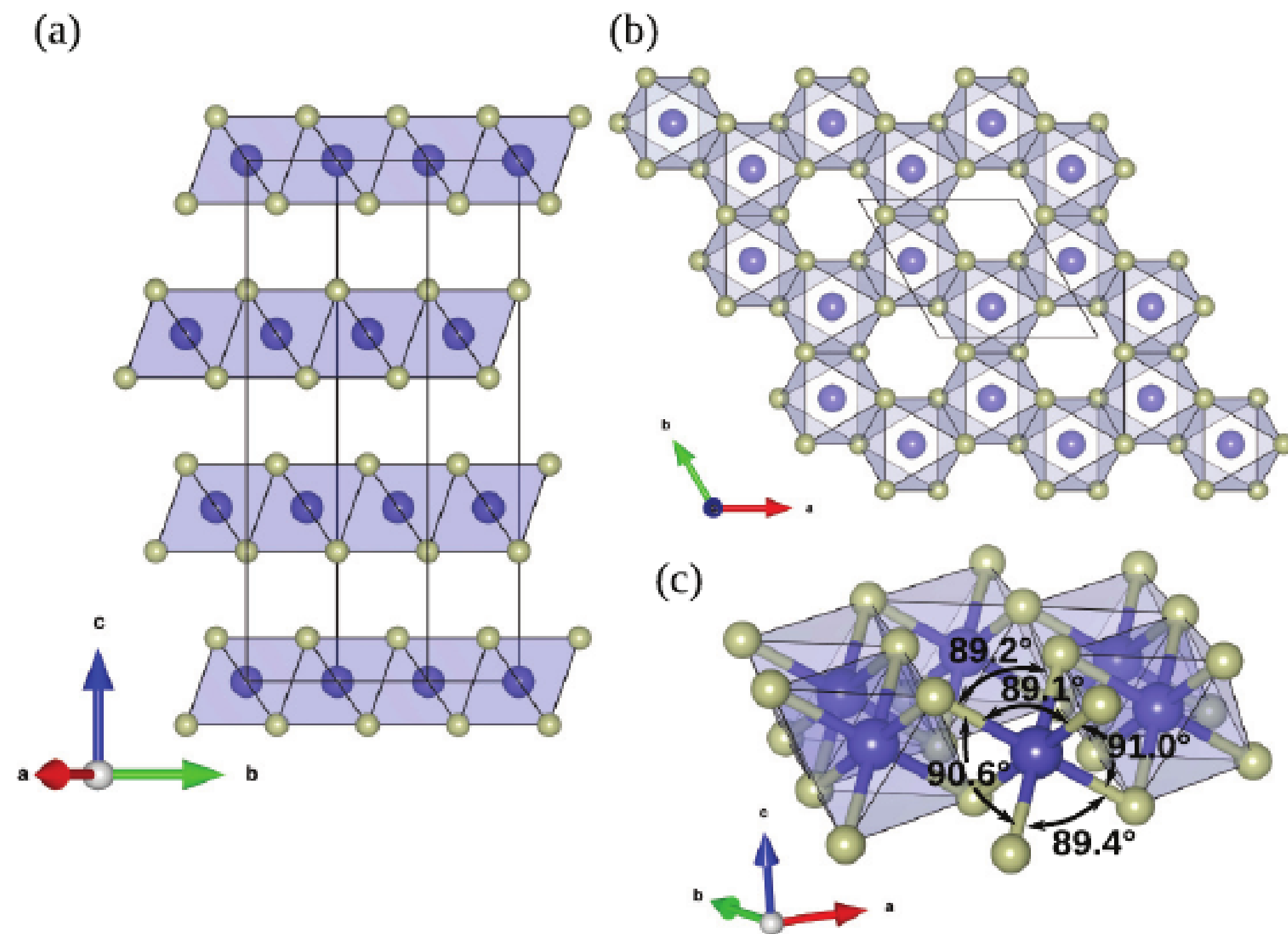
Yogesh Singh, S. Manni, J. Reuther, T. Berlijn, R. Thomale, W. Ku, S. Trebst, and P. Gegenwart, Phys. Rev. Lett. 108, 127203 (2012)

**Table 3.** Bond-averaged values of the largest magnetic interactions (in units of meV) within the plane for Na<sub>2</sub>IrO<sub>3</sub> computed using various methods. ‘Pert. Theo.’ refers to second order perturbation theory (section 3.2), ‘QC’ = quantum chemistry methods, ‘ED’ = exact diagonalization.

Method	$J_1$	$K_1$	$\Gamma_1$	$\Gamma'_1$	$K_2$	$J_3$
Pert. Theo. [44]	+3.2	-29.4	+1.1	-3.5	-0.4	+1.7
QC (2-site) [43]	+2.7	-16.9	+1.0	—	—	—
ED (6-site) [45]	+0.5	-16.8	+1.4	-2.1	-1.4	+6.7

Stephen M Winter et al, J. Phys.: Condens. Matter 29 493002 (2017)

# Kitaev materials: RuCl<sub>3</sub>



J. A. Sears, M. Songvilay, K. W. Plumb, J. P. Clancy, Y. Qiu, Y. Zhao, D. Parshall, and Young-June Kim, Phys. Rev. B 91, 144420 (2015)

Zigzag state order at 7K

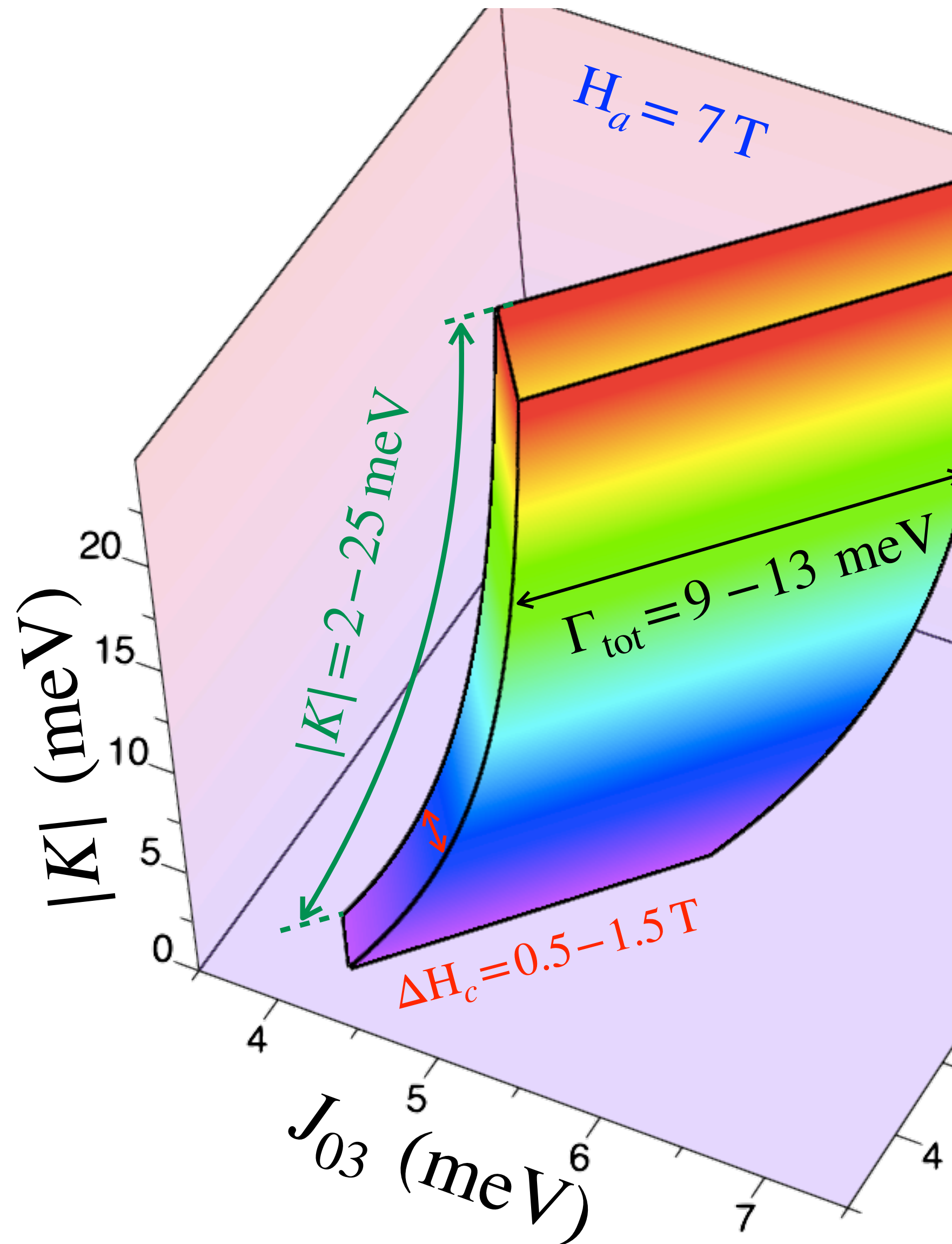
K. W. Plumb, J. P. Clancy, L. J. Sandilands, V. Vijay Shankar, Y. F. Hu, K. S. Burch, Hae-Young Kee, and Young-June Kim, Phys. Rev. B 90, 041112(R) (2014)

# RuCl<sub>3</sub> exchange parameters

Reference	Method	$K$	$\Gamma$	$\Gamma'$	$J$	$J_3$	$\Gamma+2\Gamma'$	$J+3J_3$
Banerjee et al. [22]	LSWT, INS fit	+7.0			-4.6			-4.6
Kim et al. [29]	DFT+ $t/U$ , $P3$	<b>-6.55</b>	<b>5.25</b>	-0.95	-1.53		3.35	-1.53
	DFT+SOC+ $t/U$	<b>-8.21</b>	<b>4.16</b>	-0.93	-0.97		2.3	-0.97
	same+fixed lattice	-3.55	7.08	-0.54	<b>-2.76</b>		6.01	-2.76
	same+ $U$ +zigzag	+4.6	6.42	-0.04	<b>-3.5</b>		6.34	-3.5
Winter et al. [30]	DFT+ED, $C2$	<b>-6.67</b>	6.6	-0.87	-1.67	<b>2.8</b>	4.87	6.73
	same, $P3$	+7.6	8.4	+0.2	-5.5	<b>2.3</b>	<b>8.8</b>	+1.4
Yadav et al. [24]	Quantum chemistry	<b>-5.6</b>	-0.87		+1.2		-0.87	+1.2
Ran et al. [34]	LSWT, INS fit	<b>-6.8</b>	9.5				<b>9.5</b>	
Hou et al. [31]	DFT+ $t/U$ , $U=2.5\text{eV}$	-14.43	6.43		<b>-2.23</b>	<b>2.07</b>	6.43	+3.97
	same, $U=3.0\text{eV}$	-12.23	<b>4.83</b>		<b>-1.93</b>	1.6	4.83	+2.87
	same, $U=3.5\text{eV}$	<b>-10.67</b>	<b>3.8</b>		-1.73	1.27	3.8	+2.07
Wang et al. [32]	DFT+ $t/U$ , $P3$	<b>-10.9</b>	6.1		-0.3	0.03	6.1	-0.21
	same, $C2$	<b>-5.5</b>	7.6		+0.1	0.1	7.6	+0.4
Winter et al. [35]	<i>Ab initio</i> +INS fit	<b>-5.0</b>	2.5		-0.5	0.5	2.5	+1.0
Suzuki et al. [36]	ED, $C_p$ fit	-24.41	<b>5.25</b>	-0.95	-1.53		3.35	-1.53
Cookmeyer et al. [37]	thermal Hall fit	<b>-5.0</b>	2.5		-0.5	0.11	2.5	-0.16
Wu et al. [38]	LSWT, THz fit	-2.8	2.4		-0.35	0.34	2.4	+0.67
Ozel et al. [39]	same, $K > 0$	+1.15	2.92	+1.27	-0.95		5.45	-0.95
	same, $K < 0$	-3.5	2.35		+0.46		2.35	+0.46
Eichstaedt et al. [33]	DFT+Wannier+ $t/U$	-14.3	9.8	-2.23	-1.4	0.97	5.33	+1.5
Sahasrabudhe et al. [42]	ED, Raman fit	<b>-10.0</b>	3.75		-0.75	0.75	3.75	1.5
Sears et al. [40]	Magnetization fit	<b>-10.0</b>	10.6	-0.9	<b>-2.7</b>		<b>8.8</b>	-2.7
Laurell et al. [41]	ED, $C_p$ fit	-15.1	10.1	-0.12	-1.3	0.9	<b>9.86</b>	+1.4

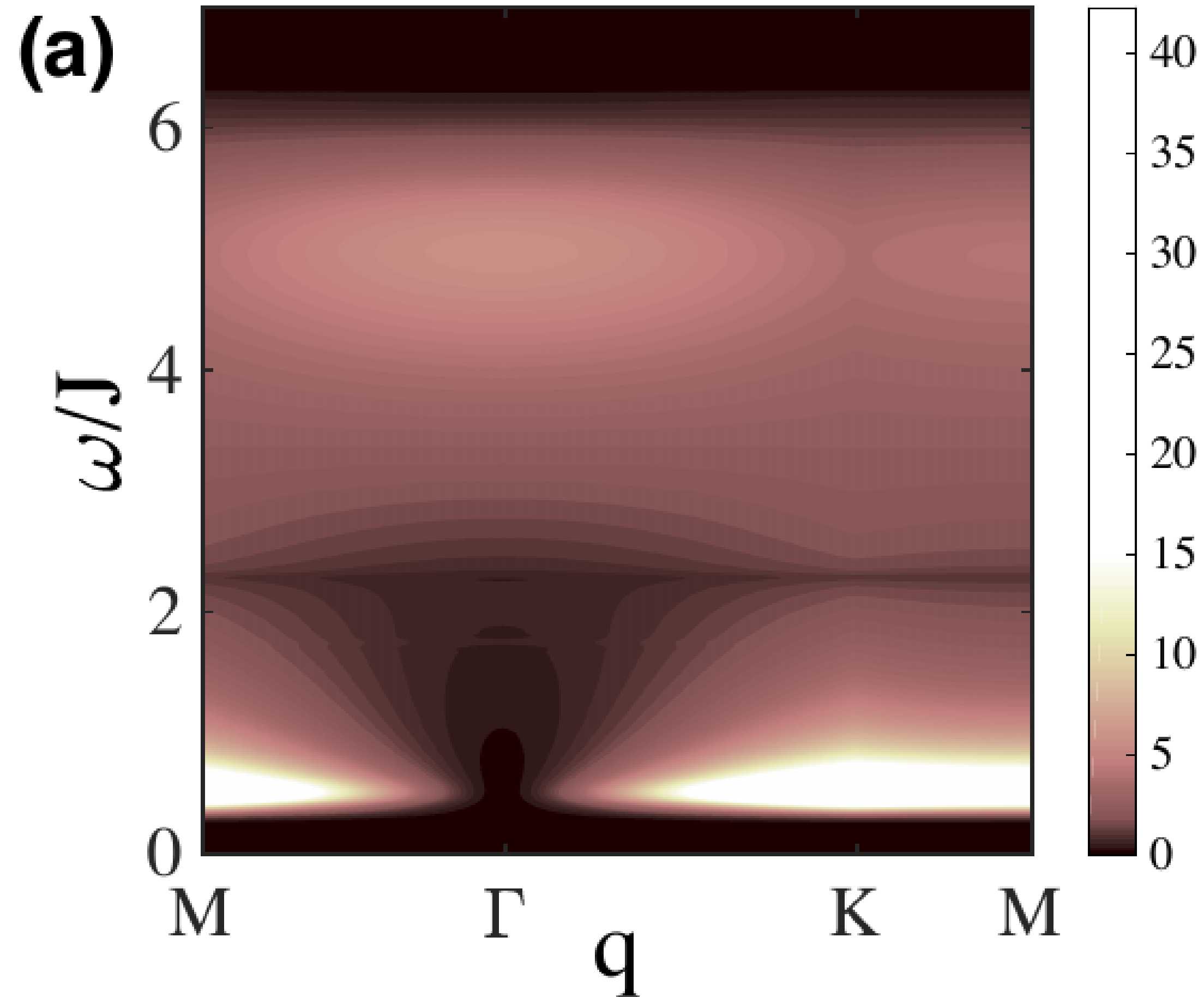
P. A. Maksimov and A. L. Chernyshev, Phys. Rev. Research 2, 033011 (2020)

# RuCl<sub>3</sub> exchange parameters



P. A. Maksimov and A. L. Chernyshev, Phys. Rev. Research 2, 033011 (2020)

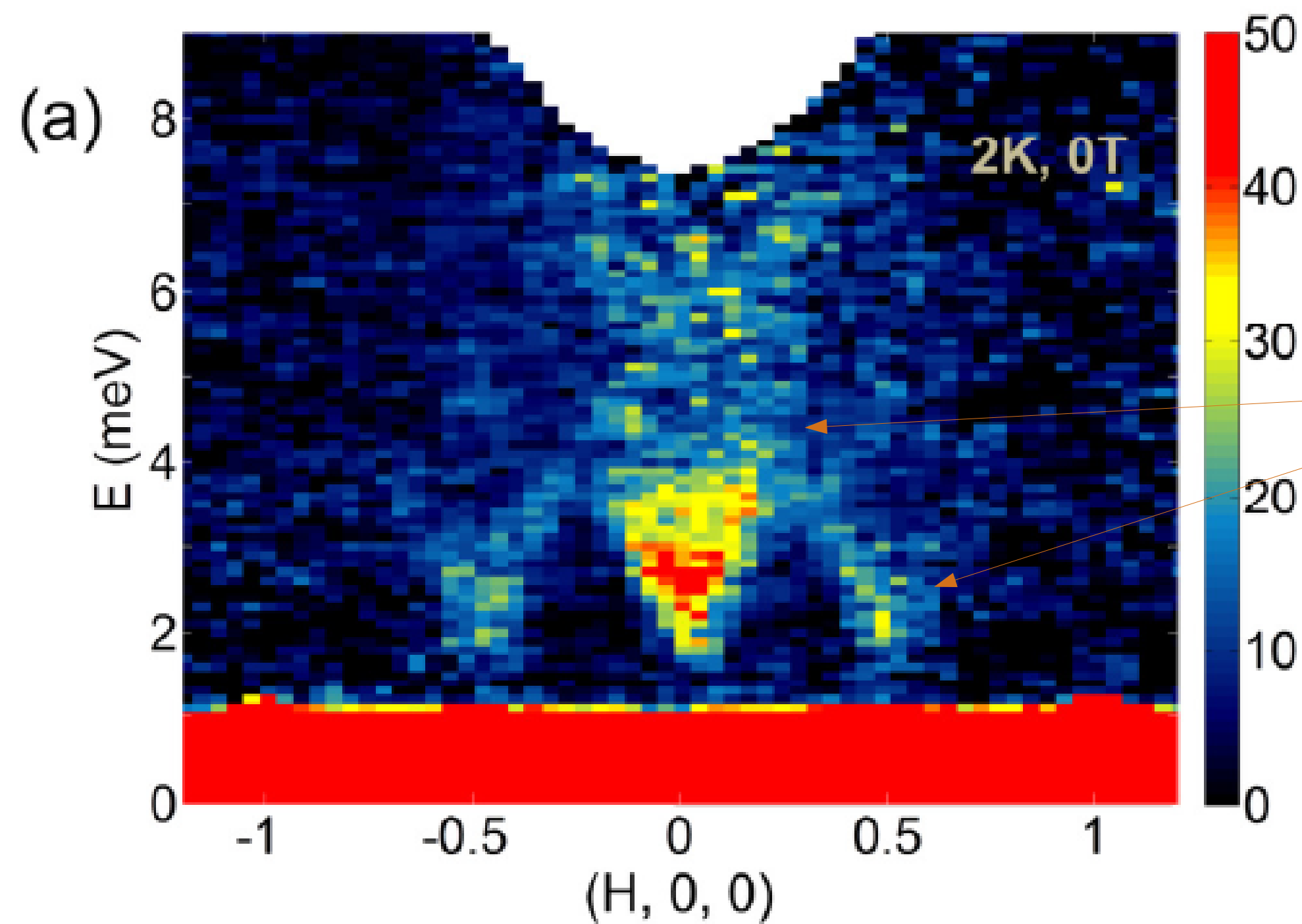
# Fractionalization of excitations



- Kitaev model excitations have  $S=1/2$
- One neutron ( $S=1$ ) creates a pair of fermions
- Energy and momentum are split between two particles  $\rightarrow$  broad continuum



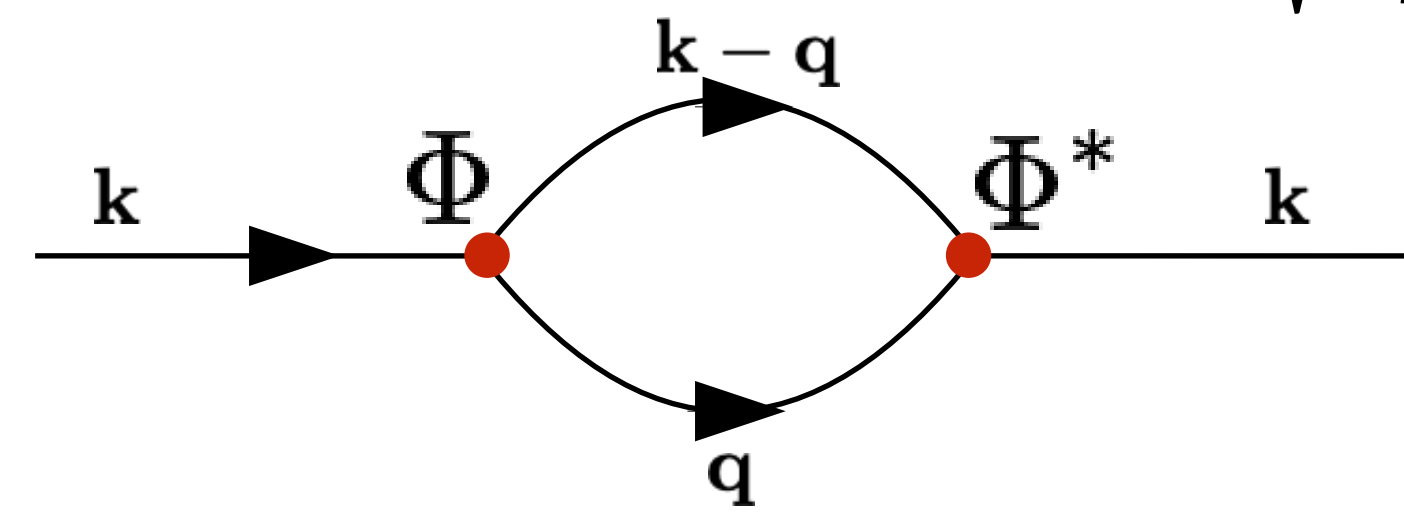
# Broadening of spectral peaks in a-RuCl<sub>3</sub>



- Coexistence of well-defined spin-waves and broad continuum
- Signatures of Majoranas

$$\mathcal{H} = \sum_{\langle ij \rangle \gamma} JS_i \cdot S_j + K S_i^\gamma S_j^\gamma + \Gamma \left( S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha \right) + \Gamma' \left( S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma \right)$$

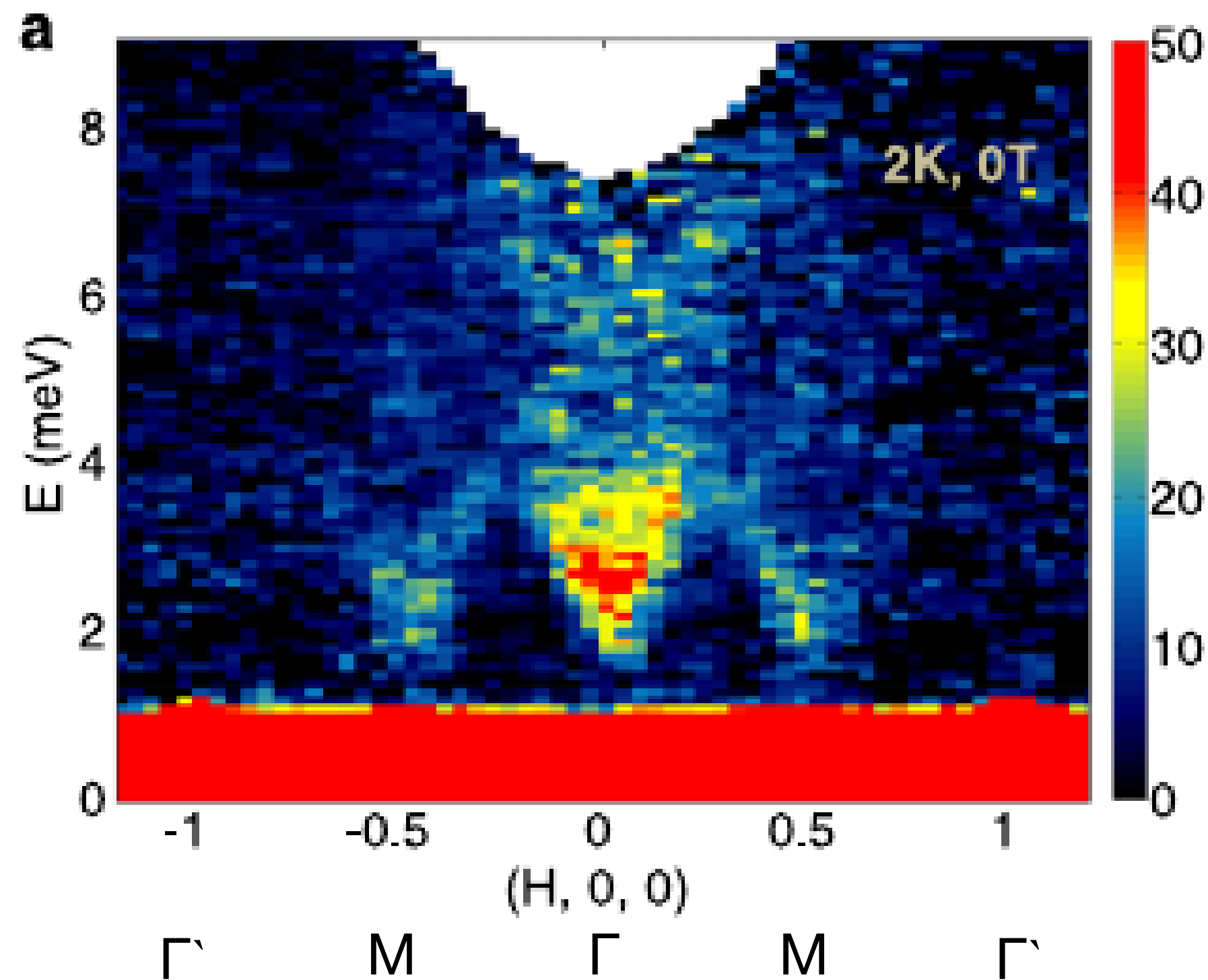
$$S^z = S - a^\dagger a \quad S^x \approx \sqrt{\frac{S}{2}} (a + a^\dagger)$$



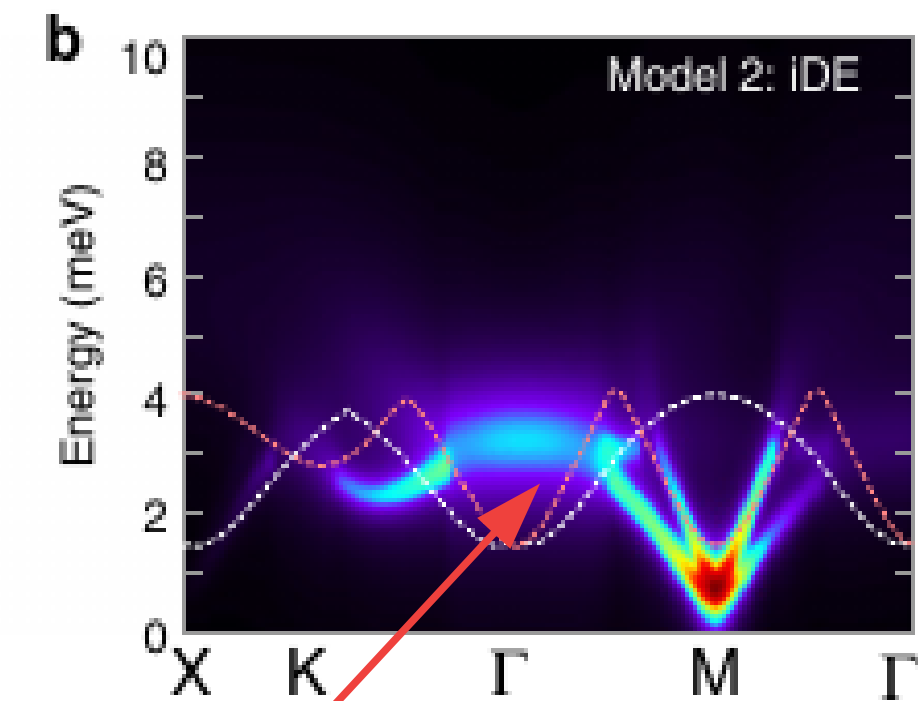
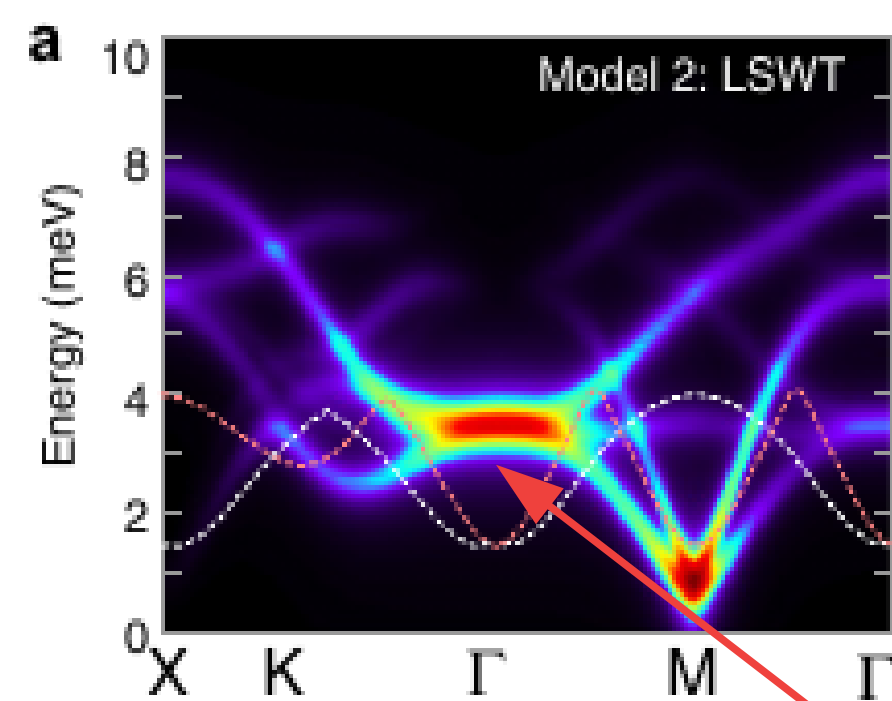
Banerjee, A., Lampen-Kelley, P., Knolle, J. et al.,  
npj Quant Mater 3, 8 (2018)

# Broadening of spectral peaks in a-RuCl<sub>3</sub>

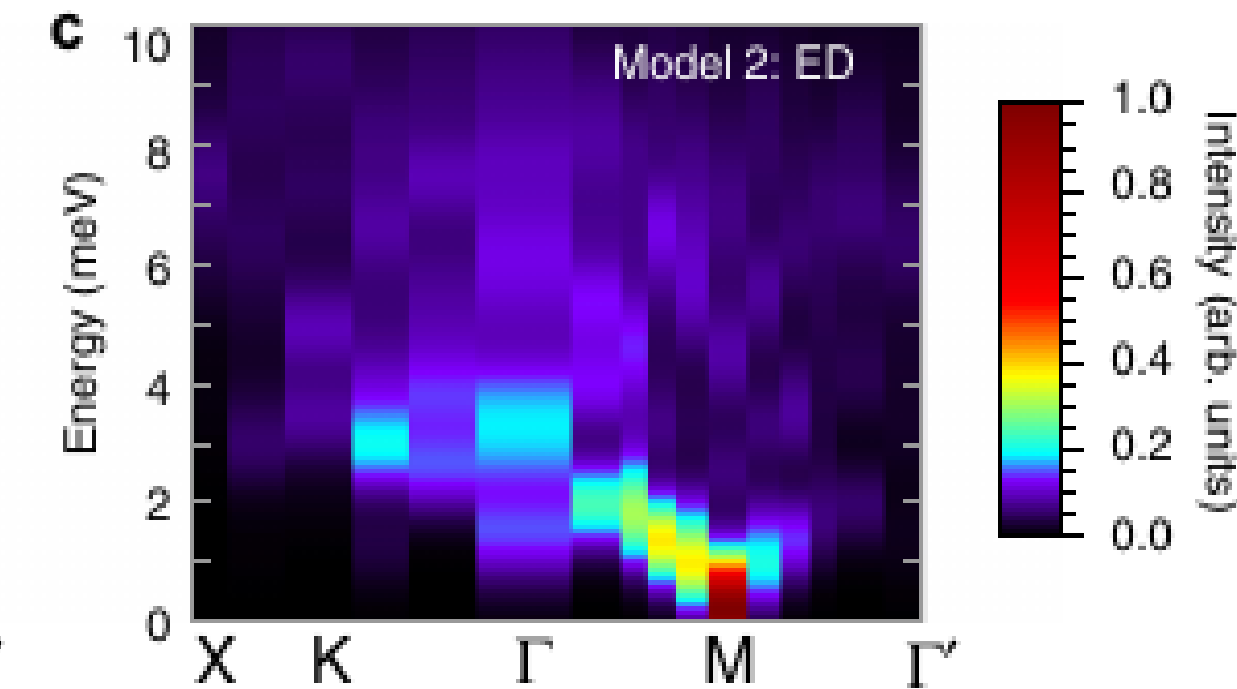
Inelastic neutron scattering



Non-interacting magnons    Interacting magnons



Exact diagonalization

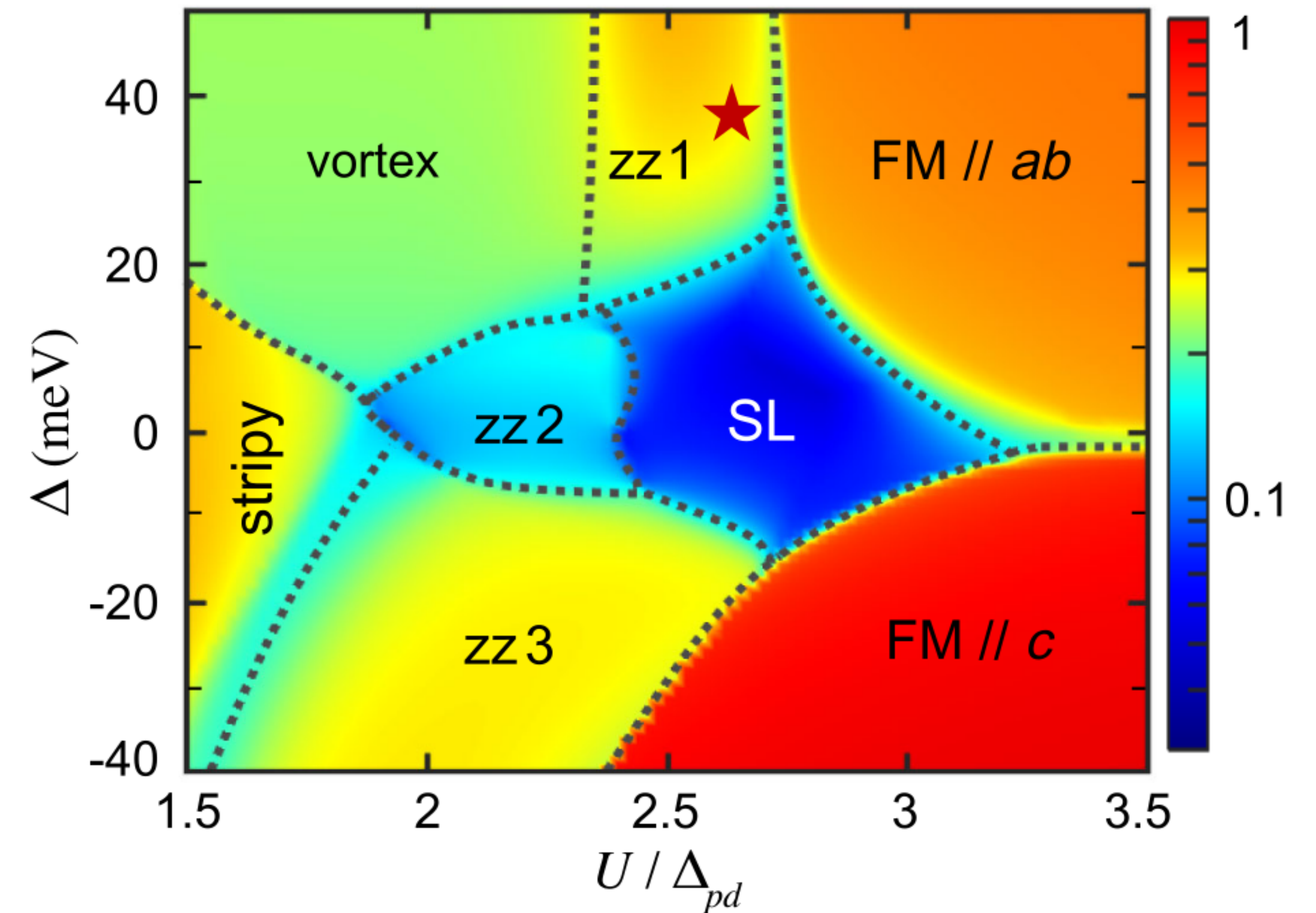
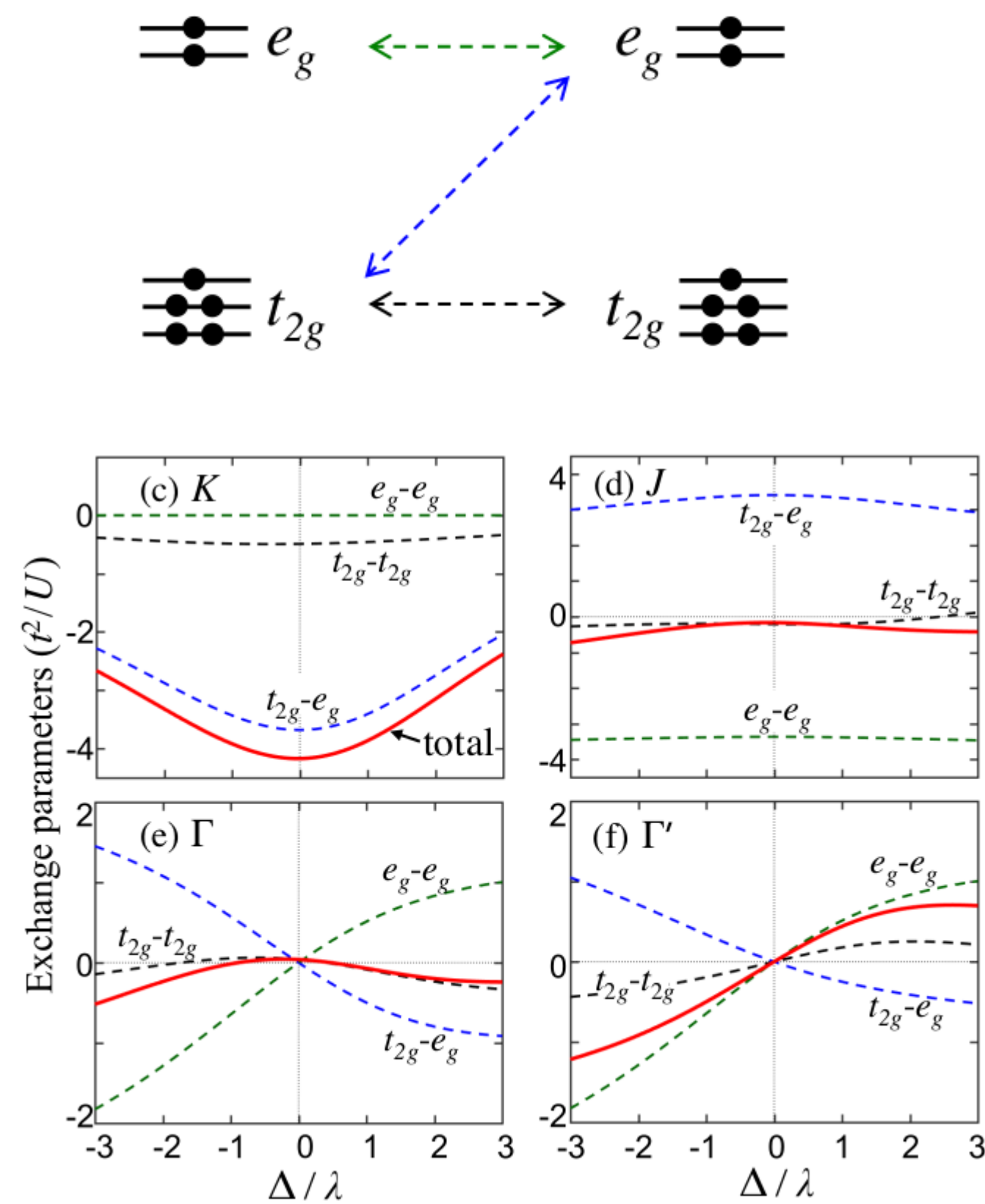


Broadening due to magnon decays

Banerjee, A., Lampen-Kelley, P., Knolle, J. et al.,  
npj Quant Mater 3, 8 (2018)

Winter, S.M., Riedl, K., Maksimov, P.A. et al., Nat. Commun 8, 1152  
(2017)

# Kitaev materials: $d^7$ ions - $\text{Co}^{2+}$

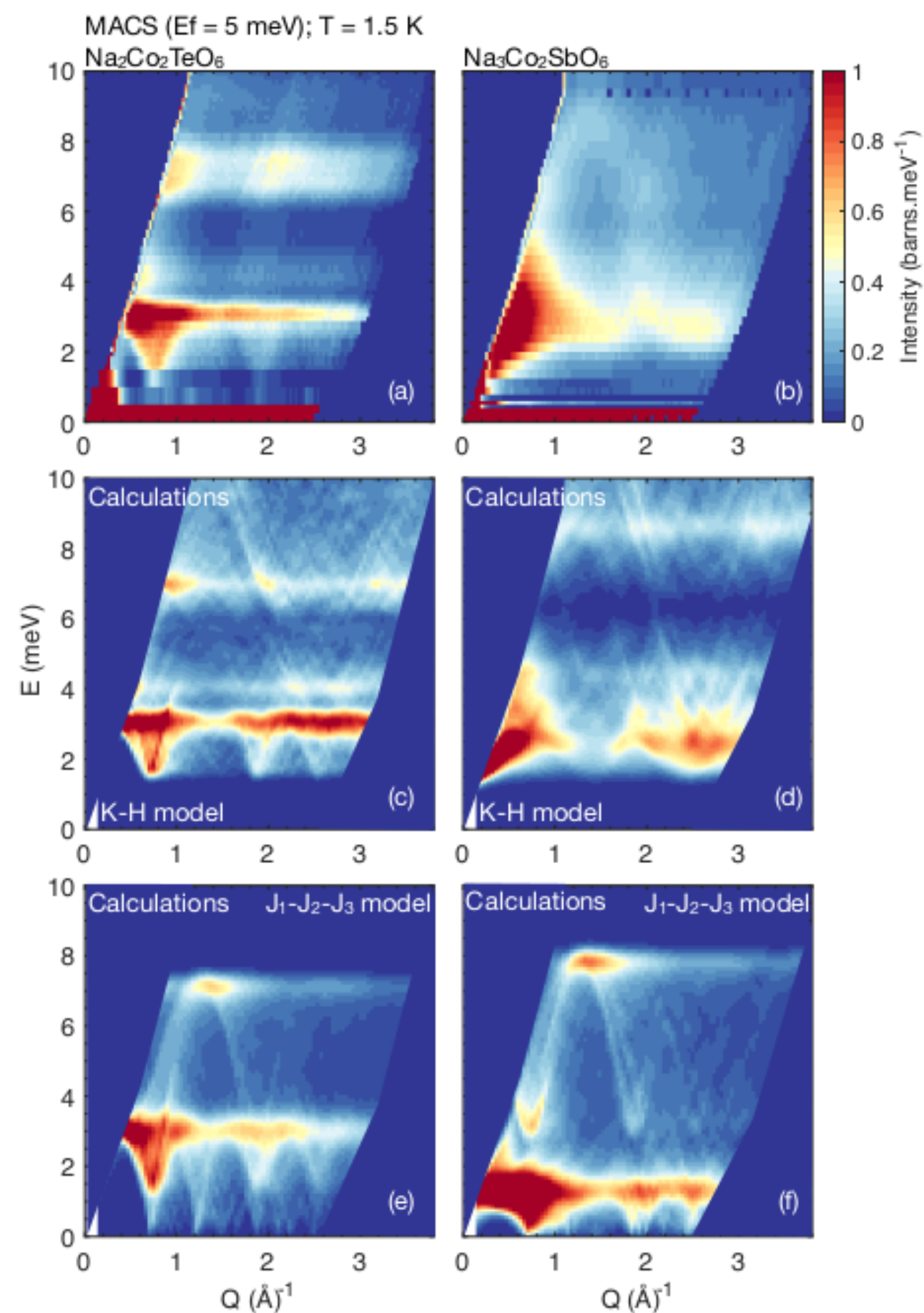


H. Liu and G. Khaliullin, Phys. Rev. B 97, 014407 (2018)

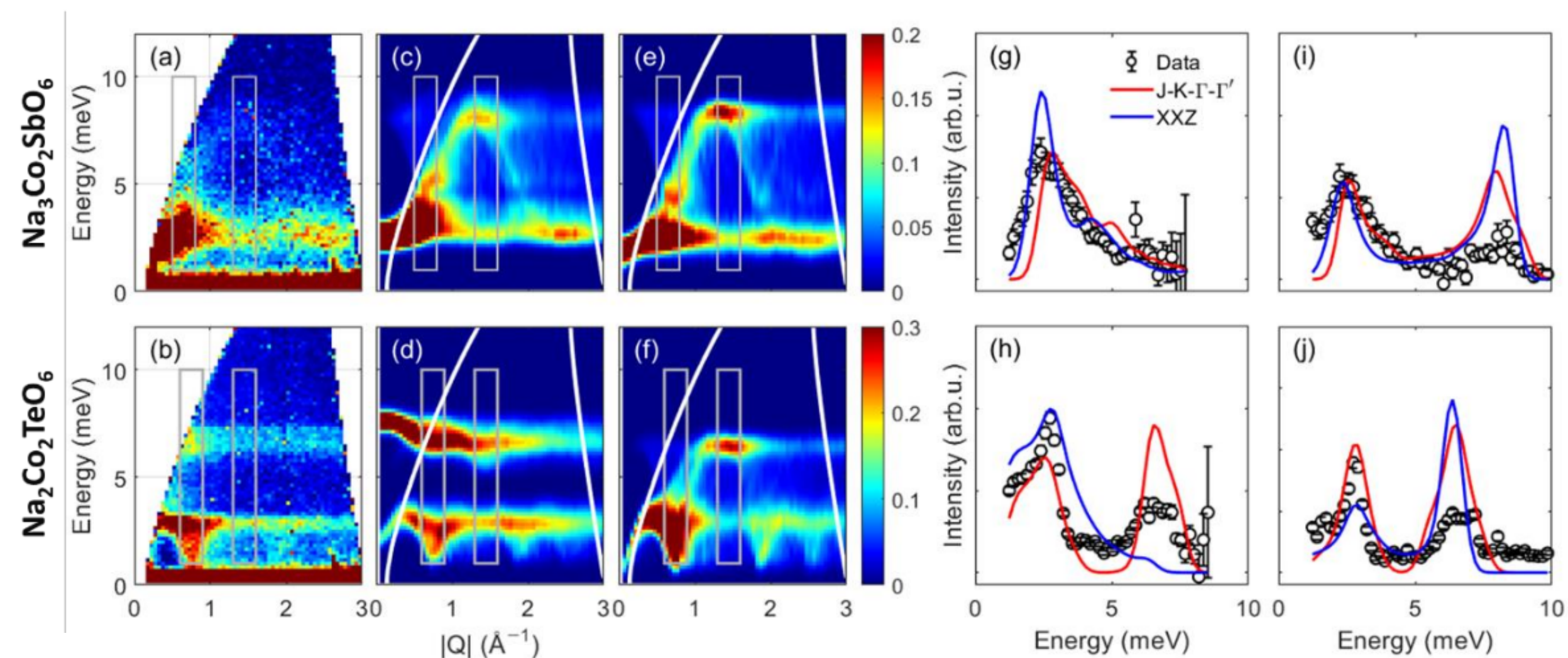
R. Sano, Y. Kato and Y. Motome, Phys. Rev. B 97, 014408 (2018)

H. Liu, J. Chaloupka and G. Khaliullin, Phys. Rev. Lett. 125, 047201 (2020)

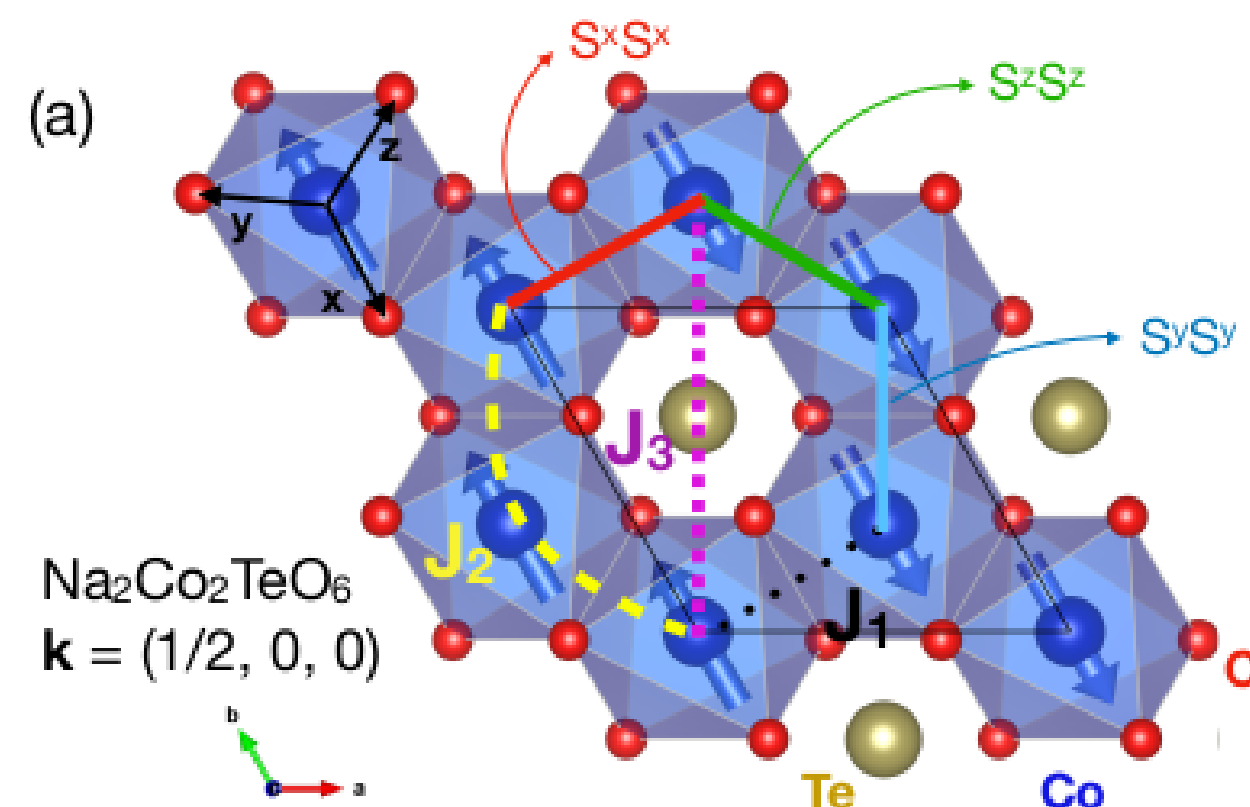
# Kitaev materials: $d^7$ ions



Several families:  
 $\text{Na}_3\text{Co}_2\text{SbO}_6$   
 $\text{Ag}_3\text{Co}_2\text{SbO}_6$   
 $\text{CoPS}_3$   
 $\text{BaCo}_2(\text{AsO}_4)_2$

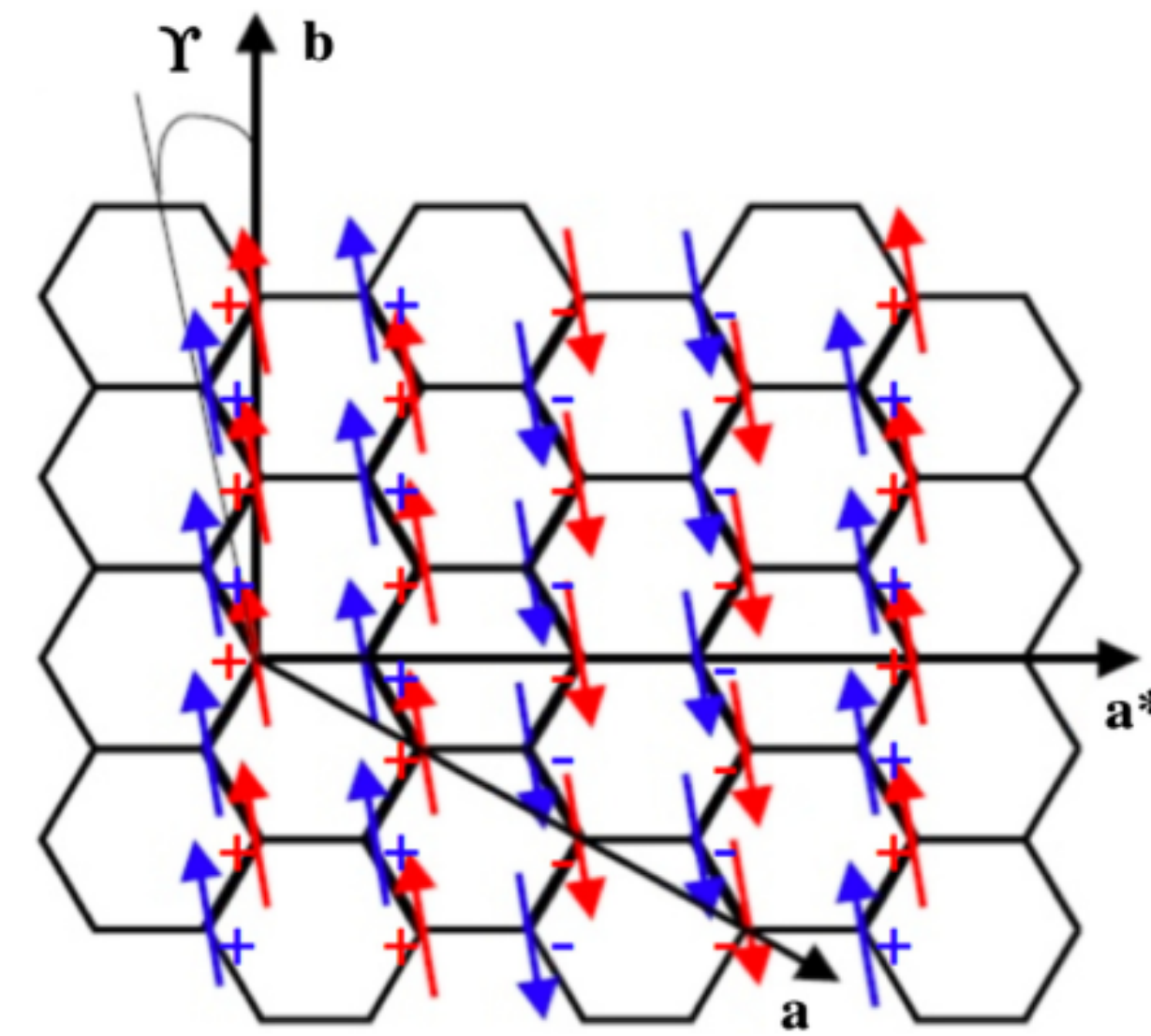
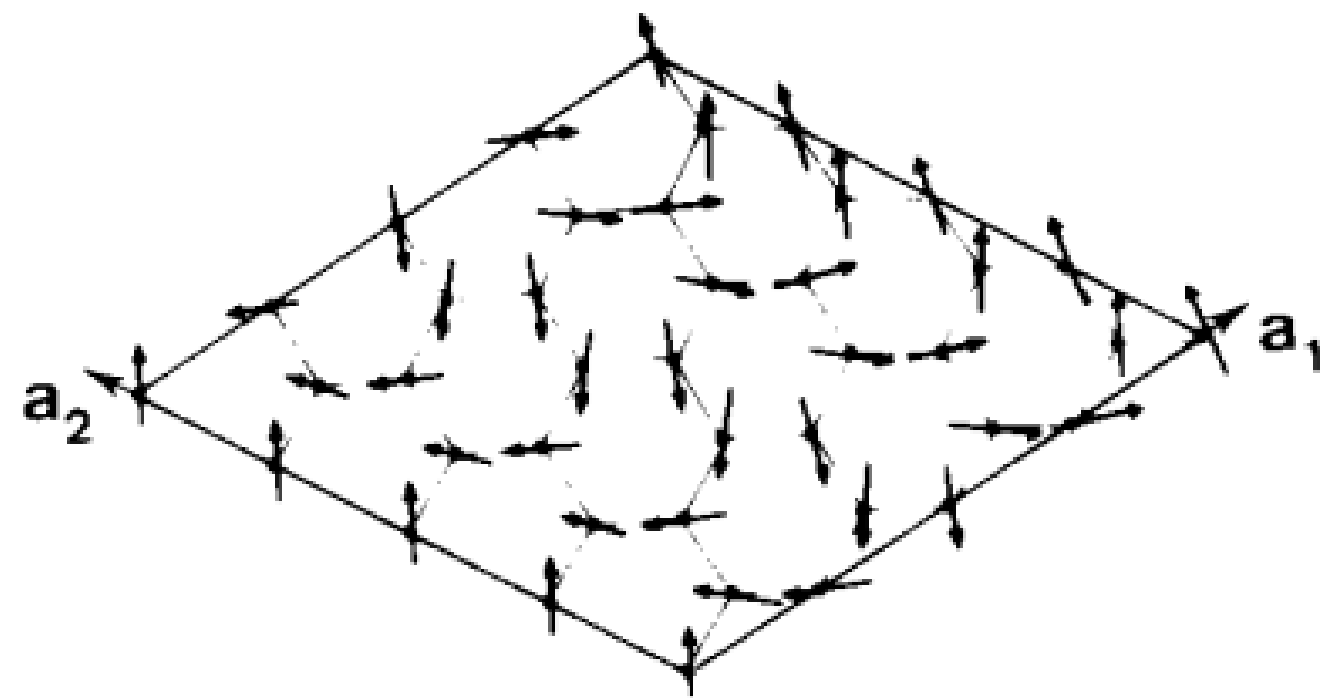
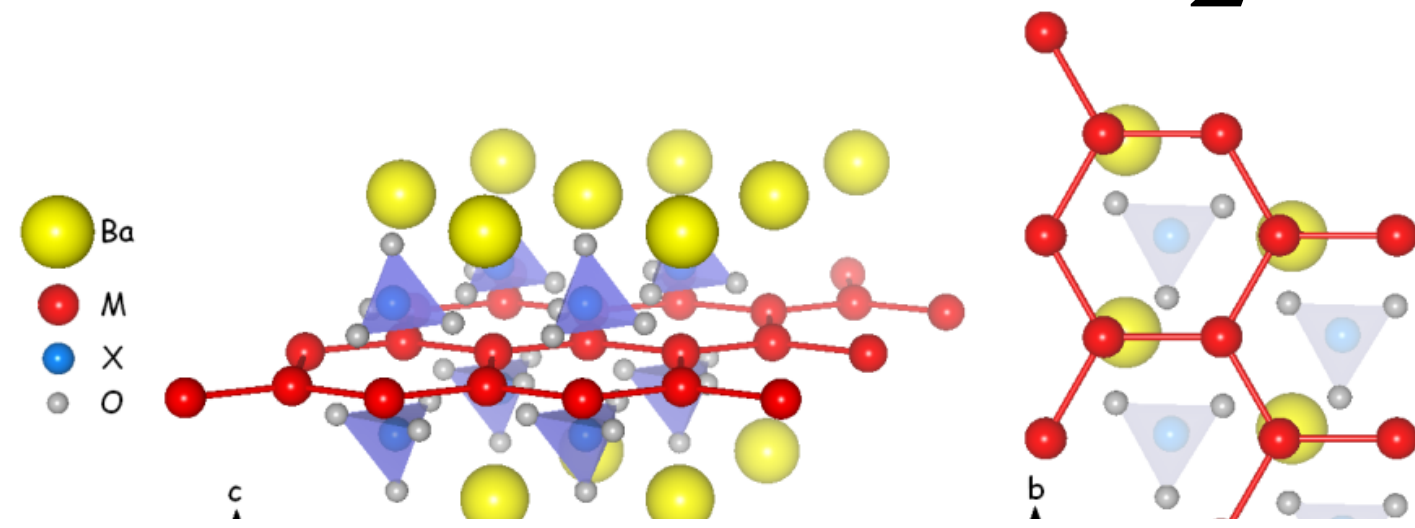


C. Kim et al., J. Phys.: Condes. Matter 34 045802 (2022)



- Zigzag ground state
- Spin-wave spectrum can be fit with Kitaev model with subleading  $\Gamma, \Gamma'$  terms

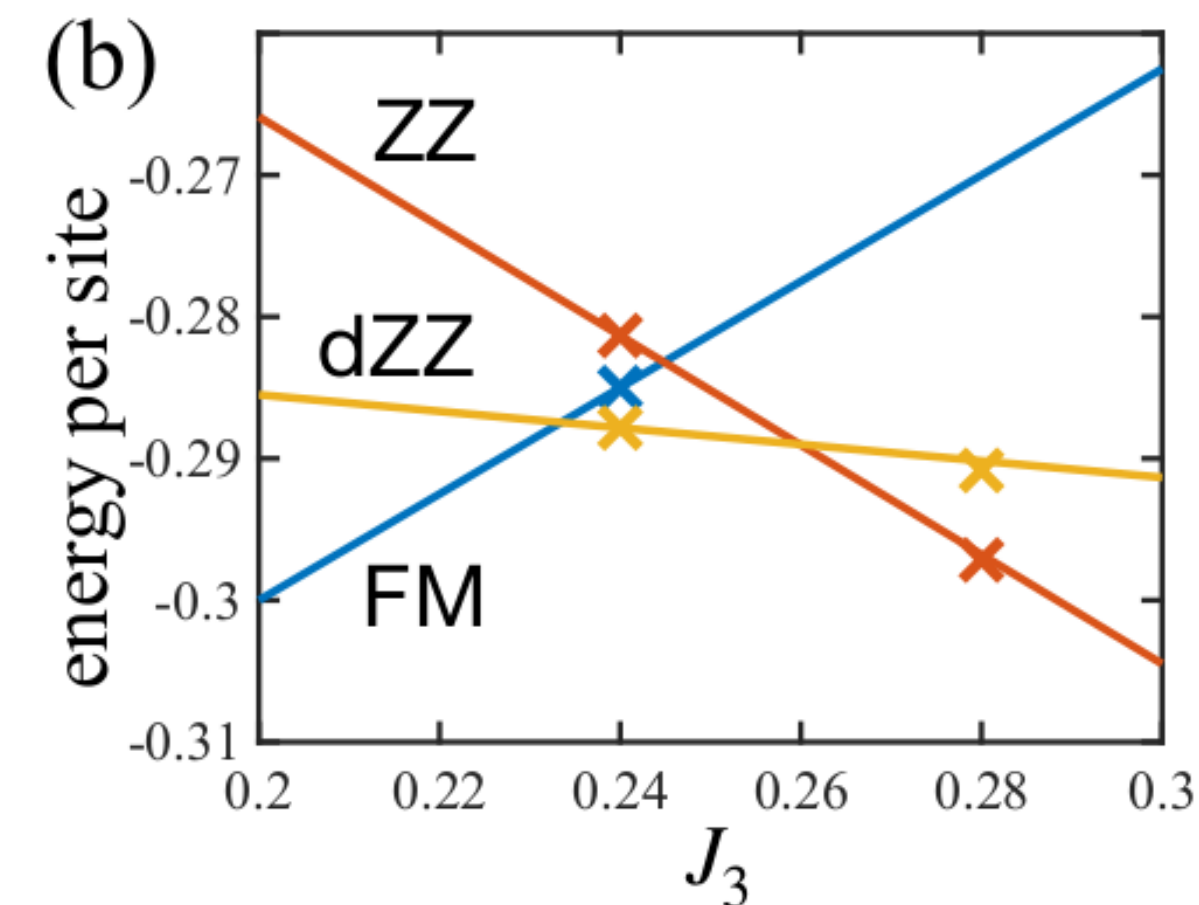
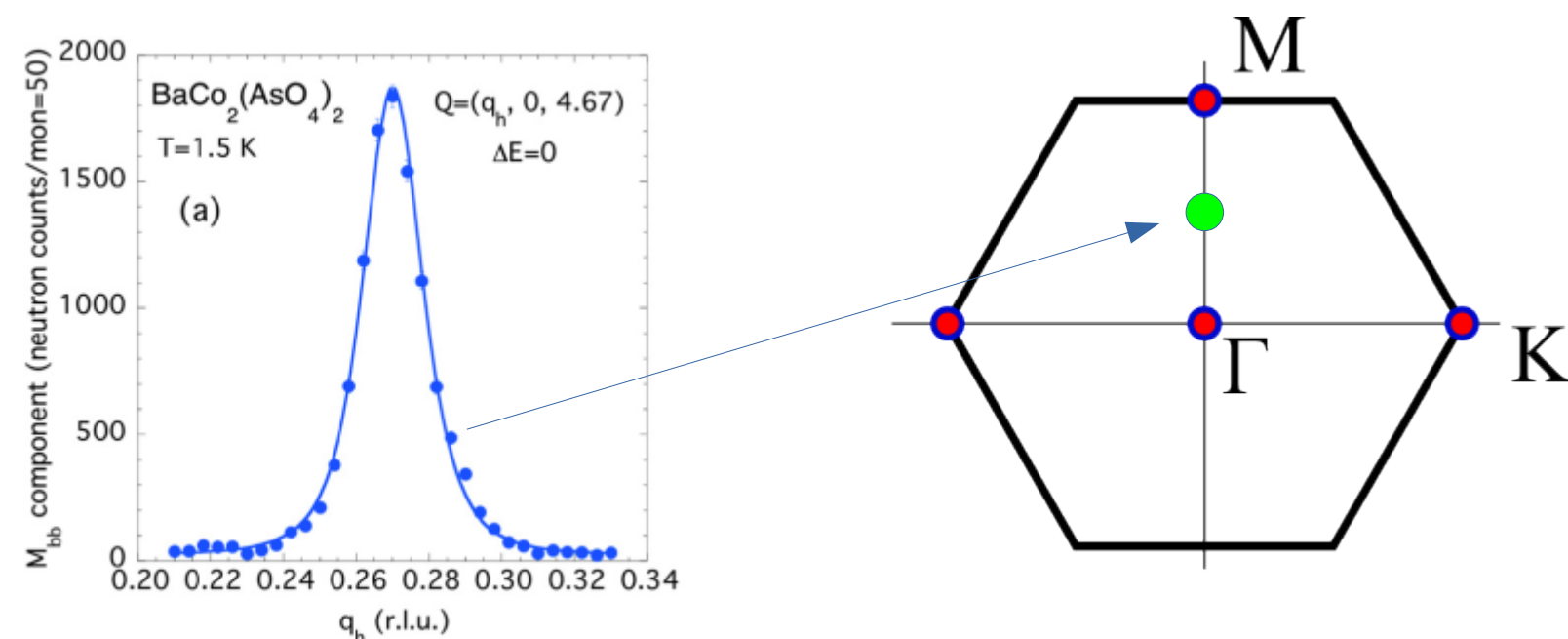
# BaCo<sub>2</sub>(AsO<sub>4</sub>)<sub>2</sub>: «double-zigzag»



- Need exchanges beyond Heisenberg
- Kitaev model?

L.P. Regnault et al., Heliyon 4, e00507 (2018)

Fig. 2. In-plane magnetic order of Co<sup>2+</sup> moments in BaCo<sub>2</sub>(AsO<sub>4</sub>)<sub>2</sub>.

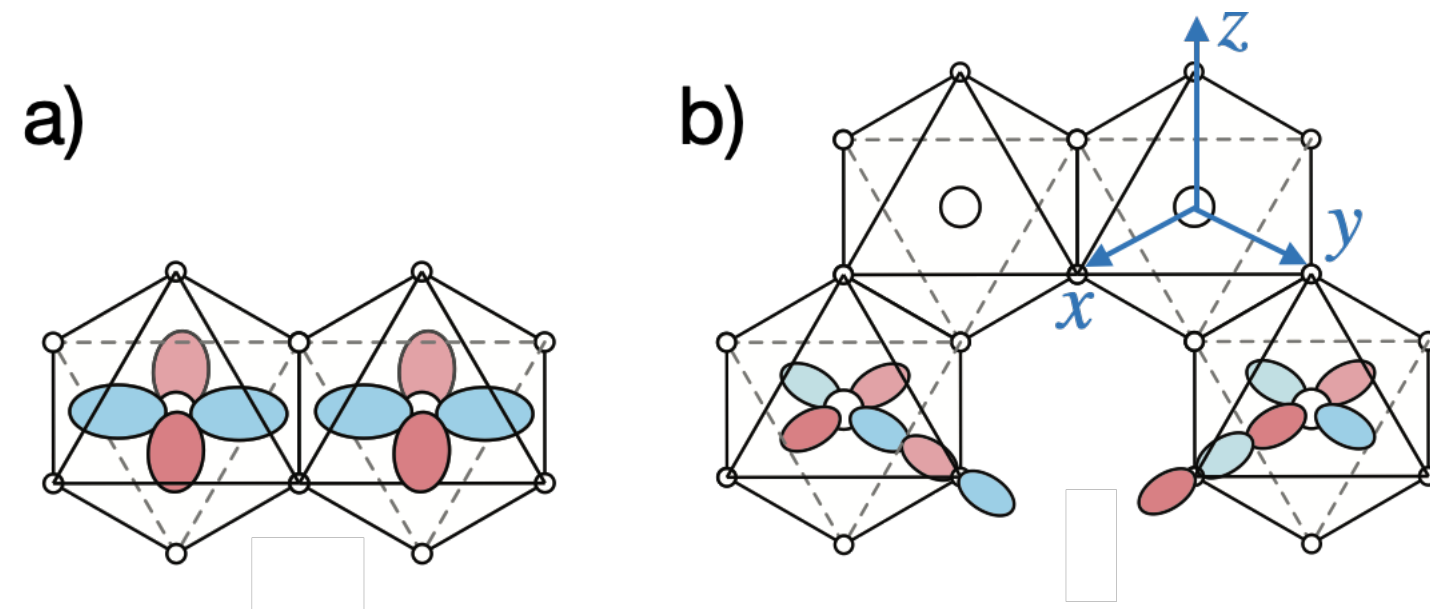


L.P. Regnault et al., Physica B+C 86, 660 (1977)

Jiang et al., arxiv 2304.06062

# BaCo<sub>2</sub>(AsO<sub>4</sub>)<sub>2</sub>: *ab initio* exchanges

DFT /A. Ushakov, Z. Pchelkina, S. Streltsov (ИФМ УрО РАН)/  
(total energy method)



Direct hopping  $t \sim -300$  meV is larger than through xz, yz orbitals,  $t \sim 50$  meV

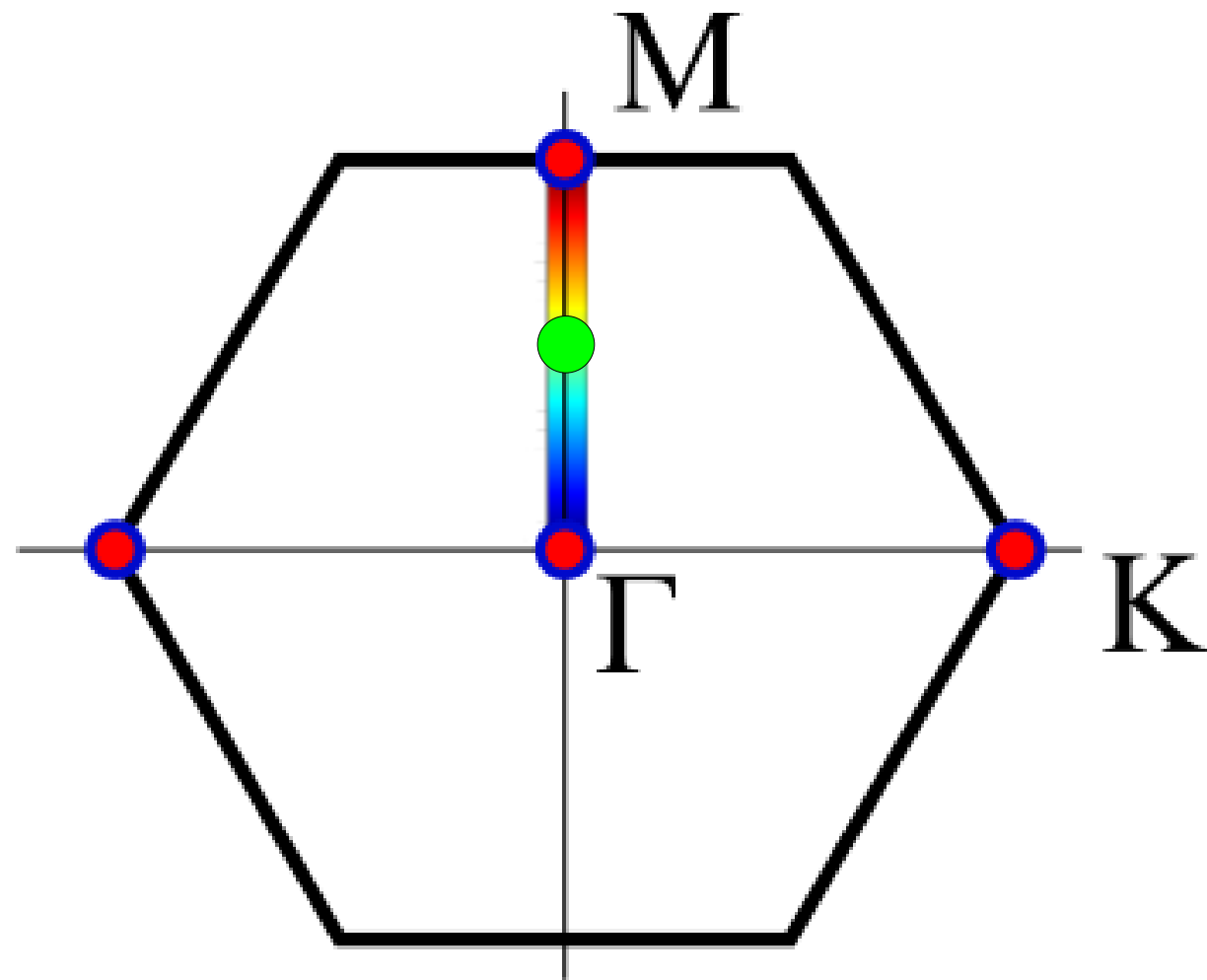
Strong overlap of third-neighbor orbitals  
 $t_3 = 124$  meV

ED /S. Winter, Y. Li (Wake Forest)/  
(2-site Exact Diagonalization)

$U$	5 eV	6 eV	7 eV
$J_1$ (K)	-61.0	<u>-40.9</u>	-37.6
$K_1$ (K)	0.3	2.2	5.3
$\Gamma_1$ (K)	-2.2	-1.7	-1.8
$\Gamma'_1$ (K)	5.1	4.0	3.2
$J_3$ (K)	31.4	<u>24.6</u>	18.7
$K_3$ (K)	-0.2	0.2	-0.2
$\Gamma_3$ (K)	-4.5	-6.0	-4.5
$\Gamma'_3$ (K)	-3.6	-2.3	-1.8

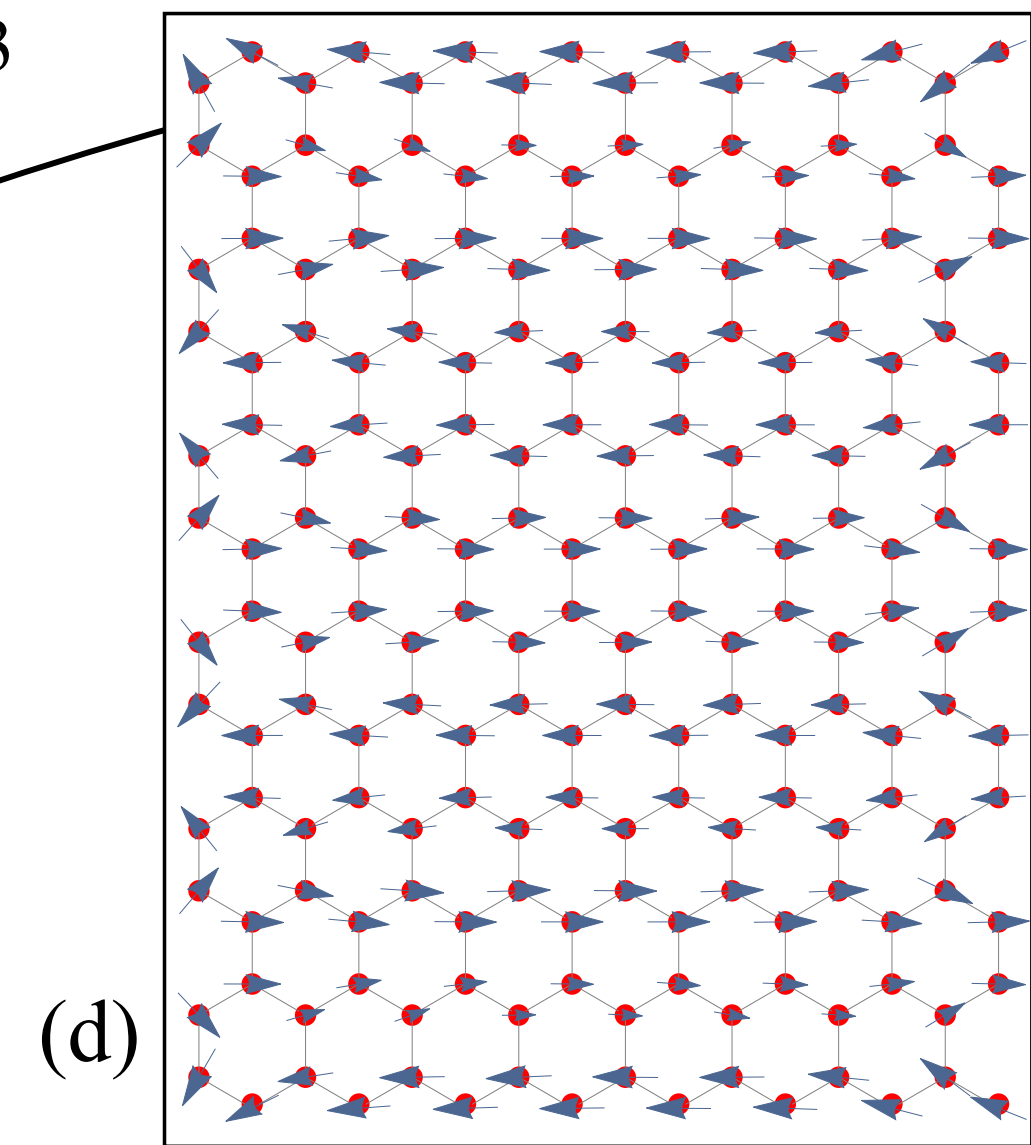
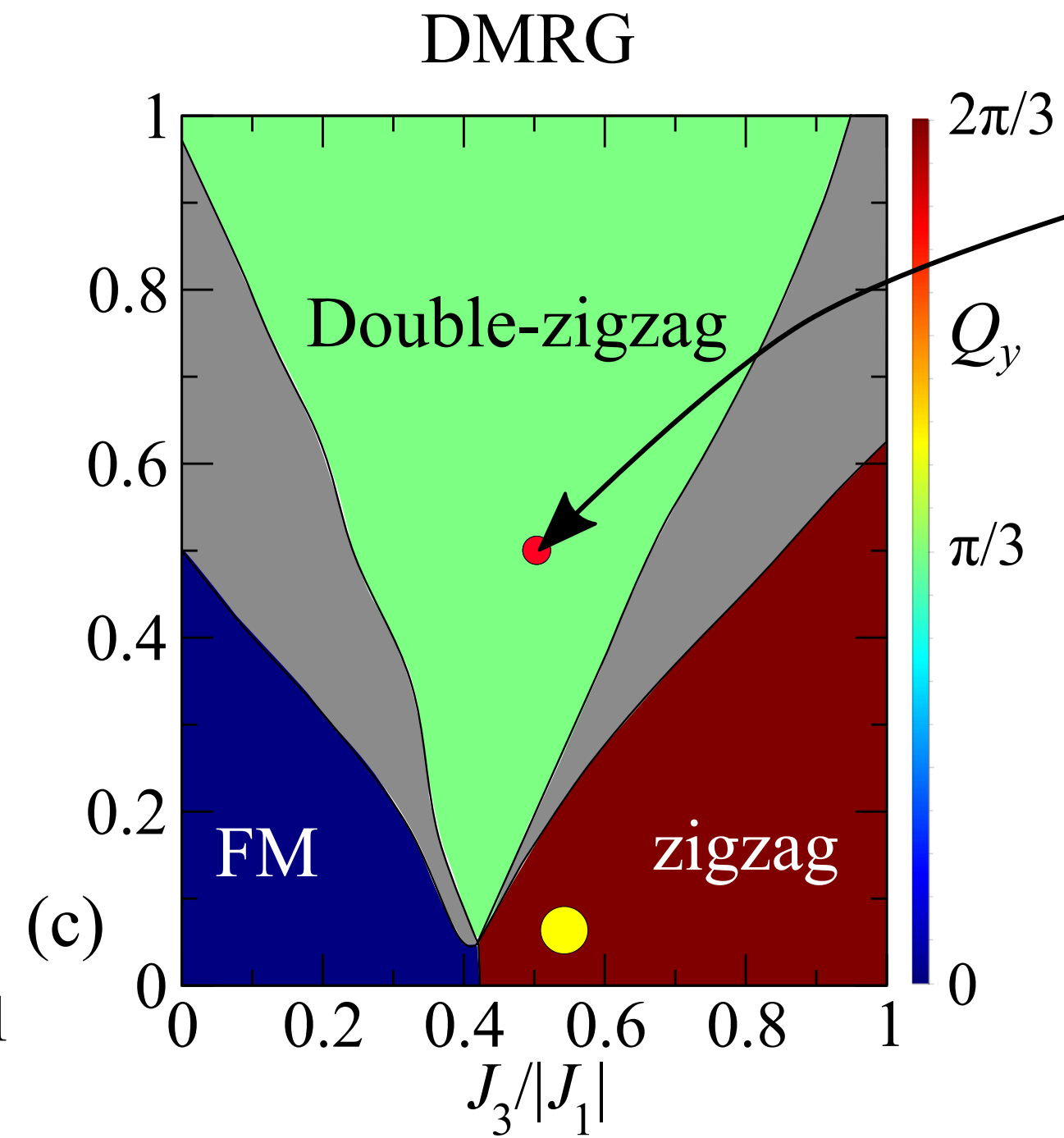
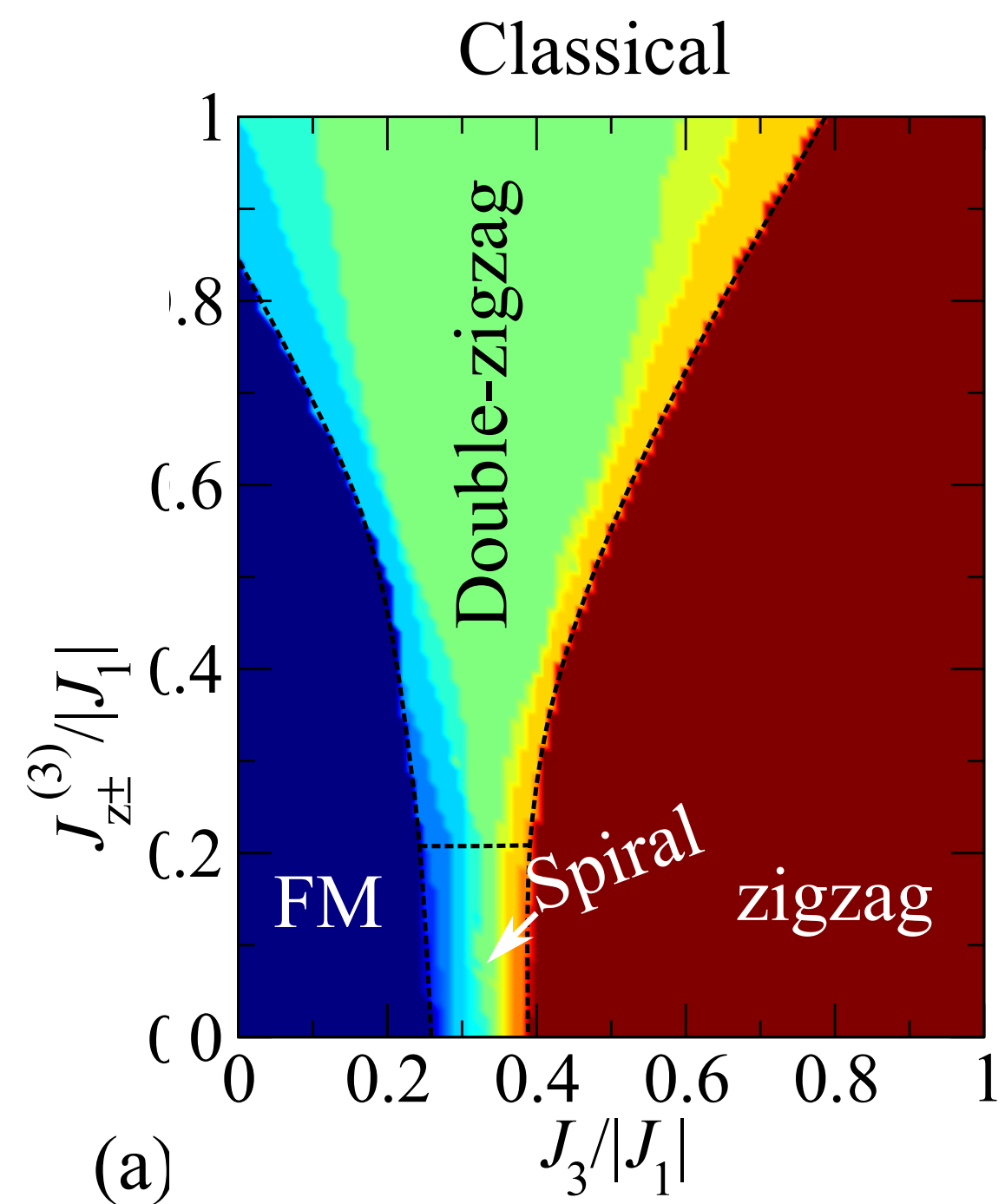
$J_{H,t_{2g}}$	0.7 eV		0.9 eV	
$U$	3.25 eV	5 eV	6 eV	7 eV
$J_1$ (K)	-107 (-127)	-37 (-57)	<u>-18 (-38)</u>	-8.8 (-29)
$K_1$ (K)	32	13	6.5	3.4
$\Gamma_1$ (K)	28 (35)	14 (21)	8.0 (15)	4.8 (12)
$\Gamma'_1$ (K)	9.4 (16)	7 (14)	4.0 (11)	2.4 (9)
$J_3$ (K)	43	30	<u>27</u>	24
$K_3$ (K)	-0.6	-0.4	-0.3	-0.3
$\Gamma_3$ (K)	-20	-12	-10	-8.9
$\Gamma'_3$ (K)	-21	-12	-11	-9.2

# Double-zigzag state in $\text{BaCo}_2(\text{AsO}_4)_2$

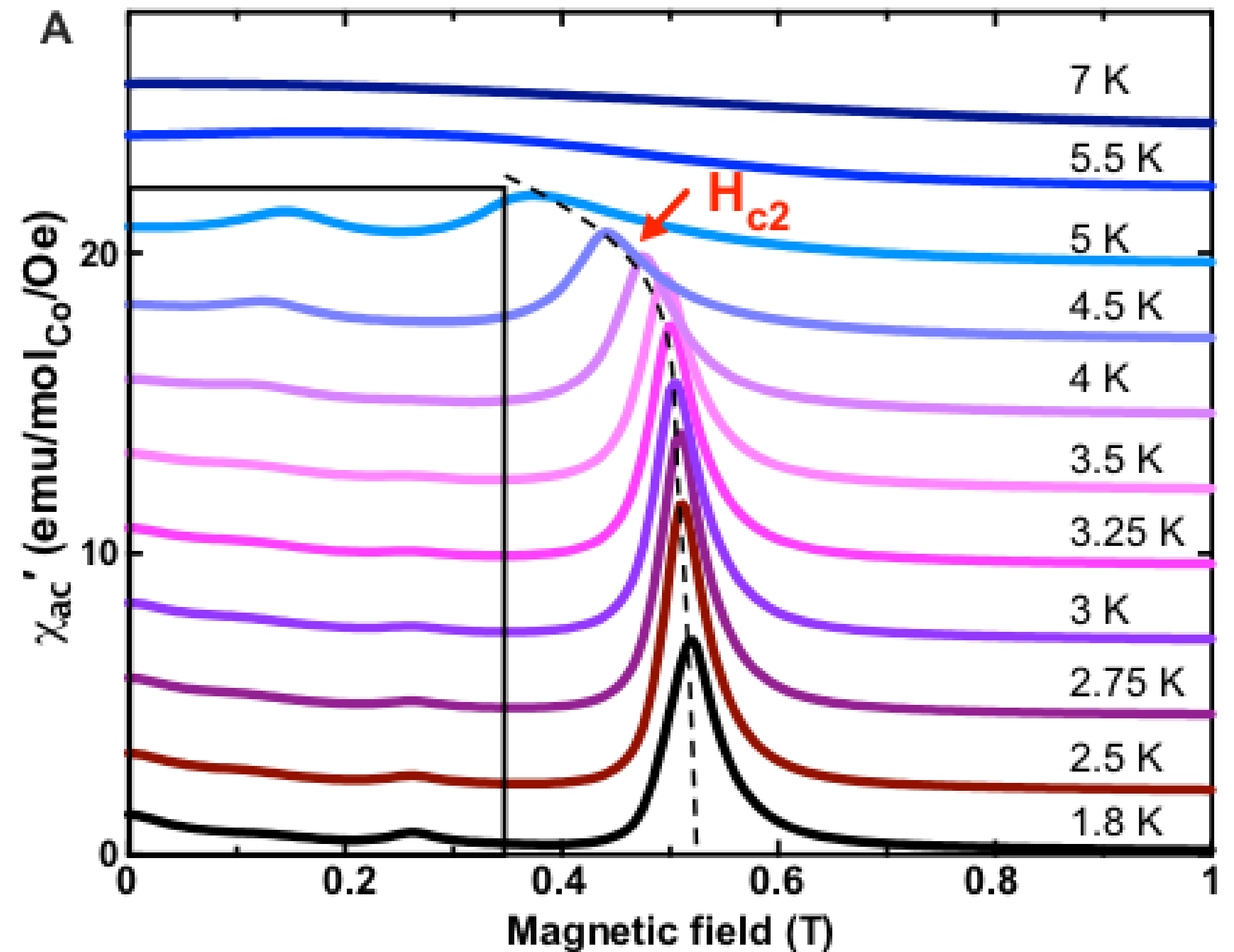
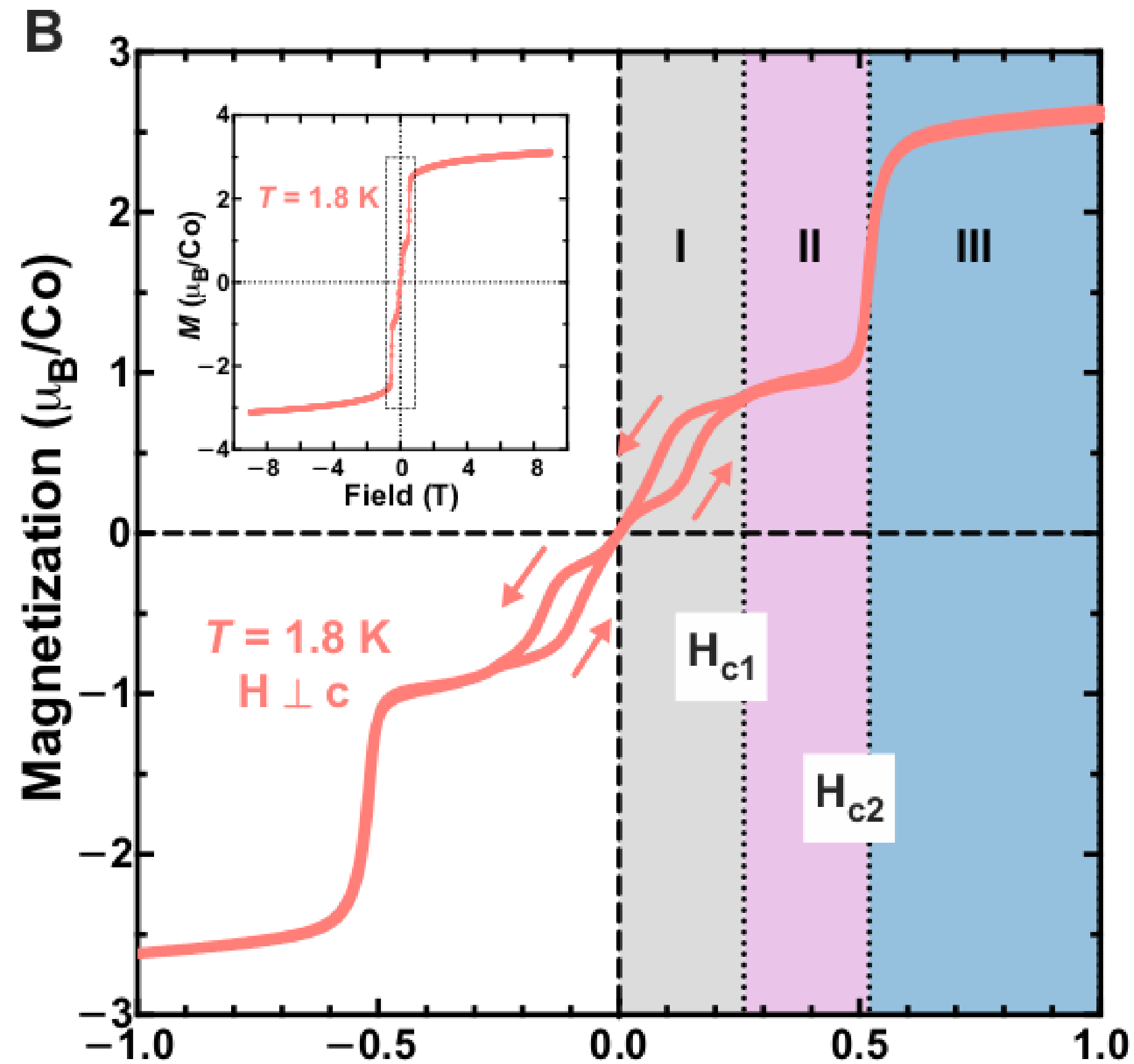


$$H_{\min} = \sum_{\langle ij \rangle_1} J_1 (S_i^x S_j^x + S_i^y S_j^y) + J_3 \sum_{\langle ij \rangle_3} (S_i^x S_j^x + S_i^y S_j^y) - J_{z\pm}^{(3)} \left( (S_i^x S_j^z + S_i^z S_j^x) c_\alpha + (S_i^y S_j^z + S_i^z S_j^y) s_\alpha \right),$$

$S=1/2$



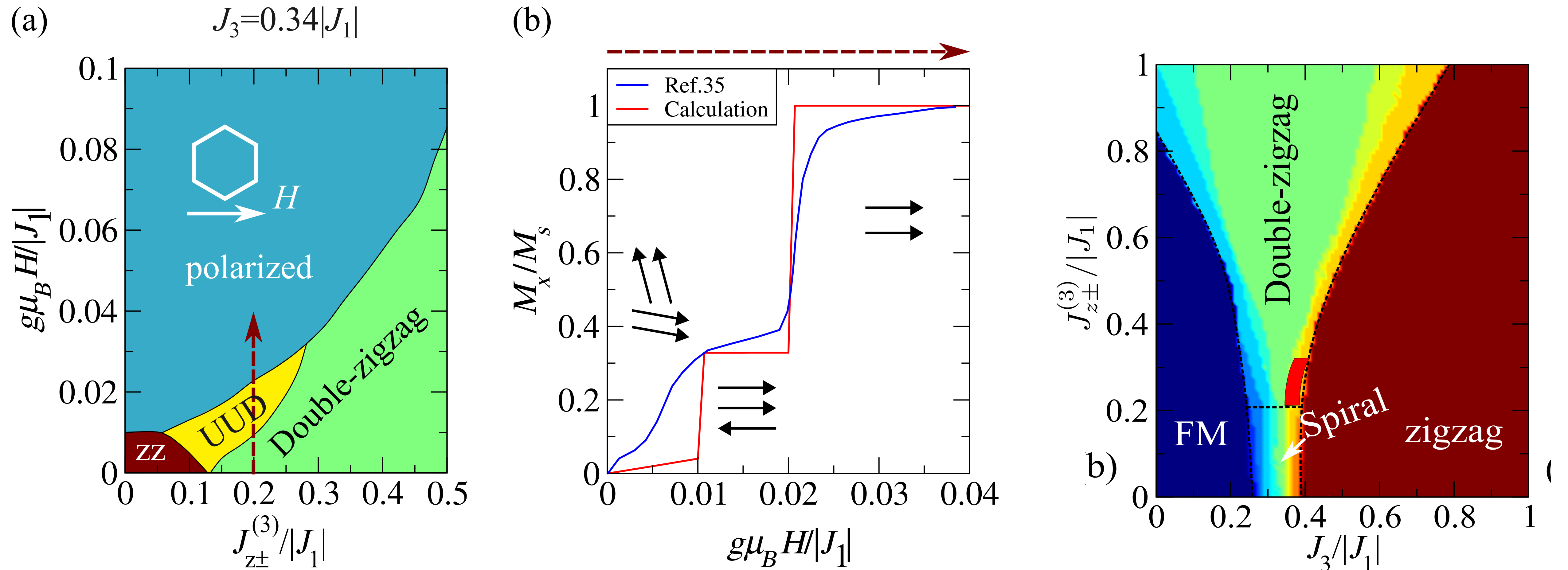
# Field-induced transitions in $\text{BaCo}_2(\text{AsO}_4)_2$



R. Zhong, T. Gao, N. P. Ong, and R. J. Cava, *Sci. Adv.* 6, eaay6953 (2020)



# Field-induced transitions in $\text{BaCo}_2(\text{AsO}_4)_2$



R. Zhong, T. Gao, N. P. Ong, and R. J. Cava, *Sci. Adv.* 6, eaay6953 (2020)

P.A. Maksimov, arxiv 2308.10672

# Conclusions

- Kitaev model is exactly solvable and its excitations can be used for topological QC
- Kitaev model can be realized in transition metal compounds with strong SOC
- Additional interactions are allowed by symmetry and lead long-range order
- Phase diagram of extended Kitaev-Heisenberg model exhibits a plethora of exotic states

$$S^x = ib^x c, \quad S^y = ib^y c, \quad S^z = ib^z c$$

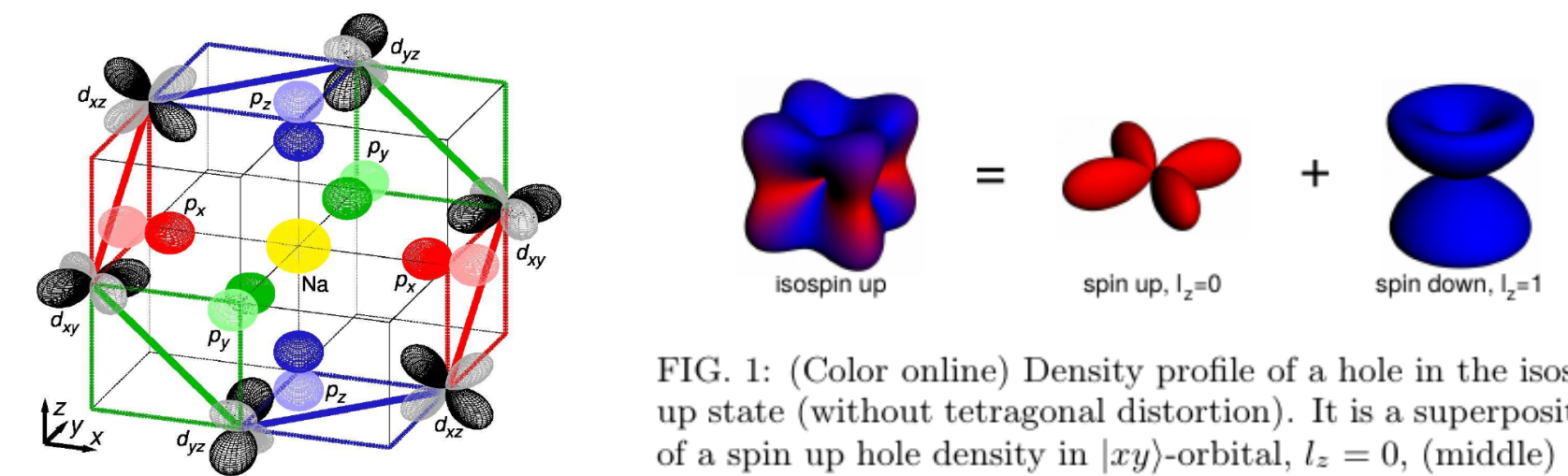
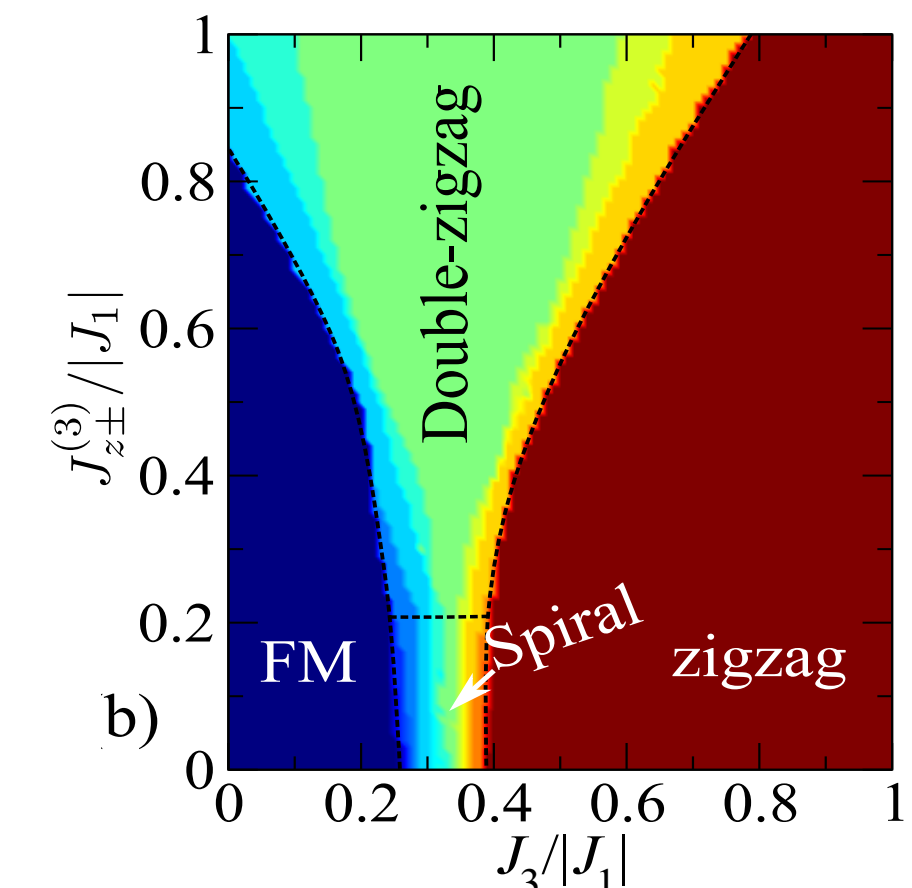
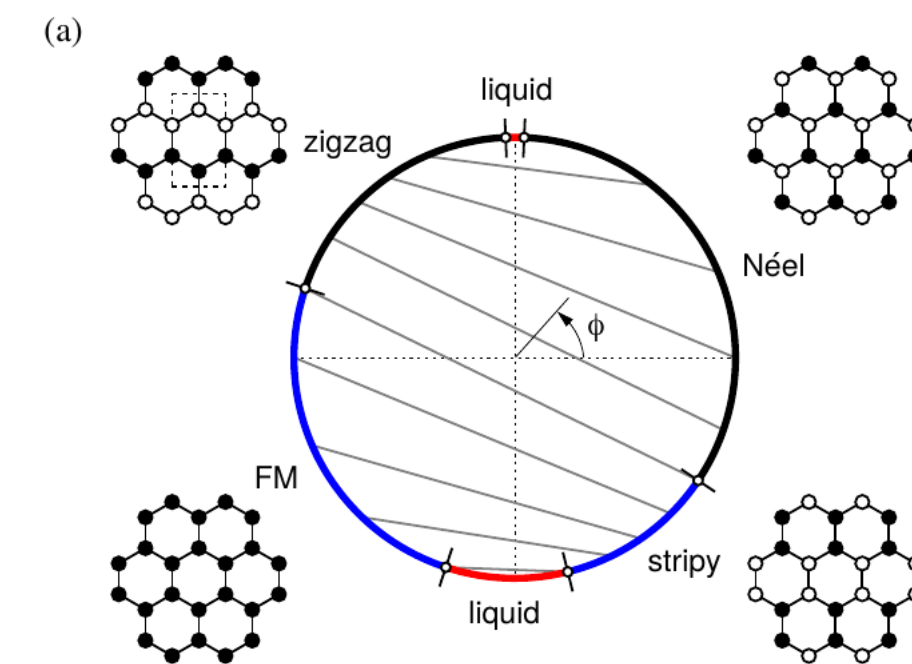
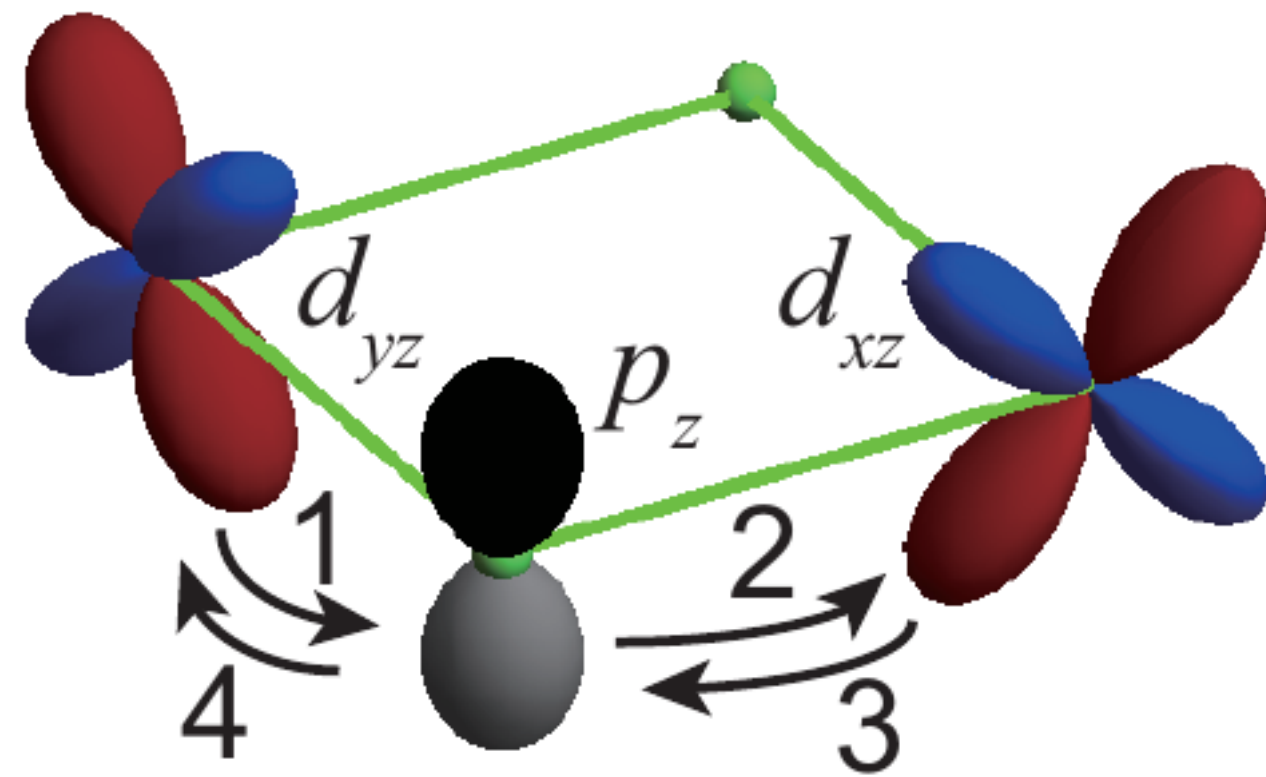


FIG. 1: (Color online) Density profile of a hole in the isospin up state (without tetragonal distortion). It is a superposition of a spin up hole density in  $|xy\rangle$ -orbital,  $l_z = 0$ , (middle) and spin down one in  $(|yz\rangle + i|xz\rangle)$  state,  $l_z = 1$ , (right).



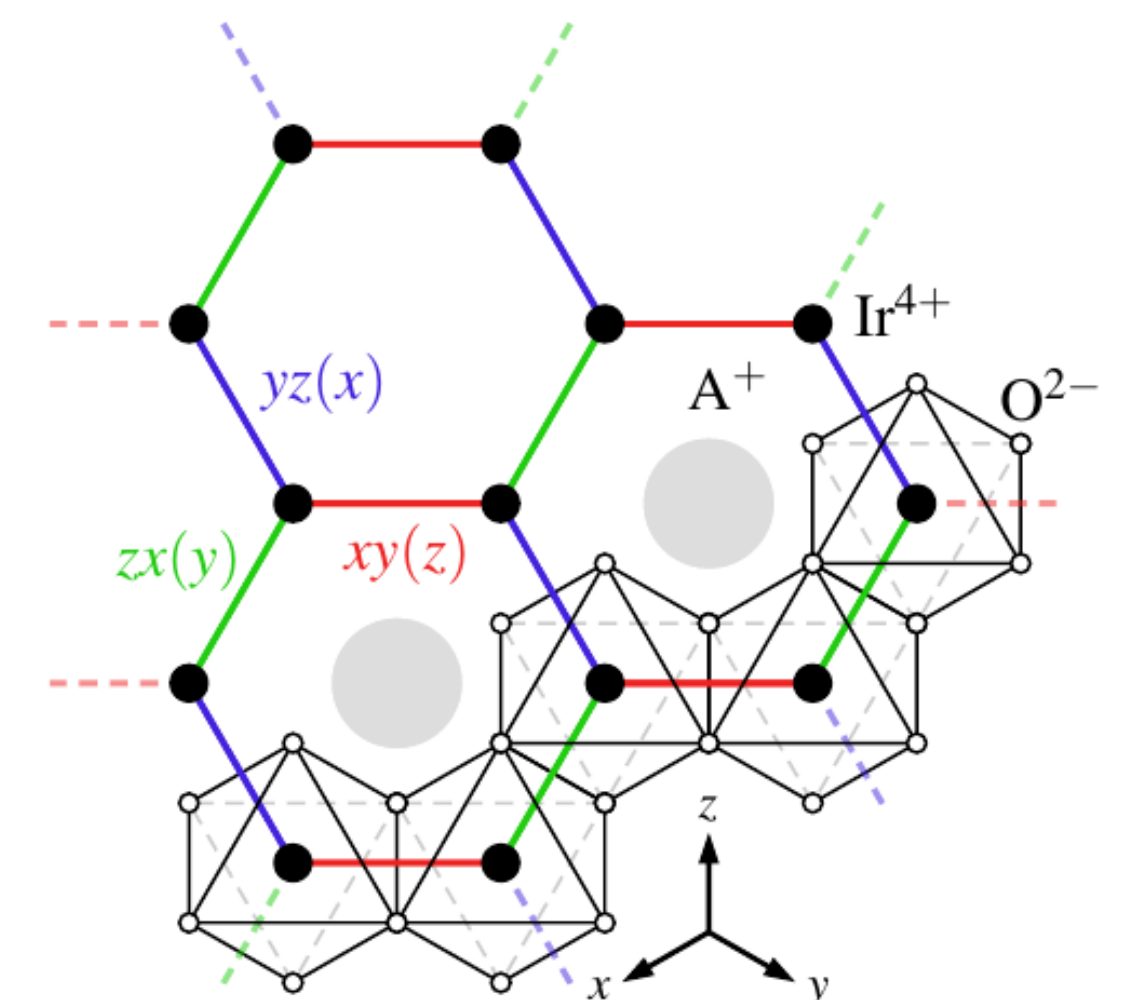
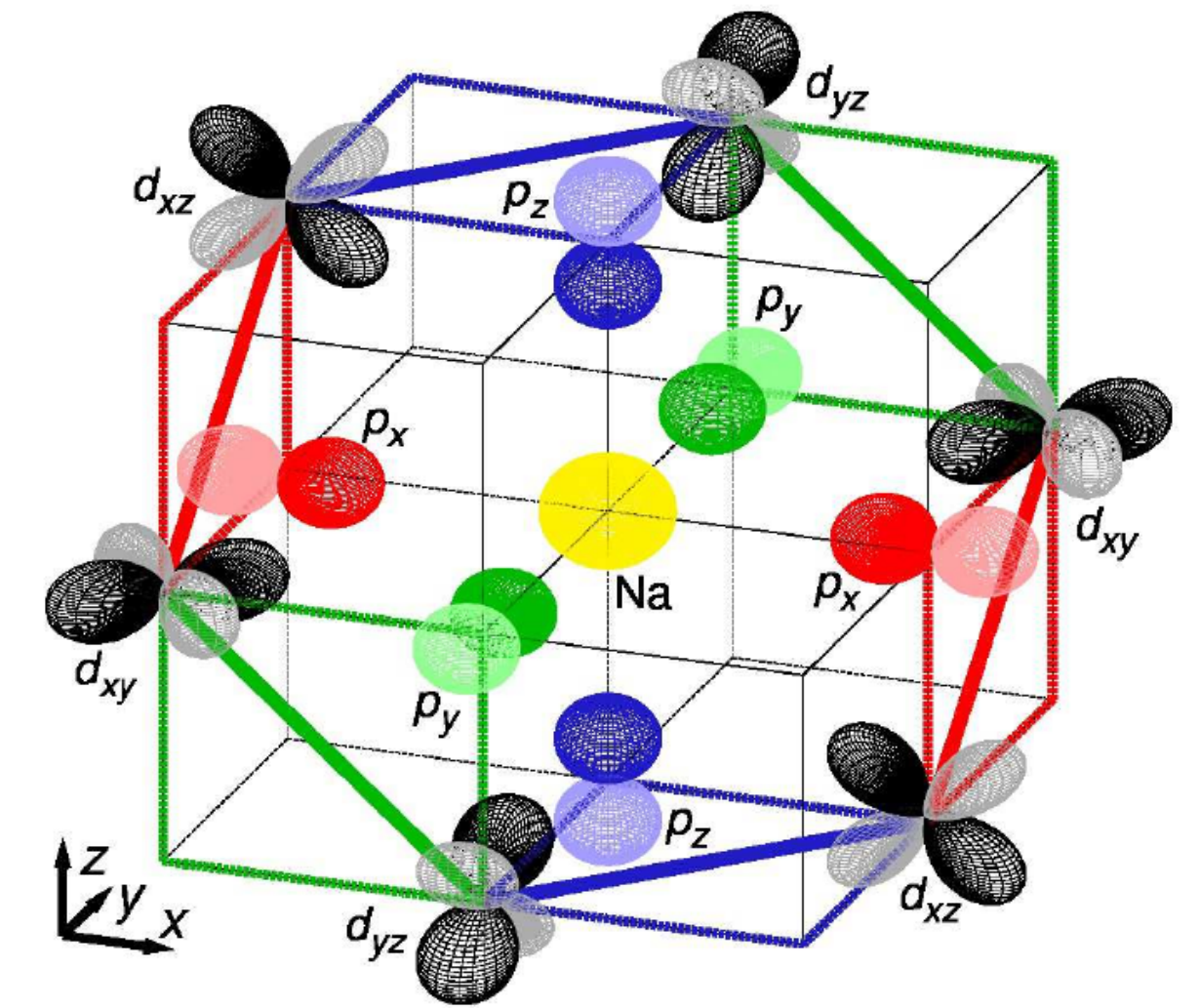
# Реализация в материалах: $d^5$ металлы



Взаимодействует только  
z-компонента  
псевдоспина

$$\mathcal{H} = K \sum_{\langle ij \rangle \gamma} S_i^\gamma S_j^\gamma$$

G. Jackeli and G. Khaliullin, Phys. Rev. Lett. 102, 017205 (2009)



# Реализация в материалах: $d^5$ металлы

$$\mathcal{H} = \sum_{\langle ij \rangle^\gamma} J \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma$$

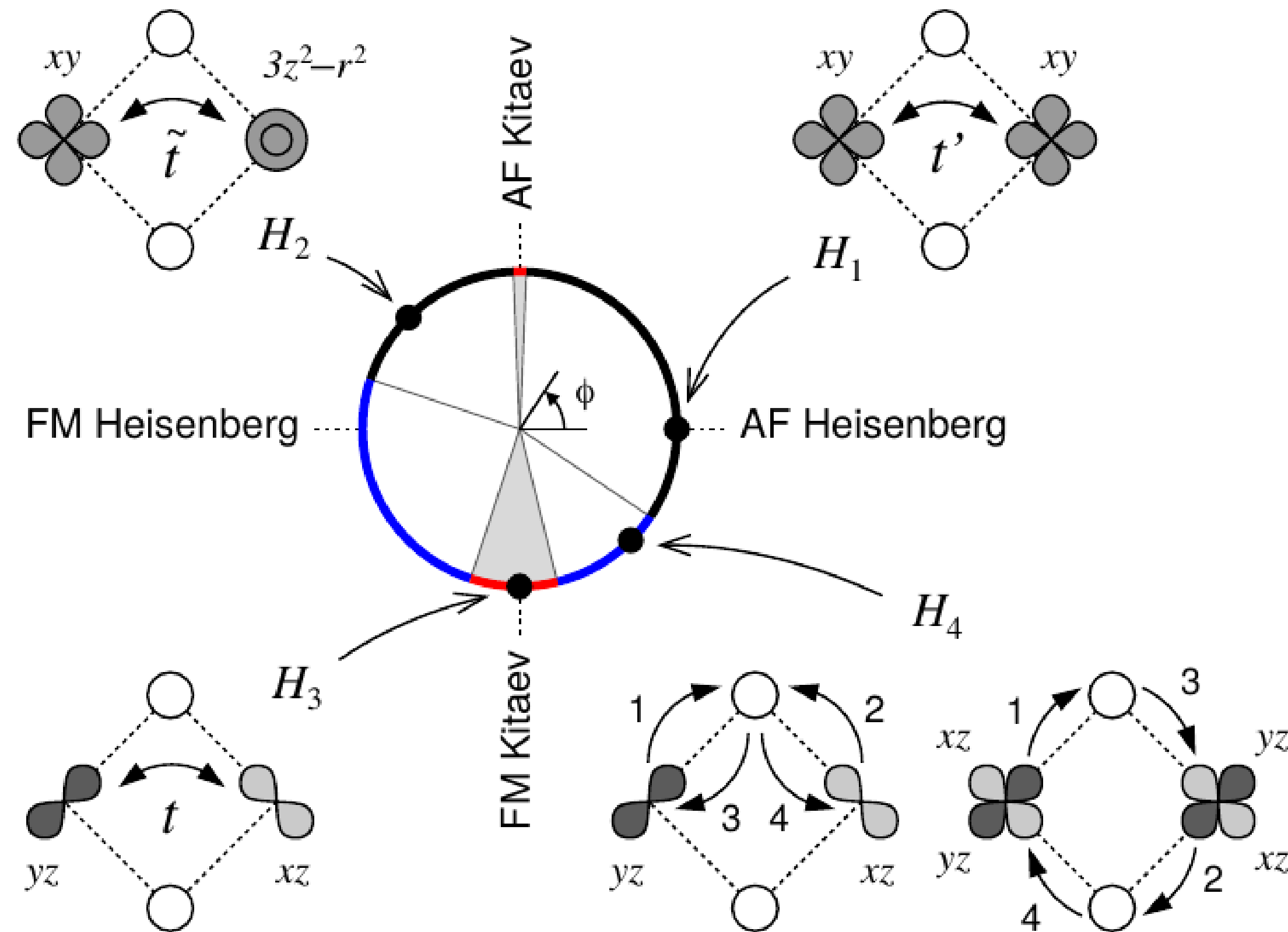
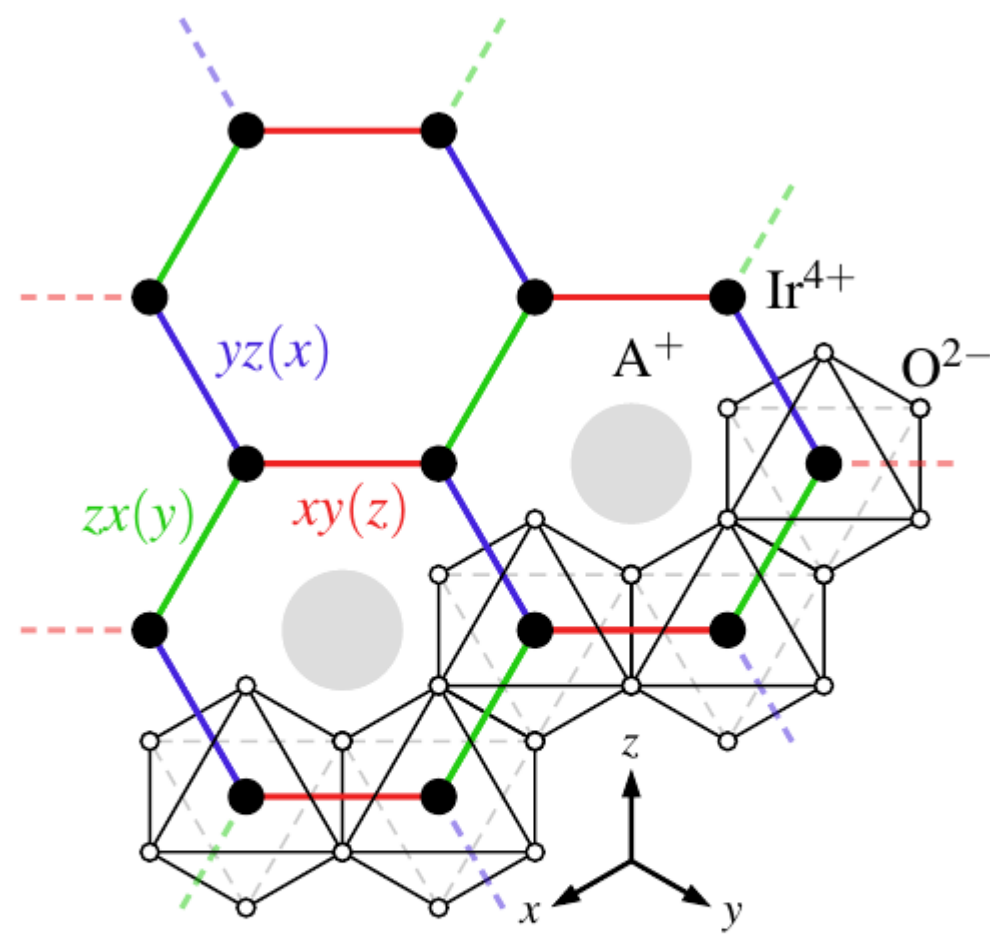
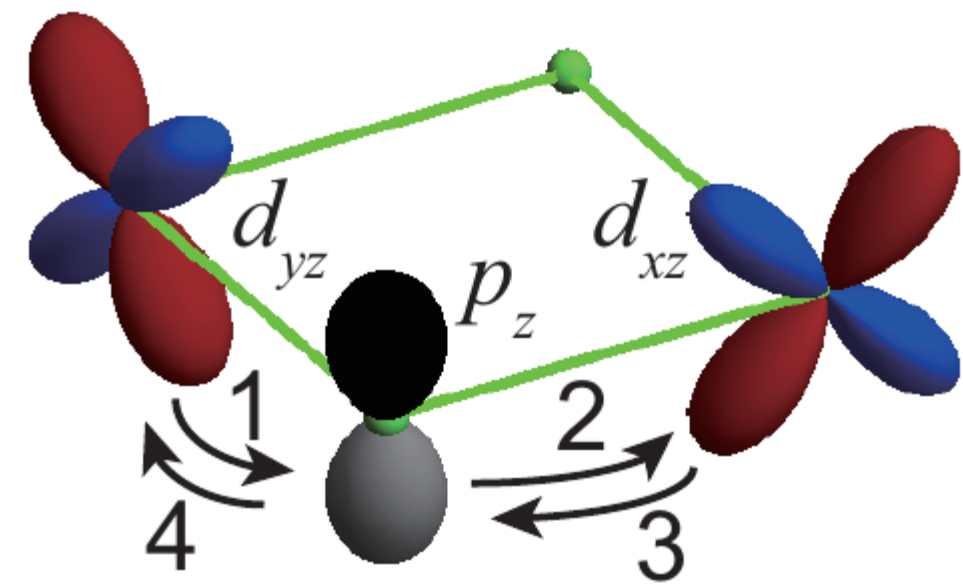


FIG. 1: Crystal structure of the honeycomb iridates  $A_2\text{IrO}_3$  with  $\text{Ir}^{4+}$  in black,  $\text{O}^{2-}$  in white, and  $A = \text{Na}^+, \text{Li}^+$  in gray. For the Kitaev and bond-dependent exchanges we have denoted the  $yz(x)$  bonds blue, the  $zx(y)$  bonds green and the  $xy(z)$  bonds red.

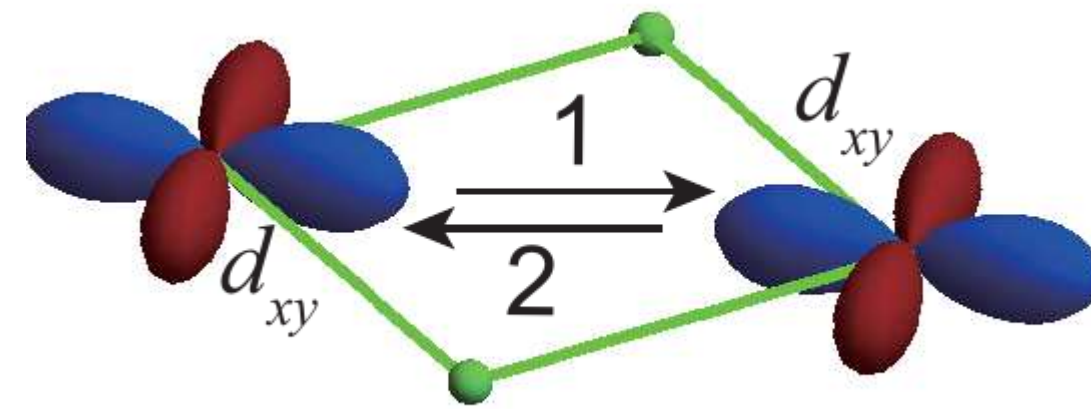
$$J = \cos \phi, \quad K = \sin \phi$$

# Расширенная модель Китаева-Гейзенберга

Китаевское взаимодействие  $K$

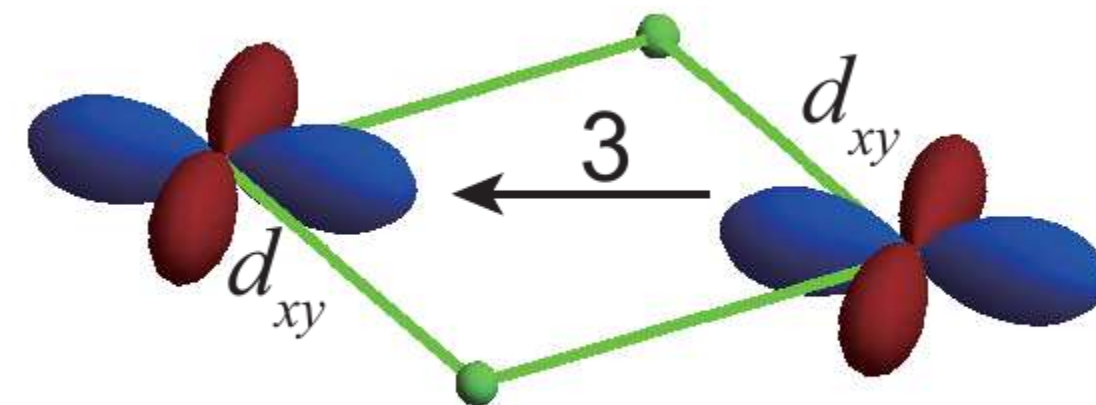
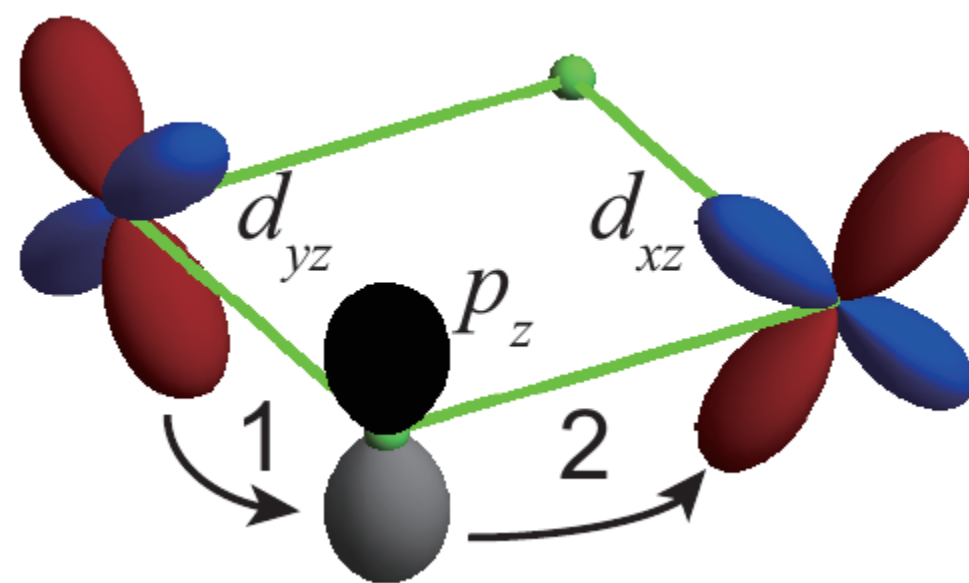


Гейзенберговское  $J$

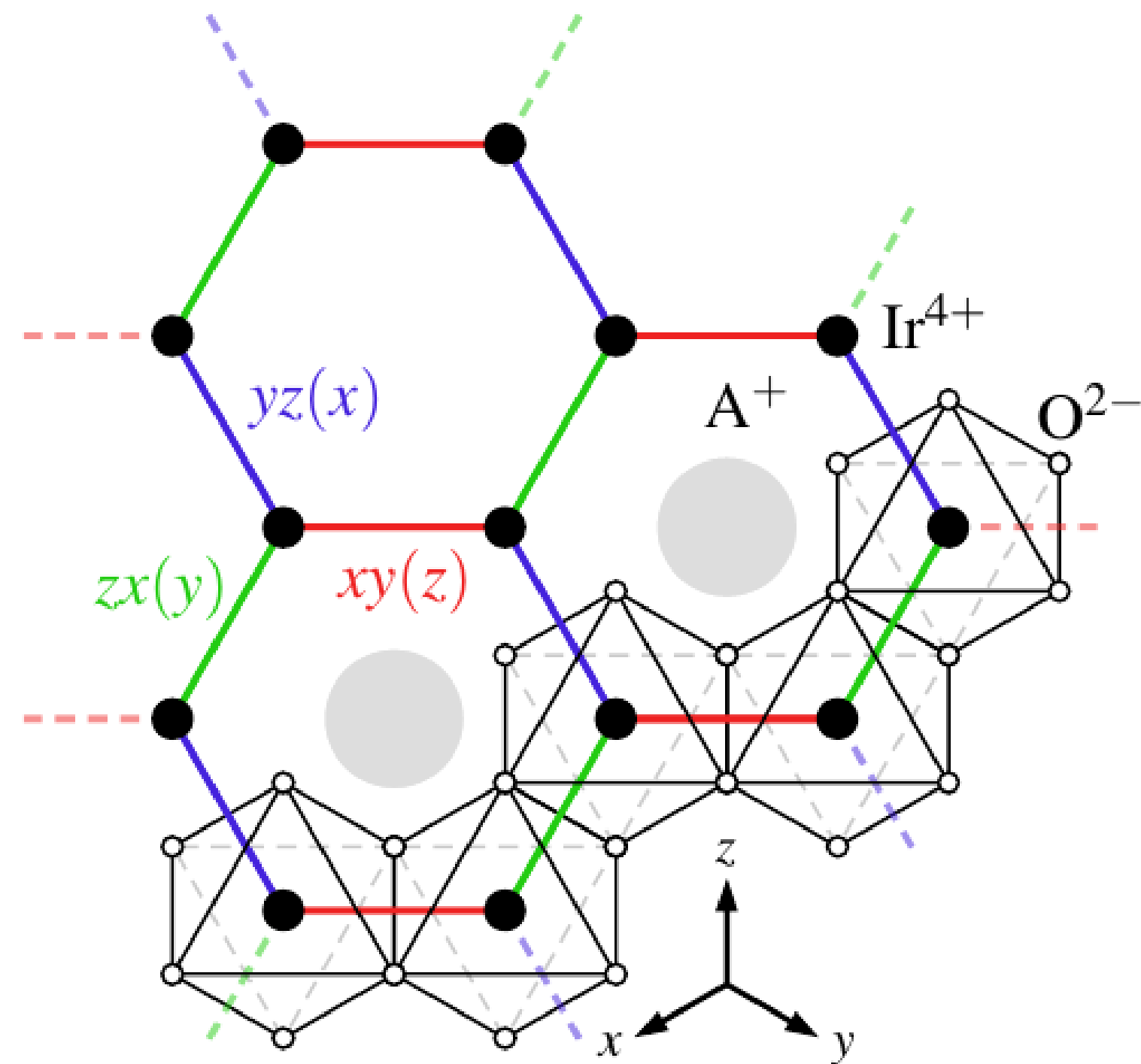


$$\mathcal{H} = \sum_{\langle ij \rangle \gamma} J \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma + \Gamma \left( S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha \right)$$

Недиагональное взаимодействие  $\Gamma$



# Реальные системы с взаимодействием Китаева



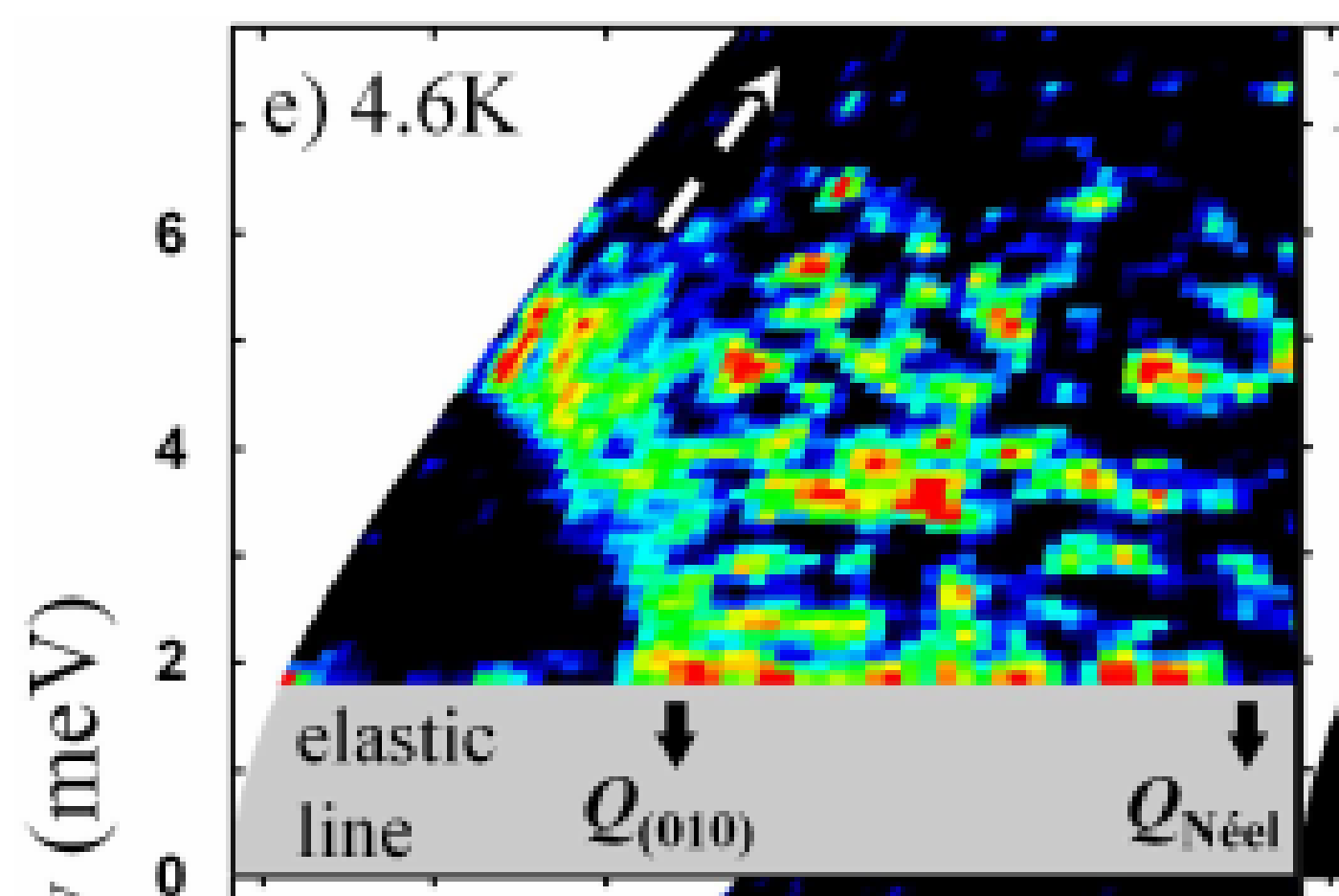
При тригональном искажении октаэдров возможен еще дополнительный член. Тогда полный гамильтониан содержит 4 члена:

$$\mathcal{H} = \sum_{\langle ij \rangle^\gamma} JS_i \cdot S_j + K S_i^\gamma S_j^\gamma + \Gamma \left( S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha \right) + \Gamma' \left( S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma \right)$$

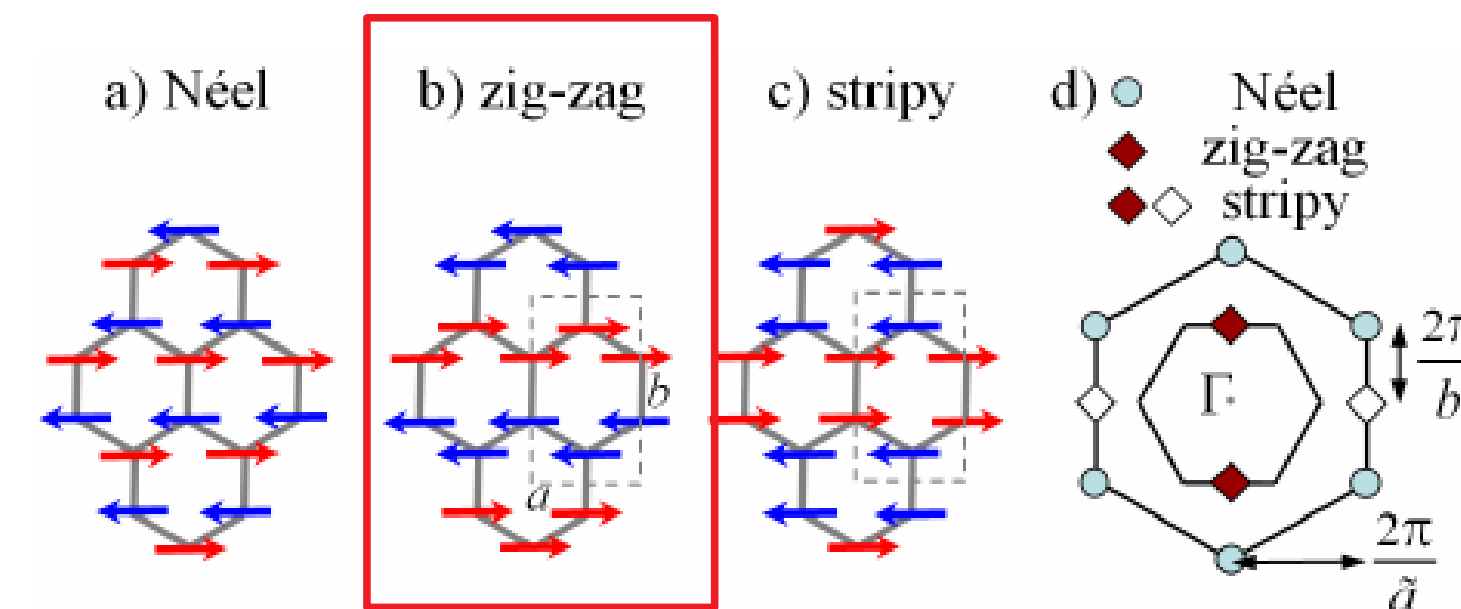
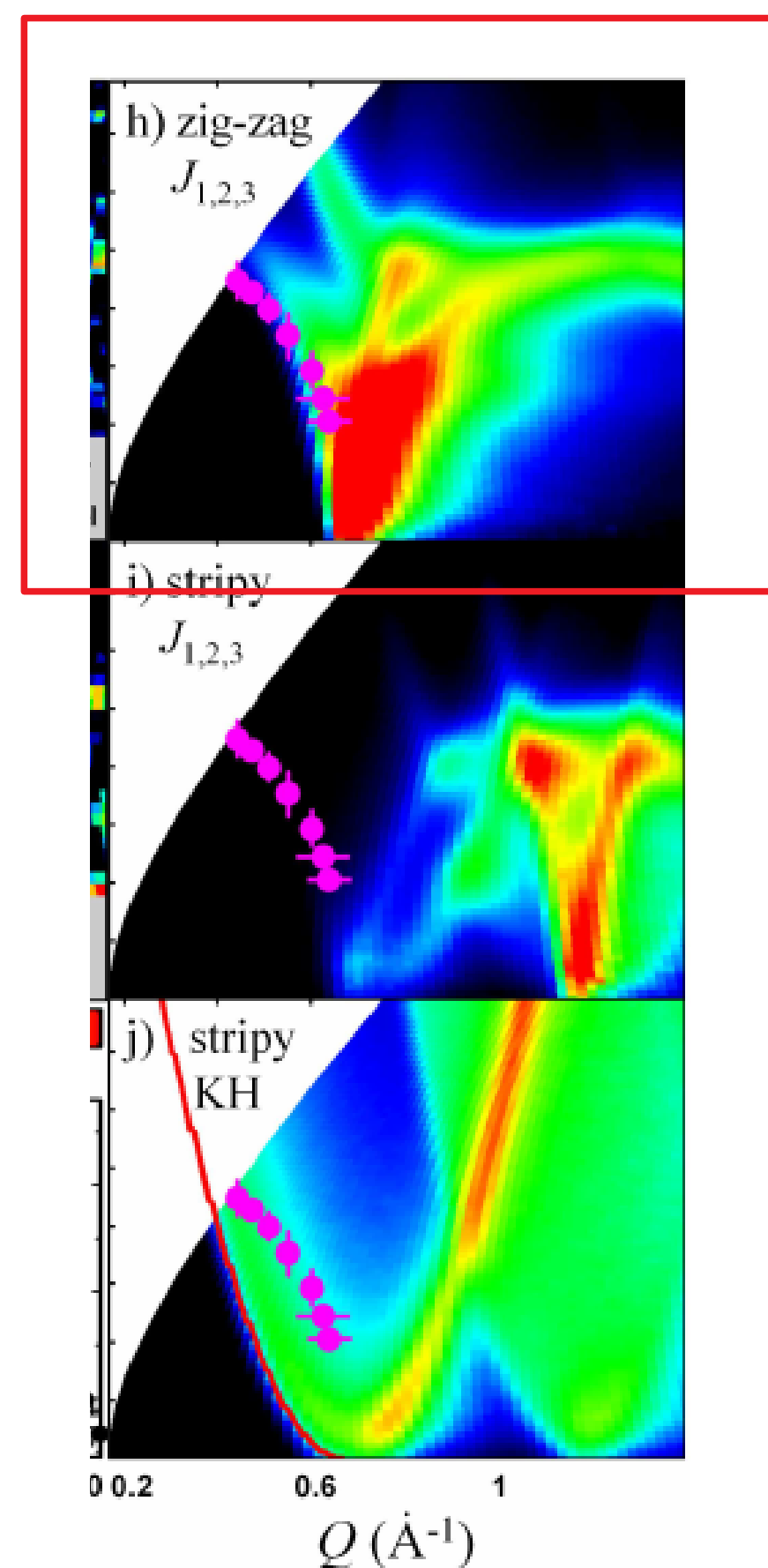
FIG. 1: Crystal structure of the honeycomb iridates  $A_2\text{IrO}_3$  with  $\text{Ir}^{4+}$  in black,  $\text{O}^{2-}$  in white, and  $A = \text{Na}^+, \text{Li}^+$  in gray. For the Kitaev and bond-dependent exchanges we have denoted the  $yz(x)$  bonds blue, the  $zx(y)$  bonds green and the  $xy(z)$  bonds red.

# Основное состояние иридата натрия

Возможные состояния

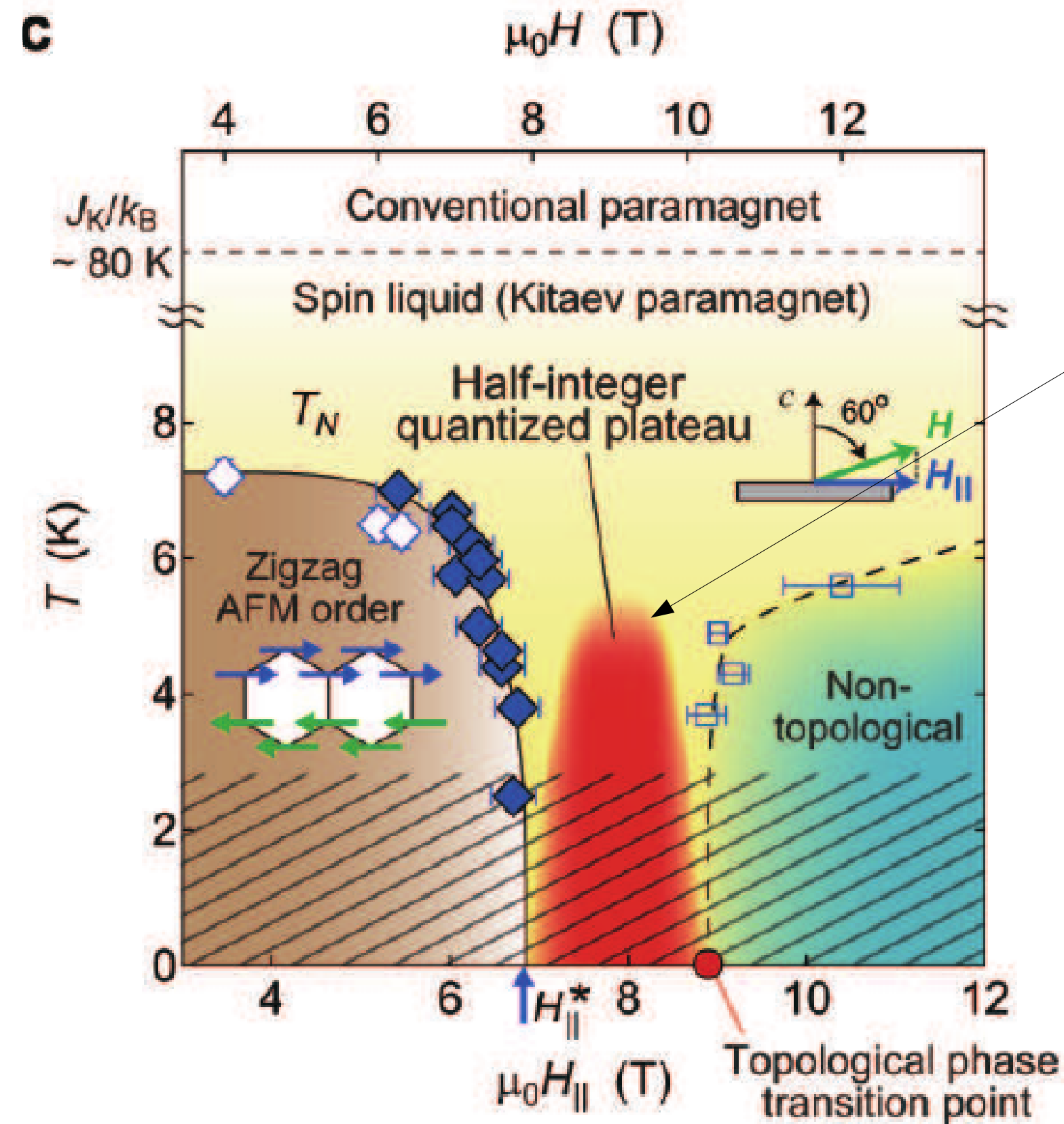
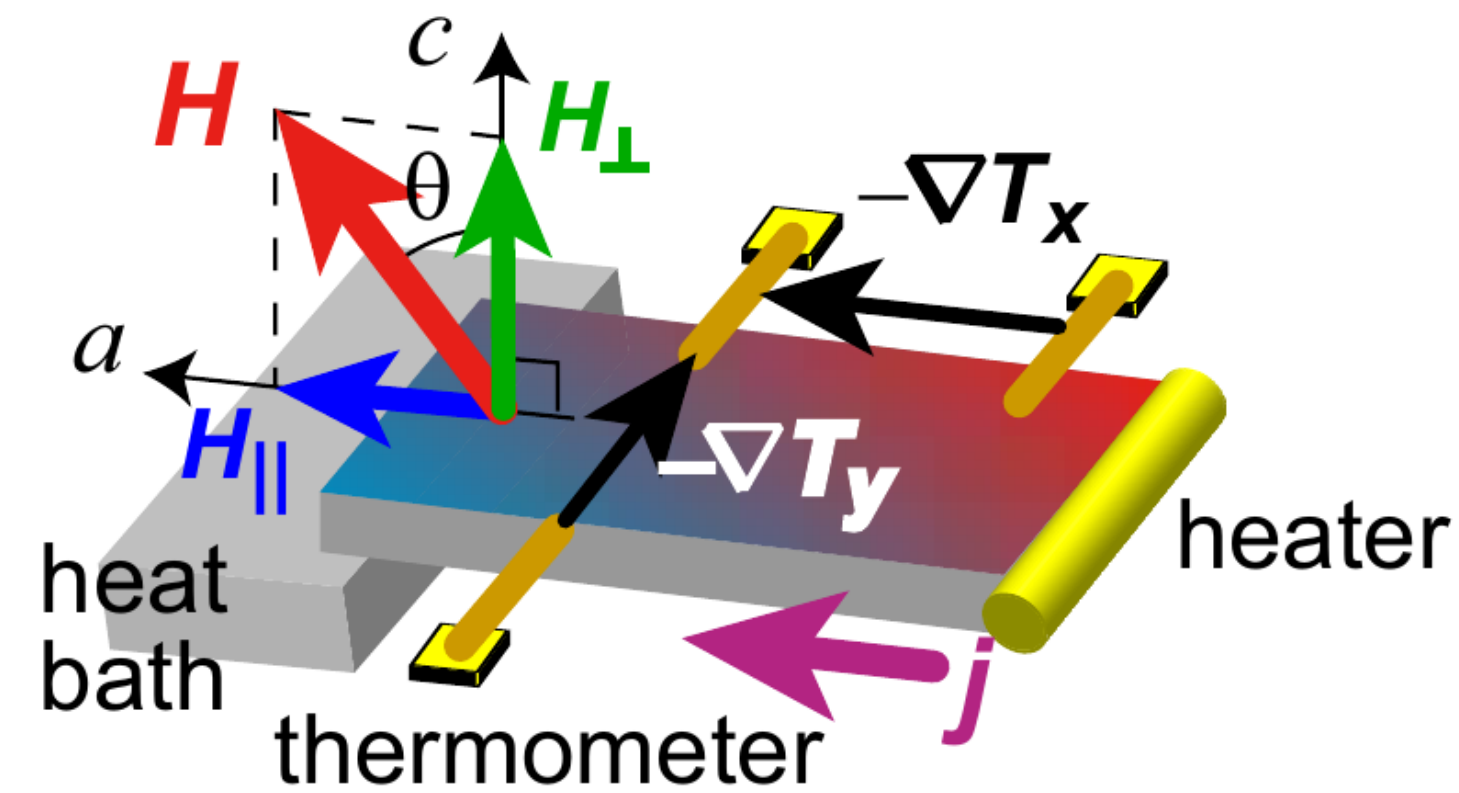


Эксперимент по неупругому рассеянию нейтронов

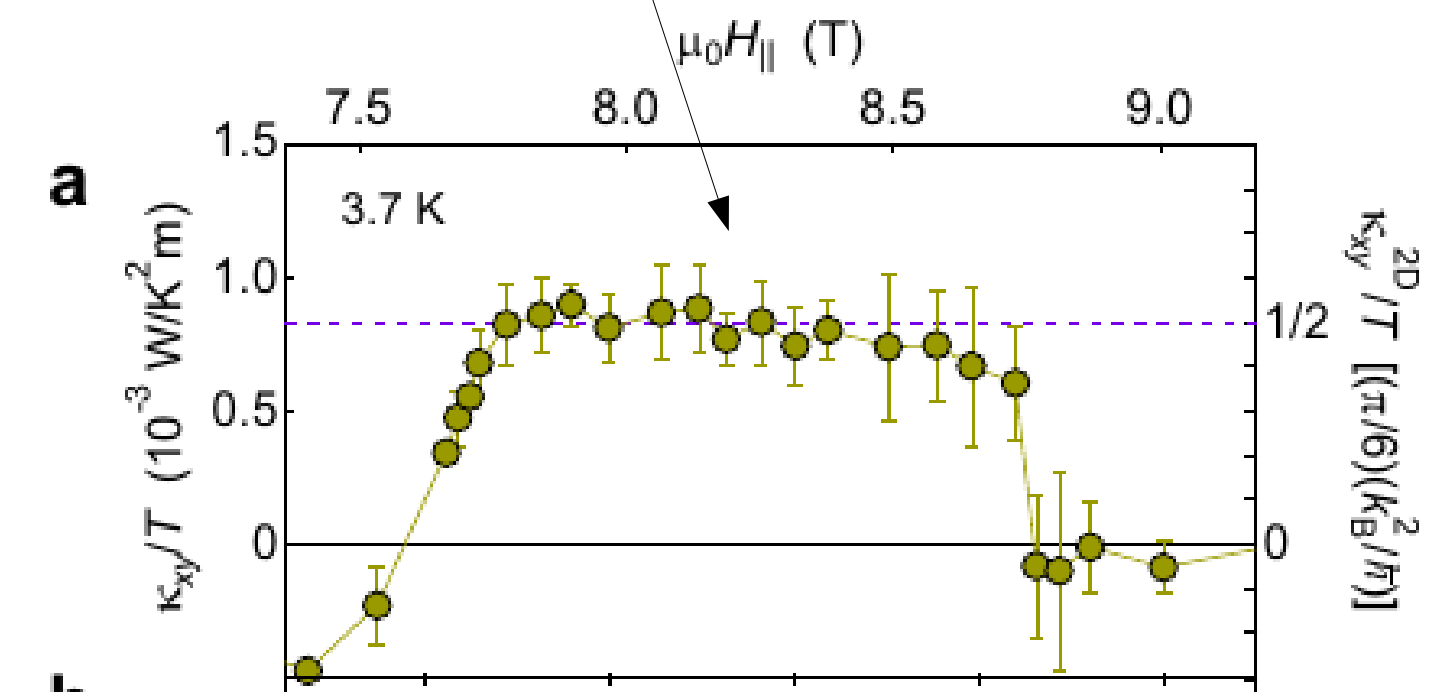


# Реальные системы с взаимодействием Китаева: хлорид рутения RuCl<sub>3</sub>

Поперечная теплопроводность



Возможная топологическая фаза в магнитном поле с квантованной поперечной теплопроводностью





# Редкоземельные системы с взаимодействием Китаева: $A_2PrO_3$

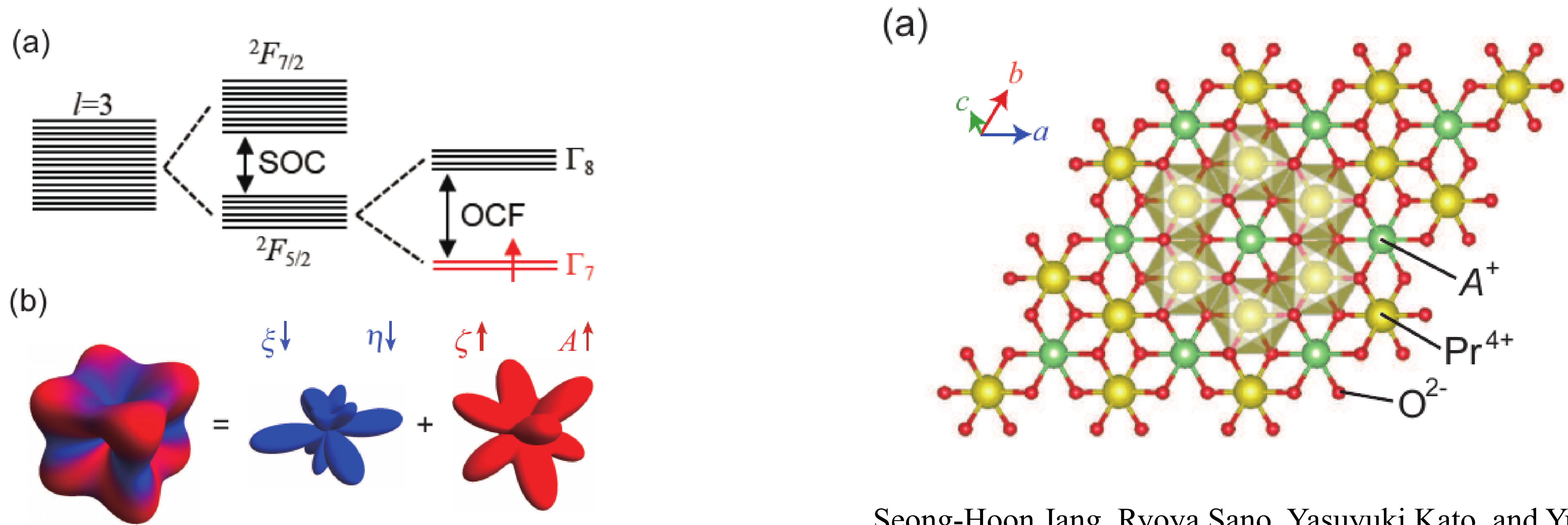
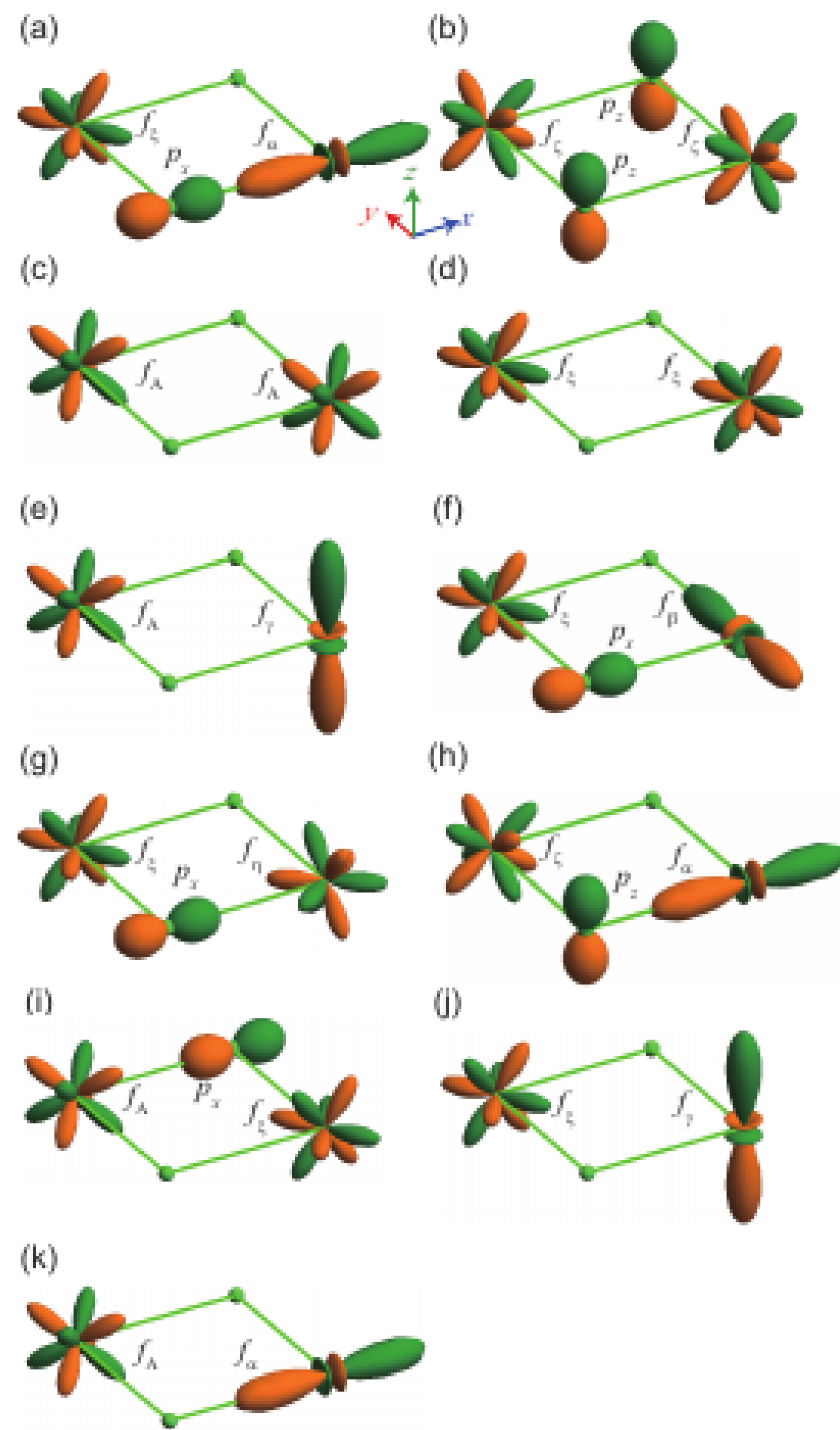


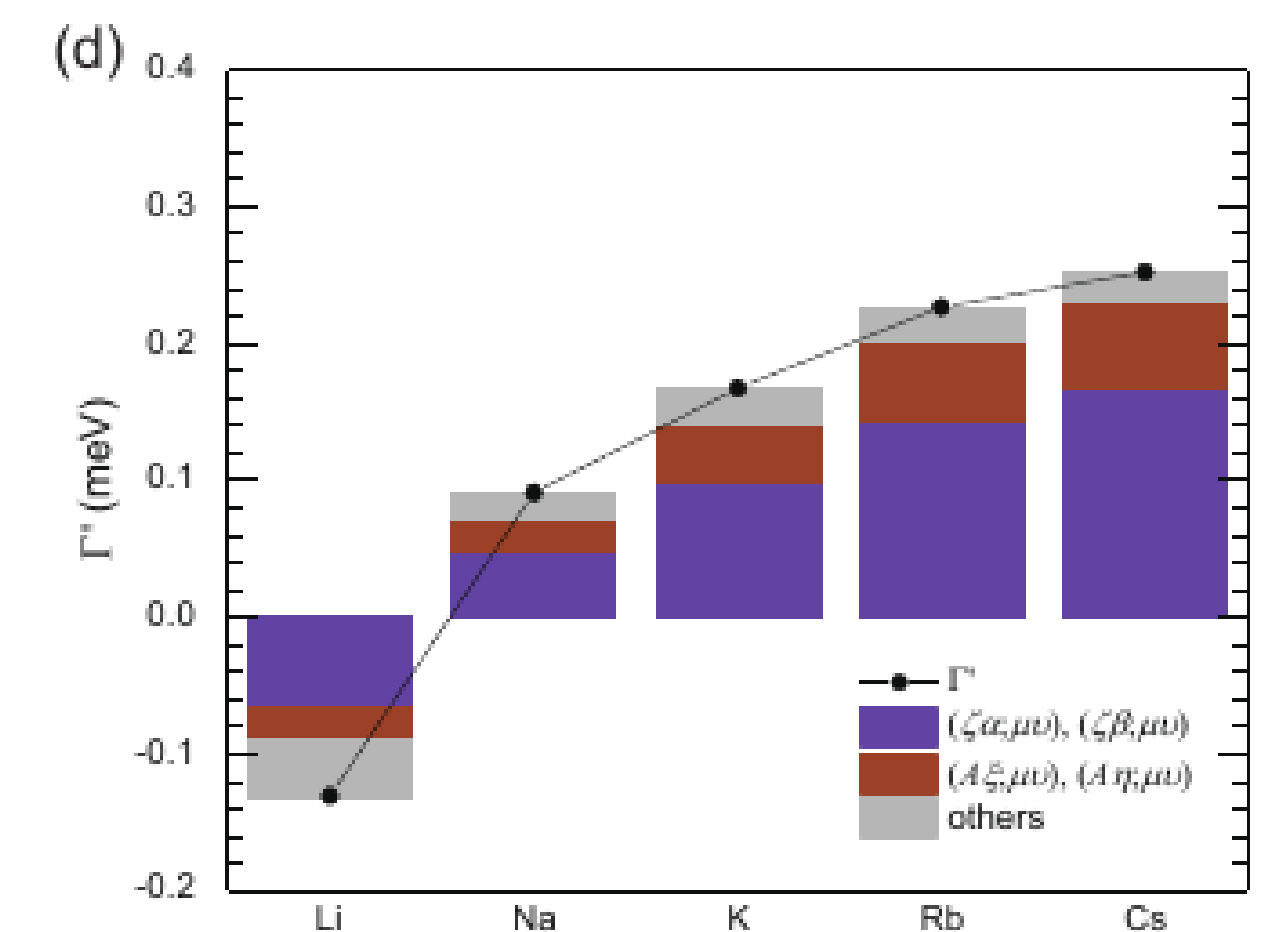
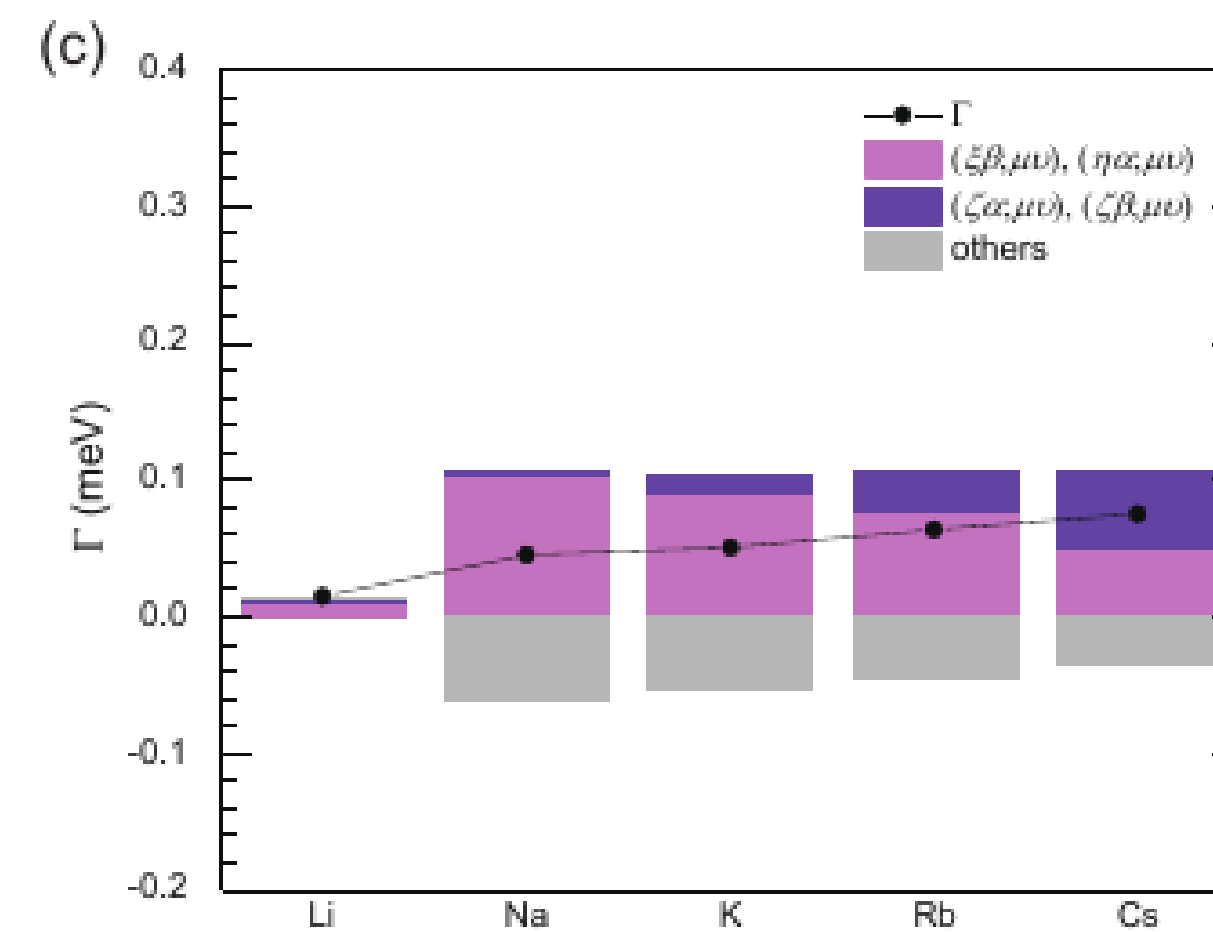
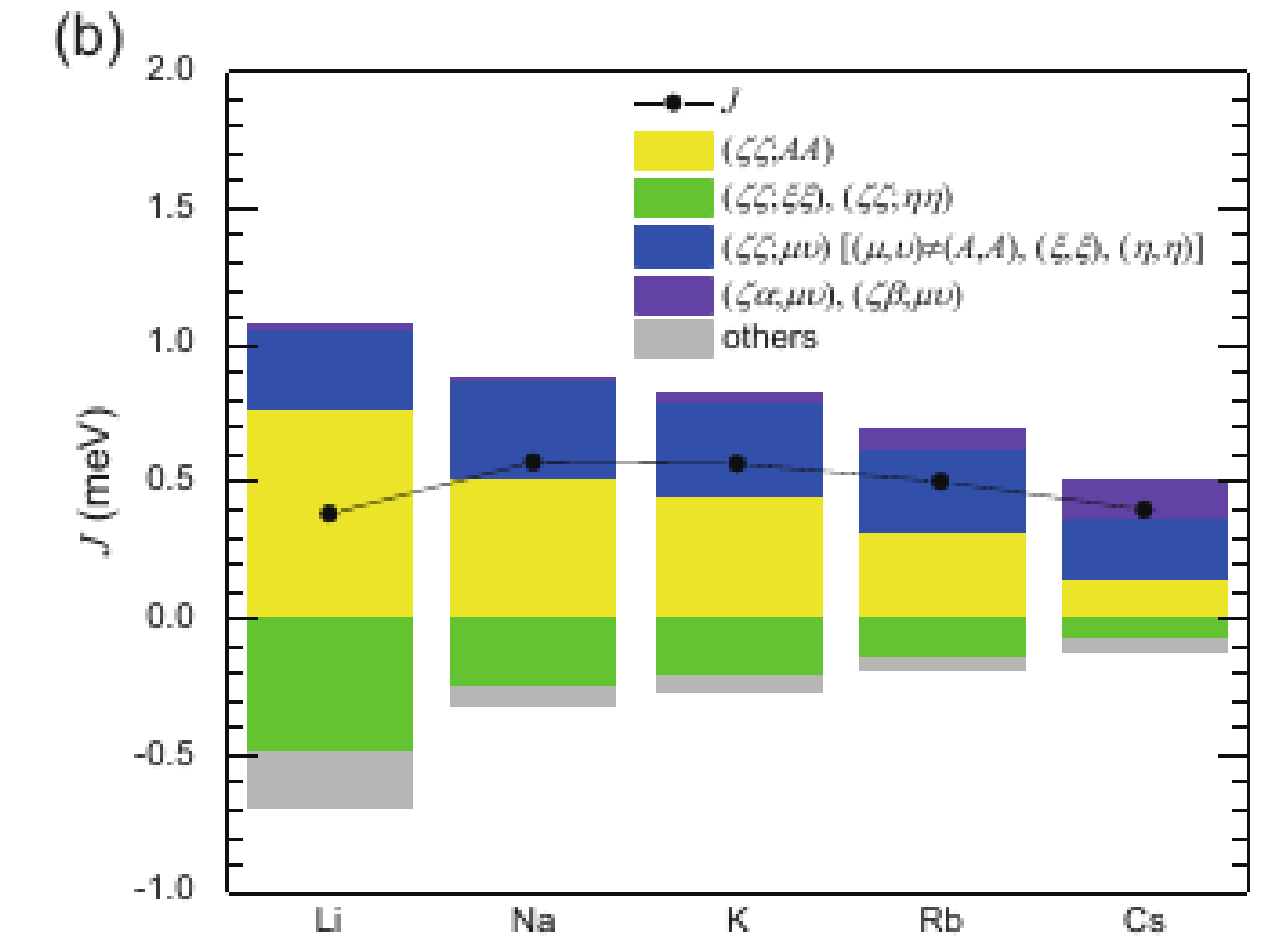
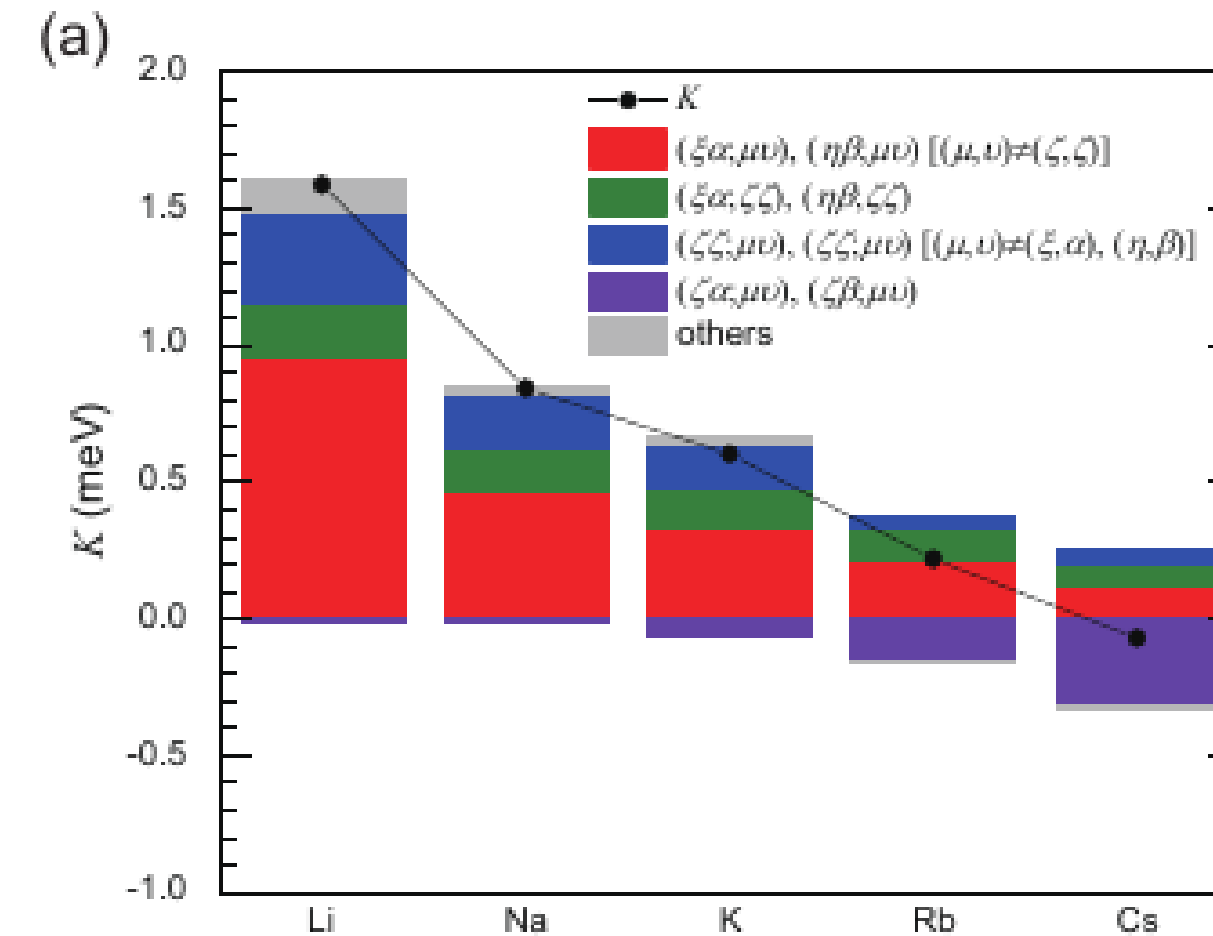
FIG. 1. (a)  $f^1$  level splitting by the spin-orbit coupling (SOC) and the octahedral crystal field (OCF). (b) Density profile of an electron in the pseudospin up state  $|+\rangle$  for the  $\Gamma_7$  doublet; see Eq. (1).

Seong-Hoon Jang, Ryoya Sano, Yasuyuki Kato, and Yukitoshi Motome, Phys. Rev. B 99, 241106(R) (2019)

# Редкоземельные системы с взаимодействием Китаева: $A_2PrO_3$



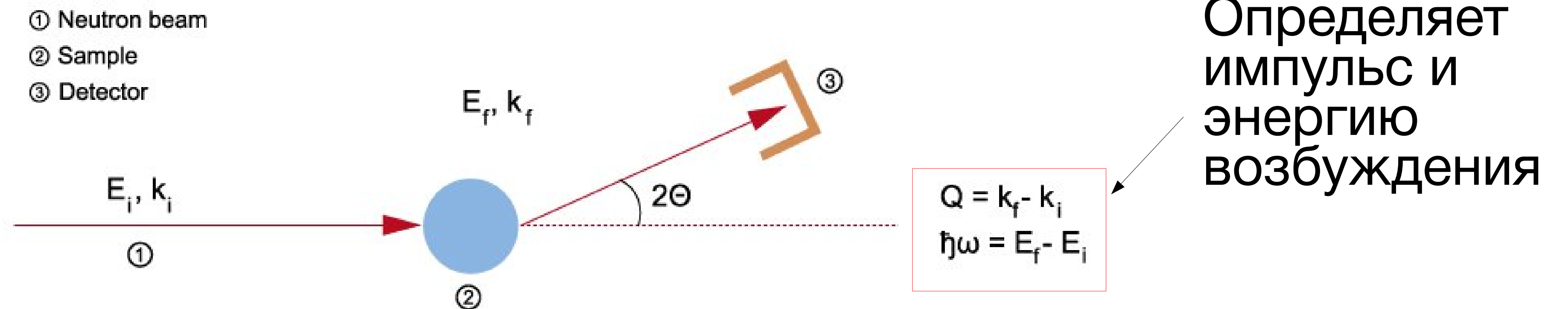
Перескоки, ведущие к анизотропному обменному Гамильтониану



Параметры Гамильтониана

# Нейтронный сигнал в упорядоченных и неупорядоченных состояниях

Neutron diffraction



# Нейтронный сигнал в упорядоченных состояниях

ФМ

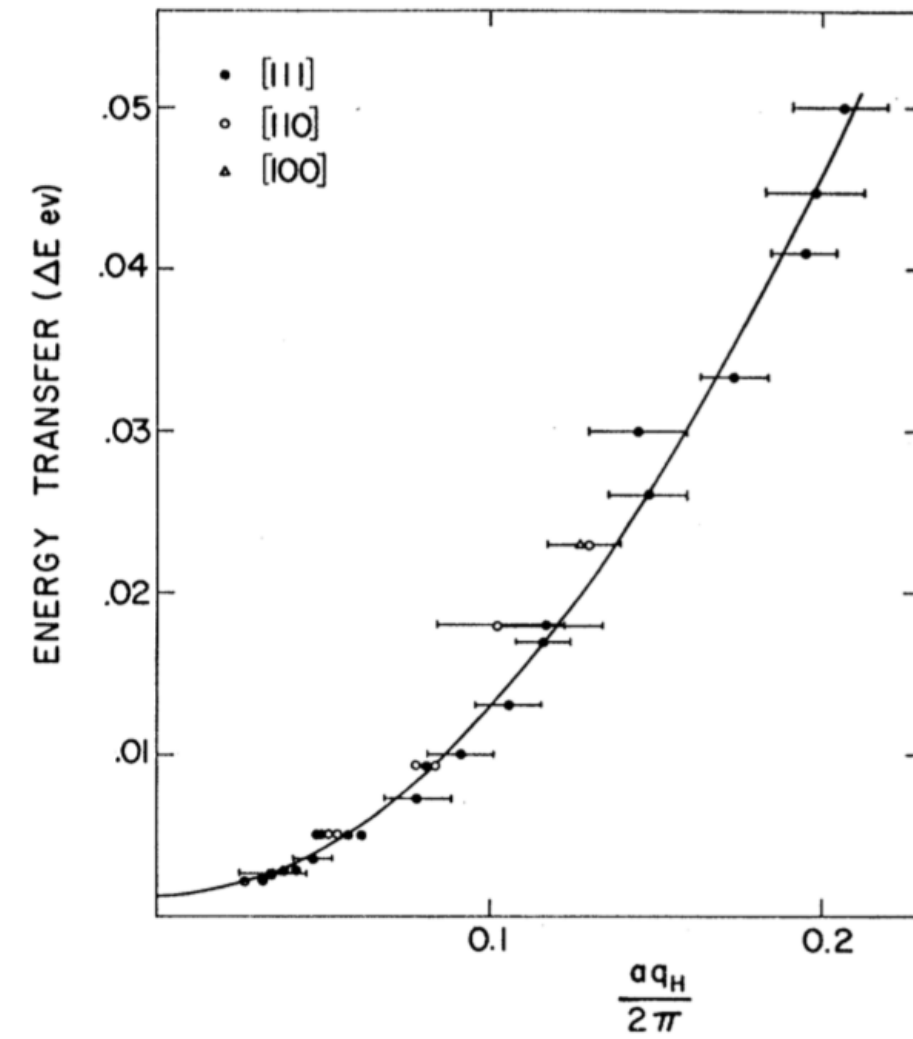
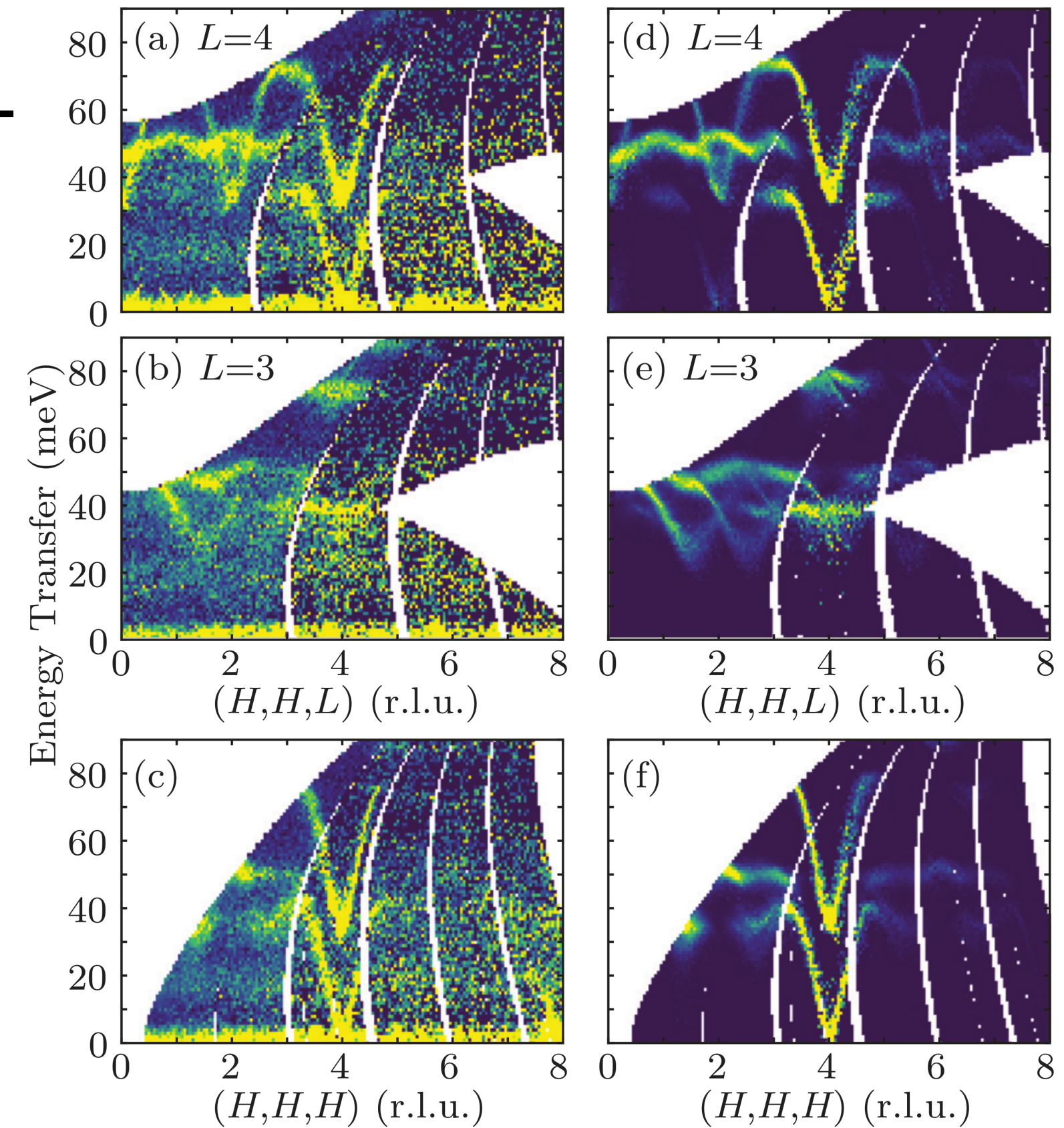
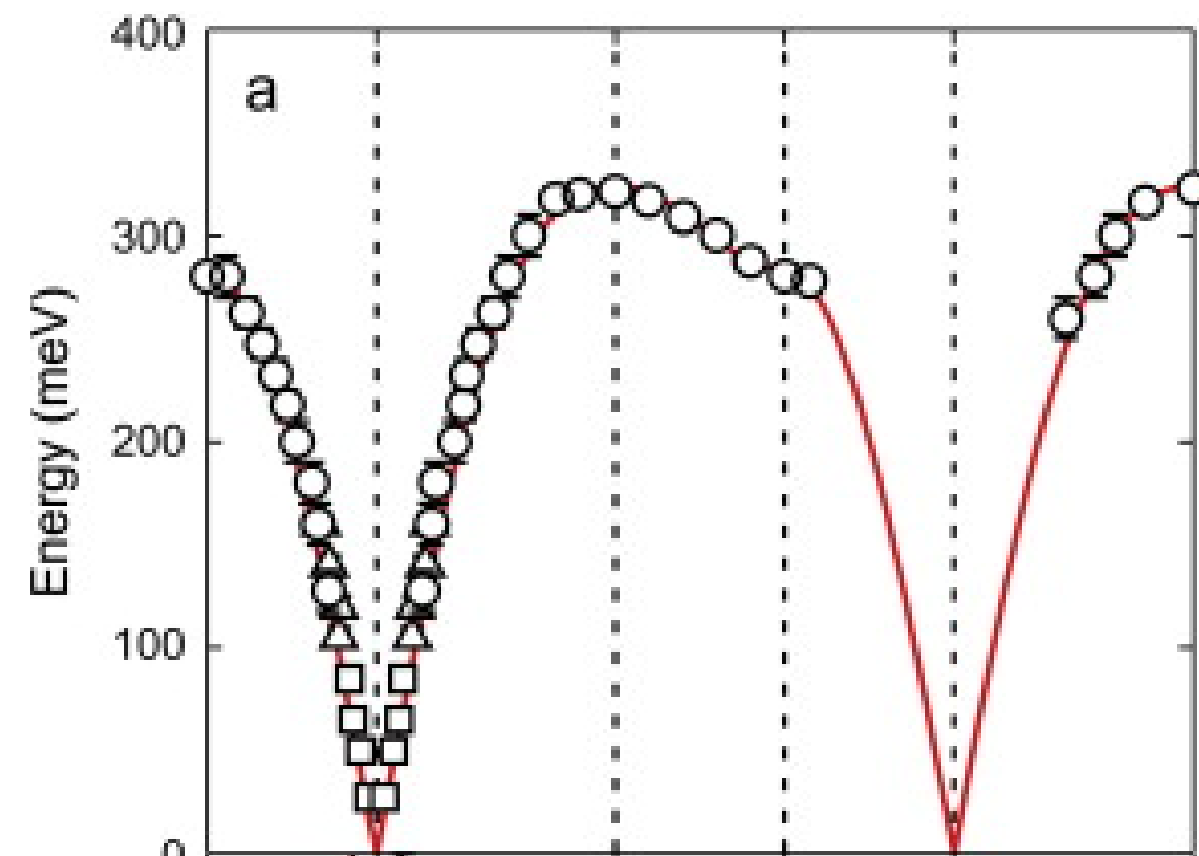


FIG. 2. The energy transfer  $\Delta E$  as a function of the reduced vector of the spin waves. The error bars are about half of the full width at half maximum of the neutron groups. The solid line is the best fit to Eq. (3).

- Магنون, переворот спина, имеет момент  $S=1$
- Нейтрон рождает один магنون

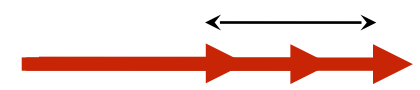
$$I(\mathbf{k}, \omega) \propto \delta(\omega - \varepsilon_{\mathbf{k}})$$

АФМ



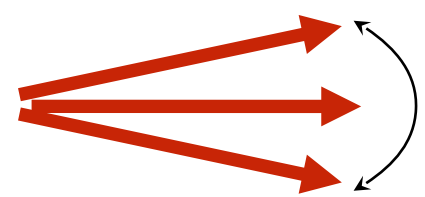
Эксперимент      Расчет

# Трехмагнонное взаимодействие



Продольные колебания (двухмагнонные, четные по вращению вокруг оси z)

$$S^z = S - a^\dagger a$$



Поперечные колебания (одномагнонные, нечетные по вращению вокруг оси z)

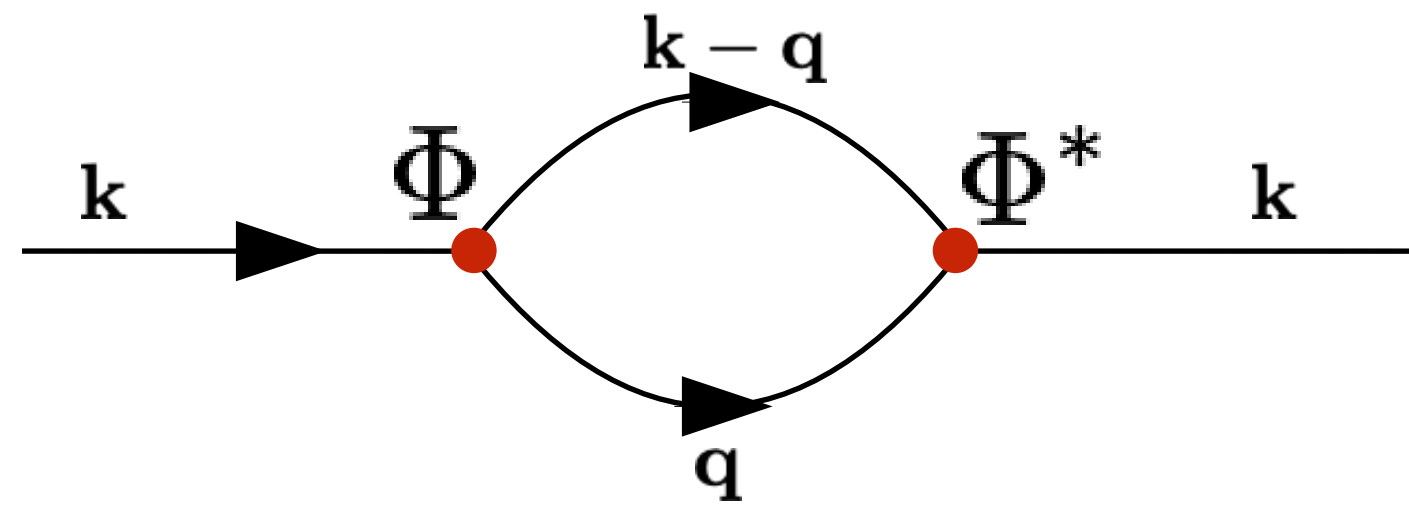
$$S^x \approx \sqrt{\frac{S}{2}} (a + a^\dagger)$$

Анизотропное взаимодействие

$$\hat{\mathcal{H}} = \dots S_i^x S_j^z + S_i^z S_j^x \dots$$

$$\mathcal{H} = \sum_{\langle ij \rangle} JS_i \cdot S_j + K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) + \Gamma' (S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma)$$

# Распад магнонов

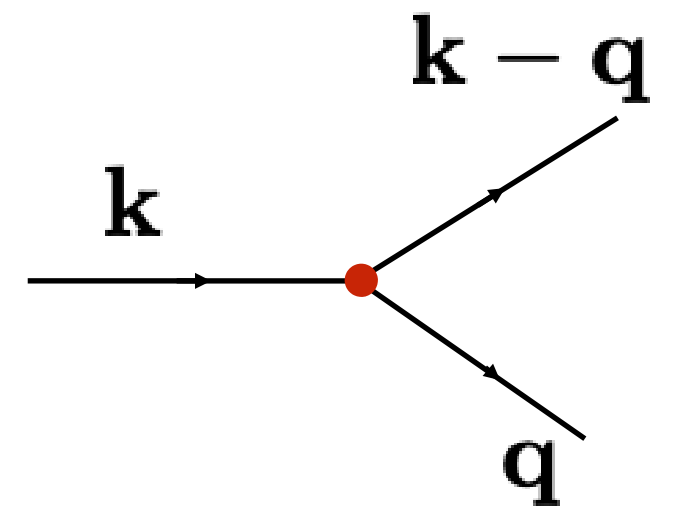


$$\Sigma(\mathbf{k}, \omega) = \frac{1}{2} \sum_{\mathbf{q}} \frac{|\Phi(\mathbf{q}, \mathbf{k})|^2}{\omega - \varepsilon_{\mathbf{k}-\mathbf{q}} - \varepsilon_{\mathbf{q}} + i0}$$

Борновское приближение:  $\Gamma_{\mathbf{k}} = \frac{\pi}{2} \sum_{\mathbf{q}} |\Phi(\mathbf{q}, \mathbf{k})|^2 \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}-\mathbf{q}} - \varepsilon_{\mathbf{q}})$

Закон сохранения энергии и импульса:

$$\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{k}-\mathbf{q}}$$



# Обзоры

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