Quasi-frozen spin concept at NICA for EDM search and its matrix analysis

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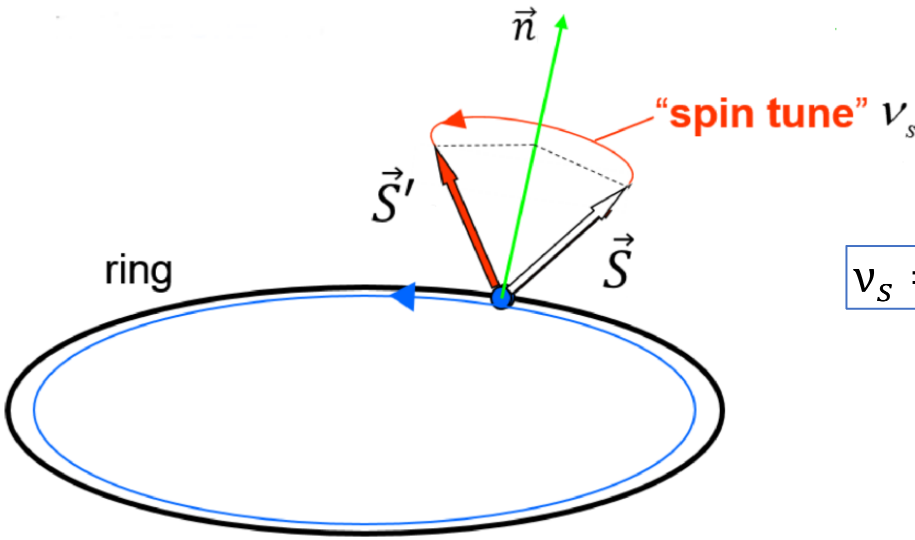
DSPIN 2023

Spin dynamics in an accelerator

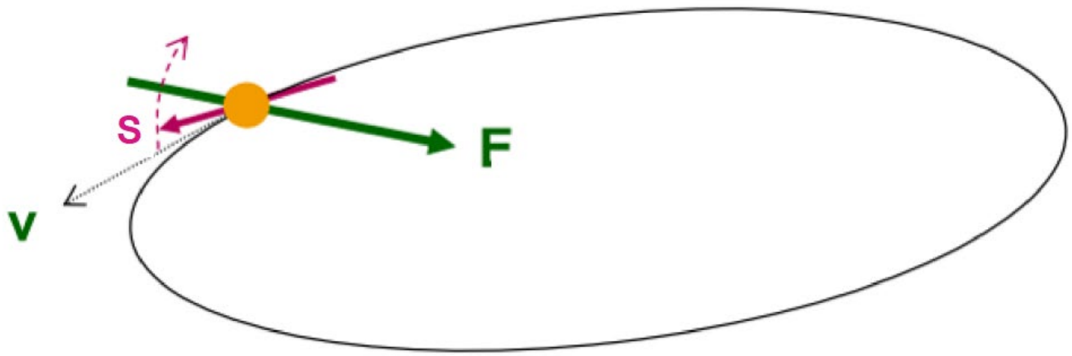


$$\nu_s = \Omega_s / \Omega_p$$

T-BMT: $$\frac{d\vec{S}}{dt} = \vec{S} \times \vec{\Omega} \qquad \vec{\Omega}_{MDM} = \frac{q}{m\gamma}\left[(\gamma G + 1)\vec{B}_{\perp} - \left(\gamma G + \frac{\gamma}{\gamma + 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

EDM in a Frozen Spin

Frozen Spin (FS) method for Electric Dipole Moment (EDM) search:



$$\frac{d\vec{S}}{dt} = \vec{S} \times (\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM})$$

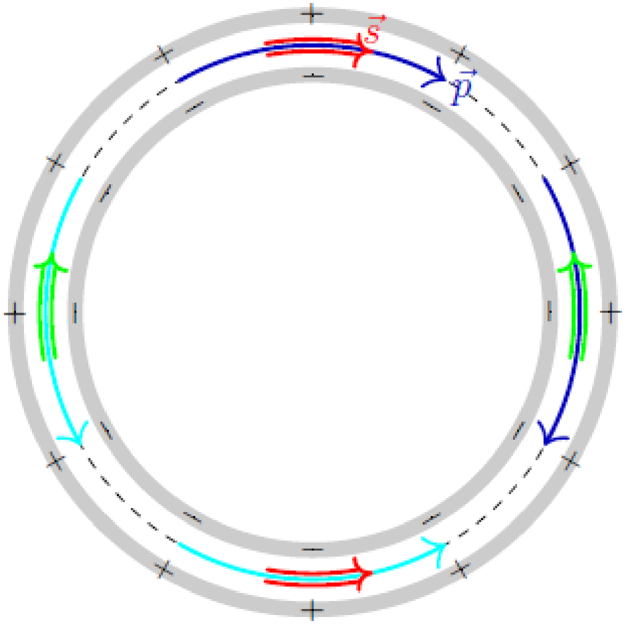
$$\vec{S} \parallel \vec{P} \rightarrow \Omega_{MDM} = 0$$

$$\vec{\Omega}_{EDM} = \frac{q\eta}{2m}\left[\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c}\right]$$

Observe the build-up of vertical spin component

FS EDM Rings

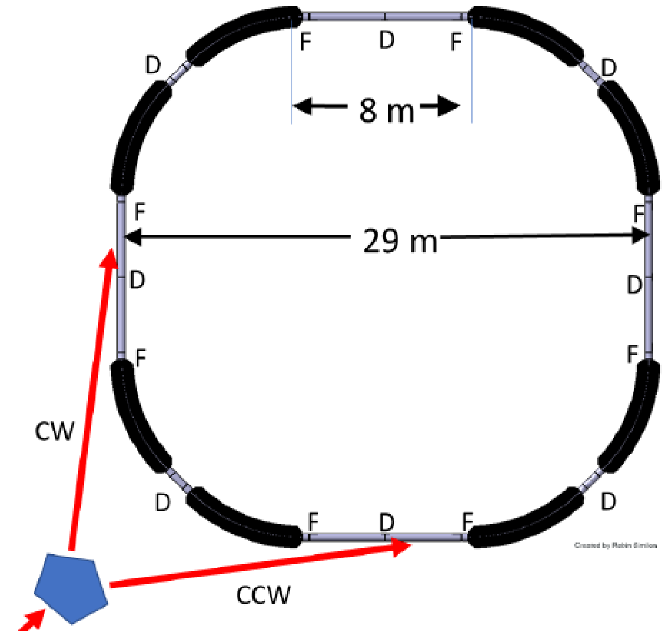
Final EDM Ring: all Electric



$$\nu_S^E = \beta^2\gamma\left(G - \frac{1}{\gamma^2 - 1}\right)$$

- $\varepsilon_{mag.}^{p} = 232.8$ МэВ, frozen spin
- **CW/CCW beams concurrently to take into account systematic effects**
- $E = \frac{8MV}{m}$, $L \sim 500m$.

Prototype EDM Ring: E+B

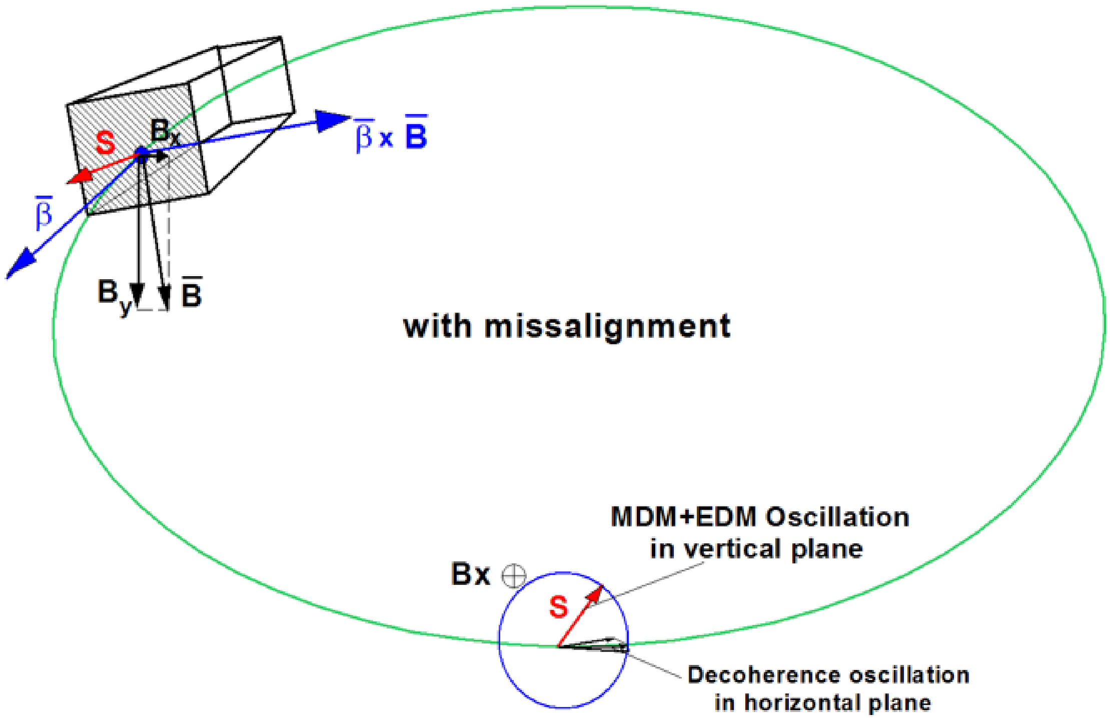


- $L \sim 100m$
- Protons 45 MeV
- E+B elements to freeze the spins
- CW/CCW injection **after the change of B field polarity**
- Direct measurement of p EDM
- Check for the ability to store 2 CW/CCW beams at one time

EDM in a Frozen Spin

Key factors limiting the accelerator EDM experiment:

1) **Systematics:** $d = 10^{-29}\ \mathrm{e*cm}$ ~ $\eta = 1.9 * 10^{-15}$ ~ $B_x^{EDM} = \mathbf{10^{-17}}\ \mathrm{T}$!!!
2) **Coherent spin rotation**



Radial field errors create fake EDM signal

The accuracy of the installation of magnets should be less than 10^{-9} meters (**super unrealistic !!!)**

EDM in a Frequency Domain

Observe the total spin-precession frequency Ω:

$$\tilde{S}_y = \sqrt{\left(\frac{\Omega_y \Omega_z}{\Omega^2}\right)^2 + \left(\frac{\Omega_x}{\Omega}\right)^2} \sin(\alpha + \phi)$$

$$\alpha = \Omega \cdot \mathrm{t}$$

$$\Omega_x = \Omega_{edm} + \Omega_{B_r} \qquad \Omega = \sqrt{\left(\Omega_{edm} + \Omega_{B_r}\right)^2 + \Omega_y^2 + \Omega_z^2}$$

We purposely make:

$$\Omega^2_{\mathrm{B_v}, E_r}, \quad \Omega^2_{\mathrm{B_z}} << \left(\Omega_{edm} + \Omega_{B_r}\right)^2$$ Then:

$$\Omega = \left(\Omega_{edm} + \Omega_{B_r}\right)\left[1 + \frac{1}{2}\frac{\Omega^2_{B_v,E_r} + \Omega^2_{B_z}}{\left(\Omega_{edm} + \Omega_{B_r}\right)^2}\right]$$ Or $$\Omega = \Omega_{B_r} + \Omega_{edm} + \frac{1}{2}\frac{\Omega^2_{B_v,E_r} + \Omega^2_{B_z}}{\Omega_{edm} + \Omega_{B_r}}$$

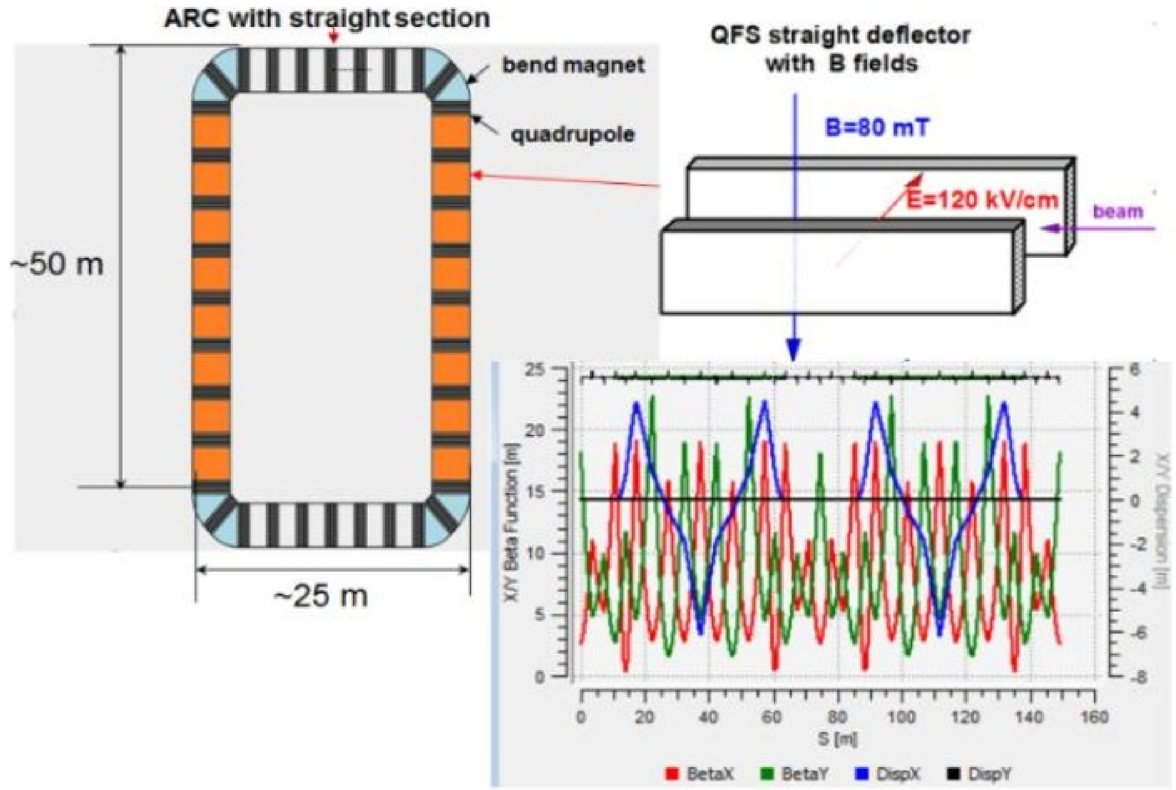
<u>Main idea</u>: to make the contribution from EDM frequency into the total frequency bigger than from MDM additions: $\Omega^2_{\mathrm{B_v},E_r}, \Omega^2_{\mathrm{B_z}}$

$$\Omega_{EDM} = (\Omega_{CW} + \Omega_{CCW})/2 + (\Omega^{CCW}_{Bx} - \Omega^{CW}_{Bx})/2$$

Misalignments now help to measure the EDM!

EDM in a Quasi-Frozen Spin

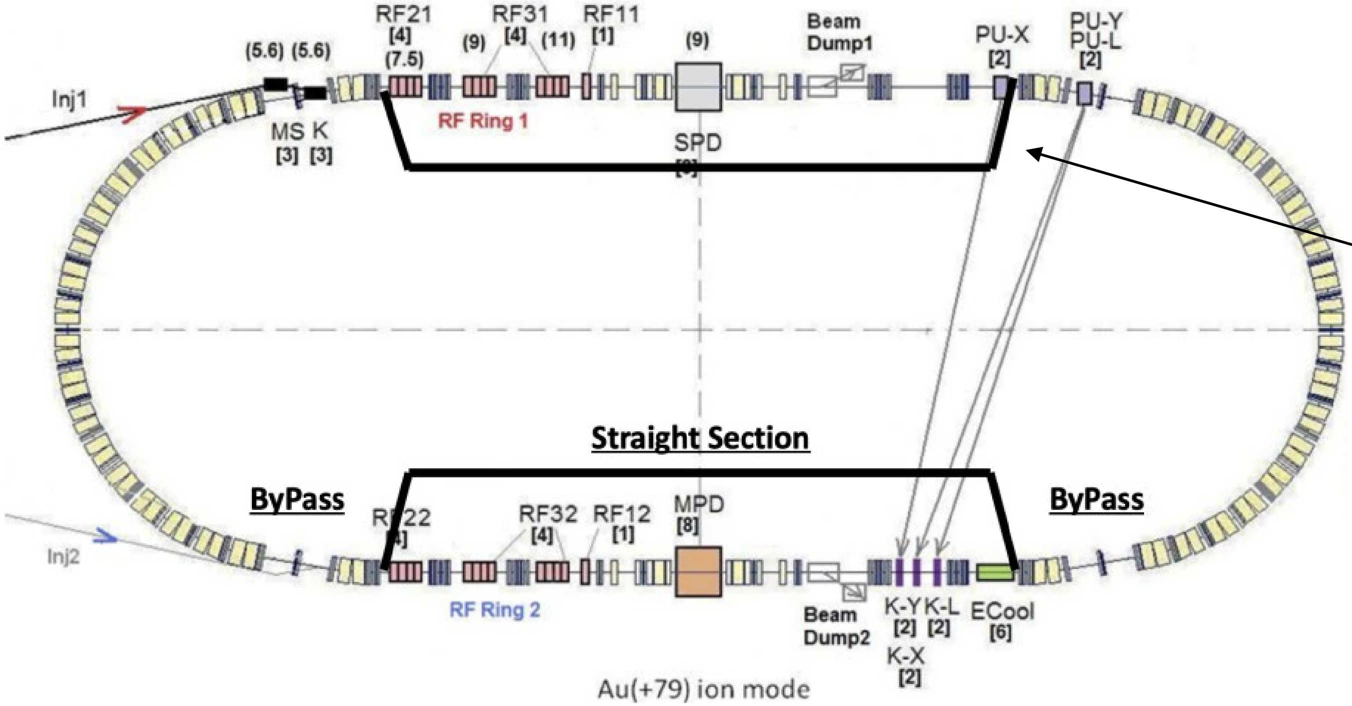
In the FS mode the special dedicated ring has to be constructed.
In the QFS mode accelerator EDM experiment can be conducted on the basis of the existing synchrotron (NICA) with adding Wien-Filters in the straight section.

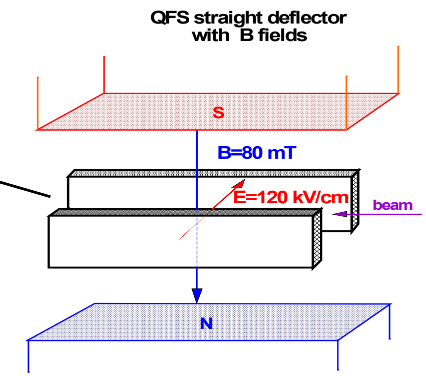


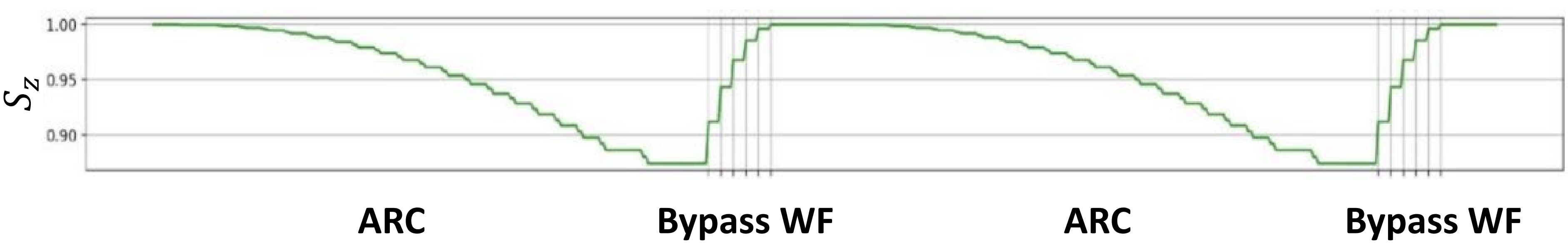
Yuri Senichev: idea of the Quasi-Frozen Spin lattice.

Spin rotates in the WF opposite to the arc and restores its rotation after one turn.

Quasi-Frozen Spin at NICA







EDM in a Frequency Domain

Questions to be investigated for the EDM measurement:

- The T-BMT equation gives the local Ω_i at each element, how are they related with the measured integral Ω at the observation point in the FS and QFS case?
- What is the direction of $\vec{n}$ in the perturbed FS and QFS lattice?
- Does the $\vec{n}$ flip its direction from CW to CCW injection in both FS and QFS lattices?
- What is the prediction for the MDM resonance strength caused by the misalignments?

Spinor Formalism

The key measured values in the experiment are the spin-tune ν_s and the direction of the invariant axis $\vec{n}$ at the observation point. These properties can be easily calculated in the spinor formalism.

After each element the $\vec{S}$ is rotated at an angle $\varphi = \Omega dt \rightarrow$
each element can be described as a matrix:

$$M = exp\left[-i(\vec{\sigma} \cdot \vec{n})\frac{\phi}{2}\right] = cos\left(\frac{\phi}{2}\right) - i(\vec{\sigma} \cdot \vec{n}) sin\left(\frac{\phi}{2}\right).$$

The coordinate frame is specified by the indices:
(1,2,3)=(radial outward, longitudinal forward, vertical up). The Pauli matrices σ_i are:

$$\vec{\sigma} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right).$$

For the ring comprising N elements the total Map is: $M = M_N M_{N-1} \ldots M_2 M_1.$

For an observation point at azimuth θ:

$$cos(\pi \nu_s) = \frac{1}{2} Tr(M(\theta)).$$

$$\vec{n}(\theta) = \frac{i/2}{sin(\pi \nu_s)} Tr(\vec{\sigma} M(\theta)).$$

Frozen Spin Lattice

The ideal unperturbed lattice has an identical spin transfer Map by intent: $M = I$. The same is true for each segment of the ring. If N radial perturbations ψ_i are present, then from the observation point in a SS the structure is equivalent to:

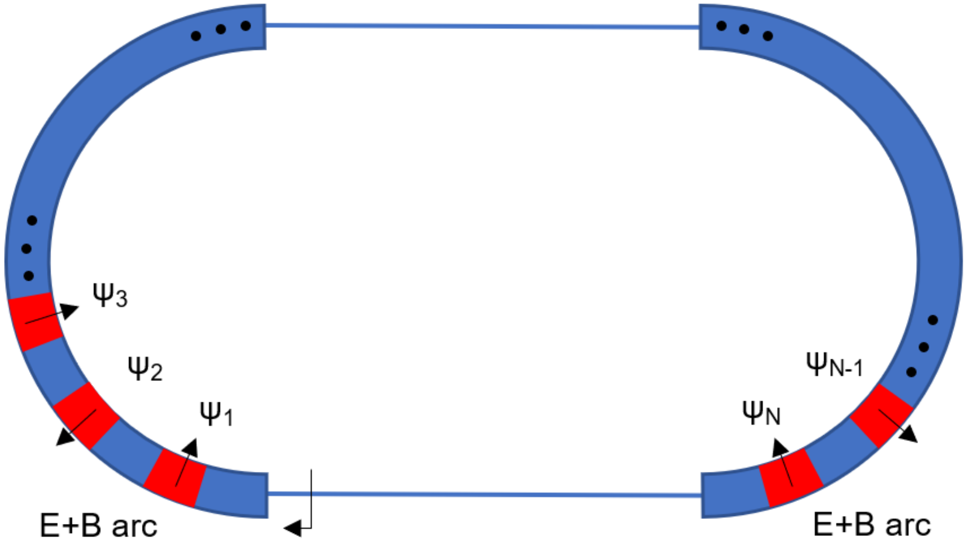
$$M = I \cdot \left[\cos\left(\frac{\psi_N}{2}\right) - i\sigma_1 \sin\left(\frac{\psi_N}{2}\right)\right] \cdot I \cdot \left[\cos\left(\frac{\psi_{N-1}}{2}\right) - i\sigma_1 \sin\left(\frac{\psi_{N-1}}{2}\right)\right] \cdot \ldots$$

$$= \cos\left(\frac{\sum_{i=1}^{N} \psi_i}{2}\right) - i\sigma_1 \sin\left(\frac{\sum_{i=1}^{N} \psi_i}{2}\right).$$

$$\nu_s = \frac{\sum_{i=1}^{N} \psi_i}{2\pi}.$$

$$\vec{n} = \vec{n}_r, unless \sum_{i=1}^{N} \psi_i = 0.$$

$$\Omega_{meas} = \frac{1}{N} \sum \Omega_i$$



Quasi-Frozen Spin Lattice

Now let us investigate the **QFS lattice** consisting of 2 magnetic arcs and 2 WF insertions in a SS, rotating $\vec{S}$ at angles φ_{Dip} and φ_{WF} respectively. The total Map of the ring is:

$$M = \left(M_{WF} \cdot M_{Dip}\right)^2 = \cos\left(\phi_{WF} + \phi_{Dip}\right) - i\sigma_3 \sin\left(\phi_{WF} + \phi_{Dip}\right).$$

$$\nu_s = \frac{\phi_{WF} + \phi_{Dip}}{\pi}.$$

$$\vec{n} = \vec{n}_{vert}, unless \quad \phi_{WF} + \phi_{Dip} = 0.$$

The formal solution for the idealized lattice omits the resonance case.

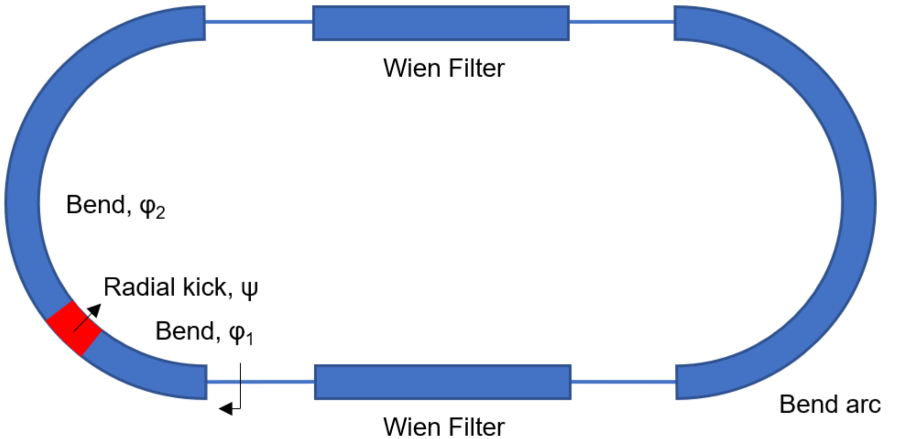
Now let us investigate the **QFS lattice with a single radial perturbation** in the arc in a QFS resonance state ($\sum \varphi_i = 0$). The total Map of the ring is:

$$M = I \cdot M_{WF} \cdot M_{Dip}(\phi_2) \cdot M_{Perturb}(\psi) \cdot M_{Dip}(\phi_1) =$$

$$\cos\left(\frac{\psi}{2}\right) - i\sigma_1 \cos\left(\phi_1\right) \sin\left(\frac{\psi}{2}\right) + i\sigma_2 \sin\left(\phi_1\right) \sin\left(\frac{\psi}{2}\right).$$

$$\nu_s = \frac{\psi}{2\pi}.$$

$$\vec{n} = \left[\cos\phi_1, -\sin\phi_1, 0\right].$$



For the CCW injection:

$M = M_{Dip}(\phi_1) \cdot M_{Perturb}(-\psi) \cdot M_{Dip}(\phi_2) \cdot M_{WF} \cdot I.$ and $\vec{n} \rightarrow -\vec{n}$

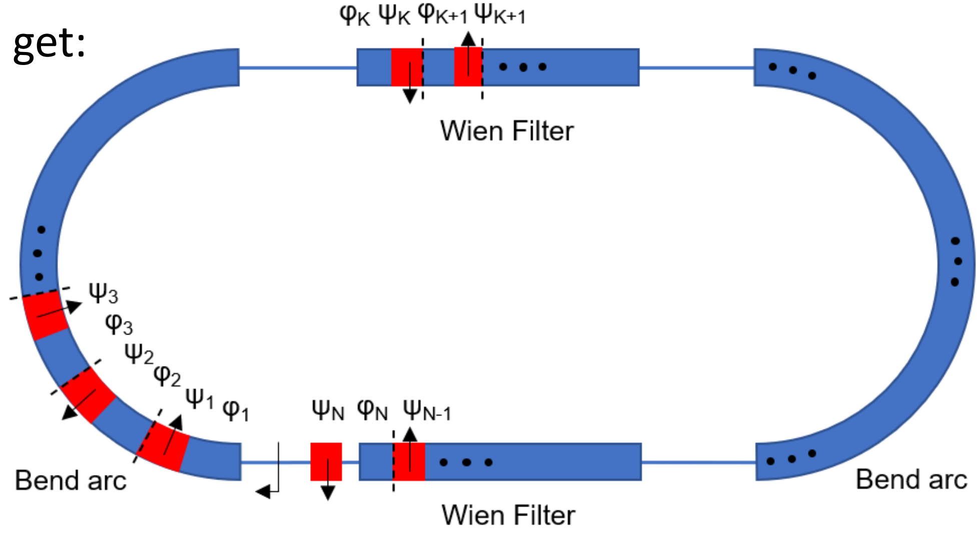
Quasi-Frozen Spin Lattice

Let us investigate the **QFS lattice with N radial perturbations** ψ_i. Here we subdivide the lattice in N sectors with sequential rotations at angles ϕ_i and ψ_i around the vertical and radial directions. We assume that radial fields act as perturbations, so that $\psi_i \ll 1$. For a lattice with a number of sectors $N >> 1$ it is also reasonable to state that $\phi_i \ll 1$. The spin transfer Map of the ring is:

$$M = \prod_{i=N}^{1} M_{Perturb}(\psi_i) \cdot M_{Dip}(\phi_i).$$

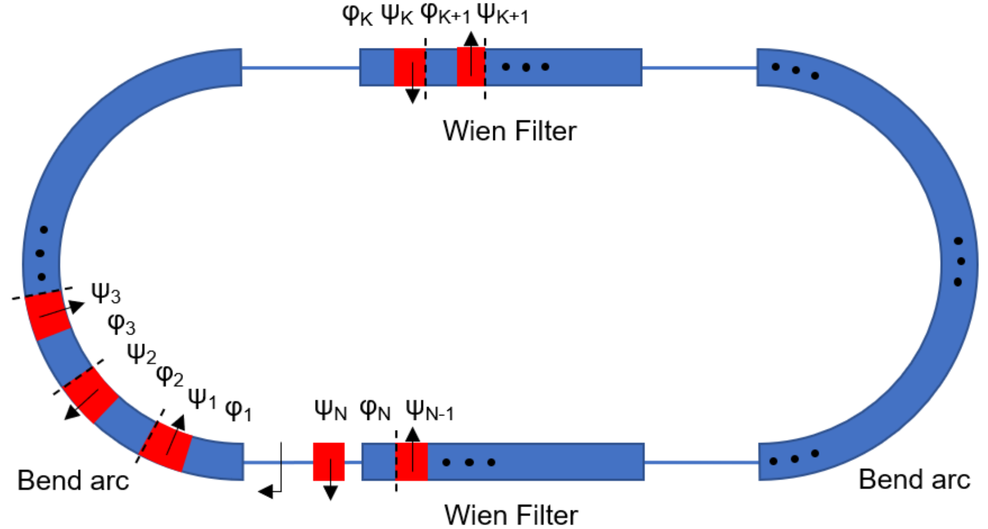
Expanding the product by induction and keeping terms up to 2nd order in (ϕ, ψ) we get:

$$M = cos\left(\frac{\sum_{i=1}^{N}\psi_i}{2}\right) \cdot cos\left(\frac{\sum_{i=1}^{N}\phi_i}{2}\right)$$
$$-i\sigma_1 cos\left(\frac{\phi_1}{2}\right) cos\left(\frac{\phi_2}{2}\right) \ldots cos\left(\frac{\phi_N}{2}\right) sin\left(\frac{\sum_{i=1}^{N}\psi_i}{2}\right)$$
$$+\frac{i}{4}\sigma_2 s_{ij}\phi_i\psi_j$$
$$-i\sigma_3 cos\left(\frac{\psi_1}{2}\right) cos\left(\frac{\psi_2}{2}\right) \ldots cos\left(\frac{\psi_N}{2}\right) sin\left(\frac{\sum_{i=1}^{N}\phi_i}{2}\right)$$
$$+o(\phi,\psi)^2, where\ s_{ij} = \begin{cases} 1, i \le j \\ -1, i > j \end{cases}.$$



Quasi-Frozen Spin Lattice

$$M = cos\left(\frac{\sum_{i=1}^{N}\psi_i}{2}\right)\cdot cos\left(\frac{\sum_{i=1}^{N}\phi_i}{2}\right)$$
$$-i\sigma_1 cos\left(\frac{\phi_1}{2}\right)cos\left(\frac{\phi_2}{2}\right)\dots cos\left(\frac{\phi_N}{2}\right)sin\left(\frac{\sum_{i=1}^{N}\psi_i}{2}\right)$$
$$+\frac{i}{4}\sigma_2 s_{ij}\phi_i\psi_j$$
$$-i\sigma_3 cos\left(\frac{\psi_1}{2}\right)cos\left(\frac{\psi_2}{2}\right)\dots cos\left(\frac{\psi_N}{2}\right)sin\left(\frac{\sum_{i=1}^{N}\phi_i}{2}\right)$$
$$+o(\phi,\psi)^2, where\ s_{ij}=\begin{cases}1, i\le j\\ -1, i>j\end{cases}.$$



Applying the QFS resonance condition ($\sum \varphi_i = 0$) we get:

$$\nu_s = \frac{\sum_{i=1}^{N}\psi_i}{2\pi} + o(\phi,\psi)^2.$$

$$\vec{n} = \left[1 + o(\phi,\psi), -\frac{s_{ij}\phi_i\psi_j}{4\sum_{i=1}^{N}\psi_i} + o(\phi,\psi), 0\right].$$

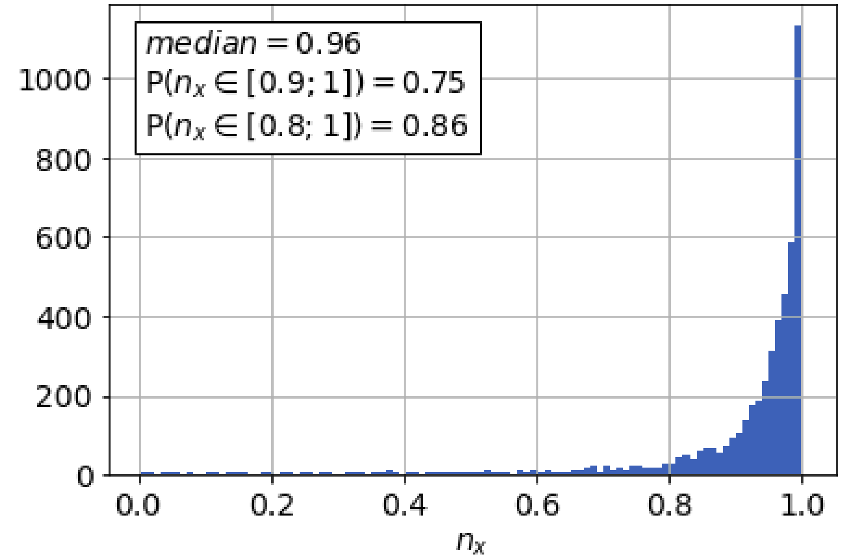
$$\Omega_{meas} = \frac{1}{N}\sum \Omega_i + o(\varphi,\psi)^2$$

The result is similar to the perturbed FS lattice, but here, due to anti-commutation of local rotations we get a **non-zero longitudinal component of $\vec{n}$**. What is more, the measured **spin-precession frequency is the arithmetic sum of the local frequencies only up to second order of Taylor expansion**.

Quasi-Frozen Spin Lattice

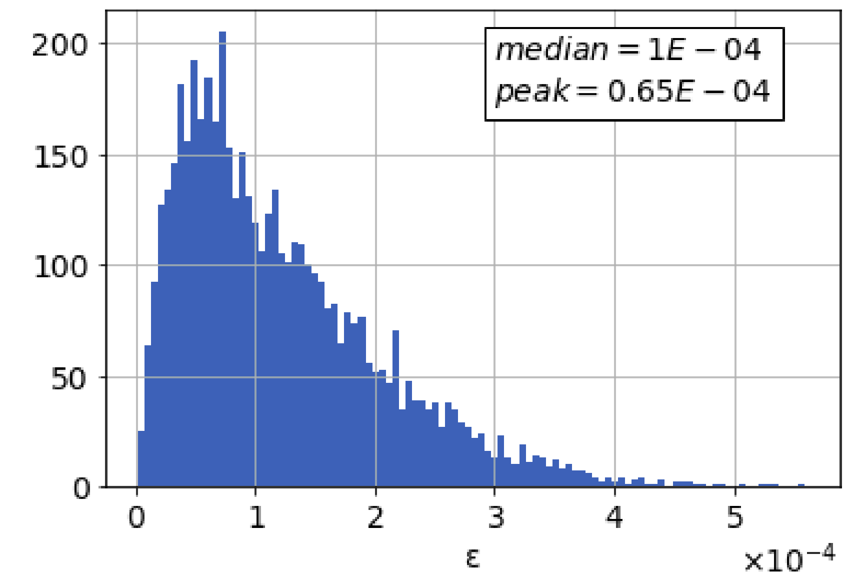
$$\nu_s = \frac{\sum_{i=1}^{N} \psi_i}{2\pi} + o(\phi,\psi)^2.$$

$$\vec{n} = \left[1 + o(\phi,\psi), -\frac{s_{ij}\phi_i\psi_j}{4\sum_{i=1}^{N}\psi_i} + o(\phi,\psi), 0\right].$$



COSY Infinity modelling for the NICA lattice. Radial perturbations: dipole rolls at angles ~ 100 um/m RMS and the closed orbit vertical deviations of 0.5 mm RMS in the quadrupoles.

The statistical distribution of the radial component of $\vec{n}$ is presented as well as the data for resonance strength.



Outcomes

QFS and FS lattices were compared in terms of spin dynamics for EDM search:

- In the perturbed QFS structure the dominant direction of $\vec{n}$ is radial but the small longitudinal component is present due to the anti-commutation of local rotations.

- The measured spin-precession frequency is the weighted sum of local T-BMT frequencies up to 2nd order of Taylor expansion of the Map. But higher order terms may arise. This question is to be investigated in terms of QFS systematic effects.

- The expected QFS resonance strength for the mentioned machine parameters is $\varepsilon \sim 10^{-4}$.

- The invariant spin axis flips its direction from CW to CCW injection. This allows to subtract EDM systematic effects.