Semi-Inclusive Deep-Inelastic Scattering (SIDIS)

$$\mu + N \rightarrow \mu' + nh + X, \qquad n = 1, 2, \ldots$$

$$a(\phi) = a^{\text{const}} + a^{\sin\phi} \sin\phi + a^{\sin 2\phi} \sin(2\phi) + a^{\sin 3\phi} \sin(3\phi) + a^{\cos\phi} \cos\phi.$$

The aim of this study is to evaluate the azimuthal asymmetries in hadron production from the longitudinally polarized target as a manifestation of the quark-spin and transverse-momentum dependent PDF and PFF

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Studies of the nucleons structure using the SIDIS azimuthal angle asymmetries calculated with modified double ratios of numbers of the hadrons

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Measurements of Semi-Inclusive Deep-Inelastic Scattering (SIDIS)

$$\mu + N \to \mu' + nh + X, \qquad n = 1, 2, \ldots \tag{1}$$

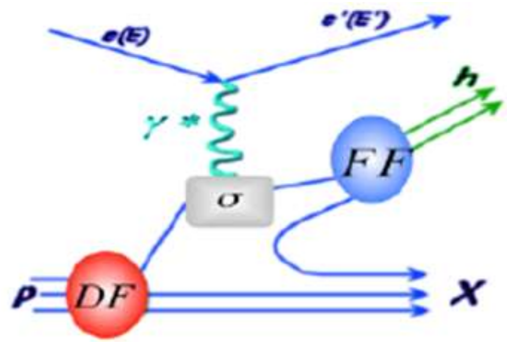
of high-energy polarised muons μ off nucleons N in the initial state and scattered muons μ', n measured

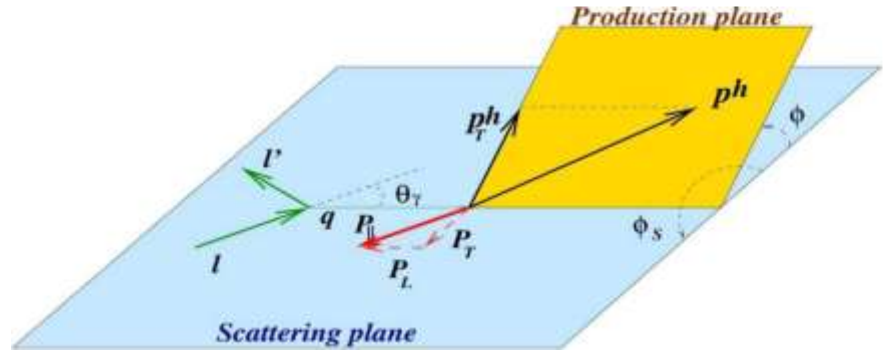
hadrons h and unobserved particles X in the final state are sensitive to the spin-dependent Parton Distribution Functions (PDFs) of nucleons. **The SIDIS cross section depends, in particular, on the azimuthal angle of each produced hadron, which leads to azimuthal asymmetries related to convolutions of the nucleon Transverse-Momentum-Dependent (TMD) PDFs and parton-to-hadron Fragmentation Functions (FFs).**

The COMPASS collaboration collected data on the related SIDIS off the ^{6}LiD (referred to as "deuteron") polarized target in 2002, 2003, 2004, 2006 and off the NH_3 (referred to as "proton") polarized target in 2007, 2010, 2011. These data provide an opportunity to measure all predicted TMD QPDFs.

M.G. Alekseev et al. [COMPASS Collaboration] // Eur. Phys. J. C 70 (2010) 39 [arXiv: 1007.1562].
2002-2004 data analisys (11 pages)
C. Adolf et al. [COMPASS Collaboration] // Eur. Phys. J. C 78 (2018) 952 [arXiv: 1609.06062].
2006 data analysis (13 pages)

Total differential SIDIS cross-section





The general expression of the total differential cross-section for the SIDIS reaction is a linear function of the muon beam polarisation P_μ and of the target polarisation components P_L and P_T

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{00} + P_\mu \mathrm{d}\sigma_{L0} + P_L \left(\mathrm{d}\sigma_{0L} + P_\mu \mathrm{d}\sigma_{LL}\right) + |P_T| \left(\mathrm{d}\sigma_{0T} + P_\mu \mathrm{d}\sigma_{LT}\right) \tag{3}$$

where the first (second) subscript of the partial cross-sections refers to the beam (target) polarisation.

The asymmetry $a(\phi)$ for hadron production from a longitudinally polarised target is defined by

$$a(\phi) = \frac{\mathrm{d}\sigma^{\leftarrow\Rightarrow} - \mathrm{d}\sigma^{\leftarrow\Leftarrow}}{|P_L|(\mathrm{d}\sigma^{\leftarrow\Rightarrow} + \mathrm{d}\sigma^{\leftarrow\Leftarrow})} = -\frac{\mathrm{d}\sigma_{0L} + P_\mu \mathrm{d}\sigma_{LL} - \tan\theta_\gamma \left(\mathrm{d}\sigma_{0T} + P_\mu \mathrm{d}\sigma_{LT}\right)}{\mathrm{d}\sigma_{00} + P_\mu \mathrm{d}\sigma_{L0}} \tag{4}$$

where $\Rightarrow$ or $\Leftarrow$ denotes the target polarisation along or opposite to the muon beam direction and $\leftarrow$ denotes the beam polarisation, which is always opposite to the beam direction. The partial cross-sections $\mathrm{d}\sigma_{00}$ and $\mathrm{d}\sigma_{L0}$ do not contribute to the numerator of the asymmetry (Eq. (4)) while $\mathrm{d}\sigma_{0T}$ and $\mathrm{d}\sigma_{LT}$ are suppressed by the small value of $|P_T|/|P_L| = \tan\theta_\gamma \approx 2(Mx/Q)\sqrt{1-y}$.

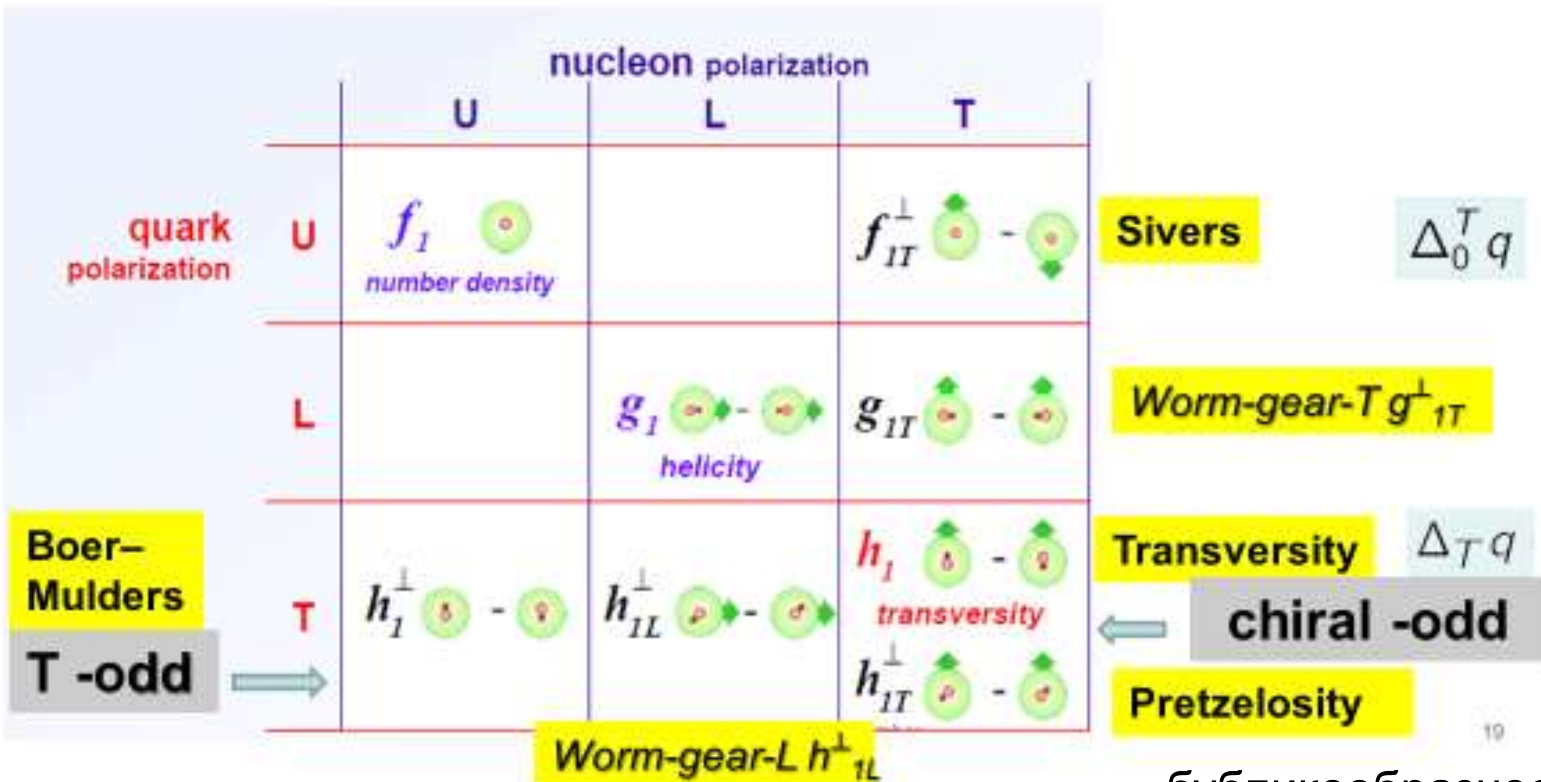
SIDIS cross section

$$\begin{aligned}
d\sigma_{00} &\propto x f_1(x)\otimes D_1(z) + \varepsilon x h_1^{\perp}(x)\otimes H_1^{\perp}(z)\cos(2\phi) \\
&+ \sqrt{2\varepsilon(1+\varepsilon)}\frac{M}{Q}x^2\left(h(x)\otimes H_1^{\perp}(z) + f^{\perp}(x)\otimes D_1(z)\right)\cos\phi\,, \\
d\sigma_{L0} &\propto \sqrt{2\varepsilon(1-\varepsilon)}\frac{M}{Q}x^2\left(e(x)\otimes H_1^{\perp}(z) + g^{\perp}(x)\otimes D_1^{\perp}(z)\right)\sin\phi\,, \\
d\sigma_{0L} &\propto \varepsilon x h_{1L}^{\perp}(x)\otimes H_1^{\perp}(z)\sin(2\phi) \\
&+ \sqrt{2\varepsilon(1+\varepsilon)}\frac{M}{Q}x^2\left(h_L(x)\otimes H_1^{\perp}(z) + f_L^{\perp}(x)\otimes D_1(z)\right)\sin\phi\,,
\end{aligned}$$

$$\begin{aligned}
d\sigma_{LL} &\propto \sqrt{1-\varepsilon^2}\,x g_{1L}(x)\otimes D_1(z) \\
&+ \sqrt{2\varepsilon(1-\varepsilon)}\frac{M}{Q}x^2\left(g_L^{\perp}(x)\otimes D_1(z) + e_L(x)\otimes H_1^{\perp}(z)\right)\cos\phi\,, \\
d\sigma_{0T} &\propto \varepsilon\left\{x h_1(x)\otimes H_1^{\perp}(z)\sin(\phi+\phi_S) + x h_{1T}^{\perp}(x)\otimes H_1^{\perp}(z)\sin(3\phi-\phi_S)\right\} \\
&+ x f_{1T}^{\perp}(x)\otimes D_1(z)\sin(\phi-\phi_S), \\
d\sigma_{LT} &\propto \sqrt{1-\varepsilon^2}\,x g_{1T}(x)\otimes D_1(z)\cos(\phi-\phi_S)\,,
\end{aligned}$$

Basic twist-2 PDFs of the nucleon

- 8 intrinsic-transverse-momentum k_T dependent PDFs at leading twist
- Azimuthal asymmetries with different angular modulations in the hadron and spin azimuthal angles, Φ_h and Φ_s
- Vanish upon integration over k_T except f_1, g_1, and h_1



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Azimuthal asymmetry of charged hadrons

The azimuthal asymmetries of charged hadron production $a_{h^\pm}(\phi)$ are defined as follows:

$$a_{h^\pm}(\phi) = \frac{d\sigma^{\leftarrow\Rightarrow} - d\sigma^{\leftarrow\Leftarrow}}{|P_L|(d\sigma^{\leftarrow\Rightarrow} + d\sigma^{\leftarrow\Leftarrow})},$$

The azimuthal asymmetries are related to transverse-momentum-dependent Parton Distribution Functions (PDF) and polarized and nonpolarized Parton Fragmentation Functions (PFF). They can depend on the transverse or longitudinal component of the quark spin.

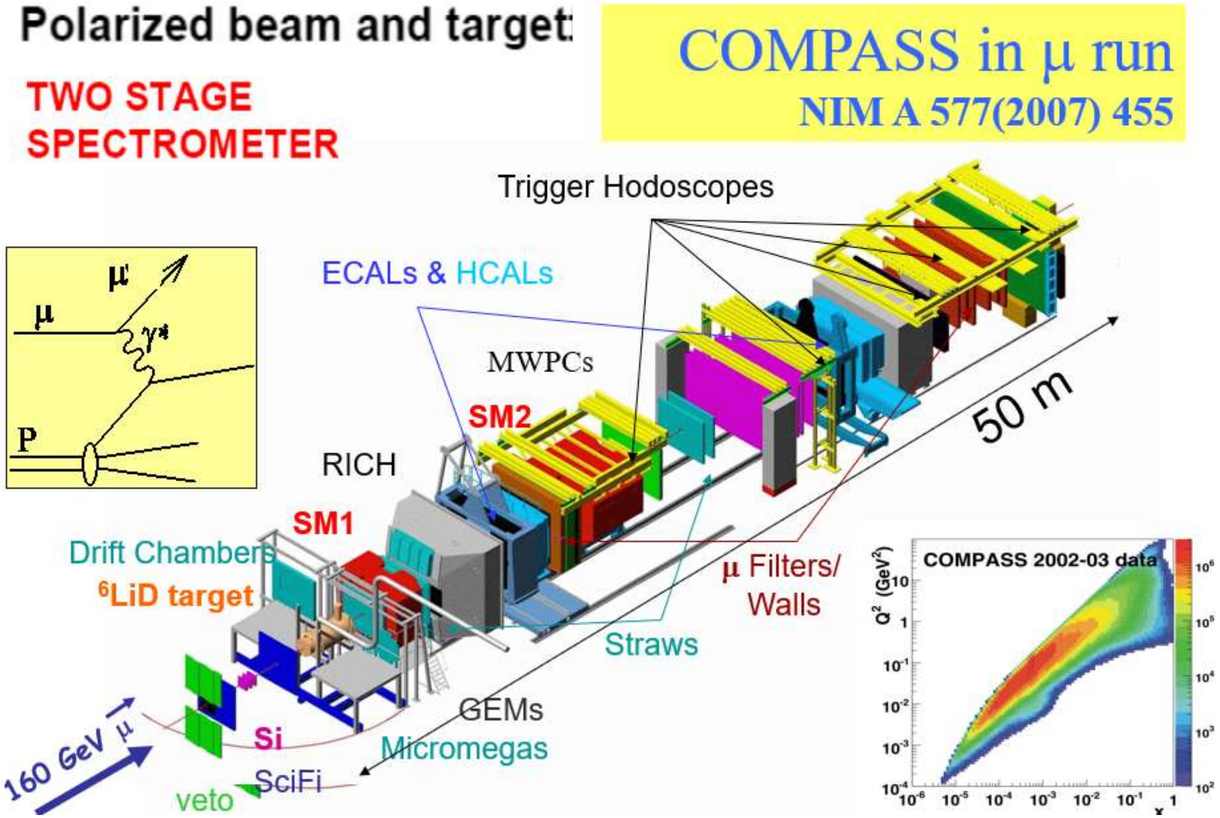
$$a_{h^\pm}(\phi) = a^{0}_{h^\pm} + a^{\sin\phi}_{h^\pm}\sin\phi + a^{\sin 2\phi}_{h^\pm}\sin 2\phi + a^{\sin 3\phi}_{h^\pm}\sin 3\phi + a^{\cos\phi}_{h^\pm}\cos\phi.$$

The sign and value of each contribution are subjects of the $a_{h\pm}(\phi)$ data analysis as functions of x, z and $\boldsymbol{p}^h{}_T$.

None of these modulations, with the exception of $a_{h\pm}{}^{0}$ and $a_{h\pm}{}^{\sin 2\phi,}$ which are connected to the twist-2 helicity PDF g_{1L} and the worm-gear-L PDF $h^{\perp}{}_{1L}$, respectively, were definitely observed

Experimental set-up: spectrometer

The COMPASS set-up is a two-stage forward spectrometer with the world's largest polarized target and various types of tracking and particle identification detectors (PID) in front and behind of two large aperture magnets SM1 and SM2. The spectrometer was operated in the high energy (160 GeV) muon beam at CERN.



Experimental set-up: target

The major modifications in 2006 were as follows: (i) the replacement of the two 60 cm long target cells by three cell of lengths 30 cm, 60 cm and 30 cm, (ii) the replacement of the target solenoid magnet by the new one with a wider aperture and (iii) the installation of the electro-magnetic calorimeter ECAL1 in front of the hadron calorimeter HCAL1. These modifications aimed to further minimizing systematic uncertainties, to increasing an acceptance of the spectrometer and improving its e/γ identification capabilities. From another side that has required reconsidering methods of data stability tests and **asymmetry calculations.**

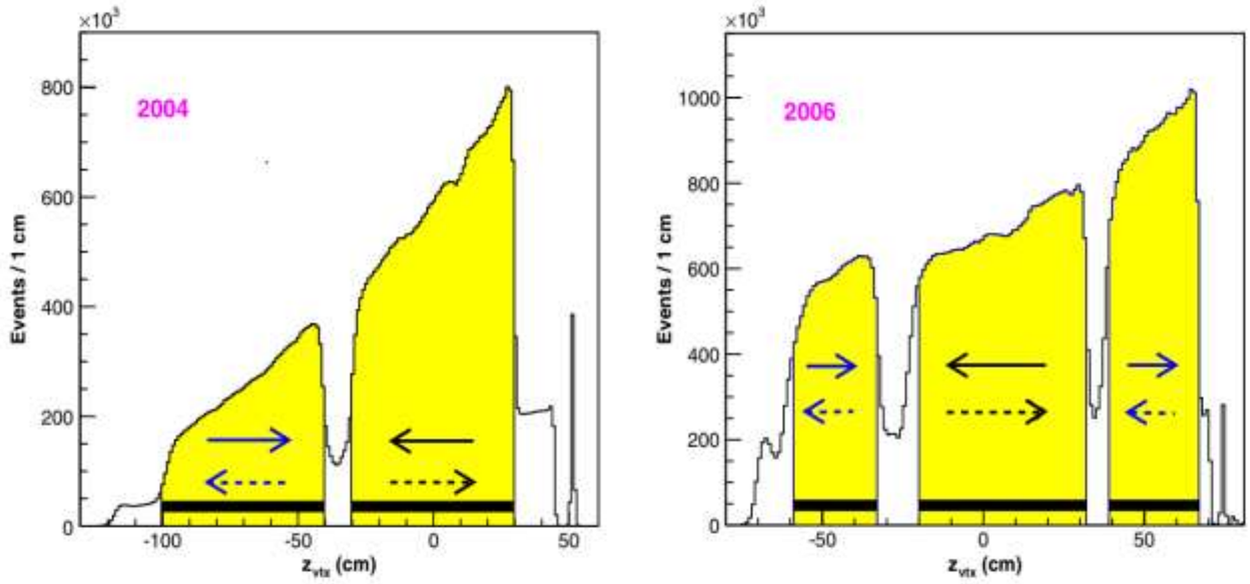


FIG. 2 (color online). Distribution of reconstructed vertex positions z_{vtx} along the beam axis for the target with two (left) and three (right) cells. Dark horizontal bars at the bottom mark the target fiducial regions, arrows denote the target polarization directions. See text for details.

Double ratios (DR)

For the azimuthal-asymmetry studies double ratios R_f of event numbers are used

$$R_f(\phi) = \left[N^U_{+,f}(\phi)/N^D_{-,f}(\phi)\right] \times \left[N^D_{+,f}(\phi)/N^U_{-,f}(\phi)\right] \qquad (6)$$

where $N^t_{p,f}(\phi)$ is the number of events in a given ϕ bin originating from the target cell t ($t = U, D$) with

the polarisation orientation p ($p = +, -$) and the solenoid field orientation f ($f = +, -$) w.r.t. to the beam direction.

Using Eqs. (3, 5) the number of events can be expressed as

$$\begin{aligned} N^t_{p,f} = C^t_f(\phi) L^t_{p,f} \Big[&(B_0 + B_1 \cos\phi + B_2 \cos 2\phi + B_3 \sin\phi + \ldots) \\ &\pm P^t_{p,f}(A_0 + A_1 \sin\phi + A_2 \sin(2\phi) + \ldots) \Big] \end{aligned} \qquad (7)$$

where $C^t_f(\phi)$ is the acceptance factor, $L^t_{p,f}$ is the luminosity, and $P^t_{p,f}$ is the absolute value of the averaged product of the measured positive or negative target polarisation and the dilution factor[2] calculated for the cell t [6]. The coefficients $B_0, B_1, \ldots$ and $A_0, A_1, \ldots$ characterise the target-spin-independent and the target-spin-dependent parts of partial cross-sections contributing to the denominator and numerator in Eq. (4), respectively.

Substituting Eq. (7) into Eq. (6), one can see that the multiplicative acceptance factors cancel out as well as the luminosity factors if the beam muons cross both cells. The ratios $R_f(\phi)$ thus depend only on the asymmetry $a(\phi)$ (Eq. (4)), which can be expressed (to first order) by

$$a_f(\phi) = \left[R_f(\phi) - 1\right] / (P^U_{+,f} + P^D_{+,f} + P^U_{-,f} + P^D_{-,f}). \qquad (8)$$

Modified double ratios (MDR)

The modified "acceptance-cancelling" double ratio method was used to calculate the ratios of the SIDIS cross sections for positive and negative target polarisations denoted as σ_+/σ_- and the 2006 asymmetries.

For the three-cell target the method was modified as follows. The target cell M was artificially divided in two sub-cells $M1$ and $M2$, each 28 cm long, and two pairs of cells (U and $M1$) and ($M2$ and D) are considered below. The cells in each pair have equal lengths, i.e. equal densities of deuterons, but opposite polarisations p ($+$ or $-$) at a given solenoid field direction f ($+$ or $-$). For each pair of cells at a given f, one can construct the double ratio using the number of selected SIDIS events or hadrons. These numbers obtained from the cell i and denoted as N^i_{pf} are usually expressed via a product of a cell luminosity (L^i_f) given by the beam intensity times the target cell material density a target cell acceptance (A^i_f) and the corresponding cross section (σ_p): $N^i_{pf} = L^i_f \times A^i_f \times \sigma_p$, i.e. luminosity, acceptance and cross section are folded in the number of events. Taking this relation into account as well as the COMPASS procedure of measurements divided in two groups of runs G1 and G2, one can construct for each pair of the target cells the "acceptance-cancelling" double ratio of events (hadrons) that provides a way to unfold the $(\sigma_+/\sigma_-)^2$. Particularly, for the polarisation settings at $f = +$, the two double ratios of event numbers constructed for the $(U,M1)$ and for the $(M2,D)$ pairs have the forms given by Eqs. (5), where events for the first (second) ratio of each pair are taken from the G1 (G2) runs.

$$\left[\frac{N^U_{++}}{N^{M1}_{-+}}\right]_{\mathrm{G1}} \times \left[\frac{N^{M1}_{++}}{N^U_{-+}}\right]_{\mathrm{G2}} = \left(\frac{\sigma_+}{\sigma_-}\right)^2_1, \qquad \left[\frac{N^D_{++}}{N^{M2}_{-+}}\right]_{\mathrm{G1}} \times \left[\frac{N^{M2}_{++}}{N^D_{-+}}\right]_{\mathrm{G2}} = \left(\frac{\sigma_+}{\sigma_-}\right)^2_2. \qquad (5)$$

$$\left[\frac{N^U_{+-}}{N^{M1}_{--}}\right]_{\mathrm{G1}} \times \left[\frac{N^{M1}_{+-}}{N^U_{--}}\right]_{\mathrm{G2}} = \left(\frac{\sigma_+}{\sigma_-}\right)^2_3, \qquad \left[\frac{N^D_{+-}}{N^{M2}_{--}}\right]_{\mathrm{G1}} \times \left[\frac{N^{M2}_{+-}}{N^D_{--}}\right]_{\mathrm{G2}} = \left(\frac{\sigma_+}{\sigma_-}\right)^2_4. \qquad (6)$$

MDR and the asymmetries $a_{h^\pm}(\phi)$

For the extraction of the azimuthal asymmetries $a_{h^\pm}(\phi)$ off the cross section ratios, the distributions of the charged hadrons h^+ and h^- were separately analysed as a function of the azimuthal angle ϕ in the region from $-180°$ to $+180°$ divided into 10 ϕ-bins. For both h^+ and h^-, the double ratios of the hadrons defined by Eqs. (5, 6), $(\sigma_+/\sigma_-)^2_k(\phi)$, $k = 1, ...4$, were calculated and combined as follows:

$$\left(\frac{\sigma_+}{\sigma_-}\right)^2_{h^\pm}(\phi)=\left[\left(\frac{\sigma_+}{\sigma_-}\right)^2_1\oplus\left(\frac{\sigma_+}{\sigma_-}\right)^2_2\oplus\left(\frac{\sigma_+}{\sigma_-}\right)^2_3\oplus\left(\frac{\sigma_+}{\sigma_-}\right)^2_4\right]_{h^\pm}(\phi)\cong 1+a_{h^\pm}(\phi)\sum_k\left[\sum_{i,f,p\in k}\mathscr{P}^i_{pf}(x)\right]_{h^\pm}\cdot W_k\,, \qquad (7)$$

where the symbol $\oplus$ means statistically weighted averaging. As it was shown in Ref. [2], in first approximation, the squared ratios of cross sections $(\sigma_+/\sigma_-)^2_{h^\pm}(\phi)$ are related to the asymmetries $a_{h^\pm}(\phi)$ multiplied by polarisation terms. For each hadron charge, the polarisation term is given by the sum of the $\mathscr{P}^i_{pf}(x)$ values, each of them being the product of target cell polarisations $|P^i_{pf}|$ and dilution factor $f^i(x)$, as defined in Refs. [2, 3], where i, p and f are those used to calculate the ratio $(\sigma_+/\sigma_-)^2_k(\phi)$, i.e. four polarisation values at each k. The weight W_k is equal to the ratio of the number of hadrons, N_k, to the total number of hadrons, $N_{\rm tot}$. Therefore, the $a_{h^\pm}(\phi)$, referred to as single-hadron asymmetries, are:

$$a_{h^\pm}(\phi)\cong\frac{(\frac{\sigma_+}{\sigma_-})^2_{h^\pm}(\phi)-1}{\sum_k[\sum_{i,f,p\in k}\mathscr{P}^i_{pf}(x)]_{h^\pm}\cdot W_k}. \qquad (8)$$

The combined data of 2002, 2003, 2004 and 2006 years

Events detected by the COMPASS spectrometer were conserved for the analysis under condition $Q^2 > 1\ (GeV/c)^2$.

The statistics of 2002-2004 was about $96 \cdot 10^6$ events.

The statistics of 2006 was about $64 \cdot 10^6$ events.

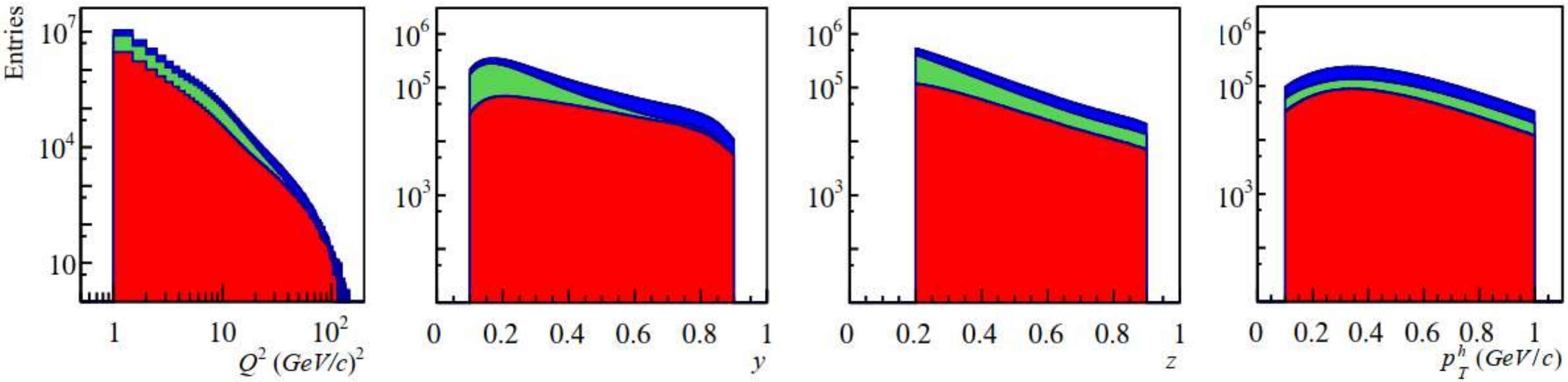


Fig. 3: Kinematic distributions of selected SIDIS events vs. Q^2 and y and of charged hadrons vs. z and p_T^h within the region shown in Table 1: 2006 (lower, red), 2002 – 2004 (middle, green) and 2002 – 2006 (upper, blue).

Integrated asymmetries

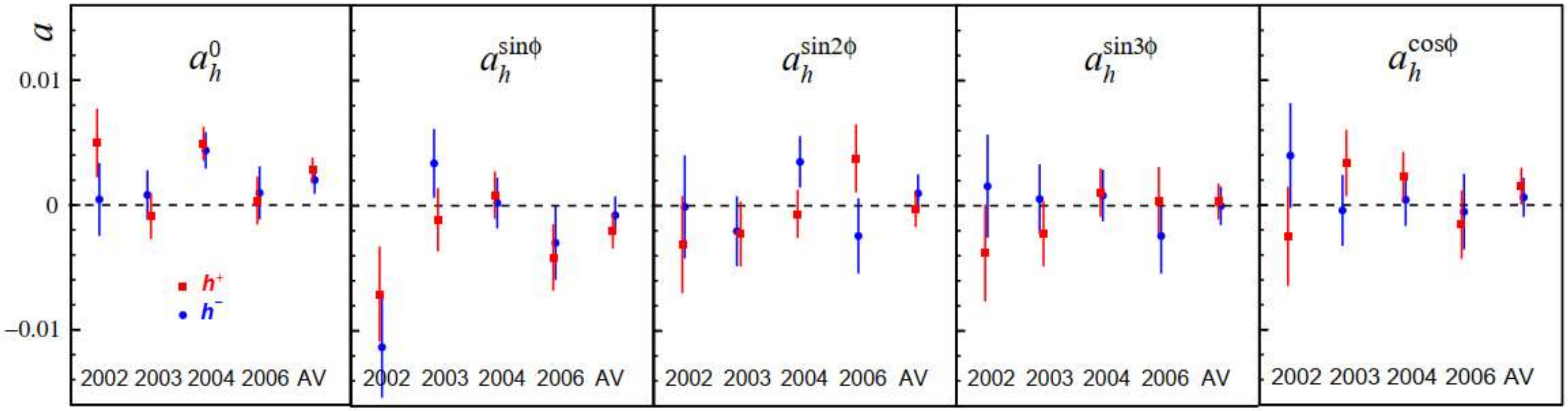


Fig. 5: The values of modulation amplitudes a together with their uncertainties obtained from the fits of the integrated asymmetries $a_{h^\pm}(\phi)$ by the function from Eq. (4) separately for the data of 2002, 2003, 2004 and 2006 as well as statistically combined modulation amplitudes for four years denoted by AV (see Section 4.1).

$$a_{h^\pm}(\phi) = a^0_{h^\pm} + a^{\sin\phi}_{h^\pm}\sin\phi + a^{\sin 2\phi}_{h^\pm}\sin 2\phi + a^{\sin 3\phi}_{h^\pm}\sin 3\phi + a^{\cos\phi}_{h^\pm}\cos\phi. \quad (4)$$

The asymmetries were fitted by the function from Eq. (4) using the standard least-square-method and extracting all asymmetry modulation amplitudes simultaneously

Asymmetries as functions of kinematic variables

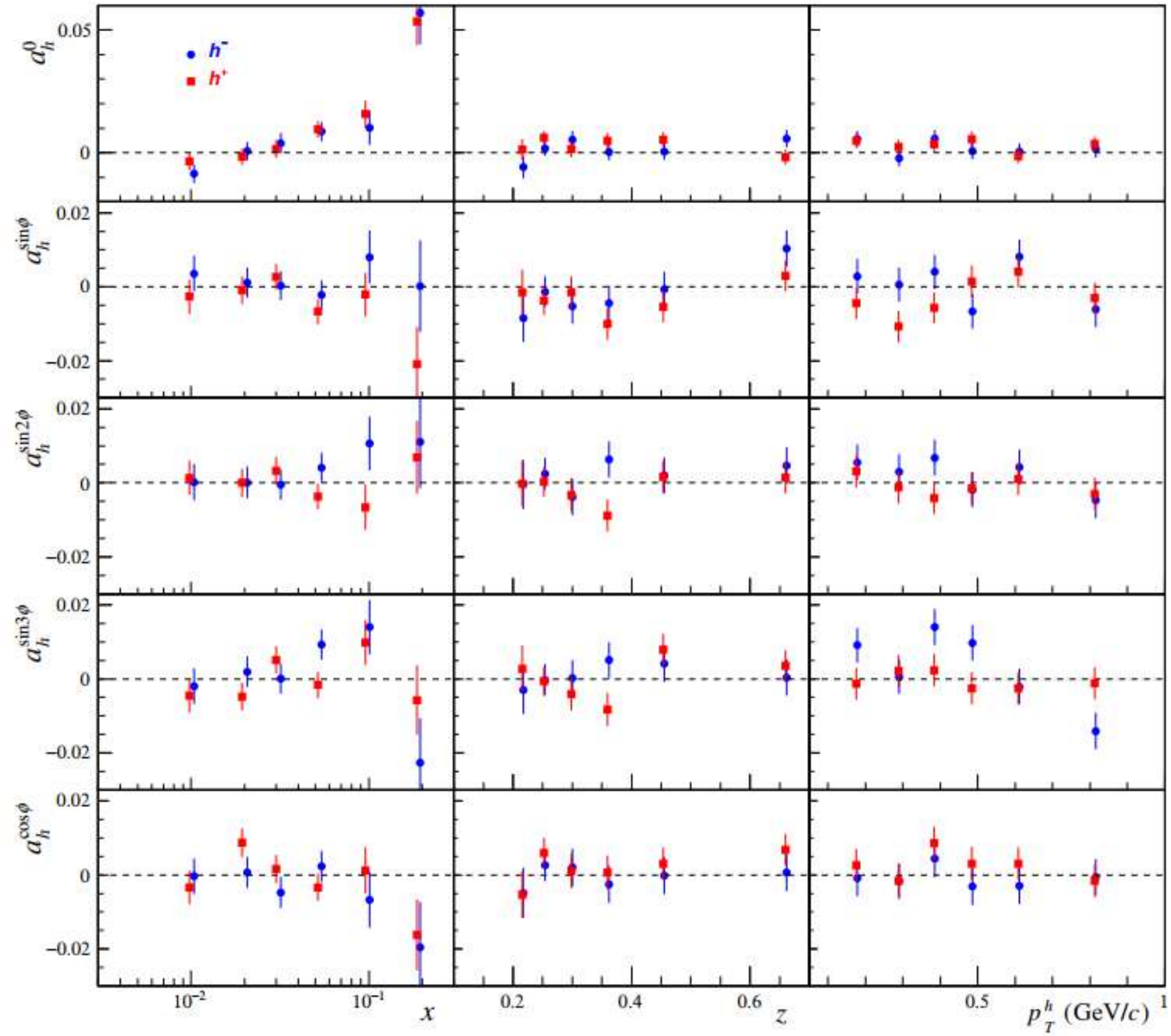


Fig. 6: The modulation amplitudes a of the h^+ and h^- azimuthal asymmetries as the function of x , z and p_T^h obtained from the combined 2002 – 2006 data on the muon SIDIS off longitudinally polarised deuterons. Only uncertainties of fits are shown.

Comparison of the hadrons symmetries as functions X

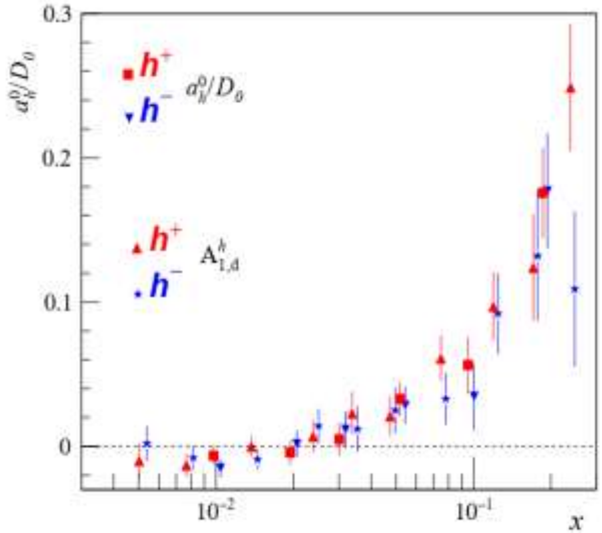
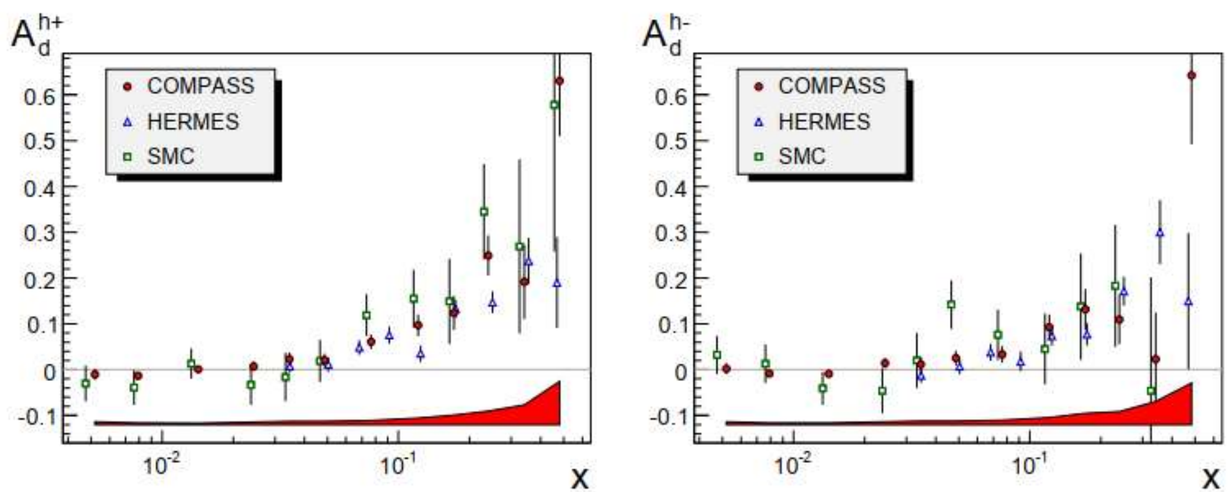


Fig. 7: The x-dependence of the values $a^0_{h^\pm}(x)/D_0(x,y)$ for 2002 – 2006 data compared to the data of Ref. [14] on the asymmetries $A^{h^\pm}_{1d}(x)$.



Conclusion (results)

<u>Altogether, contributions of TMD PDFs convoluted with PFFs to the azimuthal asymmetries in the SIDIS cross sections of hadron production by muons off longitudinally polarized deuterons are small.</u>

This could happened either due to possible cancellations of the opposite contributions to the cross section from the deuteron up and down quarks, or/and due to the smallness of the transverse component of the longitudinal target polarization and of the M/Q suppression factor.

The first assumption can be checked by analyzing the azimuthal asymmetries in the cross sections of hadron production off longitudinally polarized protons.

Conclusion (MDR)

In conclusions, the SIDIS azimuthal angle asymmetries in the hadron productions off the longitudinally polarized deuterons which are free of possible false contributions were measured using the Modified Double Ratios of numbers of the hadrons. Sources of possible false contributions to these asymmetries and their absence while using the MDRs were studied and described in this Paper. Advantages of such measurements are as following.

(1). For obtaining results on the muon SIDIS azimuthal angle asymmetries off polarized targets using the MDRs it does not required the knowledge of the set-up luminosity and acceptance, the value of the first of which is usually measured separately and the second one is calculated by MC simulations.

(2). The results on the SIDIS azimuthal angle asymmetries obtained with the MDRs have smaller statistical and systematic errors due to the absence of contributions to them from additional measurements and from the events generated by the MC simulations.

(3). For the analysis of the SIDIS azimuthal angle asymmetries from the longitudinally polarized deuterons, the MDRs of numbers of the hadrons provide measures to estimate false contributions to the SIDIS angle asymmetries related with the non perfect account of the set-up in the events reconstruction algorithm.

Acknowledgements

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