Quantum model of spinning black holes.

Quantum model of electron.

__V.P.Neznamov__, S.Yu.Sedov, V.E.Shemarulin

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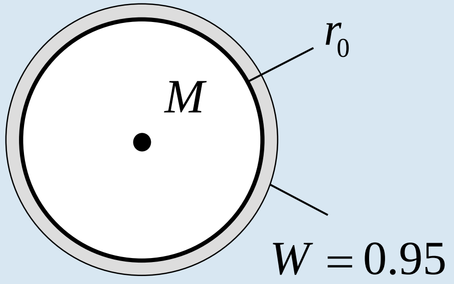
We propose a quantum model of spinning black holes. For the charged Kerr-Newman quantum metric, the complete regularization in the domain of the ring singularity $(\cos\theta = 0, r = 0)$ occurs at fixing of the maximal (cut-off) energy of gravitons $k_{UV}^{reg} = \hbar c / R_S^{reg}$.

The domains of existence of one, two and several events horizons are presented depending on parameters of the Kerr and Kerr-Newman modified metrics.

On the basis of the regular quantum Kerr-Newman metric, we propose the quantum model of the extended electron with zero self-energy.

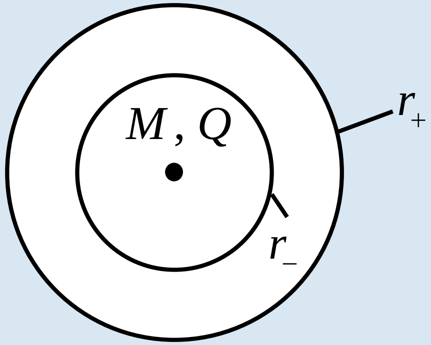
We will consider next classical black holes.

Schwarzschild black hole

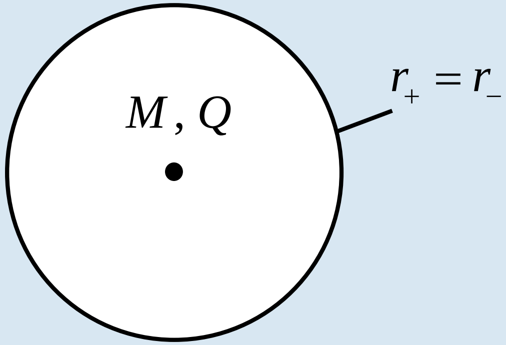


Reissner-Nordström black hole

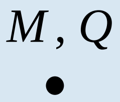
a) two event horizons



b) extreme field

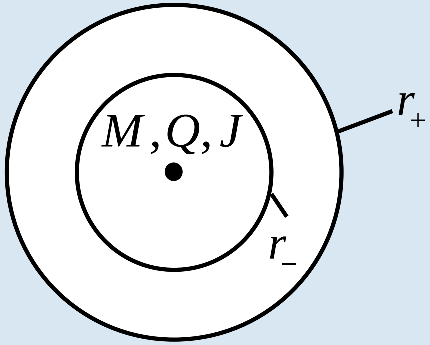


c) naked singularity

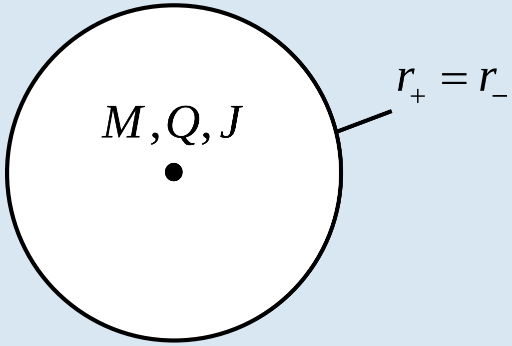


Kerr and Kerr-Newman black hole

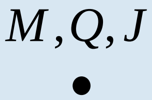
a) two event horizons



b) extreme field



c) naked singularity



Introduction

In papers [1], [2], the quantum description of black holes is presented for modified Schwarzschild (S) and Reissner-Nordström (RN) geometries. Black holes contain a quantum core described by coherent states of gravitons. Since coherent states cannot contain states with arbitrary short wave lengths of gravitons, classical central singularities become integrable singularities with finite tidal forces. In papers [1], [2], the short wave lengths are removed by cut-off of gravitation's momenta (energies). As a result, in theory, the maximum momentum (maximum energy) of gravitons appears $k_{UV} = \hbar / R_S$. Here R_S is the radius of quantum core, $c = 1$.

In the future quantum energy of gravitation, the energy cut-off will be substituted by strict integration and the absence of short wave lengths of gravitons in coherent states will be a natural result of applying a more perfect quantum theory.

[1] R. Casadio, Int. J. Mod. Phys. D 31, 2250128 (2022); arxiv: 2103.00183v4 (gr-qc).

[2] R. Casadio, A. Giusti and J. Ovalle, Phys. Rev. D 105, 124026 (2022); arxiv: 2203.03252v2 (gr-qc).

Introduction

The aim of our effort is to extend the approach of papers [1], [2] to the modified Kerr (K) and Kerr-Newman (KN) geometries describing quantum spinning black holes. Here, this notation includes either spinning black holes with quantum cores and with event horizons or spinning quantum cores without event horizons.

[1] R. Casadio, Int. J. Mod. Phys. D 31, 2250128 (2022); arxiv: 2103.00183v4 (gr-qc).
[2] R. Casadio, A. Giusti and J. Ovalle, Phys. Rev. D 105, 124026 (2022); arxiv: 2203.03252v2 (gr-qc).

Kerr and Kerr-Newman quantum space-time

We will use the elementary extension of the classical metric K (Cürses-Cürsey metric [1])

$$ds^2 = \left(1 - \frac{2r\,m(r)}{\rho^2}\right)dt^2 + \frac{4ar\,m(r)\sin^2\theta}{\rho^2}dt\,d\varphi - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \frac{\Sigma\sin^2\theta}{\rho^2}d\varphi^2.$$

Here $m(r)$ is the mass function

$$\rho^2 = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2r\,m(r) + a^2,$$

$$\Sigma = \left(r^2 + a^2\right)^2 - a^2\Delta\sin^2\theta, \qquad a = J/M.$$

The classical Kerr metric describes the geometry of an uncharged spinning black hole with a point mass *M* and an angular momentum *J*.

[1] M. Cürses and F. Cürsey, J. Math. Phys. 16, 2385(1975).

Kerr and Kerr-Newman quantum space-time

The components of the Einstein tensor* for the Kerr metric can be presented as in [1]:

$$G_0{}^0 = 2\frac{r^4 + \left(\rho^2 - r^2\right)^2 + a^2\left(2r^2 - \rho^2\right)}{\rho^6} m' - \frac{ra^2 \sin^2\theta}{\rho^4} m'', \qquad G_1{}^1 = 2\frac{r^2}{\rho^4} m',$$

$$G_2{}^2 = 2\frac{\rho^2 - r^2}{\rho^4} m' + \frac{r}{\rho^2} m'', \qquad G_3{}^3 = 2\frac{2r^2\left(\rho^2 - r^2\right) + a^2\left(\rho^2 - 2r^2\right)}{\rho^6} m' + \frac{r\left(a^2 + r^2\right)}{\rho^4} m'',$$

$$G_0{}^3 = 2\frac{a\left(2r^2 - \rho^2\right)}{\rho^6} m' - \frac{ar}{\rho^4} m'', \qquad m' = \frac{dm}{dr}, \quad m'' = \frac{d^2 m}{dr^2}.$$

The important observation (see [4]) is the fact that the Einstein tensor is linear in derivatives of a mass function $m(r)$. Any linear decomposition of the mass function $m = m_1(r) + m_2(r)$ leads to the linear decomposition of the Einstein tensor

$$G_\beta{}^\alpha(m,a) = G_\beta{}^\alpha(m_1,a) + G_\beta{}^\alpha(m_2,a).$$

*) $G_\mu{}^\nu = R_\mu{}^\nu - \delta_\mu^\nu \frac{1}{2} R = \frac{8\pi G}{c^4} T_\mu{}^\nu$ is the Einstein equation

[1] E. Conteras, J. Ovalle, R. Casadio, Phys. Rev. D 103, 044020 (2021); arxiv: 2101.08569v1 (gr-qc).

Kerr and Kerr-Newman quantum space-time

In case of $J=0, Q=0$, the Kerr metric turns into a classical Schwarzschild metric with

$$m_S = GM.$$

In case of $J=0, Q\neq 0$, the Kerr metric turns into a classical Reissner-Nordström metrics with

$$m_{RN}(r) = GM - GQ^2/2r.$$

ATTENTION: For the classical Kerr and Kerr-Newman metrics, the mass functions are not changed

$$m_K = m_S = GM, \qquad m_{KN} = m_{RN} = GM - GQ^2/2r.$$

By substitution of the mass functions m_K and m_{KN} into the Cürses-Cürsey metric, we obtain classical Kerr and Kerr-Newman metrics in Boyer-Lindquist coordinates [1]. Hence, it follows that the rotation does not change mass functions $m(r)$.

The second property of the mass function is its spherical symmetry. The mass function does not depend on θ,φ angles.

[1] R. H. Boyer, R. W. Lindquist, J. Math. Phys. 8, 265 (1967).

Kerr and Kerr-Newman quantum space-time

The above properties of the mass function allow us to fully use the mathematical apparatus for the quantum K and KN metrics, previously used for the S and RN metrics in papers [1], [2]. Let us note that in papers [1], [2], the classical potentials are

$$V_S(r) = \frac{m_S}{r}, \qquad V_{RN}(r) = \frac{m_{RN}}{r}.$$

Then, for the quantum case, the Кerr metric is corrected by the following substitutions.

The Kq metric $(Q = 0)$:

$$m_{Kq} = m_{Sq} = GM\frac{2}{\pi}\mathrm{Si}\left(\frac{r}{R_S}\right).$$

The KNq metric $(Q \neq 0)$:

$$m_{KNq} = m_{RNq} = GM\frac{2}{\pi}\mathrm{Si}\left(\frac{r}{R_S}\right) - \frac{GQ^2}{2r}\left(1 - \cos\left(\frac{r}{R_S}\right)\right).$$

[1] R. Casadio, Int. J. Mod. Phys. D 31, 2250128 (2022); arxiv: 2103.00183v4 (gr-qc).

[2] R. Casadio, A. Giusti and J. Ovalle, Phys. Rev. D 105, 124026 (2022); arxiv: 2203.03252v2 (gr-qc).

Space-time structure of the Kerr and Kerr-Newman geometries

Let us consider the metric of the classical and quantum K and KN geometries in the Kerr-Schild coordinates

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - \left(r^4 + a^2 z^2\right)^{-1} 2r^3 m(r) \times$$
$$\times \left\{ \left(r^2 + a^2\right)^{-1} \left[r\left(xdx + ydy\right) - a\left(xdy - ydx\right) \right] + r^{-1} zdz + dt \right\}^2 .$$

For the classical KN metric $m_{KN} = 0$ at $r = R^{cl}/2 = Q^2/2M$ and above metric becomes flat. In this case, the external KN solution is joined with the internal flat space-time metric (see, for example, [1]).

For the quantum K and KN metrics, the mass functions $m_q(r)$ in the range $r \in (0, \infty)$ are not zero. I. e., for the quantum K and KN metrics in the range $r \in (0, \infty)$, there is a curved space-time everywhere.

[1] C. A. Lopez, Phys. Rev. D 30, 313 (1984).

Effective energy-momentum tensor

Since the quantum Kerr metric with mass functions is already no longer a vacuum solution of GR equations, there must exist non-zero diagonal components of the energy-momentum tensor

$$T_{\mu}^{\ \nu} = \frac{G_{\mu}^{\ \nu}}{8\pi G} = \mathrm{diag}\left(\rho_{\varepsilon}, -p_{r}, -p_{\theta}, -p_{\varphi}\right).$$

Effective energy-momentum tensor

For quantum KN metric, the total energy determined by the volume integral of energy density is $T_0^{\ 0} \equiv \rho_\varepsilon(r,\theta)$

$$E = \int T_0^{\ 0}\sqrt{-g}\,dV = \frac{1}{4G}\int_0^\infty dr\int_{-1}^{1} d\mu\left(r^2+a^2\mu^2\right)G_0^{\ 0}(r,\mu) =$$

$$= \frac{1}{4G}\int_0^\infty dr\int_{-1}^{1} d\mu\left[2\frac{r^4+\left(\rho^2-r^2\right)^2+a^2\left(2r^2-\rho^2\right)}{\rho^4}m'_{KN_q} - \frac{ra^2\left(1-\mu^2\right)}{\rho^2}m''_{KN_q}\right] =$$

$$= \frac{1}{4G}\int_0^\infty dr\left\{\left[8-4\frac{r}{a}\operatorname{arctg}\frac{a}{r}\right]\left(GM\frac{2}{\pi}\frac{\sin(r/R_S)}{r}+\frac{CQ^2}{2r^2}\left(1-\cos\left(\frac{r}{R_S}\right)\right)-\frac{CQ^2}{2rR_S}\sin\left(\frac{r}{R_S}\right)\right)+\right.$$

$$+\left[2r-2\frac{r^2}{a}\operatorname{arctg}\frac{a}{r}-2a\operatorname{arctg}\frac{a}{r}\right]\left(GM\frac{2}{\pi}\frac{\cos(r/R_S)}{r/R_S}\frac{1}{R_S^2}-GM\frac{2}{\pi}\frac{\sin(r/R_S)}{(r/R_S)^2}\frac{1}{R_S^2}-\right.$$

$$\left.\left.-\frac{CQ^2}{r^3}\left(1-\cos\left(\frac{r}{R_S}\right)\right)+\frac{CQ^2}{r^2R_S}\sin\left(\frac{r}{R_S}\right)-\frac{CQ^2}{2rR_S^2}\cos\left(\frac{r}{R_S}\right)\right)\right\} = M+\frac{1}{2}\frac{|J|}{R_S}-\frac{\pi}{16}\frac{Q^2|J|}{M}\frac{1}{R_S^2} = M+\frac{1}{2}\frac{|J|}{\hbar}k_{UV}-\frac{\pi}{16}\frac{Q^2|J|}{M}\frac{k_{UV}^2}{\hbar^2}.$$

For K and KN metrics

$$\sqrt{-g}=\rho^2\sin\theta;\ \rho^2=r^2+a^2\mu^2,\ \mu=\cos\theta;\ m'\equiv\frac{dm}{dr},\ m''\equiv\frac{d^2m}{dr^2},\ m_{KNq}=GM\frac{2}{\pi}\mathrm{Si}\left(\frac{r}{R_S}\right)-\frac{GQ^2}{2r}\left(1-\cos\left(\frac{r}{R_S}\right)\right).$$

Effective energy-momentum tensor

$$E = M + \frac{1}{2}\frac{|J|}{R_S} - \frac{\pi}{16}\frac{Q^2|J|}{M}\frac{1}{R_S^2} = M + \frac{1}{2}\frac{|J|}{\hbar}k_{UV} - \frac{\pi}{16}\frac{Q^2|J|}{M}\frac{k_{UV}^2}{\hbar^2}.$$

The last equality was obtained by using the relation of $R_S = \hbar / k_{UV}$, where k_{UV} is the maximum (cut-off) energy of a graviton.

The total energy is finite. For K $(Q = 0)$ and KN $(Q \neq 0)$ metrics there is the summand proportional to modulus of an angular momentum $|J|$ and to maximum energy of graviton k_{UV}.

For KN metric there is also a summand $\sim \frac{Q^2|J|}{M}k_{UV}^2$.

In the absence of cut-off $(k_{UV}) \to \infty$, the total energy of a spinning black hole would be infinite, obviously non-physical value.

Effective energy-momentum tensor

For quantum S [1] and RN [2] metrics, the total energies of black holes are independent on R_S and are equal to $E = M$.

For the quantum KN metric, we can obtain the similar result if we assume that radius R_S is equal to

$$R_S^{reg} = \frac{\pi}{8}\frac{Q^2}{M}.$$

Let us consider the behavior of the quantum mass function $m_{KNq}(r)$ and its first and second derivatives in the neighbourhood of $r = 0$.

[1] R. Casadio, Int. J. Mod. Phys. D 31, 2250128 (2022); arxiv: 2103.00183v4 (gr-qc).

[2] R. Casadio, A. Giusti and J. Ovalle, Phys. Rev. D 105, 124026 (2022); arxiv: 2203.03252v2 (gr-qc).

Effective energy-momentum tensor

Table 1. Dependencies $m_{KNq}(r), m'_{KNq}(r), m''_{KNq}(r)$ at $r \to 0$.

	$R_S \neq R_S^{reg}$	$R_S = R_S^{reg}$
$m_{KNq}\big\|_{r\to 0}$	$\left(GM\frac{2}{\pi} - \frac{GQ^2}{4R_S}\right)\left(\frac{r}{R_S}\right)$	$\frac{1}{18}\frac{GM}{\pi}\left(\frac{r}{R_S^{reg}}\right)^3$
$m'_{KNq}\big\|_{r\to 0}$	$\left(GM\frac{2}{\pi} - \frac{GQ^2}{4R_S}\right)\left(\frac{1}{R_S}\right)$	$\frac{1}{6}\frac{GM}{\pi R_S^{reg}}\left(\frac{r}{R_S^{reg}}\right)^2$
$m''_{KNq}\big\|_{r\to 0}$	$\left(-\frac{2}{3}\frac{GM}{\pi R_S^2} + \frac{GQ^2}{8R_S^3}\right)\left(\frac{r}{R_S}\right)$	$\frac{1}{3}\frac{GM}{\pi\left(R_S^{reg}\right)^2}\left(\frac{r}{R_S^{reg}}\right)$

We see that at $R_S \neq R_S^{reg}$, the mass function $m_{KNq}(r) \sim r$, while at $R_S = R_S^{reg}$ $m_{KNq}(r) \sim r^3$.

In the first case, the singularities of the components $T_\mu^{\ \nu}$ are integrable. In the second case, the components $T_\mu^{\ \nu}$ are nonsingular.

Effective energy-momentum tensor

Really, at $R_S = R_S^{reg}$, the diagonal components of the energy-momentum tensor at $r \to 0$ are equal to

$$T_0{}^0\big|_{r\to 0} = \begin{cases} \sim\left(-r^2/\mu^2\right) & \text{at } \mu \neq 0, \\ \text{const} > 0 & \text{at } \mu = 0, \end{cases} \qquad T_1{}^1\big|_{r\to 0} = \begin{cases} \sim r^4/\mu^4 & \text{at } \mu \neq 0, \\ \text{const } 1 > 0 & \text{at } \mu = 0, \end{cases}$$

$$T_2{}^2\big|_{r\to 0} = \begin{cases} \sim r^2/\mu^2 & \text{at } \mu \neq 0, \\ \text{const } 2 > 0 & \text{at } \mu = 0, \end{cases} \qquad T_3{}^3\big|_{r\to 0} = \begin{cases} \sim r^2/\mu^2 & \text{at } \mu \neq 0, \\ \text{const } 3 > 0 & \text{at } \mu = 0. \end{cases}$$

In the Boyer-Lindquist coordinates [1], the point of $r = 0, \mu = 0$ is a ring singularity of the classical KN metric. For all components $T_\mu{}^\nu$, the integration over volume in the domain of $r = 0$ leads to finite values.

[1] R.H. Boyer, R. W. Lindquist, J. Math. Phys. 8, 265 (1967).

Kretschmann scalar

Let us consider the behavior of the Kretschmann scalar at $r \to 0$. For briefly, let us introduce the denomination of $K = R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}$.

In our case at $R_S = R_S^{reg}$

$$K\big|_{r\to 0} = \begin{cases} \sim r^4/\mu^4 & \text{at } \mu \neq 0, \\ \text{const}_K \neq 0 & \text{at } \mu = 0. \end{cases}$$

For the classical KN metric

$$K_{cl}\big|_{r\to 0} = \begin{cases} \sim 1/\mu^8 & \text{at } \mu \neq 0, \\ \sim 1/r^8 & \text{at } \mu = 0. \end{cases}$$

For the quantum KN metric with $R_S \neq R_S^{reg}$

$$K_q\big|_{r\to 0} = \begin{cases} \sim 1/\mu^4 & \text{at } \mu \neq 0, \\ \sim 1/r^4 & \text{at } \mu = 0. \end{cases}$$

Thus, the fixation of $R_S = R_S^{reg}$ leads in the neighborhood of $r = 0$, to finite values of the Kretschmann scalar, including the domain of the ring singularity of the classical KN metric $(\mu = 0, r = 0)$.

Event horizons of spinning quantum black holes

For the KN metric, let us introduce the denotation of $\Delta = r^2 f_{KN}$.

For the classical KN metric $m_{KN}(r) = GM - \frac{GQ^2}{2r}$. Then $f_{KN} = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} + \frac{a^2}{r^2}$.

Equality $f_{KN} = 0$ determines the radii of external and inner event horizons

$$R_{\pm} = GM \pm \sqrt{G^2M^2 - GQ^2 - a^2}.$$

If $GM^2 > Q^2 + a^2$, then $f_{KN} = \left(1 - \frac{R_+}{r}\right)\left(1 - \frac{R_-}{r}\right)$.

The case of $GM^2 = Q^2 + a^2$ corresponds to the extreme metric with the single event horizon

$$R_{\pm} = GM.$$

The case of $GM^2 < Q^2 + a^2$ corresponds to the naked singularity without event horizons. In this case, $f_{KN} > 0$.

Event horizons of spinning quantum black holes

For the quantum KN metric, a mass function m_{KNq} is determined as

$$m_{KNq} = m_{RNq} = GM\,\frac{2}{\pi}\mathrm{Si}\left(\frac{r}{R_S}\right) - \frac{GQ^2}{2r}\left(1-\cos\left(\frac{r}{R_S}\right)\right).$$

Then,

$$f_{KNq} = 1 - \frac{R_H}{r}\frac{2}{\pi}\mathrm{Si}\left(\frac{r}{R_S}\right) + \frac{R_Q^2}{r^2}\left(1-\cos\left(\frac{r}{R_S}\right)\right) + \frac{a^2}{r^2}, \qquad R_H = 2GM;\ R_Q^2 = GQ^2.$$

Equality $f_{KNq} = 0$ determines possible event horizons r_q

$$1 - \frac{R_H}{r_q}\frac{2}{\pi}\left(\frac{r_q}{R_H}\frac{1}{R_S/R_H}\right) + \beta_1\frac{R_H^2}{4r_q^2}\left(1-\cos\left(\frac{r_q}{R_H}\frac{1}{R_S/R_H}\right)\right) + \beta_2\frac{R_H^2}{4r_q^2} = 0.$$

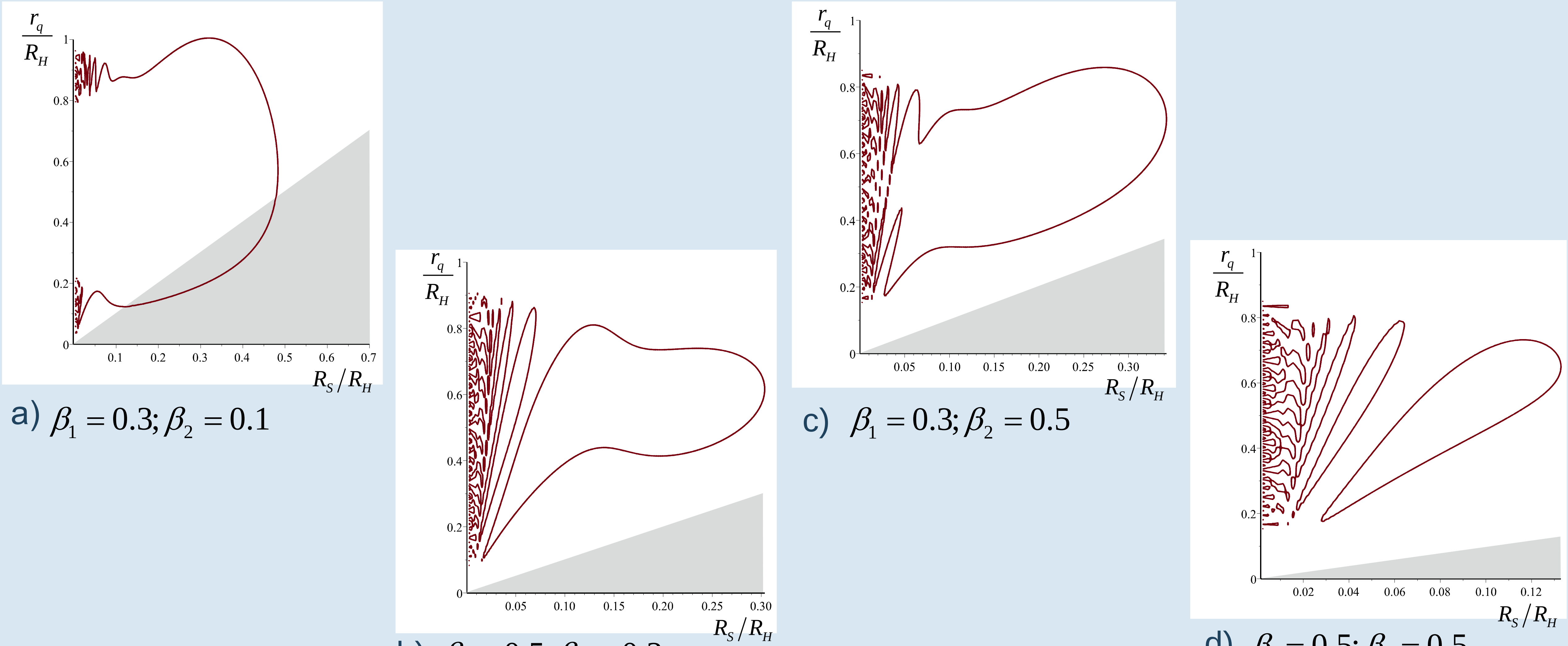
Here, $\beta_1 = \dfrac{R_Q^2}{R_H^2/4}, \beta_2 = \dfrac{a^2}{R_H^2/4}$; for the S metric $\beta_1 = \beta_2 = 0$; for the RN metric $\beta_1 \neq 0, \beta_2 = 0$;

for the K metric $\beta_1 = 0, \beta_2 \neq 0$; $\beta_1 + \beta_2 = 1$ is the analog of the extreme KN metric;

$(\beta_1 + \beta_2) > 1$ is the parametric analog of the naked singularity KN

The solution of equation $f_{KNq} = 0$ can be express as $r_q / R_H = \varphi(R_S/R_H, \beta_1, \beta_2)$.

Event horizons of spinning quantum black holes



a) $\beta_1 = 0.3; \beta_2 = 0.1$

b) $\beta_1 = 0.5; \beta_2 = 0.3$

c) $\beta_1 = 0.3; \beta_2 = 0.5$

d) $\beta_1 = 0.5; \beta_2 = 0.5$

Fig. 1. Radii r_q of quantum event horizons for different values of R_S and $(\beta_1 + \beta_2) \leq 1$.

Event horizons of spinning quantum black holes

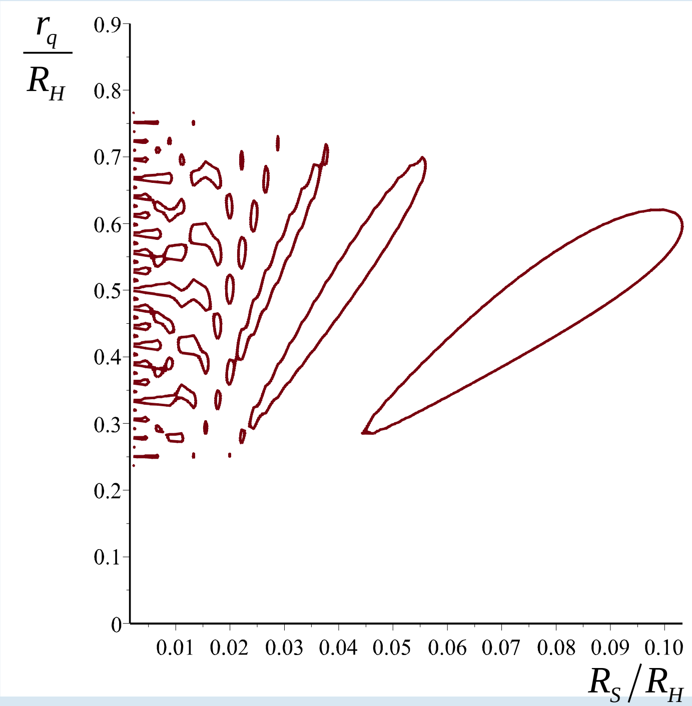
It is seen from the Fig. 1 that at small R_S / R_H values, multitude event horizons are available. With increase in R_S , the number of horizons decreases down to one $\left(r_q\right)_+$. Further increase in R_S leads to disappearance of all the event horizons.

As against the classical extreme KN black hole with a single event horizon $R_\pm = R_H$ for the quantum KN black hole with $\beta_1 + \beta_2 = 1$, the pattern describes for the values of $0 < \left(\beta_1 + \beta_2\right) < 1$ is reproduced qualitatively. In this case, the disappearance of event horizons occurs at rather small values of $R_S / R_H \approx 0.13$ (see Fig. 1d).

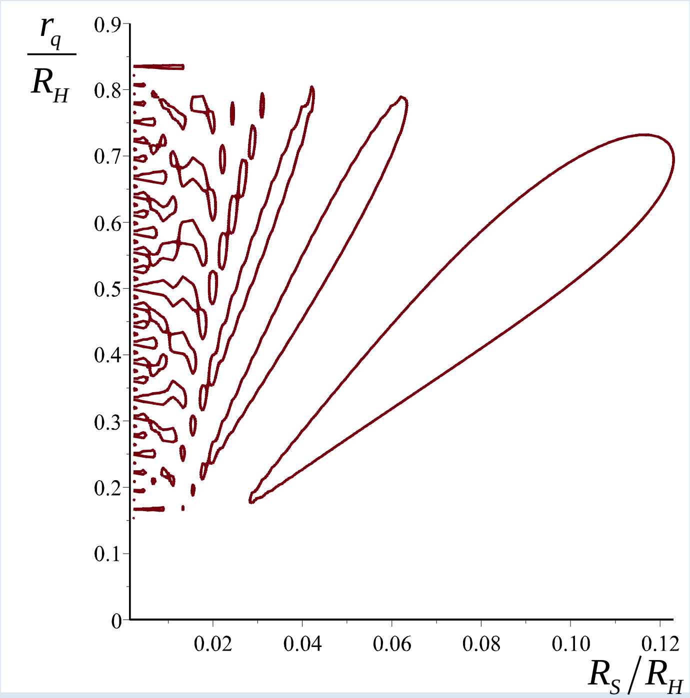
The shadowy areas in Fig. 1 corresponds to the parameters of $r_q < R_S$. Such parameters cannot be parameters of black holes.

Event horizons of spinning quantum black holes

For classical KN black holes, the case of $(\beta_1 + \beta_2) > 1$ corresponds to the naked KN singularity. For quantum KN black holes at $(\beta_1 + \beta_2) > 1$ and $R_S / R_H < 0.12$, there are multitude event horizons (see Fig. 2). With increase in R_S , the event horizons disappear.



a) $\beta_1 = 0.5; \beta_2 = 0.7$



b) $\beta_1 = 0.7; \beta_2 = 0.5$

Fig. 2. Radii r_q of quantum event horizons for different values of R_S and $(\beta_1 + \beta_2) > 1.$

Quantum model of electron

«Электрон также неисчерпаем, как и атом»

В.И.Ленин
(1908г.)

«An electron is inexhaustible as well as an atom»

V.I.Lenin
(1908)

Quantum model of electron

1. Characteristic numbers:

$$M \rightarrow m = 9.1 \cdot 10^{-28}\ \mathrm{g} \qquad Q^2 \rightarrow e^2 = 2.31 \cdot 10^{-19}\ \mathrm{erg \cdot cm}$$

$$J \rightarrow \hbar/2 = 0.5 \cdot 1.054 \cdot 10^{-27}\ \mathrm{erg \cdot sec} \qquad G = 6.67 \cdot 10^{-8}\ \mathrm{cm^3/g \cdot sec^2} \qquad c = 3 \cdot 10^{10}\ \mathrm{cm/sec}$$

$$R_H = \frac{2Gm}{c^2} = 1.35 \cdot 10^{-55}\ \mathrm{cm} \qquad \frac{Ge^2}{c^4} = \left(1.38 \cdot 10^{-34}\right)^2\ \mathrm{cm}^2 \qquad a^2 = \left(\frac{\hbar}{2mc}\right)^2 = \left(1.93 \cdot 10^{-11}\right)^2\ \mathrm{cm}^2$$

$$\beta_1 = \frac{Ge^2}{c^4}\frac{4}{R_H^2} = \frac{1.9 \cdot 10^{-68} \cdot 4}{1.35^2 \cdot 10^{-110}} = 4.2 \cdot 10^{42}$$

$$R_{cl} = \frac{e^2}{mc^2} = 2.82 \cdot 10^{-13}\ \mathrm{cm}$$

$$\beta_1 + \beta_2 \gg 1$$

$$R_S^{reg} = \frac{\pi}{8}\frac{e^2}{mc^2} = 1.11 \cdot 10^{-13}\ \mathrm{cm}$$

$$\beta_2 = \frac{a^2 \cdot 4}{R_H^2} = \frac{1.93 \cdot 10^{-22} \cdot 4}{1.35^2 \cdot 10^{-110}} = 2 \cdot 10^{88}$$

$$k_{UV}^{reg} = \frac{\hbar c}{R_S^{reg}} = 178\ MeV$$

$$\frac{R_S^{reg}}{R_H} = \frac{1.11 \cdot 10^{-13}}{1.35 \cdot 10^{-55}} = 0.74 \cdot 10^{42}$$

Quantum model of electron

2. So, our model of electron is a regular black hole with quantum KN metrics without event horizons but with rotated quantum core.

2.1 The fully regularization of the quantum Kerr-Newman metric for electron takes place at

$$R_S = R_S^{reg} = \frac{\pi}{8}\frac{e^2}{mc^2} = 1.1 \cdot 10^{-13} \text{ cm}$$

Quantum model of electron

3. The energy of the electron at rest

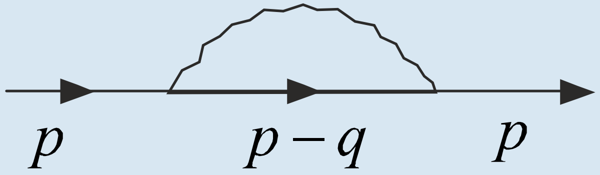
$$E = \int T_0^{\ 0} \sqrt{-g}\, dV = mc^2 + \frac{\hbar}{2mc}\left(\frac{mc^2}{2} - \frac{\pi}{16}\frac{e^2}{R_S}\right)$$

At $R_S = R_S^{reg}$ $\quad E = mc^2$!!!

Consequences: a self-energy of electron disappears.

3.1 The classical consideration $\quad E_{em} = \left.\frac{e^2}{r}\right|_{r \to 0}$

3.2 The quantum consideration



In the lowest order of the perturbation theory $\Delta m \sim me^2 \ln k_{UV}^{ph}$.

In our model we have solved a problem of self-energy of the charged particle: it is absent.

Quantum model of electron

According to the correspondence principle, in the quantum field theory, a self-energy also has to be absent after summation of the perturbation theory series.

3.3 Significant aspect:

Gravitation in the charged quantum Kerr-Newman (with rotation) and Reissner-Nordström (without rotation) metrics compensate the electromagnetic forces in the expression for particle energy.

Thank you for attention

Quantum model of electron

4. Comparison with semiclassical models of electron.

4.1 Lorentz radius $\left(mc^2 = e^2/r_0\right)$

$$r_0 = \frac{e^2}{mc^2} = 2.82 \cdot 10^{-13}\ \text{cm}$$

4.2 Israel-Lopez-Burinskii model

*(Phys. Rev. D **2**, 641 (1970), Phys. Rev. D **30**, 313 (1984), Universe **8**, 553 (2022))*

4.3 The motion of electron in repulsive Coulomb field

*(J. Exp. Theor. Phys. **128**, 672 (2019), Phys. Part. Nucl. **53**, 1126 (2022))*

$$r_b = \frac{e^2/mc^2}{1 + E/mc^2}$$

4.4 Quantum model of electron

$$R_S^{reg} = \frac{\pi}{8}\frac{e^2}{mc^2}$$

Electromagnetic structure of quantum model of electron in modified Kerr-Newman space-time

For the classical KN metric, the electromagnetic potentials A_μ are choose in the form of [1], [2]

$$A_\mu = -\frac{Qr}{\rho^2}\left(1,0,0,-a\sin^2\theta\right).$$

As $r \to \infty$, electromagnetic fields are a superposition of the Coulomb field and magnetic dipole field $\mu = Qa$. The gyromagnetic ratio $\mu/J = Q/m$ which coincides with the ratio μ/J for the Dirac electron.

The complicated internal electromagnetic structure of a classical KN metric source is presented, for example, in [3].

For our quantum KN metric, the electromagnetic potentials and electromagnetic structure of source will be different. Firstly, because of existence of a quantum core with gravitons in the coherent states and, secondly, because of the different structure of space-time.

[1] B. Carter, Phys. Rev. 174, 1559 (1968).
[2] C. A. Lopez, Phys. Rev. D 30, 313 (1984).
[3] C.L.Pekeris, K. Frankowski, Phys. Rev. A 36, 5118 (1987).

Electromagnetic structure of quantum model of electron in modified Kerr-Newman space-time

Really, the nonzero value of tensor of the electromagnetic filed $F_{\mu\nu} = \partial A_\nu / \partial x^\mu - \partial A_\mu / \partial x^\nu$ can be written as [1]

$$F_0{}^1 = -\frac{1}{a\sin^2\theta} F_3{}^1 = \frac{\Delta Q}{\rho^6}\left(\rho^2 - 2r^2\right), \; F_0{}^2 = -\frac{a}{r^2 + a^2} F_3{}^2 = \frac{Qra^2}{\rho^6} 2\cos\theta\sin\theta.$$

Further, one can obtain the components of energy-momentum tensor of the electromagnetic field (see, for example, [2], [3]):

$$\left(T_\mu{}^\nu\right)_Q = \frac{1}{4\pi}\left(-g_{\mu\lambda}F^{\lambda\beta}F_\beta{}^\nu + \frac{1}{4}g_{\mu\lambda}g^{\alpha\nu}F_{\lambda\beta}F^{\lambda\beta}\right).$$

For the part of classical mass function $m_{KN}^Q = -GQ^2/2r$, the components of the Einstein tensor divided by $8\pi G$, coincide with energy-momentum tensor $\left(G_\mu{}^\nu\right)_Q / 8\pi G = \left(T_\mu{}^\nu\right)_Q$.

But for the part of quantum mass function $m_{KN}^Q = -GQ^2\left(1 - \cos\left(r/R_S\right)\right)/2r$ this equality is not fulfilled. To recover the equality, it is necessary in addition to graviton condensate to take into account the properties of effective quantum field connected with distributed charge *e*.

[1] D. V. Galzov, Chastitsy I polia v okrestnosti chernykh dyr. Izdatelstvo Moskovskogo Universiteta (1986).

[2] L. D. Landau and E. M. Lifshits. The classical Theory of Fields. Pergamon Press, Oxford, 1975.

[3] S. Weinberg, Gravitation and Cosmology. John Wiley and Sons, Inc. (1972)

Electromagnetic structure of quantum model of electron in modified Kerr-Newman space-time

Elementary model: a quantum core with condensate of gravitons $\left(k_{UV} = 178 MeV\right)$ in coherent states and with distributed charge *e*.

1. Asymptotic as $r \to \infty$ is the Coulomb field.

2. Gyromagnetic ratio is as in the Dirac theory for a point electron.

3. At collision of relativistic beams of electrons and positrons with center-of-mass system $200 GeV$, the deviation from the point structure of electrons is not detected up to $r = 10^{-18}\ cm.$

The tasks for the nearest future

1. The study of the internal electromagnetic structure of the spinning quantum black holes (including electron).

2. The understanding how gravitation together with rotation compensates for electromagnetic forces in the quantum core of an electron in a way to ensure only the manifestation of the quasipoint structure of the electron for external electromagnetic forces

Атомные единицы таковы:

1. Единица заряда (заряд электрона) $e = 4{,}8029 \times \times 10^{-10}$ *эл.-стат. ед.* $= 1{,}6021 \cdot 10^{-20}$ *эл.-магн. ед.*

2. Единица массы (масса электрона) $m = 9{,}108_5 \cdot 10^{-28}$ *г.*

3. Единица длины (радиус первой боровской орбиты — наиболее близкой к ядру круговой орбиты атома водорода в старой квантовой теории) $a = \frac{\hbar^2}{me^2} = 5{,}2917 \cdot 10^{-9}$ *см.*

4. Единица скорости (скорость электрона на первой боровской орбите) $v_0 = \frac{e^2}{\hbar} = \alpha c = 2{,}1877 \cdot 10^8$ *см/сек.*

5. Единица импульса (момент электрона на первой боровской орбите) $p_0 = \frac{me^2}{\hbar} = mv_0 = 1{,}9926 \cdot 10^{-19}$ *г · см/сек.*

6. Единица энергии (удвоенное значение ионизационного потенциала водорода для массы ядра, равной бесконечности) $\frac{e^2}{a} = \frac{me^4}{\hbar^2} = \frac{p_0^2}{m} = 4{,}359_0 \cdot 10^{-11}$ *эрг.*

7. Единица времени $\frac{a}{v_0} = \frac{\hbar^3}{me^4} = 2{,}4189 \cdot 10^{-17}$ *сек.*

8. Единица частоты $\frac{v_0}{a} = \frac{me^4}{\hbar^3} = 4\pi\,\mathrm{Ry} = 4{,}1341 \cdot 10^{16}$ *сек*$^{-1}$ (Ry — ридбергова частота).

9. Единица электрического потенциала $\frac{e}{a} = \frac{me^3}{\hbar^2} = 0{,}09076$ *эл.-стат. ед.* $= 27{,}210$ *в.*

10. Единица напряженности электрического поля $\frac{e}{a^2} = \frac{m^2e^5}{\hbar^4} = 5{,}142 \cdot 10^9$ *в/см.*