Revisit Spin Effects by Thermal Vorticity

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Outline

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1- Observation of global polarization (GP)

LETTER

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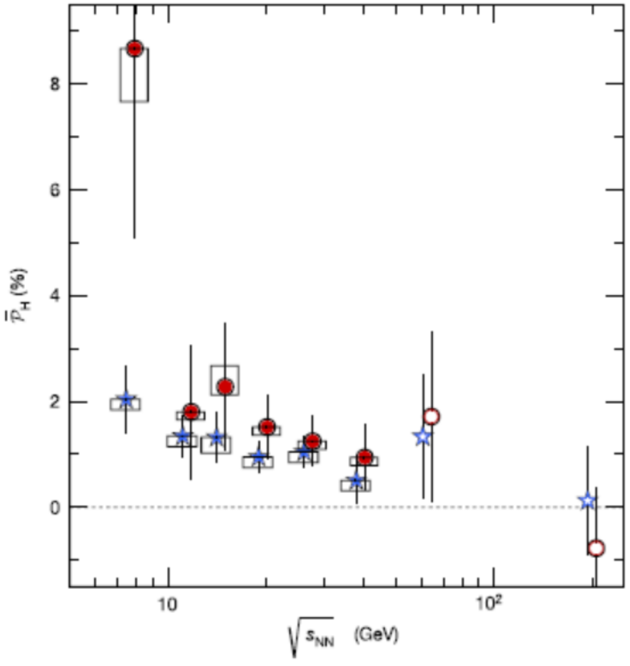
Global Λ hyperon polarization in nuclear collisions

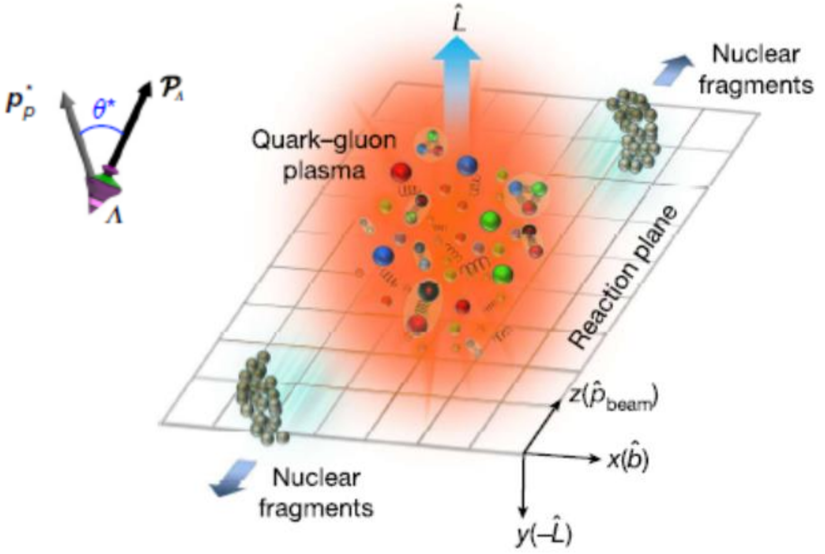
The STAR Collaboration*

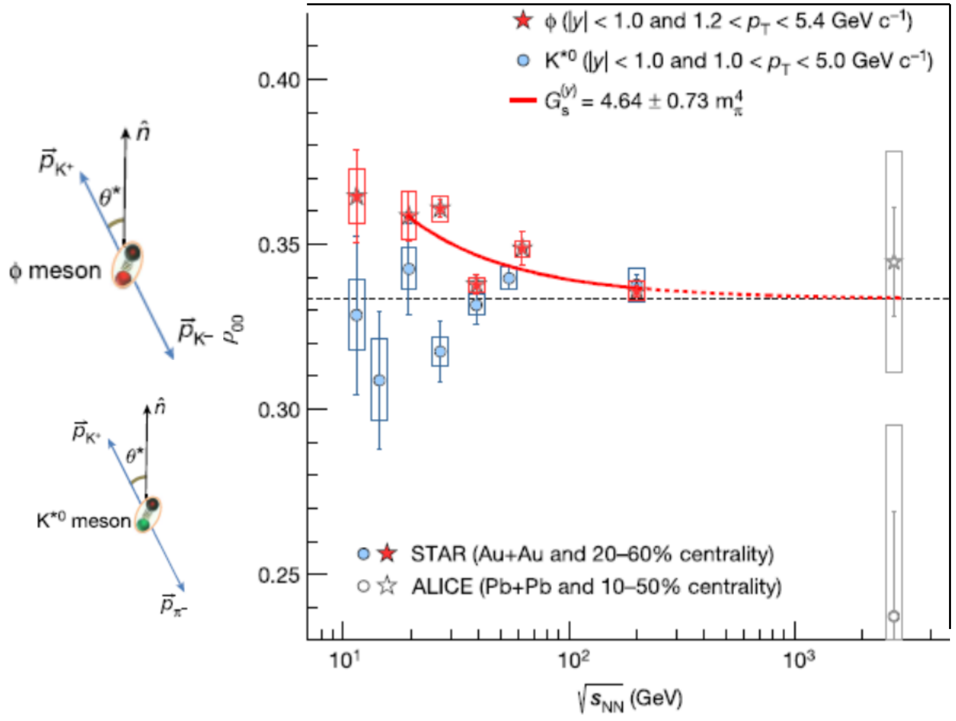
Article

Pattern of global spin alignment of ϕ and K*0 mesons in heavy-ion collisions



STAR Nature 2017





STAR Nature 2023

1- Prediction of global polarization (GP)

PRL 94, 102301 (2005) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2005

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

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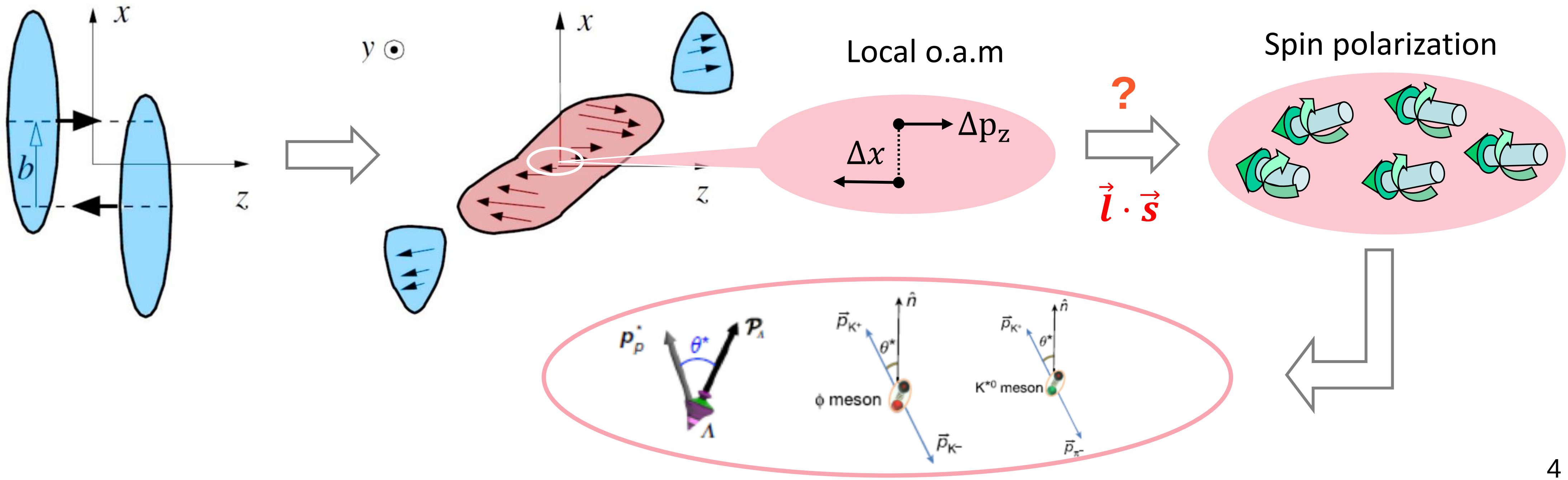
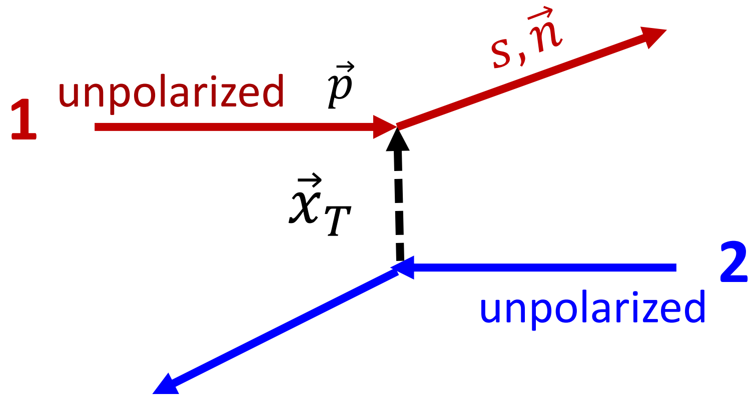


1- GP calculation from single scattering

- Single scattering at fixed impact parameter



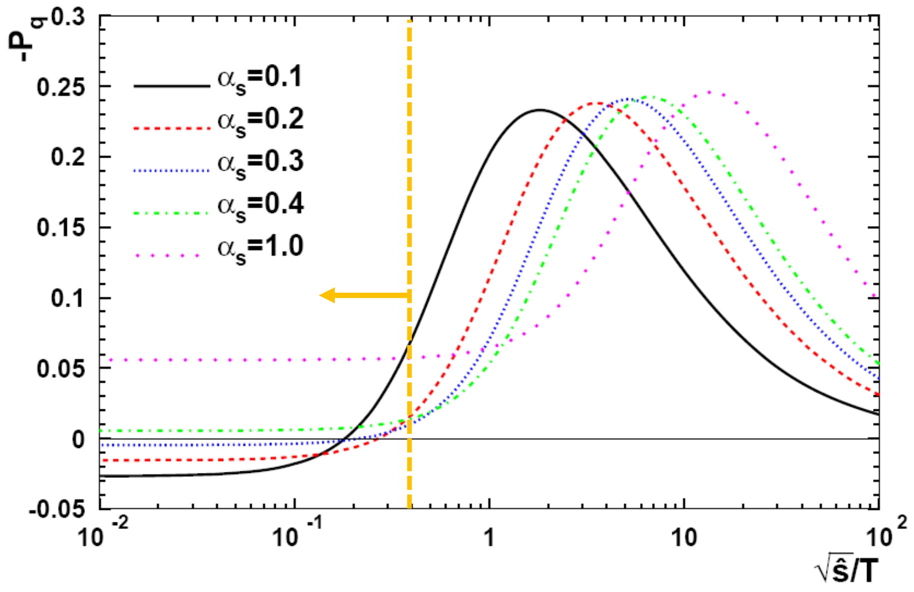
$$\frac{d\sigma_s}{d^2\vec{x}_T} = \frac{d\sigma}{d^2\vec{x}_T} + s\frac{d\Delta\sigma}{d^2\vec{x}_T}$$

Static potential model

$$P_q \approx \frac{\pi\mu p}{2E(E+m_q)}$$

Liang-Wang PRL 2005

HTL gluon propagator



T: Temperature of QGP $\sqrt{\hat{s}}$: CM energy of qq

200GeV: $\sqrt{\hat{s}}/T \sim 0.4$ (Landau) ~ 0.0025 (Bjorken)

JHG-Chen-Deng-Liang-Wang-Wang PRC 2008

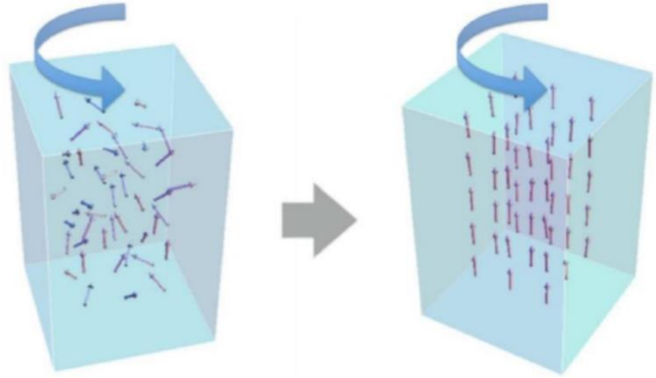
1- GP calculation from hydrodynamics

- Density operator at global equilibrium with given $J^{\mu\nu}$

$$\hat{\rho} = \frac{1}{Z}\exp\left[-\beta_\mu \hat{P}^\mu + \alpha\hat{Q} + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu}\right]$$

Thermal vorticity: $\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right)$, $\beta_\mu = u_\mu/T$

Becattini-Chandra-Del Zanna-Grossi Annals Phys. 2013



- Spin polarization vector in phase space for spin ½ particles

$$S^\mu(x,p) = -\frac{1}{8m}(1-f)\epsilon^{\mu\nu\rho\sigma}p_\nu\varpi_{\rho\sigma}$$

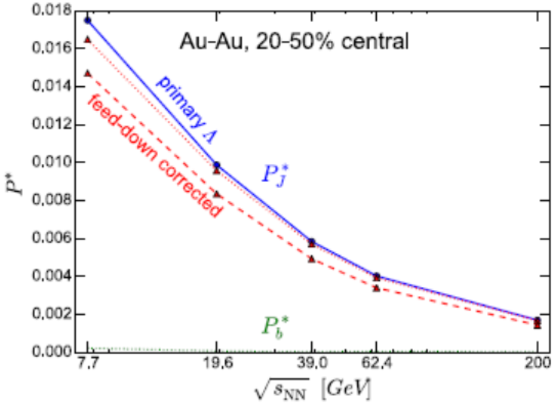
f: Fermi-Dirac distribution

- Spin polarization vector at freezeout hypersurface in heavy ion collisions:

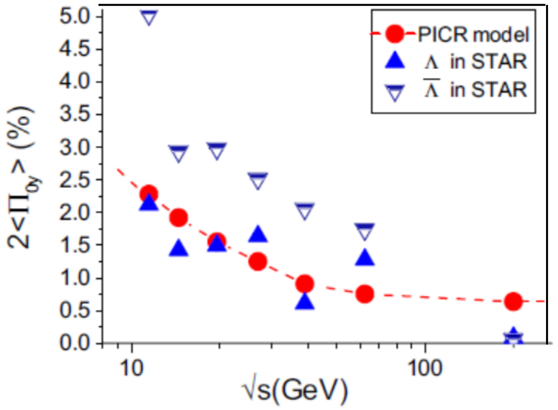
$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu\frac{\int d\Sigma_\lambda p^\lambda \boldsymbol{\varpi}_{\rho\sigma} f\,(1-f)}{\int d\Sigma_\lambda p^\lambda \varpi_{\rho\sigma} f}$$

1- Predictions for hyperon’s GP

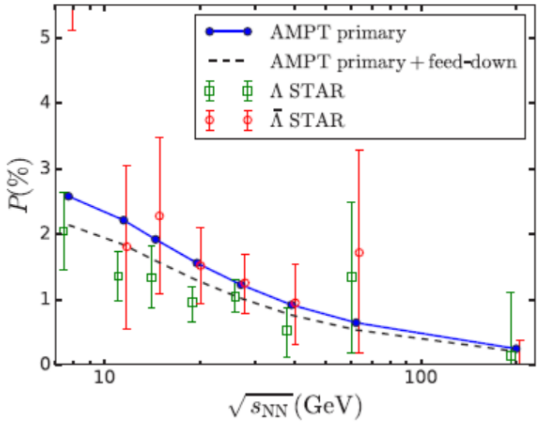
Karpenko-Becattini EPJC 2017



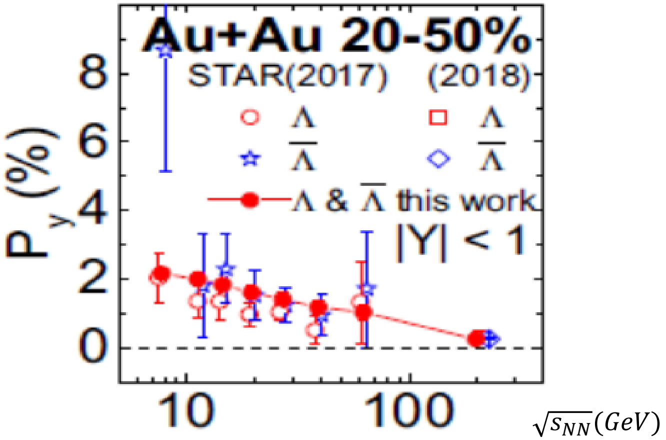
Xie-Wang-Csernai PRC 2017



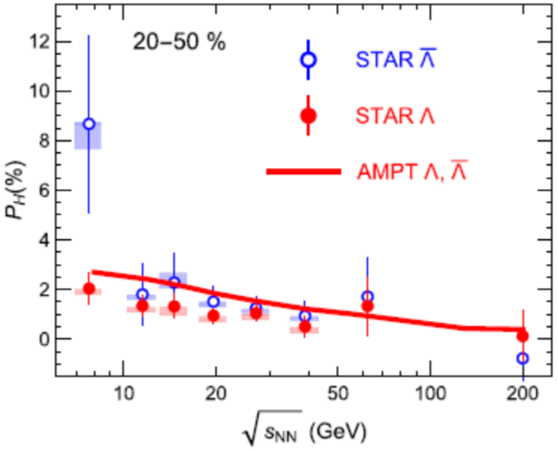
Li-Pang-Wang-Xia PRC 2017



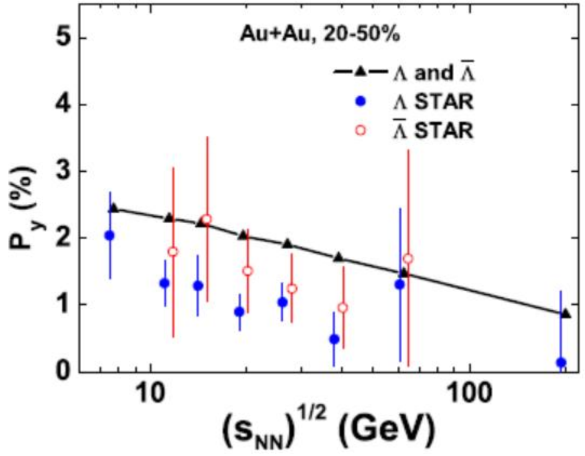
Sun-Ko PRC 2019



Shi-Li-Liao PLB 2019



Wei-Deng-Huang PRC 2019



1- Spin puzzles

- Local hyperon polarization:

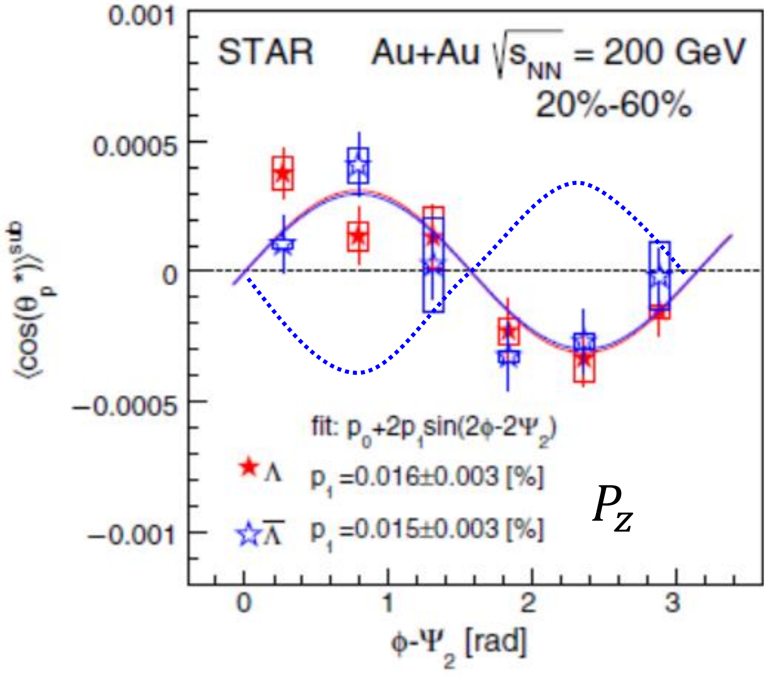
$$S^{\mu}(k) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}k_{\nu}\frac{\int d\Sigma_{\lambda}k^{\lambda}\boldsymbol{\varpi}_{\rho\sigma}n_F(1-n_F)}{\int d\Sigma_{\lambda}k^{\lambda}n_F}$$

Becattini-Karpenko PRL 2018

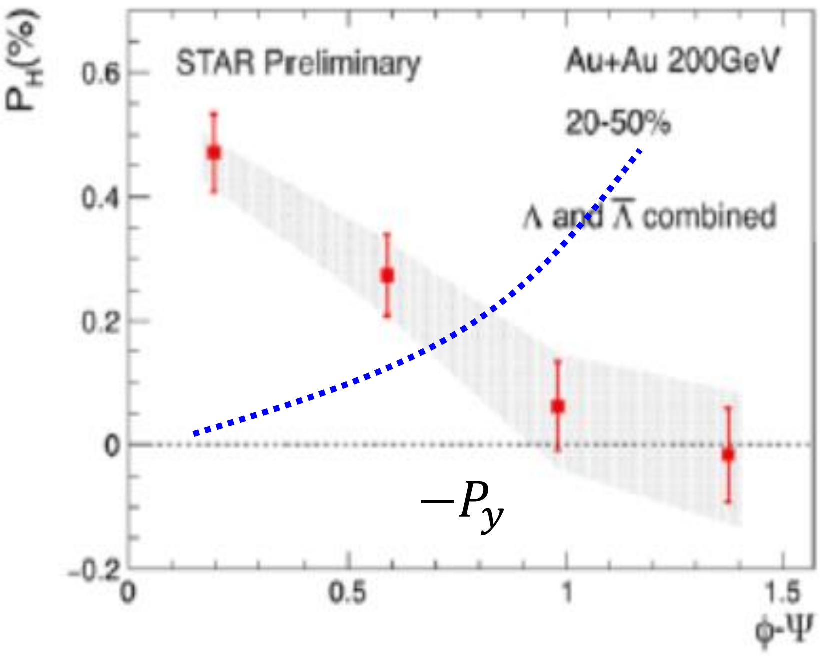
- Vector meson's spin alignment:

$$\rho_{00}^{V(\mathrm{rec})} = \frac{1-P_qP_{\bar{q}}}{3+P_qP_{\bar{q}}} \approx \frac{1}{3} - \frac{4}{9}P_qP_{\bar{q}} < \frac{1}{3}$$

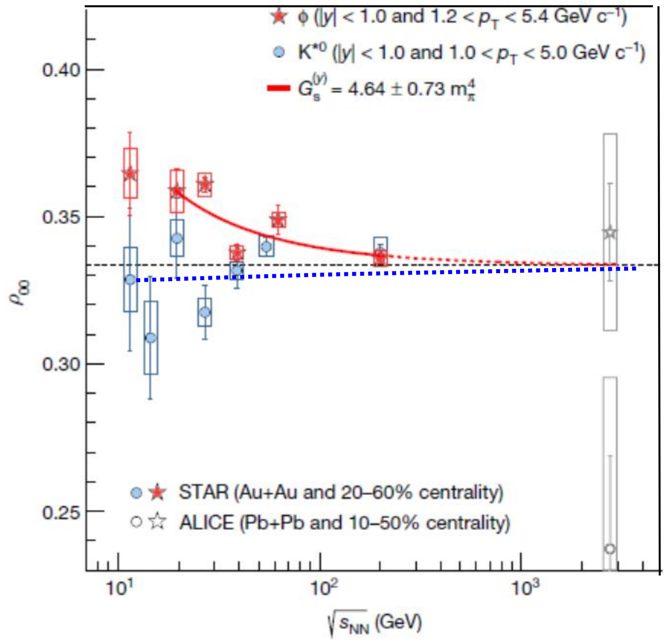
Liang-Wang PLB 2005



STAR PRL 2019



STAR NPA 2019



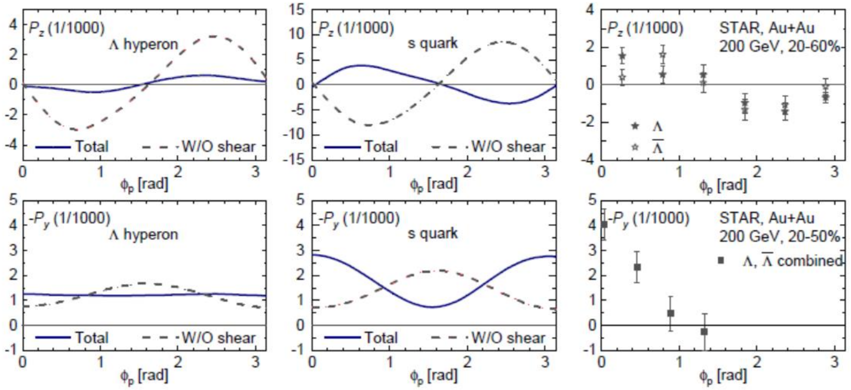
STAR Nature 2023

1. Proposal of new mechanism

- Hyperon: shear term contribution?

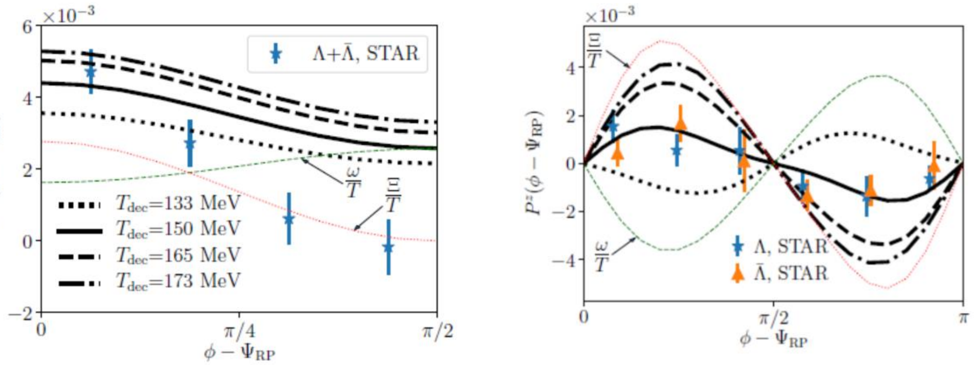
$$A^{\mu}_{\text{SIP}} = n'_F \left(\frac{p_\perp}{\varepsilon_0}\right)^2 \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha{}^{\lambda} \sigma_{\rho\lambda} \qquad \sigma_{\rho\lambda} = \frac{1}{2}\left(\partial^\perp_\rho u_\lambda + \partial^\perp_\lambda u_\rho + \frac{2}{3}\partial\cdot u\,\Delta_{\rho\lambda}\right)$$

Fu-Liu-Pang-Song-Yin PRL2021



$$S^{\mu}_{\text{ILE}} = \frac{\epsilon^{\mu\rho\sigma\tau} p_\tau}{4mT_{\text{dec}}} \frac{\int_\Sigma d\Sigma\cdot p\, n'_F \hat{t}_\rho \Xi_{\lambda\sigma}}{\int_\Sigma d\Sigma\cdot p\; n_F} \qquad \Xi_{\lambda\sigma} = \frac{1}{2}(\partial_\lambda u_\sigma + \partial_\lambda u_\sigma)$$

Becattini-Buzzegoli-Palermo-Inghirami-Karpenko PRL2021



Could be very sensitive to the EOS, the ratio η/s and freezeout temperature T_{f}!

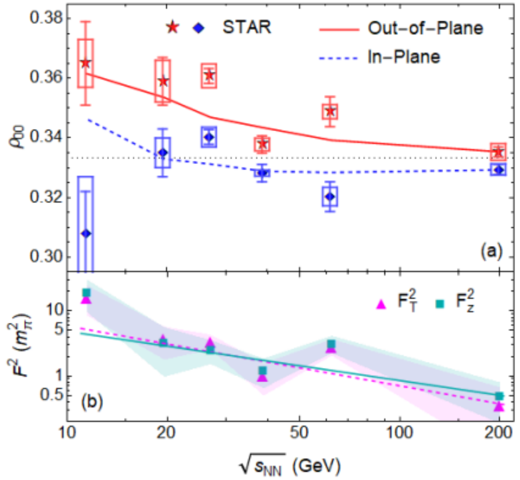
- Vector meson: strong force field?

Sheng-Oliva-Wang PRD2020, Sheng-Oliva-Liang-Wang-Wang PRL2023

$$\rho_{00}(x,\mathbf{k}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2\right] + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2\right]$$

$$-\frac{4g_\phi^2}{m_\phi^2 T_{\text{h}}^2} C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\text{h}}^2} C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right]$$

Yi-Pu-Yang 2106.00238



2- Revisiting old mechanism

- Hyperon: quark or hyperon spin polarization vector + relativistic hydrodynamics

$$S^\mu(k) = \frac{1}{2}\frac{\int d\Sigma_\alpha k^\alpha \mathrm{Tr}\left(\gamma^\mu\gamma^5 W(x,k)\right)}{\int d\Sigma_\alpha k^\alpha \mathrm{Tr}\left(W(x,k)\right)} \Longrightarrow S^{*\mu} = (0, \mathbf{S}^*) \Longrightarrow P = \mathbf{n}_3 \cdot \mathbf{S}^*$$

Definition from Wigner function — In the rest frame — Spin polarization along n_3

Can we calculate hyperon's polarization in another way?

- Vector meson: quark and antiquark polarization + coalescence model

Spin effects for vector meson without coalescence model?

2- Spin density matrix

- Spin density matrix for a particle with momentum k:

Spin 1/2 $$\rho(k)=\begin{pmatrix}\rho_{++}(k) & \rho_{+-}(k)\\ \rho_{-+}(k) & \rho_{--}(k)\end{pmatrix}$$

Spin 1 $$\rho(k)=\begin{pmatrix}\rho_{++}(k) & \rho_{+0}(k) & \rho_{+-}(k)\\ \rho_{0+}(k) & \rho_{00}(k) & \rho_{0-}(k)\\ \rho_{-+}(k) & \rho_{-0}(k) & \rho_{--}(k)\end{pmatrix}$$

- Spin density matrix in heavy ion collisions: $$\rho_{rs}(k)=\frac{\int d\Sigma_\alpha k^\alpha \rho_{rs}(x,k)f(x,k)}{\int d\Sigma_\alpha k^\alpha f(x,k)}$$ $$f(x,k)\equiv\sum_s f_{ss}(x,k)$$

- Local spin density matrix and particle distribution function: $$\rho_{rs}(x,k)=\frac{f_{rs}(x,k)}{f(x,k)}$$

- Operator definition for particle distribution function with spin: De Groot-Van Leeuwen-Van Weert 1980

$$f_{rs}(x,k)=\int\frac{d^3\mathbf{q}}{(2\pi)^3}e^{-i(E_{\mathbf{k}+\mathbf{q}/2}-E_{\mathbf{k}-\mathbf{q}/2})t}e^{i\mathbf{q}\cdot\mathbf{x}}\langle a^{s\dagger}_{\mathbf{k}-\mathbf{q}/2}a^{r}_{\mathbf{k}+\mathbf{q}/2}\rangle$$

basic unit: $\langle a^{s\dagger}_{\mathbf{p}}a^{r}_{\bar{\mathbf{p}}}\rangle$

2- Spin distribution function

- Ensemble average in global equilibrium with given $P, Q, J^{\mu\nu}$

$$\langle a_{\mathbf{p}}^{s\dagger} a_{\bar{\mathbf{p}}}^{r} \rangle = \frac{1}{Z} \mathrm{Tr} \left[\exp\left(-b\cdot P + \alpha Q + \frac{1}{2}\omega_{\mu\nu} J^{\mu\nu} \right) a_{\mathbf{p}}^{s\dagger} a_{\bar{\mathbf{p}}}^{r} \right] \qquad Z = \mathrm{Tr} e^{-b\cdot P + \alpha Q + \frac{1}{2}\omega_{\mu\nu} J^{\mu\nu}}$$

- Constant Lagrange multiplies corresponding to conserved charges: $\alpha, b_\mu, \omega_{\mu\nu}$

Spin chemical potential: $\omega_{\mu\nu}$

See: analytic continuation method: Becattini-Buzzegoli-Palermo JHEP2021; Palermo-Buzzegoli-Becattini JHEP 2021

- Commutation relations $\left[Q, a_{\mathbf{p}}^{s\dagger}\right] = a_{\mathbf{p}}^{s\dagger}, \quad \left[b\cdot P, a_{\mathbf{p}}^{s\dagger}\right] = b\cdot p\, a_{\mathbf{p}}^{s\dagger}, \quad \left[\frac{1}{2}\omega_{\mu\nu} J^{\mu\nu}, a_{\mathbf{p}}^{s\dagger}\right] = \sum_r \Lambda_{\mathbf{p}}^{sr} a_{\mathbf{p}}^{r\dagger}$

- The formal result: $\langle a_{\mathbf{p}}^{s\dagger} a_{\bar{\mathbf{p}}}^{r} \rangle = (2\pi)^3 \left[\frac{1}{e^{b\cdot p - \alpha - \Lambda_{\mathbf{p}}} \pm 1} \right]^{sr} \delta(\mathbf{p} - \bar{\mathbf{p}}) \qquad +/-$: fermion/boson

3- Distribution function with spin ½

- Dirac field with fixed spin quantization direction $\mathbf{n}_3$:

$$\langle a_{\mathrm{p}}^{s\dagger} a_{\bar{\mathrm{p}}}^{r} \rangle = (2\pi)^3 \left[\frac{1}{e^{b\cdot p - \alpha - \Lambda_{\mathrm{p}}} + 1} \right]^{sr} \delta\left(\mathbf{p} - \bar{\mathbf{p}}\right)$$

$$\Lambda_{\mathrm{p}}^{sr} = \left[\left(E_{\mathrm{p}} \boldsymbol{\varepsilon} + \boldsymbol{\omega} \times \mathrm{p}\right) \cdot i\boldsymbol{\nabla}_p + \frac{i}{2E_{\mathrm{p}}} \boldsymbol{\varepsilon} \cdot \mathrm{p} \right] \delta^{sr} + \frac{1}{2} \left(\boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon} \times \mathrm{p}}{E_{\mathrm{p}} + m} \right) \cdot \boldsymbol{\lambda}^{sr}$$

$$\boldsymbol{\lambda}^{sr} = s\delta^{s,r} \mathrm{n}_3 + is\delta^{-s,r} \mathrm{n}_2 + \delta^{-s,r} \mathrm{n}_1, \qquad \boldsymbol{\lambda} = \mathrm{n}_3 \sigma_3^T + \mathrm{n}_2 \sigma_2^T + \mathrm{n}_1 \sigma_1^T$$

Electric part: $\varepsilon^i = \omega^{i0}$, Magnetic part: $\omega^i = -\frac{1}{2}\epsilon^{ijk}\omega^{jk}$, $\mathrm{n}_2 = \frac{\hat{\mathrm{z}} \times \mathrm{n}_3}{|\hat{\mathrm{z}} \times \mathrm{n}_3|}$, $\mathrm{n}_1 = \mathrm{n}_2 \times \mathrm{n}_3$

$$f_{rs}(x,k) = \int d^3\mathrm{q} e^{-i(E_{\mathrm{k+q/2}} - E_{\mathrm{k-q/2}})t + i\mathrm{q}\cdot\mathrm{x}} \left[\frac{1}{e^{b_0 E_{\mathrm{k-q/2}} - \mathrm{b}\cdot(\mathrm{k-q/2}) - \alpha - \Lambda_{\mathrm{k-q/2}}} + 1} \right]^{sr} \delta\left(\mathrm{q}\right)$$

3- Distribution function up to first order

- Expand the distribution function as the Taylor series of $\boldsymbol{\omega}$ and $\boldsymbol{\varepsilon}$

The 0th order: $f_{rs}^{(0)}(x,k) = f_F(b\cdot p-\alpha)\delta^{sr}$ $\qquad f_F(b\cdot p-\alpha) = \dfrac{1}{e^{b\cdot p-\alpha}+1}$: Fermi-Dirac distribution

The 1st order: $f_{rs}^{(1)}(x,k) \;=\; f_F'(b\cdot p-\alpha)\left[E_{\mathbf{k}}\boldsymbol{\varepsilon}\cdot\mathbf{x}-\mathbf{k}\cdot(\boldsymbol{\varepsilon}t+\boldsymbol{\omega}\times\mathbf{x})\right]\delta^{sr}-\frac{1}{2}f_F'(b\cdot p-\alpha)\left(\boldsymbol{\omega}-\frac{\boldsymbol{\varepsilon}\times\mathbf{k}}{E_{\mathbf{k}}+m}\right)\cdot\boldsymbol{\lambda}^{sr}$

- "Resummation": $f_{rs}(x,k) = f_{rs}^{(0)}(x,k)+f_{rs}^{(1)}(x,k) \approx f_F(\beta\cdot p-\alpha)\delta^{sr}-\frac{1}{2}f_F'(\beta\cdot p-\alpha)\left(\boldsymbol{\omega}-\frac{\boldsymbol{\varepsilon}\times\mathbf{k}}{E_{\mathbf{k}}+m}\right)\cdot\boldsymbol{\lambda}^{sr}$

$$b^\mu = (b_0,\mathbf{b}) \Rightarrow \beta^\mu = \frac{u^\mu}{T} = (b_0+\boldsymbol{\varepsilon}\cdot\mathbf{x},\mathbf{b}+\boldsymbol{\varepsilon}t+\boldsymbol{\omega}\times\mathbf{x}),$$

- Spin chemical potential = thermal vorticity : $\omega^{\mu\nu} = \varpi^{\mu\nu} = -\frac{1}{2}(\partial^\mu\beta^\nu-\partial^\nu\beta^\mu)$

acceleration vector: $\boldsymbol{\varepsilon}$ vorticity vector: $\boldsymbol{\omega}$

3- Polarization along fixed $\mathbf{n}_3$

- Spin polarization is defined by:

$$P(x,k) = \frac{f_{+,+}(x,k) - f_{-,-}(x,k)}{f_{+,+}(x,k) + f_{-,-}(x,k)}$$

- Polarization from present method:

$$P(x,k) = -\frac{f_F'}{2f_F}\left(\boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon} \times \mathbf{k}}{E_{\mathrm{k}} + m}\right) \cdot \mathbf{n}_3$$

- Polarization from previous method:

$$P(x,k) = -\frac{f_F'}{2f_F} \cdot \frac{E_{\mathrm{k}}}{2m}\left[\boldsymbol{\omega} - \frac{(\boldsymbol{\omega} \cdot \mathbf{k})\mathbf{k}}{E_{\mathrm{k}}(E_{\mathrm{k}} + m)} - \frac{\boldsymbol{\varepsilon} \times \mathbf{k}}{E_{\mathrm{k}}}\right] \cdot \mathbf{n}_3$$

- Differences: No $1/m$ singularity; No $(\boldsymbol{\omega} \cdot \mathbf{k})\mathbf{k}$ term; The $\boldsymbol{\varepsilon} \times \mathbf{k}$ term suppressed

3- Polarization along $\mathrm{n}_3 \parallel \hat{\mathrm{p}}$, or $\mathrm{n}_3 \perp \hat{\mathrm{p}}$

- The polarization receives additional contribution when the spin direction depends on the momentum

$$f_{rs}(x,k) = \int \frac{d^3\mathrm{q}}{(2\pi)^3} e^{-i(E_{\mathrm{k}+\mathrm{q}/2} - E_{\mathrm{k}-\mathrm{q}/2})t} e^{i\mathrm{q}\cdot\mathrm{x}} \langle a^{s\dagger}_{\mathrm{k}-\mathrm{q}/2} a^{r}_{\mathrm{k}+\mathrm{q}/2} \rangle$$

1. Helicity polarization $\mathbf{n_3} = \mathbf{e}_p$

$$P(x,k) = -\frac{f_F'}{2f_F}\left(\boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon}\times\mathrm{k}}{E+m}\right)\cdot \mathrm{n}_3 + \frac{f_F'}{2f_F}\cdot\frac{1}{k}(E\boldsymbol{\varepsilon} + \boldsymbol{\omega}\times\mathrm{k})\cdot \mathrm{e}_\phi \cot\theta$$

2. Transverse polarization $\mathbf{n_3} = \mathbf{e}_\phi$

$$P(x,k) = -\frac{f_F'}{2f_F}\left(\boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon}\times\mathrm{k}}{E+m}\right)\cdot \mathrm{n}_3 + 0$$

3. Transverse polarization $\mathbf{n_3} = \mathbf{e}_\theta$

$$P(x,k) = -\frac{f_F'}{2f_F}\left(\boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon}\times\mathrm{k}}{E+m}\right)\cdot \mathrm{n}_3 + \frac{f_F'}{2f_F}\cdot\frac{1}{k}(E\boldsymbol{\varepsilon} + \boldsymbol{\omega}\times\mathrm{k})\cdot \mathrm{e}_\phi$$

4- Distribution function with spin 1

- Vector field with fixed spin quantization direction $\mathbf{n}_3$:

$$\langle a^{s\dagger}_{\mathrm{p}} a^{r}_{\bar{\mathrm{p}}} \rangle = (2\pi)^3 \left[\frac{1}{e^{b\cdot p - \alpha - \Lambda_{\mathrm{p}}} - 1} \right]^{sr} \delta(\mathrm{p} - \bar{\mathrm{p}}) \qquad \Lambda^{sr}_{\mathrm{p}} = \left[(E_{\mathrm{p}}\boldsymbol{\varepsilon} + \boldsymbol{\omega} \times \mathrm{p}) \cdot i\nabla_p + \frac{i}{2E_{\mathrm{p}}} \boldsymbol{\varepsilon} \cdot \mathrm{p} \right] \delta^{sr} + i \left[\boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon} \times \mathrm{p}}{E + m} \right] \cdot (\mathrm{n}_s \times \mathrm{n}_r)$$

- Orthonormal linear polarization vectors $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$:

$$\mathbf{n}_1 \times \mathbf{n}_2 = \mathbf{n}_3 \qquad \mathbf{n}_s = \mathbf{n}_s^*$$

- Orthonormal circular polarization vector $\mathbf{n}_+, \mathbf{n}_-, \mathbf{n}_0$:

$$\mathbf{n}_\pm = \frac{\mathbf{n}_1 \pm i\,\mathbf{n}_2}{\sqrt{2}} \qquad \mathbf{n}_0 = \mathbf{n}_3$$

- The spin distribution function with linear or circular polarization up to the first order :

$$f_{rs}(x,k) = f_B(\beta\cdot k - \alpha)\delta^{sr} - i f_B'(\beta\cdot k - \alpha) \left(\boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon} \times \mathbf{k}}{E_{\mathrm{k}} + m} \right) \cdot (\mathbf{n}_s \times \mathbf{n}_r^*)$$

$$f_B(\beta\cdot p - \alpha) = \frac{1}{e^{\beta\cdot p - \alpha} - 1}$$ Bose-Einstein distribution

4- Spin effects along fixed $\mathbf{n}_3$

- The spin density matrix can be measured by angular distribution of vector mesons' two-body decay:

$$\frac{dN}{d\cos\theta^* d\varphi^*} = \frac{3}{8\pi}\Big[1-\rho_{00}+(3\rho_{00}-1)\cos^2\theta^* - 2\mathrm{Re}\rho_{+-}\sin^2\theta^*\cos(2\varphi^*) - 2\mathrm{Im}\rho_{+-}\sin^2\theta^*\sin(2\varphi^*)$$
$$-\sqrt{2}\mathrm{Re}\left(\rho_{+0}-\rho_{0-}\right)\sin\theta^*\cos\varphi^* + \sqrt{2}\mathrm{Im}\left(\rho_{+0}-\rho_{0-}\right)\sin\theta^*\sin\varphi^*\Big]$$

- The 1st order: $\rho_{00}=\frac{1}{3}$ $\rho_{+-}=0$

off-diagonal element $\rho_{+0}-\rho_{0-} = -\frac{\sqrt{2}f_B'}{3f_B}\left(\boldsymbol{\omega}-\frac{\boldsymbol{\varepsilon}\times\mathbf{k}}{E_{\mathbf{k}}+m}\right)\cdot(\mathbf{n}_1+i\mathbf{n}_2)$

- The 2nd order: $f_{rr} = f+\frac{1}{2}f_B''(\beta\cdot k-\alpha)\left(\boldsymbol{\omega}-\frac{\boldsymbol{\varepsilon}\times\mathbf{k}}{E_{\mathbf{k}}+m}\right)^2-\frac{1}{2}f_B''(\beta\cdot k-\alpha)\left[\left(\boldsymbol{\omega}-\frac{\boldsymbol{\varepsilon}\times\mathbf{k}}{E_{\mathbf{k}}+m}\right)\cdot\mathbf{n}_r\right]^2$

Spin alignment $\rho_{00} = \frac{1}{3}-\frac{1}{6}\cdot\frac{f_B''}{f_B}\left\{\left[\left(\boldsymbol{\omega}-\frac{\boldsymbol{\varepsilon}\times\mathbf{k}}{E_{\mathbf{k}}+m}\right)\cdot\mathbf{n}_3\right]^2-\frac{1}{3}\left[\boldsymbol{\omega}-\frac{\boldsymbol{\varepsilon}\times\mathbf{k}}{E_{\mathbf{k}}+m}\right]^2\right\}$

4- Spin effects along $\mathbf{n}_3 \parallel \hat{\mathbf{p}}$, or $\mathbf{n}_3 \perp \hat{\mathbf{p}}$

- The polarization receives additional contribution when the spin direction depends on the momentum

$$\rho_{+0} - \rho_{0-} = -\frac{\sqrt{2}}{3}\frac{f_B'}{f_B}\left[\boldsymbol{\Omega}_{\rm k} - \frac{E_{\rm k}}{k}\left(\boldsymbol{\mathcal{E}}_{\rm k}\cdot\mathbf{e}_\theta\right)\mathbf{e}_\phi - \frac{E_{\rm k}}{k\sin\theta}\left(\boldsymbol{\mathcal{E}}_{\rm k}\cdot\mathbf{e}_\phi\right)\hat{\mathbf{z}}\right]\cdot\left(\mathbf{n}_1 + i\mathbf{n}_2\right) \qquad \boldsymbol{\Omega}_k \equiv \boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon}\times\boldsymbol{p}}{E_p + m} \qquad \boldsymbol{\mathcal{E}}_k \equiv \boldsymbol{\varepsilon} + \frac{\boldsymbol{\omega}\times\boldsymbol{p}}{E_p}$$

$$\rho_{00} = \frac{1}{3} - \frac{1}{6}\cdot\frac{f_B''}{f_B}\left\{\left[\left(\boldsymbol{\Omega}_{\rm k} - \frac{E_{\rm k}}{k}\left(\boldsymbol{\mathcal{E}}_{\rm k}\cdot\mathbf{e}_\theta\right)\mathbf{e}_\phi - \frac{E_{\rm k}}{k\sin\theta}\left(\boldsymbol{\mathcal{E}}_{\rm k}\cdot\mathbf{e}_\phi\right)\hat{\mathbf{z}}\right)\cdot\mathbf{n}_3\right]^2 - \frac{1}{3}\left[\boldsymbol{\Omega}_{\rm k} - \frac{E_{\rm k}}{k}\left(\boldsymbol{\mathcal{E}}_{\rm k}\cdot\mathbf{e}_\theta\right)\mathbf{e}_\phi - \frac{E_{\rm k}}{k\sin\theta}\left(\boldsymbol{\mathcal{E}}_{\rm k}\cdot\mathbf{e}_\phi\right)\hat{\mathbf{z}}\right]^2\right\}$$

1. Helicity polarization alone $\mathbf{e}_p$ — $\mathbf{n}_3 = \mathbf{e}_p, \quad \mathbf{n}_2 = \mathbf{e}_\phi, \quad \mathbf{n}_1 = \mathbf{e}_\theta$
2. Transverse polarization along $\mathbf{e}_\phi$ — $\mathbf{n}_3 = \mathbf{e}_\phi, \quad \mathbf{n}_2 = \mathbf{e}_\theta, \quad \mathbf{n}_1 = \mathbf{e}_p$
3. Transverse polarization along $\mathbf{e}_\theta$ — $\mathbf{n}_3 = \mathbf{e}_\theta, \quad \mathbf{n}_2 = \mathbf{e}_p, \quad \mathbf{n}_1 = \mathbf{e}_\phi$

5- Summary

- Spin effects by thermal vorticity are calculated directly from spin density matrix instead of spin polarization vector.
- The differences between these two methods are given for hyperons.
- The off-diagonal element of the spin density matrix of vector mesons receives first-order contribution.
- The spin alignment receives second-order contribution
- The spin effects receive additional contribution when the spin direction depends on the momentum.

Thanks for your attention !