

Z-boson p_T -spectrum and lepton angular coefficients in the LO high-energy factorization with the real NLO corrections

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Introduction

Drell-Yan processes as tools to search PDFs

- $p + p \rightarrow \gamma^* \rightarrow \mu^+ + \mu^-$
- $p + p \rightarrow Z \rightarrow \mu^+ + \mu^-$
- $p + p \rightarrow H \rightarrow \gamma + \gamma$

$$Q = \sqrt{(q_{\mu^+} + q_{\mu^-})^2}, \quad m_Z, \quad m_H \gg \Lambda_{QCD}$$

Final states in the DY processes are colorless \Rightarrow hard-soft factorization

Final states contain fundamental particles \Rightarrow well-known properties

Hadronization in the final state is absent \Rightarrow more simple for experimental analysis

Factorization approaches: CPM, TMD PM and HEF

Collinear parton model

- $q_{1,2T} \ll p_T$ and $\mu_F = M_T \geq M$

$$\begin{aligned} \sigma(pp \rightarrow ZX) &= \int dx_1 \int dx_2 f_q(x_1, \mu_F) f_{\bar{q}}(x_2, \mu_F) \hat{\sigma}(q + \bar{q} \rightarrow Z + g) + \\ &+ \mathcal{O}(\Lambda_{QCD}^2/\mu_F^2) \end{aligned}$$

- There are calculations in LO, NLO and NNLO in the strong constant α_S
- k_T -ordering during QCD evolution of partons and DGLAP evolution equations

Factorization approaches: CPM, TMD PM and HEF

TMD PM by Collins, Soper, Stermann

- $q_{1,2T} \sim p_T$ and $\Lambda_{QCD} \sim p_T \ll \mu_F$

$$\sigma^{TMD}(pp \rightarrow ZX) = \int dx_1 d^2 q_{1T} \int dx_2 d^2 q_{2T} F_q(x_1, q_{1T}, \mu_F, \mu_Y) \times \\ \times F_{\bar{q}}(x_2, q_{2T}, \mu_F, \mu_Y) \hat{\sigma}(q + \bar{q} \rightarrow Z) + \mathcal{O}(\langle q_T^2 \rangle / \mu_F^2) +$$

-

$$\frac{d\sigma^{TMD}(p + p \rightarrow ZX)}{dp_T dy} = \sigma_0(s, M_Z, \mu_F) \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \times \\ \times \tilde{F}_{q/P_1}(x_1, \mathbf{b}, \mu, \zeta_1) \tilde{F}_{\bar{q}/P_2}(x_2, \mathbf{b}, \mu, \zeta_2) + \\ + Y(M_Z, y, \mathbf{p}_T) + \text{suppressed corrections}$$

where $\tilde{F}_{q/P}(x, \mathbf{b}, \mu, \zeta)$ is universal TMD PDFs with evolution.

$$\tilde{F}_{g/P}(x, \mathbf{b}, \mu, \zeta) \Leftrightarrow F_{g/P}(x, \mathbf{k}_T, \mu, \zeta)$$

Factorization approaches: CPM, TMD PM and HEF

High-Energy Factorization \rightarrow Parton Reggeization ApproachParton Reggeization Approach is based on HEF and smoothly interpolate between regions $p_T \ll \mu$ and $p_T \geq \mu$

$$\begin{aligned} \sigma(pp \rightarrow ZX) &= \int \frac{dx_1}{x_1} \frac{d^2 q_{1T}}{\pi} \int \frac{dx_2}{x_2} \frac{d^2 q_{2T}}{\pi} \Phi_Q(x_1, \vec{q}_{1T}, \mu^2) \\ &\quad \times \Phi_{\bar{Q}}(x_2, \vec{q}_{2T}, \mu^2) \hat{\sigma}^{PRA}(Q\bar{Q} \rightarrow Z) \\ q_{1,2}^\mu &= x_{1,2} P_{1,2}^\mu + q_{1,2T}^\mu, \quad q_{1,2T} \neq 0, \quad q_{1,2}^2 = q_{1,2T}^2 = -\vec{q}_{1,2T}^2 \end{aligned}$$

Instead of TMD PM, initial partons in HEF are off-mass-shell

For details, see following publications:

- M.A. Nefedov, N.N. Nikolaev and V.A. Saleev, Drell-Yan lepton pair production at high energies in the Parton Reggeization Approach, Phys. Rev. D **87** (2013) no.1, 014022
- M. Nefedov and V. Saleev, Off-shell initial state effects, gauge invariance and angular distributions in the Drell-Yan process // Phys. Lett. B **790**, 551-556 (2019)
- M. A. Nefedov and V. A. Saleev, High-Energy Factorization for Drell-Yan process in pp and $p\bar{p}$ collisions with new Unintegrated PDFs // Phys. Rev. D **102** (2020), 114018

Parton Reggeization Approach

MRK factorization, $z_{1,2} \ll 1$ and $\tilde{q}_2^+ \ll \tilde{q}_2^-$, $\tilde{q}_1^- \ll \tilde{q}_1^+$, q_T^2/Q^2 can be arbitrary, $Q^2, Q_T^2 \ll S$

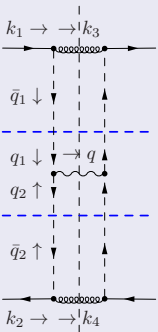


Figure: Diagrammatic representations of squared (M)MRK amplitudes for $q + \bar{q} \rightarrow \gamma^* + 2g$ process. The “small” light-cone momentum components \tilde{q}_1^- and \tilde{q}_2^+ are neglected beyond blue dashed lines. Solid dots denote Fadin-Sherman vertices.

MMRK factorization, from $2 \rightarrow 3$ to $2 \rightarrow 1$ with Reggeized off-shell quarks

We consider auxiliary hard CPM subprocesses:

$$q + \bar{q} \rightarrow \gamma^* + g + g \Rightarrow Q + \bar{Q} \rightarrow \gamma^*$$

$$g + g \rightarrow \gamma^* + q + \bar{q} \Rightarrow Q + \bar{Q} \rightarrow \gamma^*$$

$$q + g \rightarrow \gamma^* + q + g \Rightarrow Q + \bar{Q} \rightarrow \gamma^*$$

$$\Gamma_\mu^{(+ -)}(q_1, q_2) = \gamma_\mu - \hat{q}_1 \frac{n_\mu^-}{q_2^-} - \hat{q}_2 \frac{n_\mu^+}{q_1^+} \quad (1)$$

The Fadin-Sherman vertex [V. S. Fadin and V. E. Sherman, JETP Lett. 23, 599 (1976)] which we have applied to the case of Drell-Yan process for the first time in [M. Nefedov, N. Nikolaev, and V. Saleev, Phys. Rev. D 87, 876 014022 (2013)]

LO MMRK factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)]; M.A. Nefedov, V.A. Saleev, Phys. Rev. **102**, 114018 (2020) for details.

Auxiliary hard CPM subprocess:

$$q(k_1) + \bar{q}(k_2) \rightarrow g(k_3) + \gamma^*(q) + g(k_4),$$

where $k_1^2 = 0$, $k_1^- = 0$, $k_2^2 = 0$, $k_2^+ = 0$.

Kinematic variables ($0 < z_{1,2} < 1$):

$$z_1 = \frac{k_1^+ - k_3^+}{k_1^+}, \quad z_2 = \frac{k_2^- - k_4^-}{k_2^-},$$

Two limits where $|\overline{\mathcal{M}}|^2$ factorizes:

- 1 **Collinear limit:** $\mathbf{k}_{T3,4}^2, \mathbf{q}_T^2 \ll Q^2$, $z_{1,2}$ – arbitrary,
- 2 **Multi-Regge limit:** $z_{1,2} \ll 1$, $\mathbf{k}_{T3,4}^2, \mathbf{q}_T^2$ – arbitrary.

LO MMRK factorization formula

Multi-Regge limit: $z_{1,2} \ll 1$, $\mathbf{k}_{T3,4}^2$ – arbitrary:

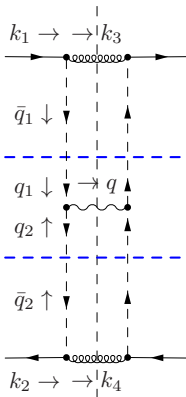
$$|\overline{\mathcal{M}}|^2_{\text{MRK}} \simeq \frac{4g_s^4}{\mathbf{k}_{T3}^2 \mathbf{k}_{T4}^2} \tilde{P}_{qq}(z_1) \tilde{P}_{qq}(z_2) \frac{|\overline{\mathcal{A}_{PRA}}|^2}{z_1 z_2},$$

where $|\overline{\mathcal{A}_{PRA}}|^2$ is the **gauge-invariant** amplitude $Q_+(q_1) + \bar{Q}_-(q_2) \rightarrow \gamma^*(q)$ with **Reggeized (off-shell)** partons in the initial state.

In MRK regime we can use Lipatov's Effective Field Theory of Reggeized gluons and Reggeized quarks to obtain \mathcal{A}_{PRA}

- L. N. Lipatov, Nucl. Phys. B452, 369 (1995).
- L. N. Lipatov and M. I. Vyazovsky, Nucl. Phys. B597, 399 (2001).
- V. S. Fadin and V. E. Sherman, JETP Lett. 23, 599 (1976)
- E. N. Antonov, L.N. Lipatov, E.A. Kuraev, and I. O. Cherednikov, Nucl. Phys. B721, 111 (2005).
- M.A. Nefedov, ReggeQCD model-file for FeynArts (2017-2019).

LO MMRK factorization formula



Modified MRK approximation: $z_{1,2}$ and $\mathbf{k}_{T3,4}^2$ – arbitrary

:

$$|\overline{\mathcal{M}}|^2_{\text{MMRK}} \simeq \frac{4g_s^4}{\tilde{q}_1^2 \tilde{q}_2^2} P_{qq}(z_1) P_{qq}(z_2) \frac{|\overline{\mathcal{A}_{PRA}}|^2}{z_1 z_2},$$

where $\tilde{q}_{1,2}^2 = \mathbf{q}_{T1,2}^2 / (1 - z_{1,2})$, $P_{qq}(z)$ – DGLAP $q \rightarrow q$ splitting function. This factorization formula has correct **collinear** and **Multi-Regge** limits!

Factorization formula

Substituting the $|\overline{\mathcal{M}}|^2_{\text{MMRK}}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_q^{(\text{tree-level})}(x_1, t_1, \mu_Y^2) \times \\ \times \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_{\bar{q}}^{(\text{tree-level})}(x_2, t_2, \mu_Y^2) \cdot d\hat{\sigma}_{\text{PRA}},$$

where $x_1 = q_1^+ / P_1^+$, $x_2 = q_2^- / P_2^-$, $\Phi^{(\text{tree-level})}(x, t, \mu_Y^2)$ – “tree-level” **unintegrated PDFs**. The partonic cross-section in PRA is written as:

$$d\hat{\sigma}_{\text{PRA}} = \frac{|\overline{\mathcal{A}_{\text{PRA}}}|^2}{2Sx_1x_2} \cdot (2\pi)^4 \delta \left(\frac{1}{2} \left(q_1^+ n_- + q_2^- n_+ \right) + q_{T1} + q_{T2} - q(\gamma^*) \right) d\Phi(\gamma^*).$$

Note that **flux-factor** $2x_1x_2S$ for **off-shell** initial-state partons should be used.

Parton Reggeization Approach

Tree-level unintegrated PDFs:

$$\Phi_i^{(\text{tree-level})}(x, t, \mu_F, \mu_Y^2) = \frac{\alpha_s(\mu_R)}{2\pi} \frac{1}{t} \sum_{j=q, \bar{q}, g} \int_x^1 dz P_{ij}(z) F_j\left(\frac{x}{z}, \mu_F^2\right) \theta(\Delta(t, \mu_Y^2) - z),$$

where $F_i(x, \mu_F^2) = x f_j(x, \mu_F^2)$. The θ -functions enforce the rapidity-ordering between particles in the final-state $y_3 > y_{\gamma^*} > y_4$, for our MMRK approximation for the squared amplitude and kinematics to be applicable. The KMRW cutoff function defines the region of applicability of MMRK:

$$\Delta(t, \mu) = \frac{\sqrt{\mu}}{\sqrt{\mu} + \sqrt{t}}.$$

Parton Reggeization Approach

Reggeized hadronic tensor

$$\begin{aligned}
 W_{\mu\nu}^{MMRK} &= \sum_{k,l} \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_k^{(\text{tree-level})}(x_1, \mathbf{q}_{T1}, \mu_Y) \\
 &\times \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_l^{(\text{tree-level})}(x_2, \mathbf{q}_{T2}, \mu_Y) \\
 &\times (2\pi)^4 \delta(q_1 + q_2 - q) \frac{w_{\mu\nu}^{MRK}}{2Sx_1x_2}
 \end{aligned}$$

Reggeized partonic tensor, $Q(\tilde{q}_1) + \bar{Q}(\tilde{q}_2) \rightarrow \gamma^*(q)$

$$w_{\mu\nu}^{MRK} = \frac{(4\pi\alpha)}{4N_c} e_q^2 \text{tr} \left[\left(\frac{q_1^+}{2} \hat{n}_- \right) \Gamma_\mu^{(+)}(q_1, q_2) \left(\frac{q_2^-}{2} \hat{n}_+ \right) \Gamma_\nu^{(-)}(q_1, q_2) \right]$$

Unintegrated PDFs with exact normalization

To resolve a divergence problem of $\Phi_i^{(\text{tree-level})}(x, t, \mu_Y^2)$

we follow the standard definition of the UPDF in BFKL formalism and require that:

$$\int_0^{\mu^2} dt \Phi_i(x, t, \mu^2) = F_i(x, \mu^2),$$

which is equivalent to:

$$\Phi_i(x, t, \mu^2) = \frac{d}{dt} [T_i(t, \mu^2, x) F_i(x, t)],$$

where $T_i(t, \mu^2, x)$ is usually referred to as *Sudakov formfactor*, satisfying the boundary conditions $T_i(t = 0, \mu^2, x) = 0$ and $T_i(t = \mu^2, \mu^2, x) = 1$.

We ask exact equivalence between last ones and following (*KMRW*) prescription:

$$\Phi_i(x, t, \mu_Y^2) = \frac{\alpha_s(t)}{2\pi} \frac{T_i(t, \mu^2, x)}{t} \sum_{j=q, \bar{q}, g} \int_x^1 dz P_{ij}(z) F_j\left(\frac{x}{z}, t\right) \theta(\Delta(t, \mu_Y^2) - z),$$

Unintegrated PDFs with exact normalization

The solution for Sudakov formfactor

$$T_i(t, \mu^2, x) = \exp \left[- \int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} (\tau_i(t', \mu^2) + \Delta\tau_i(t', \mu^2, x)) \right]$$

with

$$\tau_i(t, \mu^2) = \sum_j \int_0^1 dz z P_{ji}(z) \theta(\Delta(t, \mu^2) - z),$$

$$\Delta\tau_i(t, \mu^2, x) = \sum_j \int_0^1 dz \theta(z - \Delta(t, \mu^2)) \left[z P_{ji}(z) - \frac{F_j(\frac{x}{z}, t)}{F_i(x, t)} P_{ij}(z) \theta(z - x) \right].$$

MMRK PDFs versus KMRW PDFs

The Sudakov formfactor without the $\Delta\tau_i$ -term in the exponent is similar to the Sudakov formfactor of LO KMRW UPDF but with a numerically-important difference that in our MMRK approach, **the rapidity-ordering condition is imposed both on quarks and gluons, while in KMRW-approach it is imposed only on gluons.**

Z-boson production in the LO PRA

M. A. Nefedov and V. A. Saleev, High-Energy Factorization for Drell-Yan process in pp and $p\bar{p}$ collisions with new Unintegrated PDFs // Phys. Rev. D **102** (2020), 114018

$$Q + \bar{Q} \rightarrow Z(\gamma^*) \rightarrow \mu^+ + \mu^-$$

$$\begin{aligned} |\overline{\mathcal{M}}_{q\bar{q}}|^2 = & \frac{(4\pi\alpha)^2}{4N_c} \left\{ \frac{e_q^2}{\hat{s}^2} M_{VV,VV}^2 + \frac{1}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \left[2 \left(C_{AA}^{(q)} C_{VV}^{(q)} + C_{AV}^{(q)} C_{VA}^{(q)} \right) M_{VV,AA}^2 \right. \right. \\ & + \left. \left((C_{AA}^{(q)})^2 + (C_{AV}^{(q)})^2 + (C_{VA}^{(q)})^2 + (C_{VV}^{(q)})^2 \right) M_{VV,VV}^2 + \left((C_{AA}^{(q)})^2 + (C_{VA}^{(q)})^2 \right) \Delta M^2 \right] \\ & \left. + \frac{4(\hat{s} - M_Z^2)e_q}{\hat{s}((\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)} \left[C_{VV}^{(q)} M_{VV,VV}^2 + C_{AA}^{(q)} M_{VV,AA}^2 \right] \right\}, \end{aligned}$$

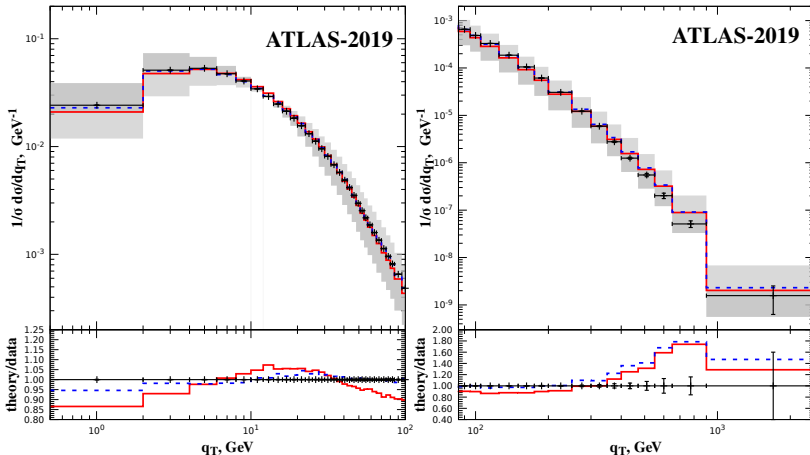
where $C_{ab}^{(q)} = C_q^{(a)} C_{Zl}^{(b)}$ is the product of quark and lepton coupling-factors (17), while

$$\begin{aligned} M_{VV,VV}^2 = & \frac{8}{q_1^+ q_2^-} [2(p_1^- q_1^+)^2 (q_1^+ q_2^- - t_1) - 2p_1^- q_1^+ q_2^- (q_1^+ (m_l^2 - \hat{t} - t_1) + p_1^+ (2q_1^+ q_2^- + \hat{t} + \hat{u} - 2m_l^2)) \\ & + q_2^- (2(p_1^+)^2 q_2^- (q_1^+ q_2^- - t_2) + 2p_1^+ q_1^+ q_2^- (t_2 + \hat{u} - m_l^2) + q_1^+ (2m_l^2 (q_1^+ q_2^- + \hat{s}) - \hat{s}(\hat{t} + \hat{u})))], \\ M_{VV,AA}^2 = & 8\hat{s}(\hat{u} - \hat{t} + 2(p_1^+ q_2^- - p_1^- q_1^+)), \\ \Delta M^2 = & 32m_l^2(\hat{t} + \hat{u} - 2m_l^2), \end{aligned}$$

Z-boson production in the LO PRA

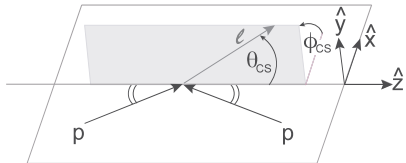
The total cross section, $80 < M_{l\bar{l}} < 100$ GeV

$$K_{LO} = \frac{\sigma^{exp}}{\sigma_{LO}^{theory}} \simeq 1.7$$



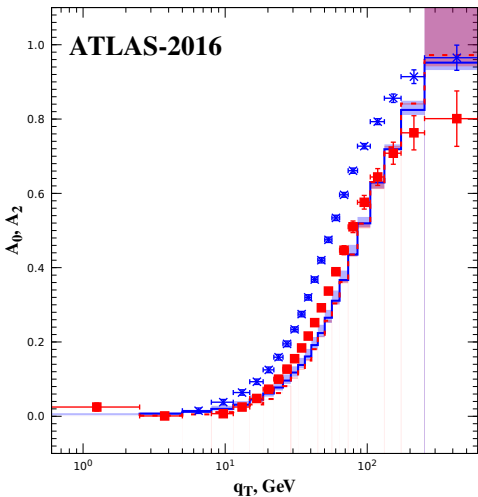
Z-boson production in the LO PRA

To obtain angular coefficients A_n we use the analytical harmonic-projectors method

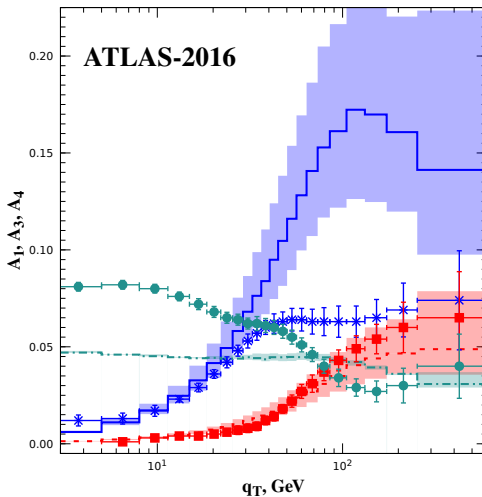


$$\begin{aligned}
 \frac{d\sigma}{dQd\mathbf{q}_T^2 dy d\Omega_l} &= \frac{3}{16\pi} \frac{d\sigma}{dQd\mathbf{q}_T^2 dy} \left\{ (1 + \cos^2 \theta_l) + \frac{A_0}{2} (1 - 3 \cos^2 \theta_l) \right. \\
 &+ A_1 \sin 2\theta_l \cos \phi_l + \frac{A_2}{2} \sin^2 \theta_l \cos 2\phi_l + A_3 \sin \theta_l \cos \phi_l + A_4 \cos \theta_l \\
 &+ \left. A_5 \sin^2 \theta_l \sin 2\phi_l + A_6 \sin 2\theta_l \sin \phi_l + A_7 \sin \theta_l \sin \phi_l \right\},
 \end{aligned}$$

Z-boson production in the LO PRA

Angular coefficients A_0 (blue) and A_2 (red)

Z-boson production in the LO PRA

Angular coefficients A_1 (blue), A_3 (red) and A_4 (green)

Z-boson production in the NLO CPM

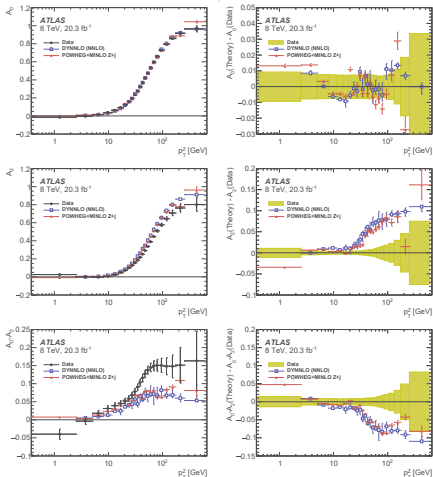
Angular coefficients $A_{0,2}$ 

Figure 13: Distributions of the angular coefficients A_0 (top), A_2 (middle) and $A_0 - A_2$ (bottom) as a function of p_T^Z . The results from the y^Z -integrated measurements are compared to the DYNNLO and POWHEG MiNLO predictions (left). The differences between the two calculations and the data are also shown (with the shaded band)

Z-boson production in the NLO CPM

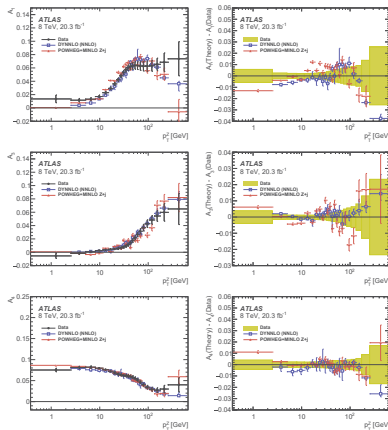
Angular coefficients $A_{1,3,4}$ 

Figure 14: Distributions of the angular coefficients A_1 (top), A_3 (middle) and A_4 (bottom) as a function of p_T^Z . The results from the p_T^Z -integrated measurements are compared to the DYNLO and Powheg MINLO predictions (left). The differences between the two calculations and the data are also shown (right), with the shaded band around zero representing the total uncertainty in the measurements. The error bars for the calculations show the total uncertainty for DYNLO, but only the statistical uncertainties for Powheg MINLO (see text).

Z-boson production in the NLO* PRA

Quark-gluon (Compton-like) scattering is studied

$$R + Q \rightarrow Z + q \rightarrow q + \mu^+ + \mu^-$$

$$R + \bar{Q} \rightarrow Z + \bar{q} \rightarrow \bar{q} + \mu^+ + \mu^-$$

Quark-antiquark annihilation is neglected due to smallness (+ necessity of virtual corrections)

$$\bar{Q} + Q \rightarrow Z + g \rightarrow g + \mu^+ + \mu^-$$

KaTie

MC parton-level event generator KaTie

- A. van Hameren, “KaTie : For parton-level event generation with k_T -dependent initial states,” *Comput. Phys. Commun.* **224** (2018), 371-380
- TMD PDFs from TMDlib: <https://tmdlib.hepforge.org/>
- Output files are in LHEF format (Les Houches Event File)

Abstract

KATIE is a parton-level event generator for hadron scattering processes that can deal with partonic initial-state momenta with an explicit transverse momentum dependence causing them to be space-like. Provided with the necessary transverse momentum dependent parton density functions, it calculates the tree-level off-shell matrix elements and performs the phase space importance sampling to produce weighted events, for example in the Les Houches Event File format. It can deal with arbitrary processes within the Standard Model, for up to at least four final-state particles. Furthermore, it can produce events for single-parton scattering as well as for multi-parton scattering.

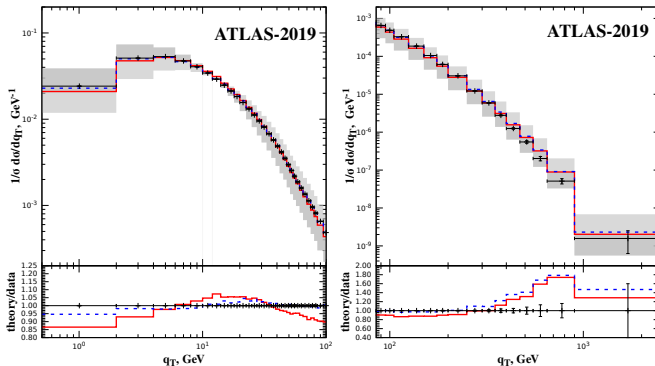
KaTie

MC parton-level event generator KaTie

- 1 The approach to obtaining gauge invariant amplitudes with off-shell initial state partons in scattering at high energies was proposed in the Ref. [A. van Hameren, P. Kotko, and K. Kutak, JHEP 01, 078 (2013), 1211.0961]. The method is based on the use of spinor amplitudes formalism and recurrence relations of the Britto-Cachazo-Feng-Witten (BCFW) type.
- 2 This formalism for numerical amplitude generation is equivalent to amplitudes built according to Feynman rules of the Lipatov EFT at the level of tree diagrams. It has been tested numerically at least for $2 \rightarrow 2$ and $2 \rightarrow 3$.
- 3 The accuracy of our numerical calculations using KaTie for total proton-proton cross sections is equal to 0.1%.

Results in NLO* PRA

$$K_{NLO} = \frac{\sigma^{exp}}{(\sigma_{LO}^{theory} + \sigma_{NLO}^{theory})} \simeq 1.1$$



Results in NLO* PRA

$$\begin{aligned}
E^\ell(CS) &= \frac{M}{2} \\
p_x^\ell(CS) &= \frac{M}{2} \sin \theta_{CS}^\ell \cos \phi_{CS}^\ell = \frac{(p_T^\ell)^2 - (p_T^{\bar{\ell}})^2}{2p_T^\ell \sqrt{1+r^2}} \equiv \frac{(p_x^\ell - p_x^{\bar{\ell}})}{2\sqrt{1+r^2}} \\
p_y^\ell(CS) &= \frac{M}{2} \sin \theta_{CS}^\ell \sin \phi_{CS}^\ell = p_y^\ell \equiv -p_y^{\bar{\ell}} \\
p_z^\ell(CS) &= \frac{M}{2} \cos \theta_{CS}^\ell = \frac{(p_z^\ell E^{\bar{\ell}} - p_z^{\bar{\ell}} E^\ell)}{M\sqrt{1+r^2}}.
\end{aligned}$$

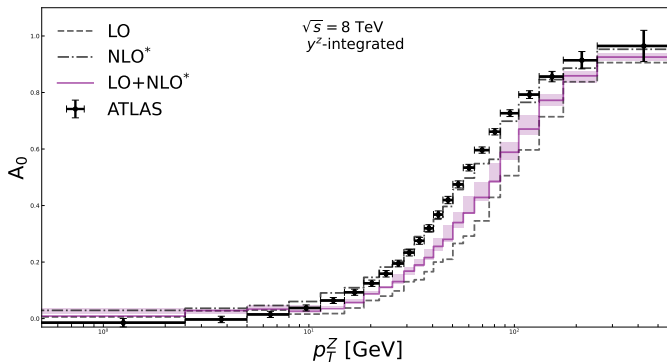
$$\langle P(\theta_l, \phi_l) \rangle = \frac{1}{N_{ev}} \sum_k^{N_{ev}} P(\theta_l^{(k)}, \phi_l^{(k)})$$

$$\langle \frac{1}{2}(1 - 3 \cos^2 \theta) \rangle = \frac{3}{20}(A_0 - \frac{2}{3}); \quad \langle \sin 2\theta \cos \phi \rangle = \frac{1}{5}A_1; \quad \langle \sin^2 \theta \cos 2\phi \rangle = \frac{1}{10}A_2;$$

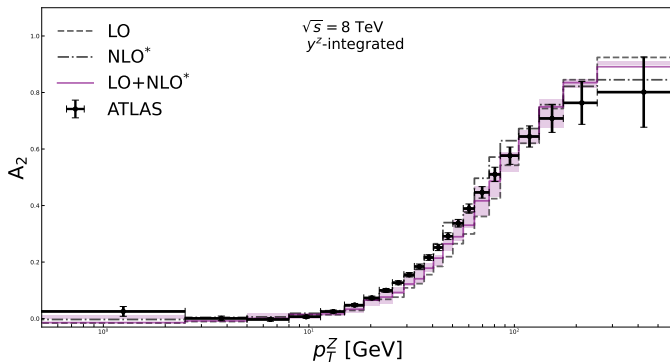
$$\langle \sin \theta \cos \phi \rangle = \frac{1}{4}A_3; \quad \langle \cos \theta \rangle = \frac{1}{4}A_4; \quad \langle \sin^2 \theta \sin 2\phi \rangle = \frac{1}{5}A_5;$$

$$\langle \sin 2\theta \sin \phi \rangle = \frac{1}{5}A_6; \quad \langle \sin \theta \sin \phi \rangle = \frac{1}{4}A_7.$$

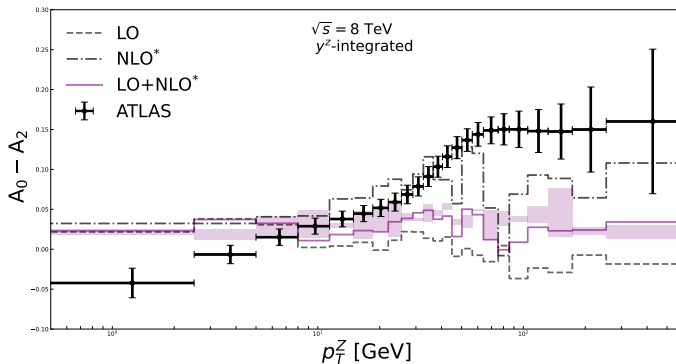
Results in NLO* PRA



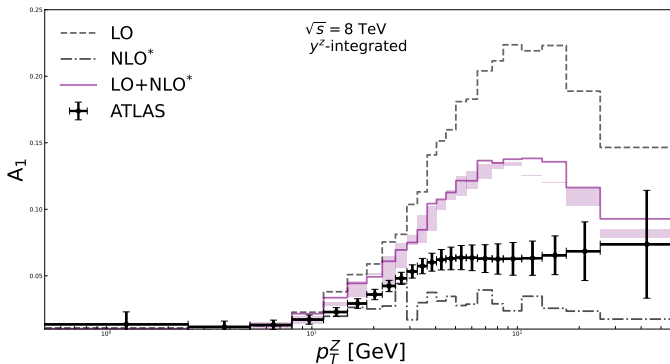
Results in NLO* PRA



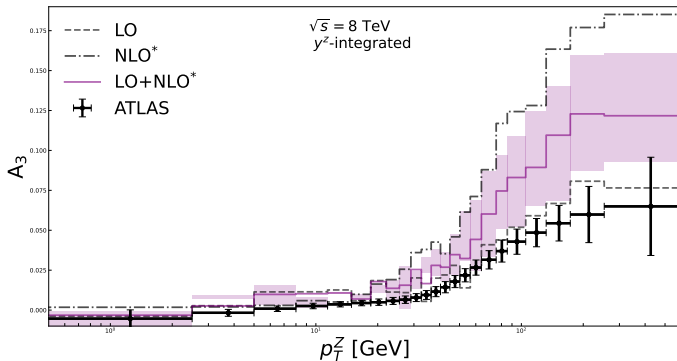
Results in NLO* PRA



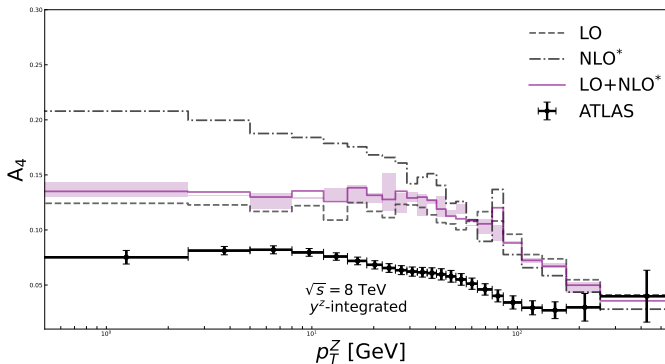
Results in NLO* PRA



Results in NLO* PRA



Results in NLO* PRA



Conclusions

- Z-boson production is studied in NLO* approximation of PRA for the first time
- MC generator KaTie is used to calculate cross section and lepton angular coefficients A_n for the first time
- In the PRA, $K_{LO} = 1.7$ and $K_{NLO} = 1.1$
- NLO* correction improves theoretical predictions for $A_{0,1,2}$
- NLO* correction make worse theoretical predictions for $A_{3,4}$
- More accurate study is needed, y^Z -dependence

Thank you for your attention!