Axial Current and Theory of Equilibrium

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Outline of the talk

The talk is intended to justify summary:

there exists a kind of theory which unifies effects, or interactions, surviving in the limit of the Newton constant tending to zero, $G_N \to 0$.

It includes, preliminary :
– theory of equilibrium
– using accelerated frames, or covariant derivatives ∇_α
–Feynman graphs for loop effects for matter in external grav. field, calculated to lowest order in gravity and in appproximation of flat space

Outline of the Talk (cont'd)

Part I. Gauge Chiral Anomaly and Equilibrium.

Part II. Gravitational Chiral Anomaly and Equilibrium.

III Conclusions: Theory in the limit of vanishing Newton Constant.

Original papers in collaboration
with G.Yu.Prokhorov and O.V. Teryaev

Theory of Equilibrium

First University lecture on physics:

Three types of equilibrium: stable, unstable, neutral

Respectively, equilibrium position: on bottom of the potential, on top of potential, flat potential

(+steady motion)

Lecture of I.V. Obreimov

Density Operator

In quatum statistics matrix elements are averaged with density operator $\hat{\rho}$

$$\hat{\rho} \;=\; \exp\left(-\hat{H}_{eff}/T\right)$$

where $\hat{H}_{eff}$ is built on conserved quantities: charges $\hat{Q}_i$, angular momentum $\hat{\vec{J}}$ (Landau-Lifshitz) + boost $\hat{\vec{K}}$ (F. Becattini)

$$\hat{H}_{eff} \;=\; \hat{H}_0 \;-\; \Sigma_i \mu_i \hat{Q}_i - \vec{\Omega}\hat{\vec{J}} - \vec{a}\hat{\vec{K}}$$

where $\vec{\Omega}$ angular velocity, $\vec{a}$ is acceleration
$\hat{H}_{eff}$ picks up maximum entropy state
while in case of $\hat{H}_0$ we look for for minimum of energy

Adaptation to hydrodynamics

In case there is single chiral current conserved:

$$\delta \hat{H} = -\mu_V \hat{Q}_V - \mu_A \hat{Q}_A \quad \rightarrow \quad \delta L(x) \;=` \mu_V u_\alpha j^\alpha_V(x) + \mu_A j^\alpha_A(x)$$

where $\partial_\alpha j^\alpha(x) = 0$

$$eA^V_\alpha \rightarrow eA^V_\alpha + \mu_V u_\alpha; \qquad g_A A^A_\alpha \rightarrow g_A A_\alpha + \mu_A u_\alpha$$

where velocity u_α is treated as an external field.

We are all set to evaluate one-loop graphs
in theory of equilibrium
in terms of fundamental (i.e. field-theoretic) currents

Chiral magnetic effect

Chiral anomaly on fundamental level :

$$Q^A = Q^A_{naive} + \frac{e^2}{2\pi^2}\mathcal{H}; \qquad \frac{dQ^A}{dt} = 0$$

where $\mathcal{H} = \int d^3x \vec{A}\vec{B}$

Using the rules above convert it to:

$$\vec{J}^{el} = \frac{e^2\mu_5}{2\pi^2}\vec{B}$$

where μ_5 is axial chemical potential, $\vec{B}$ is magnetic field,

Chiral vortical effect

There are further terms generated in hydrodynamics. in particular, chiral vortical effect (CVE)

$$\vec{J}_5 = \frac{\mu_V^2}{2\pi^2}\vec{\Omega}$$

which is most interesting since it survives in absence of $\vec{E}, \vec{B}$ and is to be conserved. No place for a new Noether current (no free symmetry in field theory). The way out: conservation is specific for absence of dissipation or for ideal fluid

Extension of symmetry

$$Q^A_{fluid\ \ helicity} = \frac{1}{4\pi^2}\int j^0_{fluid\ \ helicity}$$

$$j^\alpha_{fluid\ \ helicity} = \mu_V^2 \epsilon^{\alpha\beta\gamma\delta} u_\beta \partial_\gamma u_\delta$$

The current is indeed conserved for ideal fluid.

The conservation of the current is manifestation of diffeomorphism

Remarkable: started from chiral symmetry of field theory. For consistency of radiative corrections the symmetry of the system is to be enhanced.

To get CME back from CVE replace $\partial_\alpha \;\rightarrow\; \nabla_\alpha$
Amusing non-relativistic analogs to chiral anomalies

$$\partial_\alpha J^\alpha_5 \;\sim\; \vec{E}\cdot\vec{B} \quad (as\text{``}usual'')$$

$$\partial_\alpha J^\alpha_{el} \;\sim\; \vec{E}_5\cdot\vec{B} \equiv \; \vec{\nabla}\mu_{5,non-rel}\cdot\vec{B}$$

where $\vec{E}\equiv\vec{\nabla}\mu$, $\vec{E}_5\equiv\vec{\nabla}\mu_5$
Here we summarize our findings:

- The newly discovered anomalies are not related to any ultraviolet divergences.
- We started with a non-relativistic picture for fluid and match standard UV anomalies only up to a factor
- non-conservation of electric charge is (unusual wording)

A few references on ideal fluid and "anomalies"

"On consistency of hydrodynamic approximation for chiral media", A. Avdoshkin , V.P. Kirilin , A.V. Sadofyev , V.I. Zakharov, e-Print: 1402.3587 [hep-th].
A. G. Abanov and P. B. Wiegmann, "Anomalies in fluid dynamics: flows in a chiral background via variational principle," [arXiv:2207.10195 [hep-th]] + 3 other papers (2021)
"Divergence anomaly and Schwinger terms: Towards a consistent theory of anomalous classical fluids", Arpan Krishna Mitra and Subir Ghosh, 2111.00473 [hep-th].

Conclusions on Part I

What is chiral fluid of massless quarks at short distances

might look as ideal non-relativistic fluid at large distances

since anomalies are similar
(variation of' 't Hooft consistency condition)

There are two similar chains of anomalies,
IR and UV sensitive, respectively,
and starting frrom different symmetries (chiral vs diffeomorphism)

Remarkable: QGP is both chiral and (close to) ideal

Gravitational anomaly and hydrodynamics

General Relativity is built on non-trivial metric tensor

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$$

Gauge transformation, analog of

$$\delta A_\mu = \partial_\mu \Lambda$$

is

$$\delta h_{\mu\nu} = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$$

which is also a signature of diffeomorphism

Gauge invariant field, analog of $\vec{E}, \vec{B}$, is Riemann tensor $R_{\alpha\beta\gamma\delta}$ which involves $(\partial_\alpha \partial_\beta h_{\mu\nu})$ *and* $(\partial_\alpha h_{\mu\nu})^2$

Gravitational anomaly is built on the curvature $R_{\alpha\beta\gamma\delta}$

$$\nabla_\alpha J^\alpha_A = (const) R\tilde{R}$$

Generalities (cont'd)

Equivalence principle: physics in accelerated frame is imitated by a nontrivial gravitational field
Two basic non-inertial frames, considered by Einstein:
accelerated and rotated frames
(corresponding $h_{\mu\nu}$ are easy to identify)
But only considering acceleration brought a new principle.
Rotation is reduced to Lorentz

In case of fluids acceleration and rotation 4-vectors are given by:

$$a_\mu = u^\alpha \partial_\alpha u_\mu, \quad \omega_\mu = (1/2)\epsilon_{\mu\nu\rho\sigma} u^\nu \partial^\rho u^\sigma \quad (u^\nu \text{ is } 4\text{-velocity}) \tag{1}$$

Plasma produced in heavy-ion collisions
both accelerated and rotated

Duality of statistical and gravitational approaches

Properties of fluids in equilibrium are evaluated statistically in terms of density operator, or effective interaction:

$$\hat{H}_{eff} = -\vec{\Omega}\hat{\vec{J}} - \vec{a}\hat{\vec{K}}$$

where $\hat{\vec{K}}, \hat{\vec{J}}$ are operators of boost and angular momentum and $\vec{a}$, $\vec{\Omega}$ are acceleration and angular velocity.

Cont'd

In field theory, gravitational interaction is described by fundamental interaction Lagrangian:

$$\delta L = -\frac{1}{2}\theta^{\alpha\beta} h_{\alpha\beta}$$

where $\theta^{\alpha\beta}$ is the energy-momentum tensor of matter, $h_{\alpha\beta}$ is the gravitational potential, also accommodating $\vec{\Omega}_{grav}, \vec{a}_{grav}$.

Cont'd

Furthermore, one evaluates "external probes", $< \theta^{\alpha\beta} >, < J_5^{\alpha} >$ within both approaches, statistical and gravitational. The results compared for the same values of a, Ω. However, within statistical approach $\vec{a}, \vec{\Omega}$ are kinematic. In gravity they are dynamic.

As a generalization of the equivalence principle, the results are to be the same.

Cont'd

The duality is confirmed on a number of examples.
The reason for similarity of results is that both theories are entirely built on conserved quantities

Note, equivalence of entropic and gravitational forces was promoted by E. Verlinde (2011), (see also D.V. Fursaev) within a much more ambitious framework. Our results seem to be safer, on the other hand.

Cont'd

Independent reason for existence of bulk/surface duality was given, in particular, by E.Witten, provided that bulk is diffeomorphic. Applied mostly to de Sitter space. Generically, the results are similar. In particular the "surface"description is in terms of entropic state, "bulk" - in terms of field-theoretic state.

We found direct analogs of this approach in our case.

Anomalous currents in absence of anomaly

Recollect, we derived chiral vortical current ($\vec{j}^A \sim \mu^2 \vec{\Omega}$) proportinal to anomaly in absence of external e-m fields. We found it applying perturbative equilibrium theory. It was crucial also to have dissipation-free fluid, so that the current woud flow as determined by initial conditions.

The same approach works in the gravity-related statistical approach. The result, e.g., is:

$$\vec{j}_5 \; = \; (const) a^2 \vec{\Omega}$$

It is also crucial that it is in accelerated frame.

Unruh effect (a reminder)

Observer moving with acceleration a with respect to Minkowskian vacuum sees thermal distribution of particles with temperature

$$T_{Unruh} = \frac{a}{2\pi} \qquad (\textit{quantum effect})$$

while an observer at rest sees no particles.

For many, it sounds disappointing that the Unruh effect is a kind of observer-dependent. However, it is dynamical: By virtue of the equivalence principle, accelerated frame equivalent to vacuum in strong gravitational field resulting in the same acceleration a. Naturally, such a field produces particles

Removing ultraviolet

Currents in absence of external gravity refer in fact to accelerated frames describing results of measurements on the Unruh sample of particles

We start with Minkowski space, have no particles, no currents,

go to accelerated frame and work with UV safe sample.

Apply covariant derivatives, calculate divergence of axial current. Everything is UV finite.
Find same anomaly as evaluated in UV sensitive way.

Conclusions

there exists a kind of theory which unifies effects, or interactions, surviving in the limit of the Newton constant tending to zero, $G_N \to 0$.

It includes, preliminary :

– theory of equilibrium

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This approach might have interesting applications