

Influence of relativistic rotation on QCD properties

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6 September, 2023

In collaboration with

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- ▶ A. Roenko
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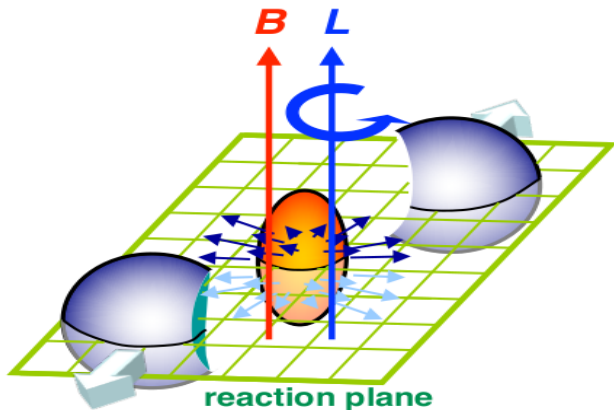
JETP Lett. 112, 9–16 (2020); JETP Lett. 117, 9, 639-644 (2023); Phys.Rev.D 103 (2021) 9, 094515; PoS LATTICE2022 (2023) 190; PoS LATTICE2021 (2022) 125; e-Print: 2303.03147

Outline:

- ▶ Introduction
- ▶ Critical temperatures
- ▶ Moment of inertia of QGP
- ▶ Inhomogeneous phase transitions in QGP
- ▶ Conclusion

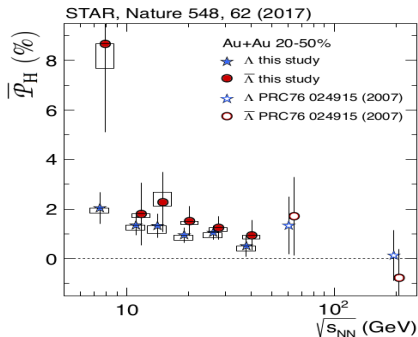
More details can be found in the talk of A. Roenko

Rotation of QGP in heavy ion collisions



- ▶ QGP is created with non-zero angular momentum in non-central collisions

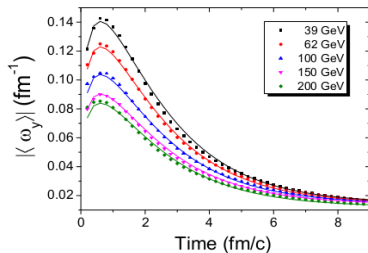
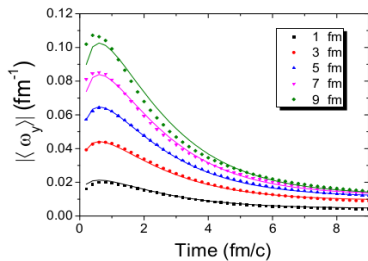
Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- ▶ $\Omega = (P_\Lambda + P_{\overline{\Lambda}}) \frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- ▶ $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 9 \times 10^{21} s^{-1}$)
- ▶ Relativistic rotation of QGP

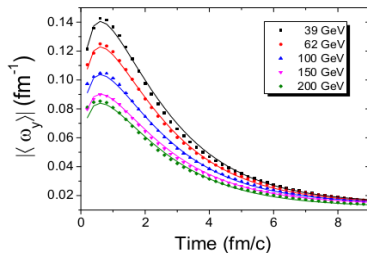
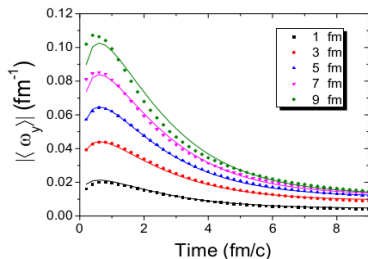
Rotation of QGP in heavy ion collisions



Hydrodynamic simulations (Phys.Rev.C 94, 044910 (2016))

- ▶ Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- ▶ $\Omega \sim (4 - 28)$ MeV
- ▶ Relativistic rotation of QGP

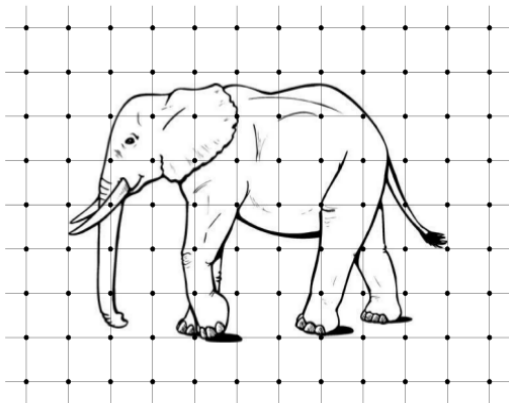
Rotation of QGP in heavy ion collisions



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How relativistic rotation influences QCD?



Lattice simulation

- ▶ Allows to study strongly interacting systems
- ▶ Based on the first principles of quantum field theory
- ▶ Powerful due to modern supercomputers and algorithms

Lattice simulation of QCD

- ▶ We study QCD in thermodynamical equilibrium
- ▶ The system is in the finite volume
- ▶ Calculation of the partition function

$$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s,\dots} \det(\hat{D}_i(U) + m_i)$$

- ▶ Monte Carlo calculation of the integral
- ▶ Carry out continuum extrapolation $a \rightarrow 0$
- ▶ Uncertainties (discretization and finite volume effects) can be systematically reduced
- ▶ The first principles based approach. No assumptions!
- ▶ Parameters: g^2 and masses of quarks

Study of rotating QGP

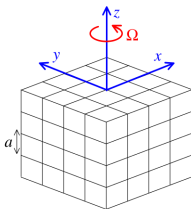
- ▶ Our aim: study rotating QCD within lattice simulations
- ▶ Rotating QCD at thermodynamic equilibrium
 - ▶ At the equilibrium the system rotates with some Ω
 - ▶ The study is conducted in **the reference frame which rotates with QCD matter**
 - ▶ QCD in external gravitational field
- ▶ **Boundary conditions are very important!**

Details of the simulations

- ▶ Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- ▶ The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- ▶ Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- ▶ Partition function (\hat{H} is conserved)

$$Z = \text{Tr} \exp [-\beta \hat{H}] = \int DA \exp [-S_G]$$

- ▶ Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2} \int d^4x \sqrt{g_E} g_E^{\mu\nu} g_E^{\alpha\beta} F_{\mu\alpha}^{(a)} F_{\nu\beta}^{(a)}$$

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \text{Tr} [(1 - r^2 \Omega^2) F_{xy}^a F_{xy}^a + (1 - y^2 \Omega^2) F_{xz}^a F_{xz}^a +$$

$$+(1 - x^2 \Omega^2) F_{yz}^a F_{yz}^a + F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz}^a F_{zy}^a]$$

Details of the simulations

- ▶ *Ehrenfest–Tolman effect*: In gravitational field the temperature is not constant in space at thermal equilibrium

$$T(r)\sqrt{g_{00}} = \text{const} = 1/\beta$$

$$T(r)\sqrt{1 - r^2\Omega^2} = 1/\beta$$

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- ▶ **One could expect that rotation decreases the critical temperature**

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- ▶ Rotation effectively heats the system from the rotation axis to the boundaries $T(r) > T(r = 0)$
- ▶ **One could expect that rotation decreases the critical temperature**
- ▶ We use the designation $T = T(r = 0) = 1/\beta$

Details of the simulations

Boundary conditions

▶ Periodic b.c.:

▶ $U_{x,\mu} = U_{x+N_i,\mu}$

▶ Not appropriate for the field of velocities of rotating body

▶ Dirichlet b.c.:

▶ $U_{x,\mu}|_{x \in \Gamma} = 1, \quad A_\mu|_{x \in \Gamma} = 0$

▶ Violate Z_3 symmetry

▶ Neumann b.c.:

▶ Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$

▶ *The dependence on boundary conditions is the property of all approaches*

▶ *One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening*

Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \text{Tr} \left[(1 - r^2 \Omega^2) F_{xy}^a F_{xy}^a + (1 - y^2 \Omega^2) F_{xz}^a F_{xz}^a + \right. \\ \left. + (1 - x^2 \Omega^2) F_{yz}^a F_{yz}^a + F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a - \right. \\ \left. - 2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz}^a F_{zy}^a \right]$$

- ▶ The Euclidean action has imaginary part (**sign problem**)
- ▶ Simulations are carried out at imaginary angular velocities $\Omega \rightarrow i\Omega_I$
- ▶ The results are analytically continued to real angular velocities
- ▶ This approach works up to sufficiently large Ω

Details of the simulations: critical temperatures

Confinement/deconfinement phase transition

- ▶ Polyakov line

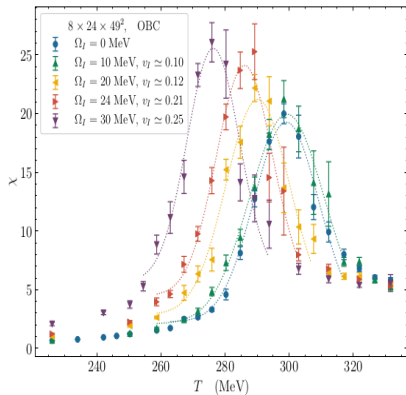
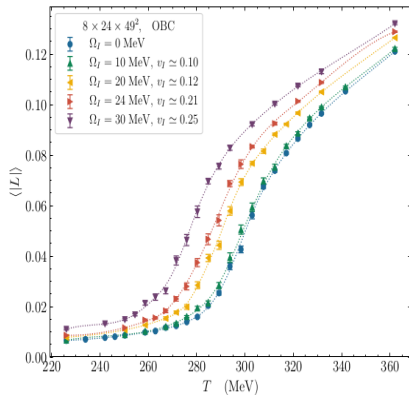
$$L = \left\langle \text{Tr} \mathcal{T} \exp \left[ig \int_{[0,\beta]} A_4 dx^4 \right] \right\rangle$$

- ▶ Susceptibility of the Polyakov line

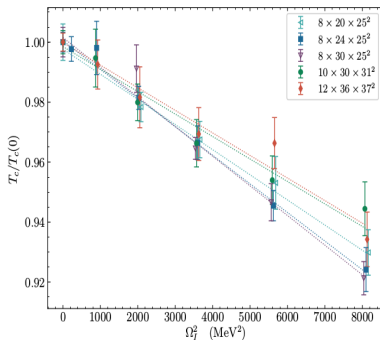
$$\chi = N_s^2 N_z (\langle |L|^2 \rangle - \langle |L| \rangle^2)$$

- ▶ T_c is determined from Gaussian fit of the $\chi(T)$

Results of the calculation (Neumann b.c.)



Results of the calculation

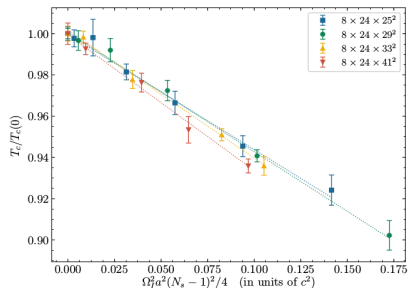
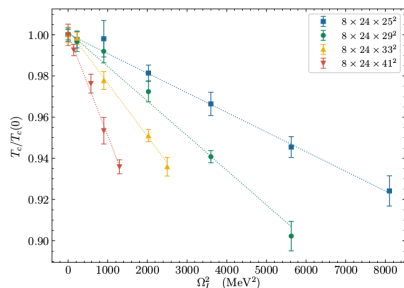


- ▶ The results can be well described by the formula ($C_2 > 0$)

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2 \Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2 \Omega^2$$

- ▶ **The critical temperature rises with angular velocity**
- ▶ The results weakly depend on lattice spacing and the volume in z -direction

Dependence on the transverse size

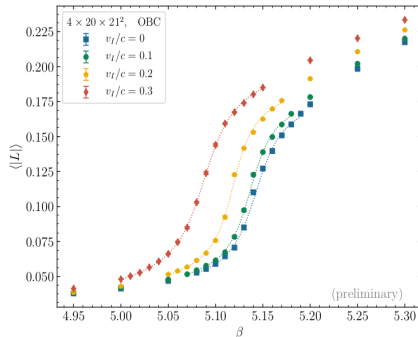
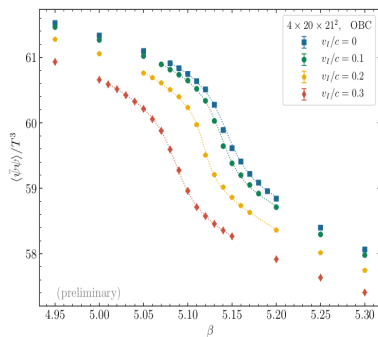


- ▶ The results can be well described by the formula

$$\frac{T_c(\Omega)}{T_c(0)} = 1 - B_2 v_I^2, \quad v_I = \Omega_I(N_s - 1)a/2, \quad C_2 = B_2(N_s - 1)^2 a^2/4$$

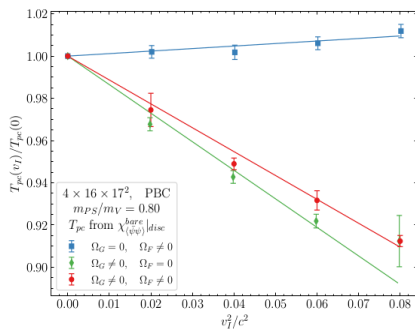
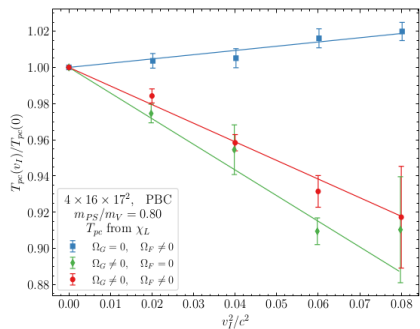
- ▶ **Periodic b.c.:** $B_2 \sim 1.3$
- ▶ **Dirichlet b.c.:** $B_2 \sim 0.5$
- ▶ **Neumann b.c.:** $B_2 \sim 0.7$
- ▶ **Good variable is $v = \Omega R$, rather than Ω**

Simulation with fermions



- ▶ Lattice simulation with Wilson fermions
- ▶ Critical couplings of both transitions coincide
- ▶ Critical temperatures are increased

Simulation with fermions



- ▶ QCD action: $S = S_f(\Omega_F) + S_g(\Omega_G)$
- ▶ One can introduce velocities for gluons Ω_G and fermions Ω_F
- ▶ $\Omega_F \neq 0, \Omega_G = 0$ decreases critical temperatures
- ▶ $\Omega_F = 0, \Omega_G \neq 0$ increases critical temperatures
- ▶ $\Omega_G = \Omega_F \neq 0$ pull system to opposite directions but **gluons win**

EoS of rotating gluodynamics

- ▶ Free energy of rotating QGP

$$F(T, R, \Omega) = F_0(T, R) + C_2 \Omega^2 + \dots$$

- ▶ The **moment of inertia**

$$C_2 = -\frac{1}{2} I_0(T, R), \quad I_0(T, \Omega) = -\frac{1}{\Omega} \left(\frac{\partial F}{\partial \Omega} \right)_{T, \Omega \rightarrow 0}$$

- ▶ Instead of $I_0(T, R)$ we calculate $K_2 = -\frac{I_0(T, R)}{F_0(T, R) R^2}$
- ▶ Sign of K_2 coincides with the sign of $I_0(T, R)$

EoS of rotating gluodynamics

- ▶ Classical moment of inertia

$$I_0(R) = \int_V d^3x x_\perp^2 \rho_0(x_\perp)$$

- ▶ Related to the trace of EMT $T_\mu^\mu = \rho_0(x_\perp)c^2$
- ▶ Generation of mass scale in QCD and scale anomaly

$$T_\mu^\mu \sim \langle G^2 \rangle \sim \langle H^2 + E^2 \rangle$$

- ▶ In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- ▶ *One could anticipate: $\rho_0 \sim \langle H^2 + E^2 \rangle$?*

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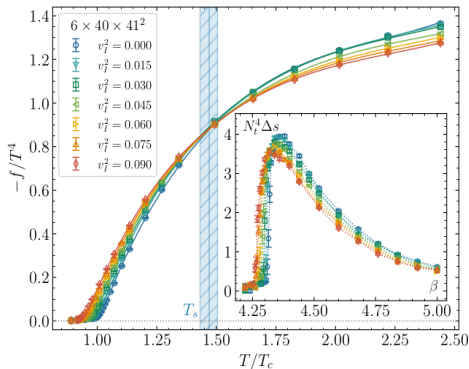
- ▶ In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- ▶ *One could anticipate: $\rho_0 \sim \langle H^2 + E^2 \rangle$?*
- ▶ $I_0 = I_{fluct} + I_{cond}$ *valid for QCD!*

$$I_{fluct} = \langle J_3^2 \rangle - (\langle J_3 \rangle)^2$$

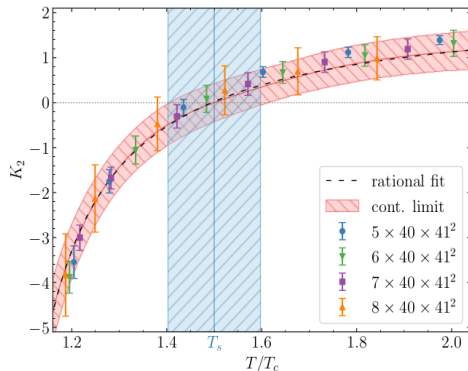
$$I_{cond} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle$$

Calculation of free energy on the lattice

- ▶ $F = -T \log Z$ impossible to calculate on the lattice
- ▶ $\frac{\partial F}{\partial \beta} \sim \langle \Delta s(\beta) \rangle = s(\beta)_T - s(\beta)_{T=0}, \quad \beta = \frac{6}{g^2}$
- ▶ $\frac{F(T)}{T^4} \sim \int_{\beta_0}^{\beta_1} d\beta' \langle \Delta s(\beta') \rangle$

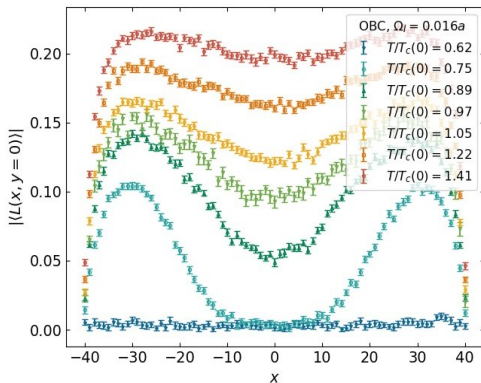


Moment of inertia of gluon plasma



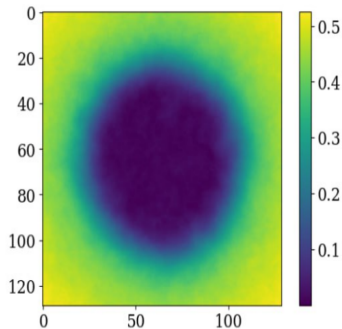
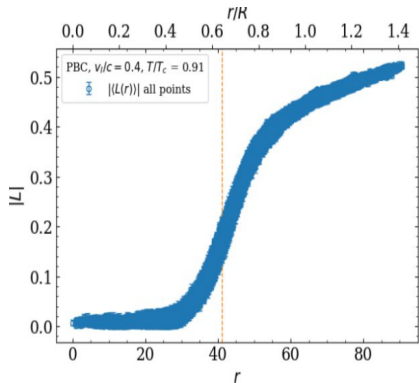
- ▶ $I(T, R) = -F_0(T, R)K_2R^2$
- ▶ $I < 0$ for $T < 1.5T_c$ and $I > 0$ for $T > 1.5T_c$
- ▶ **Negative moment of inertia indicates a thermodynamic instability of rigid rotation**
- ▶ The region of $I < 0$ is related to magnetic condensate and the scale anomaly
- ▶ **We believe that the same is true for QCD**

Polyakov loop in rotating QGP (preliminary!)



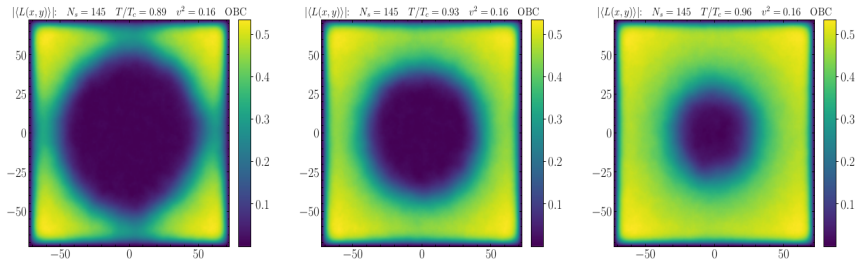
- ▶ Week dependence at large temperatures
- ▶ No dependence at low temperatures
- ▶ Strong inhomogeneity of Polyakov loop close to $\sim T_c$

Inhomogeneous phase transitions in QGP



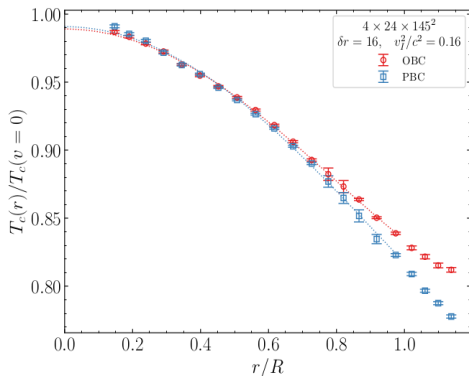
- ▶ Confinement in the center and deconfinement close to boundary
- ▶ Such configurations can be found close to T_c
- ▶ Vortex?

Inhomogeneous phase transitions in QGP



- ▶ As temperature is increased, vortex penetrates closer to center
- ▶ Deconfinement appears on the boundaries and captures all volume

Inhomogeneous phase transitions in QGP



- ▶ Critical temperature can be well described by:

$$T_c(r)/T_c(0) = 1 - C(\Omega_I r)^2$$

- ▶ Local thermalization?

Conclusion

- ▶ Lattice study of rotating gluodynamics and QCD have been carried out
- ▶ Critical temperatures rise with rotation
- ▶ We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- ▶ Negative moment of inertia indicates a thermodynamic instability of rigid rotation
- ▶ We observed inhomogeneous phase transitions in GP
- ▶ We believe that all observed effects remain in QCD

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THANK YOU!