# PHENOMENOLOGY OF VECTOR-MESON ELECTROPRODUCTION ON SPINLESS TARGETS

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## Contents

# Introduction

- Vector-meson electroproduction provides information on the reaction mechanism and target structure.

- Electroproduction of vector mesons is one of two basic processes for extraction of Generalized Parton Distributions (GPD).
  Radyushkin, Ji

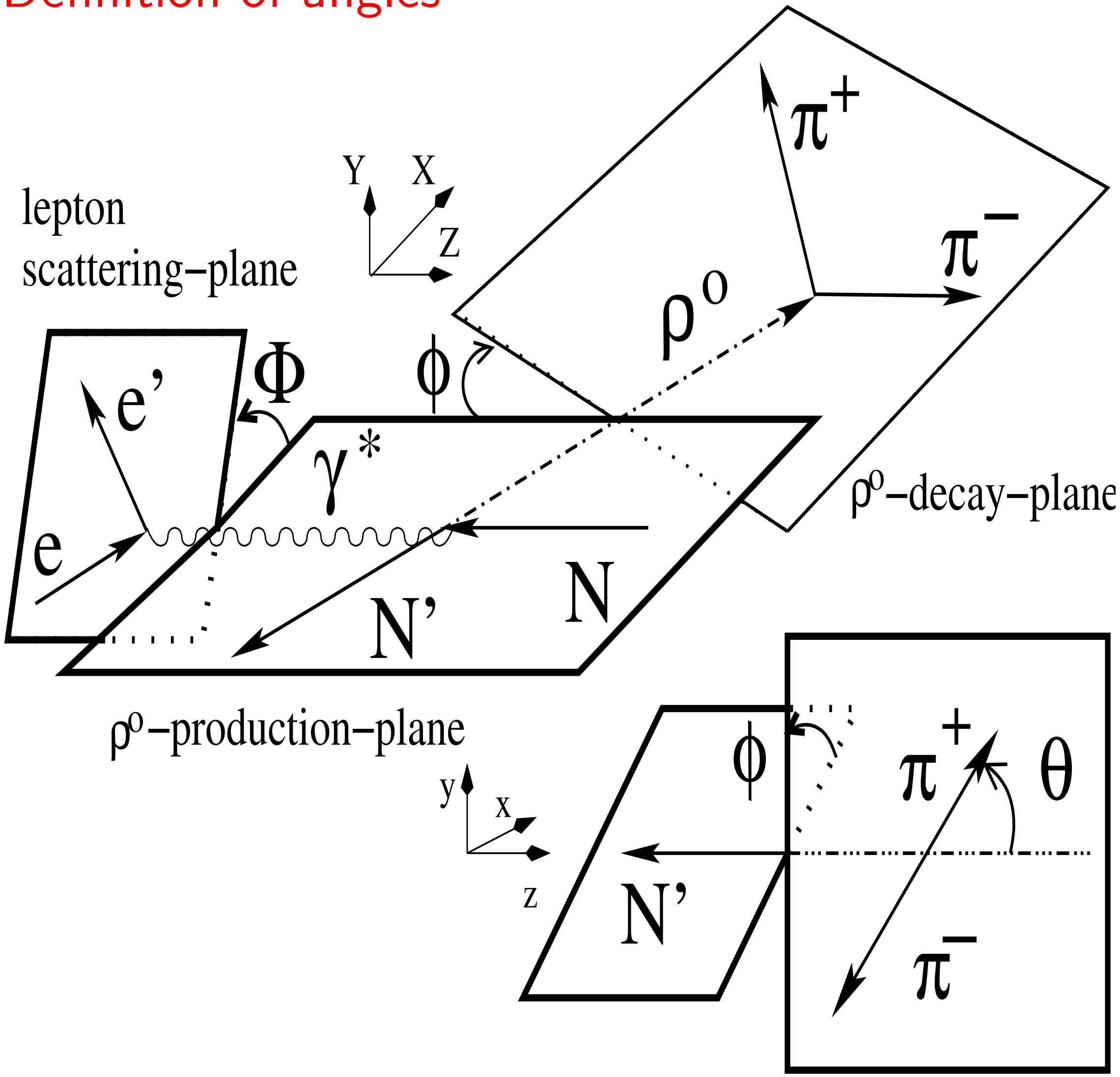
- Amplitudes of vector-meson production on nucleons at high $Q^2$ and small $x_B$ in leading-logarithm approximation are proportional to gluon distributions $G(x_B, Q^2)$, $\Delta G(x_B, Q^2)$ (Ryskin).

- Usual method of data treatment is Spin-Density-Matrix-Element (SDME) method.

- The alternative and more economical method is amplitude method of data processing when the amplitude ratios are extracted from data..

- Not all helicity amplitudes of vector-meson production can be calculated in theoretical models since the factorization theorem is not proved for all the amplitudes.
  Therefore individual amplitudes are to be extracted from the data.

- Definition of angles




$\Phi$ is angle between production plane $(\vec{q}, \vec{v})$ and lepton-scattering plane $(\vec{k}_e, \vec{k}'_e)$.
$\theta$ and $\phi$ are polar and azimuthal angles of $\pi^+$-momentum in $\rho^0$-meson rest frame

# Formalism

- Reactions $e \to e' + \gamma^*$, $\gamma^*(\lambda_\gamma) + S \to V(\lambda_V) + S$, $V = \rho^0 \to \pi^+ + \pi^-$.
  $S$ is spinless nucleus.
  $\gamma^*$ is virtual photon with helicity $\lambda_\gamma$ in center-of-mass (CM) $\gamma^* S$ system.
  $V$ denotes produced vector meson with helicity $\lambda_V$ in CM system.
  Amplitudes $F_{\lambda_V \lambda_\gamma}$ in CM system obey relations $F_{-\lambda_V -\lambda_\gamma} = (-1)^{\lambda_V - \lambda_\gamma} F_{\lambda_V \lambda_\gamma}$
  due to parity conservation. Independent amplitudes are: $F_{11}$, $F_{10}$, $F_{1-1}$, $F_{01}$, $F_{00}$.

- The von Neumann Formula
  $\mathcal{N} \rho_{\lambda_V \tilde{\lambda}_V} = \sum_{\lambda_\gamma, \tilde{\lambda}_\gamma} F_{\lambda_V \lambda_\gamma} F^*_{\tilde{\lambda}_V \tilde{\lambda}_\gamma} \varrho_{\lambda_\gamma \tilde{\lambda}_\gamma}$
  $\varrho_{\lambda_\gamma \tilde{\lambda}_\gamma} = \varrho^U_{\lambda_\gamma \tilde{\lambda}_\gamma} + P_b \varrho^L_{\lambda_\gamma \tilde{\lambda}_\gamma}$
  is spin-density matrix of virtual photon $(\mathrm{tr}\{\varrho\} = 1)$ known from QED.
  $\rho_{\lambda_V \tilde{\lambda}_V}$ is spin-density matrix of vector meson $(\mathrm{tr}\{\rho\} = 1)$.

$\mathcal{N} = \mathcal{N}_T + \epsilon \mathcal{N}_L$ is normalized factor.
$\mathcal{N}_T = |F_{11}|^2 + |F_{01}|^2 + |F_{-11}|^2$,
$\mathcal{N}_L = |F_{10}|^2 + |F_{00}|^2 + |F_{-10}|^2$,
$\epsilon$ is flux ratio of longitudinally polarized $(\lambda_\gamma = 0)$ virtual photons
to transversely polarized $(\lambda_\gamma = \pm 1)$ photons produced by lepton beam.

# Formalism

- Conservation of angular momentum in decay $\rho^0 \to \pi^+ + \pi^-$

  $|\rho^0; J = 1, J_z = \lambda_V\rangle \to |\pi^+\pi^-; L = 1, L_z = \lambda_V\rangle \to Y_{1\lambda_V}(\theta, \phi)$

- Angular distribution of decay pions

$$\mathcal{W}(\Phi, \theta, \phi) = \sum_{\lambda_V, \tilde{\lambda}_V} Y_{1\lambda_V}(\theta, \phi) Y^*_{1\tilde{\lambda}_V}(\theta, \phi) \rho_{\lambda_V \tilde{\lambda}_V}(\Phi, \epsilon)$$

- Angular distribution of decay pions for unpolarized beam and spin-dependent distribution for longitudinally polarized lepton beam, $P_b$

$$\mathcal{W}(\Phi, \theta, \phi) = \mathcal{W}^U(\Phi, \theta, \phi) + P_b \mathcal{W}^L(\Phi, \theta, \phi) \quad (1)$$

- Angular distribution of decay pions for unpolarized beam

$$\mathcal{W}^U(\Phi,\theta,\phi) = \frac{3}{4\pi}\Bigg[\frac{1}{2}(1-r^{04}_{00}) + \frac{1}{2}(3r^{04}_{00}-1)\cos^2\theta - \sqrt{2}\mathrm{Re}\{r^{04}_{10}\}\sin 2\theta\cos\phi -$$
$$-r^{04}_{1-1}\sin^2\theta\cos 2\phi - \epsilon\cos 2\Phi\Big(r^1_{11}\sin^2\theta + r^1_{00}\cos^2\theta - \sqrt{2}\mathrm{Re}\{r^1_{10}\}\sin 2\theta\cos\phi -$$
$$-r^1_{1-1}\sin^2\theta\cos 2\phi\Big) - \epsilon\sin 2\Phi\Big(\sqrt{2}\mathrm{Im}\{r^2_{10}\}\sin 2\theta\sin\phi + \mathrm{Im}\{r^2_{1-1}\}\sin^2\theta\sin 2\phi\Big) +$$
$$+\sqrt{2\epsilon(1+\epsilon)}\cos\Phi\Big(r^5_{11}\sin^2\theta + r^5_{00}\cos^2\theta - \sqrt{2}\mathrm{Re}\{r^5_{10}\}\sin 2\theta\cos\phi - r^5_{1-1}\sin^2\theta\cos 2\phi\Big) +$$
$$+\sqrt{2\epsilon(1+\epsilon)}\sin\Phi\Big(\sqrt{2}\mathrm{Im}\{r^6_{10}\}\sin 2\theta\sin\phi + \mathrm{Im}\{r^6_{1-1}\}\sin^2\theta\sin 2\phi\Big)\Bigg],$$

All "unpolarized" Schilling-Wolf SDMEs $r^{\alpha}_{\lambda_V\tilde{\lambda}_V}$ can be extracted from data.

# Formalism

- Angular distribution of decay pions for polarized beam

$$\mathcal{W}^L(\Phi,\theta,\phi)=\frac{3}{4\pi}\Bigg[\sqrt{1-\epsilon^2}\Big(\sqrt{2}\mathrm{Im}\{r^3_{10}\}\sin 2\theta\sin\phi+\mathrm{Im}\{r^3_{1-1}\}\sin^2\theta\sin 2\phi\Big)+$$
$$+\sqrt{2\epsilon(1-\epsilon)}\cos\Phi\Big(\sqrt{2}\mathrm{Im}\{r^7_{10}\}\sin 2\theta\sin\phi+\mathrm{Im}\{r^7_{1-1}\}\sin^2\theta\sin 2\phi\Big)+$$
$$+\sqrt{2\epsilon(1-\epsilon)}\sin\Phi\Big(r^8_{11}\sin^2\theta+r^8_{00}\cos^2\theta-\sqrt{2}\mathrm{Re}\{r^8_{10}\}\sin 2\theta\cos\phi-r^8_{1-1}\sin^2\theta\cos 2\phi\Big)\Bigg].$$

All "polarized" Schilling-Wolf SDMEs can be extracted from data.

# Basic Equations

- Schilling-Wolf SDMEs describe vector-meson productions on nucleons

  Formulas for SDMEs in terms of helicity amplitudes (K. Schilling, G. Wolf, 1973)
  $r^{\alpha}_{\lambda_V \tilde{\lambda}_V} = f^{\alpha}_{\lambda_V \tilde{\lambda}_V}(T_{\lambda_V \mu_N \lambda_\gamma \lambda_N}, U_{\lambda_V \mu_N \lambda_\gamma \lambda_N})$
  where $T_{\lambda_V \mu_N \lambda_\gamma \lambda_N}$ are Natural-Parity-Exchange (NPE) amplitude,
  $U_{\lambda_V \mu_N \lambda_\gamma \lambda_N}$ are Unnatural-Parity-Exchange (UPE) amplitude;
  $\lambda_N$ and $\mu_N$ are respectively helicities of initial and final nucleon,
  $\lambda_\gamma$ is virtual photon helicity, while $\lambda_V$ denoted vector-meson helicity in process

  $\gamma^*(\lambda_\gamma) + N(\lambda_N) \to V(\lambda_V) + N(\mu_N)$

- Simplification of algebraic expressions for spinless ($S$) targets
  Reaction $\gamma^*(\lambda_\gamma) + S \to V(\lambda_V) + S$
  All UPE-amplitudes become zero.

  $\frac{1}{2}\sum_{\lambda_N, \mu_N}\{T_{\lambda_V \mu_N \lambda_\gamma \lambda_N} T^*_{\lambda'_V \mu_N \lambda'_\gamma \lambda_N}\} \to F_{\lambda_V \lambda_\gamma} F^*_{\lambda'_V \lambda'_\gamma}$

- Basic equations
  $r^{\alpha}_{\lambda_V \tilde{\lambda}_V} = f^{\alpha}_{\lambda_V \tilde{\lambda}_V}(F_{\lambda_V \lambda_\gamma}, U_{\lambda_V \mu_N \lambda_\gamma \lambda_N} \equiv 0)$

  Is it possible to find all helicity amplitudes $F_{\lambda_V \lambda_\gamma}$ for spinless targets?

  Yes!

# Solution of Basic Equations

- Property of solutions
  Angular distribution normalized to uniry and Schilling-Wolf SDMEs are dimensionless quantities. Therefore they depend on amplitude ratios rather than on amplitudes themselves.
- Ratios of amplitudes to $F_{00}$. Formulas can be obtained for polarized beam only.

$$\frac{F_{11}}{F_{00}} = \epsilon\sqrt{8}\frac{\left(r^1_{1-1} - \mathrm{Im}\{\mathrm{r}^2_{1-1}\}\right)\left(\mathrm{Im}\{\mathrm{r}^6_{10}\} - \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^7_{10}\}\right)}{\left(r^{04}_{00} + r^1_{00}\right)\left(\mathrm{Im}\{\mathrm{r}^2_{1-1}\} - \mathrm{r}^1_{11} - \mathrm{r}^1_{1-1} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^3_{1-1}\}\right)}.$$

$$\frac{F_{01}}{F_{00}} = \frac{\epsilon}{\sqrt{2}}\frac{\left(r^5_{00} + i\cdot r^8_{00}\right)}{\left(r^{04}_{00} + r^1_{00}\right)}.$$

$$\frac{F_{10}}{F_{00}} = -\frac{\mathrm{Im}\{\mathrm{r}^6_{1-1}\} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^7_{1-1}\}}{2\left(\mathrm{Im}\{\mathrm{r}^6_{10}\} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^7_{10}\}\right)}.$$

$$\frac{F_{1-1}}{F_{00}} = \epsilon\sqrt{8}\frac{\left(r^1_{11} - i\cdot\mathrm{Im}\{\mathrm{r}^3_{1-1}\}\right)\left(-\mathrm{Im}\{\mathrm{r}^6_{10}\} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^7_{10}\}\right)}{\left(r^{04}_{00} + r^1_{00}\right)\left(r^1_{1-1} - \mathrm{Im}\{\mathrm{r}^2_{1-1}\} + \mathrm{r}^1_{11} - \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^3_{1-1}\}\right)}.$$

# Solution of Basic Equations

- Ratios of helicity amplitudes to $F_{11}$ (limit $Q^2 \to 0$)

$$\frac{F_{00}}{F_{11}} = \frac{1}{\epsilon\sqrt{8}} \frac{\left(r^{04}_{00} + r^{1}_{00}\right)\left(\mathrm{Im}\{\mathrm{r}^{2}_{1-1}\} - \mathrm{r}^{1}_{11} - \mathrm{r}^{1}_{1-1} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^{3}_{1-1}\}\right)}{\left(r^{1}_{1-1} - \mathrm{Im}\{\mathrm{r}^{2}_{1-1}\}\right)\left(\mathrm{Im}\{\mathrm{r}^{6}_{10}\} - \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^{7}_{10}\}\right)},$$

$$\frac{F_{01}}{F_{11}} = \frac{2\left(\mathrm{Im}\{\mathrm{r}^{2}_{10}\} - \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^{3}_{10}\}\right)}{r^{1}_{1-1} - \mathrm{Im}\{\mathrm{r}^{2}_{1-1}\} + \mathrm{r}^{1}_{11} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^{3}_{1-1}\}}.$$

$$\frac{F_{10}}{F_{11}} = \frac{\mathrm{Im}\{\mathrm{r}^{6}_{1-1}\} - \mathrm{r}^{5}_{1-1} - \mathrm{i}\cdot\left(\mathrm{r}^{8}_{11} - \mathrm{Im}\{\mathrm{r}^{7}_{1-1}\}\right)}{\sqrt{2}\left(r^{1}_{1-1} - \mathrm{Im}\{\mathrm{r}^{2}_{1-1}\}\right)}.$$

$$\frac{F_{1-1}}{F_{11}} = \frac{r^{1}_{11} - i\cdot\mathrm{Im}\{\mathrm{r}^{3}_{1-1}\}}{r^{1}_{1-1} - \mathrm{Im}\{\mathrm{r}^{2}_{1-1}\}}.$$

# Comparison of SDME and Amplitude Methods

- SDME method
  Properties of Schilling-Wolf SDMEs $r^{\alpha}_{\lambda_V \tilde{\lambda}_V}$
  All SDMEs $r^{\alpha}_{\lambda_V \tilde{\lambda}_V}$ are considered as independent in a fit of experimental angular distribution. Total number of independent SDMEs is 23.
  Since Schilling-Wolf SDMEs can be expressed through 4 complex amplitude ratio, hence they are mutually dependent. There are 15 equations of constraints.

  The region for SDME values which guaranties positive definiteness of angular distribution is unknown. If $\mathcal{W} \leq 0$ maximum likelihood method is inapplicable.
  Likelihood function is linear with respect to $r^{\alpha}_{\lambda_V \tilde{\lambda}_V}$.

- Amplitude method
  The number of real fit functions is 8 (4 real parts and 4 imaginary parts of helicity-amplitude ratios).
  Four complex helicity-amplitude ratios are independent.
  Angular distribution is positive for any non-zero helicity amplitudes.
  Likelihood function is nonlinear with respect to amplitude ratios.

- If Monte-Carlo codes describe detector good enough results of both methods should be in agreement.

# Virtual-photon Longitudinal-to-transverse Cross-section Ratio

- Exact formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio for vector-meson productions on spinless targets

  $$R \equiv \frac{d\sigma_L}{dt} / \frac{d\sigma_T}{dt} \equiv \frac{|F_{10}|^2+|F_{00}|^2+|F_{-10}|^2}{|F_{11}|^2+|F_{01}|^2+|F_{-11}|^2} = \frac{r^{04}_{00}+r^{1}_{00}+2(r^{1}_{11}-r^{04}_{1-1})}{\epsilon(2r^{1}_{1-1}-r^{1}_{00})}.$$

  Rosenbluth deconposition is not needed to obtain $R$ ($\frac{d\sigma}{dt} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt}$).

- New approximate formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio for scattering from nucleons

  $$R \approx R_U = \frac{r^{04}_{00}+r^{1}_{00}+2(r^{1}_{11}-r^{04}_{1-1})}{\epsilon\{1-r^{04}_{00}-r^{1}_{00}-2(r^{1}_{11}-r^{04}_{1-1})\}}.$$

  Correction to $R_U$ are proportional to contributions of UPE-amplitudes only.
  Correction does not contain contribution of the largest UPE-amplitude $|U_{1\frac{1}{2}1\frac{1}{2}}|^2$ at $W \sim$ a few GeV.

- Standard approximate formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio for scattering from nucleons

  $$R \approx R_{04} = \frac{r^{04}_{00}}{\epsilon\{1-r^{04}_{00}\}}.$$

  $|R-R_U| \ll |R-R_{04}|$ at high energies since $|R-R_{04}|$ contains contributions of NPE-amplitudes, which are greater than those of UPE-amplitudes at $W \to \infty$.

# Summary

- Exact formulas for helicity-amplitude ratios in terms of spin-density-matrix elements (SDMEs) of vector mesons produced on spinless targets are obtained. It is shown that single-valued formulas for helicity-amplitude ratios can be obtained only if the beam is longitudinally polarized.

- Making use of the amplitude ratios instead of SDMEs as free fit parameters in maximum likelihood method reduces the number of free real parameters from 23 to 8.

- A comparison between SDMEs directly extracted from the experimental data and calculated from the helicity-amplitude ratios permits to estimate systematic uncertainties of description with Monte Carlo codes of the applied detector properties.

- Exact formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio, $R$ in terms of SDMEs for vector-meson production on spinless targets is established. There is no need in Rosenbluth decomposition to obtain $R$.

- A new approximate formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio, $R_U$ in terms of SDMEs for vector-meson production on nucleons is proposed. Value of $R_U$ is more precise than $R_{04}$ for high energies.

# Back-up Slides. Spin-density Matrix of Virtual Photon

- General Formula for Spin-density Matrix of Virtual Photon

  $\varrho_{\lambda_\gamma \tilde{\lambda}_\gamma} = \varrho^U_{\lambda_\gamma \tilde{\lambda}_\gamma} + P_b \varrho^L_{\lambda_\gamma \tilde{\lambda}_\gamma}$

- Spin-density Matrix of Virtual Photon for Unpolarized Beam

$$\varrho^U_{\lambda_\gamma \tilde{\lambda}_\gamma}(\epsilon, \Phi) =$$

$$= \frac{1}{2} \begin{pmatrix} 1 & \sqrt{\epsilon(1+\epsilon)} e^{-i\Phi} & -\epsilon e^{-2i\Phi} \\ \sqrt{\epsilon(1+\epsilon)} e^{i\Phi} & 2\epsilon & -\sqrt{\epsilon(1+\epsilon)} e^{-i\Phi} \\ -\epsilon e^{2i\Phi} & -\sqrt{\epsilon(1+\epsilon)} e^{i\Phi} & 1 \end{pmatrix},$$

- Additive Term to Spin-density Matrix of Virtual Photon due to Longitudinal Polarization of Beam

$$\varrho^L_{\lambda_\gamma \tilde{\lambda}_\gamma}(\epsilon, \Phi) =$$

$$= \frac{\sqrt{1-\epsilon}}{2} \begin{pmatrix} \sqrt{1+\epsilon} & \sqrt{\epsilon} e^{-i\Phi} & 0 \\ \sqrt{\epsilon} e^{i\Phi} & 0 & \sqrt{\epsilon} e^{-i\Phi} \\ 0 & \sqrt{\epsilon} e^{i\Phi} & -\sqrt{1+\epsilon} \end{pmatrix}.$$