SANC: processes $\gamma\gamma \to ZH$ at one-loop level including polarization effects

L. Rumyantsev,
on behalf of SANC group

JINR, Dubna

DSPIN

8 September 2023

Motivation, SANC, $\gamma\gamma$ initial state

- Photon-photon collisions have been a topic of some interest for many decades.
- Investigated both on electron and hadron colliders.
- Search for anomalous neutral gauge boson self coupling.
- Process $\gamma\gamma \to W^+W^-$ - pair W-boson production at the photon-photon collisions.

Future lepton colliders projects

Linear collider (e^+e^-)

- ILC; CLIC
- ILC: technology at hand,

realization in Japan

E_{cm}

- 250GeV – 1TeV, 91GeV (ILC)
- 500GeV – 3TeV (CLIC)

$L \approx 2 \times 10^{34} cm^{-2} s^{-1} (\sim 500 fb^{-1}/year)$

$\Rightarrow$ Stat. uncertainty $\sim 10^{-3}$ ... 10^{-2}

Beam polarization

e^- beam P = 80-90%
e^+ beam
ILC: P = 30% baseline;
60% upgrade

CLIC: P $\geq$ 60% upgrade

Circular collider

- FCC-ee
- CEPC, μ Collider

Projects under study

E_{cm}

- 91 GeV, 160GeV, 240GeV, 350GeV

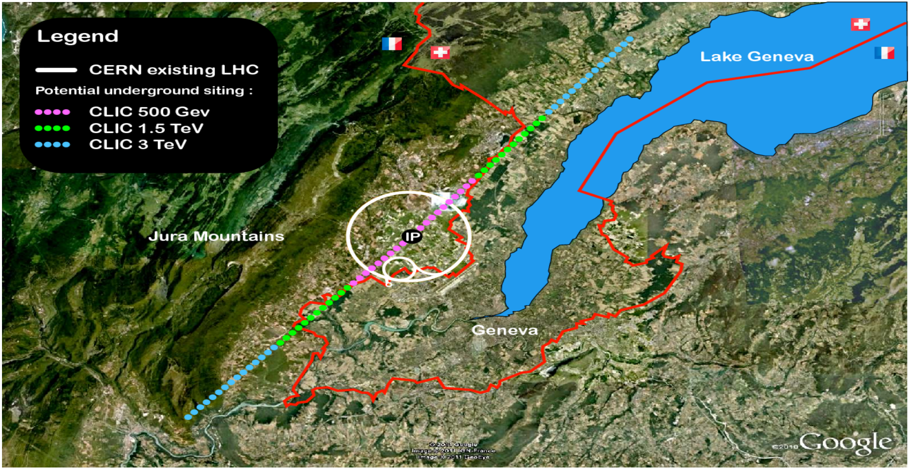
$L \approx 10^{36} cm^{-2} s^{-1}$ (4 experiments)

$\Rightarrow$ Stat. uncertainty $\leq 10^{-3}$

Beam polarization

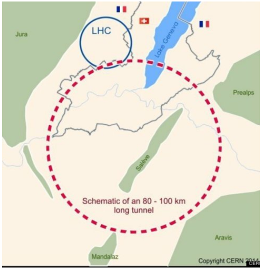
Future lepton collider projects

CLIC - Compact linear collider



Future lepton collider projects

FCC-ee - Future Circular Collider



Results of the ARIeL group was presented at the the 1st Mini workshop: Precision EW and QCD calculations for the FCC studies: methods and tools, 12-13 January 2018, CERN, Geneva, proceeding " Standart Model Theory for the FCC-ee: The Tera-Z ".

arXiv 1809.01830v2 [hep-ph] 22 Sept 2018, https://indico.cern.ch/event/669224.

SANC: $\gamma\gamma$ polarized scattering

- $\gamma\gamma \to \nu\bar{\nu}$
- $\gamma\gamma \to \mathbf{e^+e^-}$
- $\gamma\gamma \to \gamma\gamma, \mathbf{Z}\gamma, \mathbf{ZZ}, \mathbf{ZH}$
- $\gamma\gamma \to \mathbf{W^+W^-}$
- $\gamma\gamma \to \mathbf{HH}$

SANC: basics, procedures

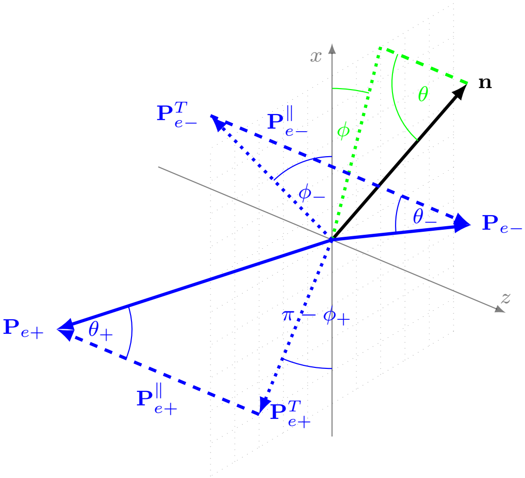
Calculations for the unpolarized cross-section:

- **Covariant amplitudes (CA)** — $\mathcal{CA}$
- **Scalar Form Factors (FF)** — $\mathcal{F}_i$
- **Helicity Amplitudes (HA)** — $\mathcal{H}_{\{\lambda_i\}}(\mathcal{F}_i)$

  standard approach: $\sigma \;\propto\; |\mathcal{CA}|^2$

  while in terms of HA: $\sigma \;\propto\; \sum_{\{\lambda_i\}} |\mathcal{H}_{\{\lambda_i\}}|^2$
- **Bremsstrahlung using HA: (BR)**
- **MCSANC, ReneSANCe**

Decomposition of the $e^{\pm}$ polarization vectors



Matrix element squared

$$|\mathcal{M}|^2 = L^{\shortparallel}_{e-} R^{\shortparallel}_{e+} |\mathcal{H}_{-+}|^2 + R^{\shortparallel}_{e-} L^{\shortparallel}_{e+} |\mathcal{H}_{+-}|^2 + L^{\shortparallel}_{e-} L^{\shortparallel}_{e+} |\mathcal{H}_{--}|^2$$

$$+ R^{\shortparallel}_{e-} R^{\shortparallel}_{e+} |\mathcal{H}_{++}|^2$$

$$- \frac{1}{2} P^{\perp}_{e-} P^{\perp}_{e+} \,\mathrm{Re}\Big[e^{i(\Phi_+ - \Phi_-)} \mathcal{H}_{++} \mathcal{H}^*_{--} + e^{i(\Phi_+ + \Phi_-)} \mathcal{H}_{+-} \mathcal{H}^*_{-+} \Big]$$
$$+ P^{\perp}_{e-} \,\mathrm{Re}\Big[e^{i\Phi_-} \Big(L^{\shortparallel}_{e+} \mathcal{H}_{+-} \mathcal{H}^*_{--} + R^{\shortparallel}_{e+} \mathcal{H}_{++} \mathcal{H}^*_{-+} \Big) \Big]$$
$$- P^{\perp}_{e+} \,\mathrm{Re}\Big[e^{i\Phi_+} \Big(L^{\shortparallel}_{e-} \mathcal{H}_{-+} \mathcal{H}^*_{--} + R^{\shortparallel}_{e-} \mathcal{H}_{++} \mathcal{H}^*_{+-} \Big) \Big],$$

where
$L^{\shortparallel}_{e\pm} = \frac{1}{2}(1 - P^{\shortparallel}_{e\pm})$, $R^{\shortparallel}_{e\pm} = \frac{1}{2}(1 + P^{\shortparallel}_{e\pm})$, $\Phi_\pm = \phi_\pm - \phi$,
$\mathcal{H}_{--}$, $\mathcal{H}_{++}$,$\mathcal{H}_{-+}$,$\mathcal{H}_{+-}$ — helicity amplitudes.

SANC: basics, scheme of FF calculation

- **The renormalization scheme on the mass shell in the R_ξ calibration with the three calibration parameters ξ_{A}, ξ_{Z} and $\xi \equiv \xi_{\mathsf{W}}$ let us achieve the one-loop accuracy level.**
- **The dimensional regularization was used to parametrize the ultraviolet divergences.**
- **Loop integrals were obtained in terms of standard scalar Passarino-Veltman functions: A_0, B_0, C_0, D_0.**
- **Our assurance in the correctness of our calculations is based on the absence of ξ dependencies.**

SANC: basics, analytical calculations

- Predictions: calculate in advance all the necessary single-loop diagrams and related quantities (for example, renormalization constants, etc.) and save the results as files.
- Covariant amplitudes (CA)
- Scalar Form Factors ($\mathcal{F}$)
- Helicity Amplitudes (HA)
- Related Photon Radiation (BR)

Analytical calculations – FORM4.2 *

*) J.A.M. Vermaseren, New features of FORM, math-ph/0010025

Foundation stone: $bbbb \to 0$ and channel reversal

The covariant amplitude for the channel $\gamma\gamma \to ZH$ can be obtained from annihilation to the vacuum with the following permutation of the 4-momenta:

$$
\begin{aligned}
p_1 &\to p_1,\\
p_2 &\to p_2,\\
p_3 &\to -p_3,\\
p_4 &\to -p_4
\end{aligned}
$$

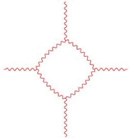
Precomputation level, channel $\gamma\gamma ZH \to 0$

In the `SANC` system the basic concept of the analytical calculations is precomputation of vacuum building blocks, namely diagrams, in which **all external particles** are considered to be **incoming** and **not lying on the mass shell**.
These are the building blocks that can be used as the elements in the calculation of real processes in relevant channels by means of transformation of external particles momenta and replacing the squares of momenta by the squares of masses. Consider this concept on the example of the process $\gamma\gamma ZH \to 0$.

Precomputation level, channel $\gamma\gamma ZH \to 0$

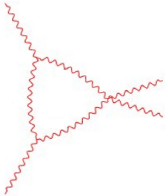
The process at the one-loop level of accuracy is described by two blocks of the diagrams with a **fermionic** and **bosonic** propagators, respectively.
Their calculation can be made **independently**.

Block of bosonic diagrams consists of three box diagrams, Fig.a,

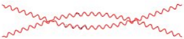


a)

six triangular graphs — pinches, Fig.b, and three diagrams of the "fish" type — self energies, Fig.c.

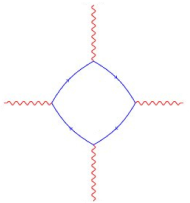


b)



c)

Block of fermionic diagrams consists of only three box diagrams, Fig.*d*. Each diagram is characterized by different order of 4-momenta of the incoming particle p_1, p_2 for photons, p_3 for Z boson and p_4 for H boson, respectively.



d)

Рис.: Fermionic diagrams $\gamma\gamma ZH \to 0$ process

Precomputation level, channel $\gamma\gamma \to ZH$

In the computation of the diagrams — the application of Feynman rules, the Passarino–Veltman (PV) reduction of one-loop integrals, scalarizing and separation of the poles — the expressions of the amplitude can be represented in the form of the sum of products of structures (including gamma-matrices, for example) and relevant scalar form factors.

In terms of structures we write the expression for the covariant amplitude $\gamma\gamma \to ZH$

$$\mathcal{A}_{\gamma\gamma\to ZH} = \sum_{i=1}^{14} \Big[\mathcal{F}_i^{\rm b}\,(s\,,t\,,u) + \mathcal{F}_i^{\rm f}\,(s\,,t\,,u)\Big] T_i^{\alpha\beta\nu}.$$

Structures may be written as a three rank tensor, with imposing the conditions of physical transversality and zero mass of the photons ($p_1^2 = p_2^2 = 0$).

Scalar Form-factors

- $\mathcal{F}_i$ – scalar coefficients at each basis structure of the covariant amplitude — the projection of the amplitude onto the full expression of the basis $T_i^{\alpha\beta\nu}$.
- They are some combination of scalar Passarino-Veltman functions A_0, B_0, C_0, D_0, and depend on the invariants $s\,,t\,,u$, as well as on the masses of fermions and bosons.
- At the same time, they do not contain ultraviolet (UV) poles.

The number of terms in $\mathcal{F}_i$ is thousands in the case of non-zero masses of of loop particles, but this number is greatly reduced at the zero limit of fermion masses.

In the helicity amplitude approach

$$
\begin{aligned}
\mathcal{A} &= \sum_{\text{spins}} \left[C^{\text{bosons}} \times \mathcal{H}^{\text{bosons}}_{\text{spins}} + C^{\text{fermions}} \times \mathcal{H}^{\text{fermions}}_{\text{spins}} \right], \\
\left|\mathcal{A}_{\gamma\gamma\to ZH}\right|^2 &= \sum_{\text{spins}} \left[C^2_{\text{bosons}} |\mathcal{H}^{\text{bosons}}_{\text{spins}}|^2 + C^2_{\text{fermions}} |\mathcal{H}^{\text{fermions}}_{\text{spins}}|^2 \right. \\
&+ C^{\text{bosons}} C^{\text{fermions}} \left(\mathcal{H}^{*\text{bosons}}_{\text{spins}} \times \mathcal{H}^{\text{fermions}}_{\text{spins}} \right. \\
&+ \left. \left. \mathcal{H}^{\text{bosons}}_{\text{spins}} \times \mathcal{H}^{*\text{fermions}}_{\text{spins}} \right) \right].
\end{aligned}
$$

Short Basis, six structures

$$
\begin{aligned}
T^{\alpha\beta\nu}(1) &= p_{1\beta}p_{1\nu}\left[\frac{m_Z^4 - um_Z^2 - tm_Z^2 + tu}{s^2}p_{2\alpha} + \frac{m_Z^2 - t}{s}p_{3\alpha}\right] \\
&\quad + p_{1\nu}p_{3\beta}\left[\frac{m_Z^2 - u}{s}p_{2\alpha} + p_{3\alpha}\right], \\
T^{\alpha\beta\nu}(2) &= T^{\alpha\beta\nu}(1)\Big[p_{1\nu} \to p_{2\nu}\Big], \\
T^{\alpha\beta\nu}(3) &= p_{1\nu}\left[\frac{2}{s}p_{1\beta}p_{2\alpha} + \delta_{\alpha\beta}\right], \\
T^{\alpha\beta\nu}(4) &= T^{\alpha\beta\nu}(3)\Big[p_{1\nu} \to p_{2\nu}\Big], \\
T^{\alpha\beta\nu}(5) &= p_{1\nu}p_{2\alpha}\left[\frac{2(m_Z^2 - t)}{s^2}p_{1\beta} + \frac{2}{s}p_{3\beta}\right] + \delta_{\alpha\nu}\left[\frac{m_Z^2 - t}{s}p_{1\beta} + p_{3\beta}\right], \\
T^{\alpha\beta\nu}(6) &= p_{1\beta}p_{2\nu}\left[\frac{2(m_Z^2 - u)}{s^2}p_{2\alpha} + \frac{2}{s}p_{3\alpha}\right] + \delta_{\beta\nu}\left[\frac{m_Z^2 - u}{s}p_{2\alpha} + p_{3\alpha}\right].
\end{aligned}
$$

$\gamma\gamma \to$ ZH, Helicity amplitudes

$$
\begin{aligned}
\mathcal{H}_{+++} &= \frac{1}{2}k_0\left[-\frac{1}{8}c^-c^+k_1\left(\mathcal{F}_1-\mathcal{F}_2\right)+s\left(\mathcal{F}_3-\mathcal{F}_4\right)+\frac{1}{2}\sqrt{\lambda(s,m_Z^2,m_H^2)}\left(c^-\mathcal{F}_5+c^+\mathcal{F}_6\right)\right],\\
\mathcal{H}_{---} &= -\mathcal{H}_{+++},\\
\mathcal{H}_{++-} &= -\frac{1}{2}k_0\left[-\frac{1}{8}c^-c^+k_1\left(\mathcal{F}_1-\mathcal{F}_2\right)+s\left(\mathcal{F}_3-\mathcal{F}_4\right)+\frac{1}{2}\sqrt{\lambda(s,m_Z^2,m_H^2)}\left(c^+\mathcal{F}_5+c^-\mathcal{F}_6\right)\right],\\
\mathcal{H}_{--+} &= -\mathcal{H}_{++-},\\
\mathcal{H}_{++0} &= \frac{1}{8m_Z}\left(\frac{1}{4}c^-c^+\left[\left(\frac{k_1}{s}\sqrt{\lambda(s,m_Z^2,m_H^2)}-k_2\cos\vartheta_Z\right)\mathcal{F}_1+\left(\frac{k_1}{s}\sqrt{\lambda(s,m_Z^2,m_H^2)}\right.\right.\right.\\
&+ \left.\left. k_2\cos\vartheta_Z\right)\mathcal{F}_2\right]\\
&- 2\left[\sqrt{\lambda(s,m_Z^2,m_H^2)}-\cos\vartheta_Z(s+m_Z^2-m_H^2)\right]\mathcal{F}_3\\
&- 2\left[\sqrt{\lambda(s,m_Z^2,m_H^2)}+\cos\vartheta_Z(s+m_Z^2-m_H^2)\right]\mathcal{F}_4\\
&+ \left.\frac{1}{s}\sqrt{\lambda(s,m_Z^2,m_H^2)}c^-c^+(s+m_Z^2-m_H^2)(\mathcal{F}_5+\mathcal{F}_6)\right),\\
\mathcal{H}_{--0} &= \mathcal{H}_{++0},
\end{aligned}
$$

$\gamma\gamma \to$ ZH, Helicity amplitudes

$$
\begin{aligned}
\mathcal{H}_{-+0} &= -\frac{1}{8m_Z} c^- c^+ \Bigg(\frac{1}{4} \Bigg[\Bigg(\frac{k_1}{s} \sqrt{\lambda(s, m_Z^2, m_H^2)} - k_2 cos\, \vartheta_Z \Bigg) \mathcal{F}_1 + \Bigg(\frac{k_1}{s} \sqrt{\lambda(s, m_Z^2, m_H^2)} \\
&+ k_2 cos\, \vartheta_Z \Bigg) \mathcal{F}_2 \Bigg] \\
&+ \frac{1}{s} \sqrt{\lambda(s, m_Z^2, m_H^2)}\, (s + m_Z^2 - m_H^2)(\mathcal{F}_5 + \mathcal{F}_6) \Bigg), \\
\mathcal{H}_{+-0} &= \mathcal{H}_{-+0}, \\
\mathcal{H}_{+-+} &= \frac{1}{4} k_0 c^- \left[\frac{1}{4} c^+ k_1 (\mathcal{F}_1 - \mathcal{F}_2) - \sqrt{\lambda(s, m_Z^2, m_H^2)}\, (\mathcal{F}_5 + \mathcal{F}_6) \right], \\
\mathcal{H}_{-+-} &= -\mathcal{H}_{+-+}, \\
\mathcal{H}_{+--} &= -\frac{1}{4} k_0 c^+ \left[\frac{1}{4} c^- k_1 (\mathcal{F}_1 - \mathcal{F}_2) - \sqrt{\lambda(s, m_Z^2, m_H^2)}\, (\mathcal{F}_5 + \mathcal{F}_6) \right], \\
\mathcal{H}_{-++} &= -\mathcal{H}_{+--}.
\end{aligned}
$$

Meaning of the notations is the following:
$c^- = \cos\theta_Z - 1, c^+ = \cos\theta_Z + 1, k_0 = \frac{\sin\theta_Z}{\sqrt{2}\sqrt{s}}, k_1 = (s + m_Z^2 - m_H^2)^2 - 4m_Z^2 s,$

$k_2 = \Big((s + m_Z^2 - m_H^2)^2 - (m_Z^2 + m_H^2)(s - 2m_H^2 + 2m_Z^2) - 2m_Z^2 s\Big) + \frac{1}{s}(m_Z^2 - m_H^2)^3.$

THANKS FOR YOUR ATTENTION!