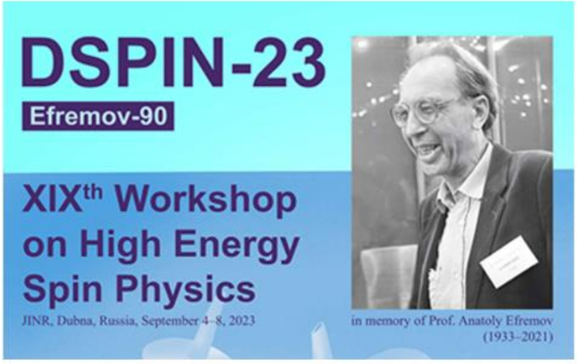

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Analog of Weyl meson as analog of the dark photon

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Abstract

The Lagrangian for conformal gravitation with two scalar fields, which is linear in scalar curvature, is considered. In this Lagrangian, the Weyl vector is replaced by a Weyl-gauge vector, which transforms like the Weyl vector, but does not enter into the Weyl connection. The space of such a model is an integrable Weyl space. Based on this Lagrangian, the relationship between the Weyl-gauge vector and the electromagnetic potential vector is considered. The probability of conversion of quanta of both types of vectors is considered. A possible variant is given for the case when such conversion can be observed under astrophysical conditions.

An introduction to the problem from Wikipedia

Hermann Weyl

Born	Hermann Klaus Hugo Weyl 9 November 1885 Elmshorn, German Empire
Died	8 December 1955 (aged 70) Zürich, Switzerland

Hermann Weyl in an attempt to unify general relativity and electromagnetism, conjectured that Eichinvarianz or invariance under the change of scale (or "gauge") might also be a local symmetry of general relativity. After the development of quantum mechanics, Weyl, Vladimir Fock and Fritz London modified gauge by replacing the scale factor with a complex quantity and turned the scale transformation into a change of phase, which is a U(1) gauge symmetry. This explained the electromagnetic field effect on the wave function of a charged quantum mechanical particle. This was the first widely recognised gauge theory, popularised by Pauli in 1941.

Sanomiya, Lobo, Formiga, Dahia and Romero. An Invariant Approach to Weyl's unified field theory. arXiv:2002.00285v1 [gr-qc] 1 Feb 2020.

For reasons of consistency of his physics with the new geometry, Weyl required his theory to be completely invariant with respect to change between gauges (or frames). On the other hand, he chose the simplest of all possible invariant actions, namely,

$$S = \int d^4x\sqrt{|g|}[R^2 + \omega F_{\mu\nu}F^{\mu\nu}], \tag{8}$$

where ω is a constant [5]. This action describes the gravitational-electromagnetic sector only.

$$F_{\mu\nu} = \partial_\nu\sigma_\mu - \partial_\mu\sigma_\nu. \quad \Gamma^{\alpha}_{\beta\lambda} = \{^{\alpha}_{\beta\lambda}\} - \frac{1}{2}g^{\alpha\mu}[g_{\mu\beta}\sigma_\lambda + g_{\mu\lambda}\sigma_\beta - g_{\beta\lambda}\sigma_\mu],$$

Let us now consider the set of all closed curves $\alpha : [a,b] \in R \to M$, i.e, with $\alpha(a) = \alpha(b)$. Then, either from (6) or (7) it follows that

$$L = L_0 e^{\frac{1}{2}\oint \sigma_\alpha dx^\alpha},$$

where L_0 and L denotes the values of $L(\lambda)$ at a and b, respectively. From Stokes's theorem we then can write[2]

$$L = L_0 e^{-\frac{1}{4}\int\int F_{\mu\nu}dx^\mu \wedge dx^\nu},$$

Mark Israelit. A Weyl-Dirac cosmological model with DM and DE. General Relativity and Gravitation. Volume 43, pages 751–775 (2011)

Dirac [19] presented a revised version of Weyl's theory. He introduced a scalar field $\beta(x^\nu)$, which under WGT changes as

$$\beta \to \tilde{\beta} = e^{-\lambda}\beta \tag{7}$$

As the scalar β defines uniquely the gauge, it is called the Dirac gauge function. With the help of β, and making use of $K^{\lambda}_{\sigma\mu\nu}$ and $W_{\mu\nu}$, Dirac wrote the action integral as

$$I = \int \left[W^{\lambda\rho}W_{\lambda\rho} - \beta^2 R + \sigma\beta^2 w^\lambda w_\lambda + 2\sigma\beta\, w^\lambda \beta,_\lambda + (\sigma+6)\beta,_\rho \beta,_\lambda g^{\lambda\rho} + \right. \tag{8}$$
$$\left. + 2\Lambda\beta^4 + L_{\mathrm{matter}} \right]\sqrt{-g}\; d^4x$$

In (8) σ is the Dirac parameter, Λ is the cosmological constant and L_{matter} is the Lagrangian density of matter (introduced by Rosen [20]). Varying in (8) $g_{\mu\nu}$, w_ν, β and choosing $\sigma = 0$, Dirac obtained a geometrically based theory of gravitation and electromagnetism that in the Einstein gauge, $\beta = 1$, results in Einstein's general relativity theory and Maxwell's electrodynamics.

S. Yu. Sedov. About Weyl-Dirac theory of gravitation and its development. ABSTRACTS of PHYSICAL INTERPRETATIONS OF RELATIVITY THEORY (PIRT–2023) XXIII International Scientific Conference Moscow, July 3–6, 2023.

One hundred years ago H. Weyl proposed a theory of gravitation, which uses local symmetry with regard to measurement gauging. The ideas of H. Weyl are still urgent today. The number of papers on the basis of these ideas permanently grows.

During recent years different ways to apply Weyl (local conformal) symmetry in gravitation theory have been considered from the point of view of modification of general relativity (GR) to describe dark matter, dark energy, and evolution of early Universe. Modifications of GR with local conformal invariance have been examined for the long time as the attempts in solving different problems; in particular, these are methods of renormalization in quantum gravitation, renormalization of energy-momentum tensor, dynamics of inflation in early Universe and the origin of masses of elementary particles.

Models of conformal gravitation that contain lagrangians, which are linear on scalar curvature and with non-minimal connection with the scalar field, are discussed in this report. Theory of Weyl-Dirac gravitation has been reported in detail. The new version of conformal lagrangian with two scalar fields is proposed, in which the Weyl vector is replaced with the vector which is transformed as a Weyl vector, but is not contained in Weylian connection. Weyl integrable space is the space of such model [1].

The problem of description of conformal stage in the evolution of the Universe on the basis of Friedmann metrics is considered within Weyl-Dirac gravitation theory with nonminimal connection with the real scalar field. Conformal invariant solutions for the scale factor are presented. It is demonstrated that quantum corrections for the trace of energy-momentum tensor are partially compensated by gauging the Dirac function, which results into the lagrangian of the General Relativity theory [2].

References:

1. S. Yu. Sedov, Conformal Lagrangians in Weyl Gravitation, VANT. Ser.: Theoret. i prikl. fizika. 2022. No 1. P. 13-28.

2. S. Yu. Sedov, Weyl-Dirac gravitation and Friedmann´s cosmology, VANT. Ser.: Theoret. i prikl. fizika.. 2022. No 1. P. 40-54.

Our model with local scale invariance

In the proposed model of conformal gravity with a scalar field, there are: scalar Riemannian curvature R, Dirac real scalar field $\beta(x)$ from [1], complex scalar field $\varphi(x)$, real Weyl-gauge vector $B_\mu(x) = e_{Weyl} B^\mu_{Weyl}(x)$, electromagnetic potential $A^\mu(x) = e_{QED} A^\mu_{QED}$.

Here the fine-structure constant $\alpha_{QED} \sim e^2_{QED}$,

the thick-structure constant $\alpha_{Weyl} \sim e^2_{Weyl}$ (it's a joke).

Let's introduce parameters α, μ, λ, ρ, ξ, ε, δ. The action is $S_\beta = -\frac{M_P^2}{2}\int d^4x\sqrt{-g}\cdot L_\beta$. The Lagrangian of our model [2] is written as:

$$\begin{aligned} L_\beta &= \beta^2 R + 6\beta_\lambda\beta^\lambda + \alpha g^{\mu\nu}\left(\beta_\mu - B_\mu\cdot\beta\right)\cdot\left(\beta_\nu - B_\nu\cdot\beta\right) + \mu\left(\varphi\varphi^* - \rho\cdot\beta^2\right)^2 \\ &+2\lambda\beta^4 + \xi g^{\mu\nu}\left(\partial_\mu\varphi - (B_\mu + iA_\mu)\cdot\varphi\right)^*\cdot\left(\partial_\nu\varphi - (B_\nu + iA_\nu)\cdot\varphi\right) \\ &+E^{\mu\nu}E_{\mu\nu} + \delta^2\cdot F^{\mu\nu}F_{\mu\nu} - 2\varepsilon\delta\cdot F^{\mu\nu}E_{\mu\nu} \end{aligned}$$

Vector field strengths A^μ and B^μ are defined as usually: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $E_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. Note that the Lagrangian has an interaction proportional to the product $F^{\mu\nu}E_{\mu\nu}$. The affine connection is written in terms of the integrable Weyl geometry, that is, the Weyl vector here is gradient and is equal to $\frac{\partial\beta_\nu}{\beta}$. In this model, the metric of the gravitational field $g_{\mu\nu}$ and vector B^α (as the analog of Weyl vector, see [3]) are changed under local conformal transformations according to the rule:

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2(x)\cdot g_{\mu\nu}\ ,\ B_\mu \to B_\mu - \frac{\partial\ln\Omega(x)}{\partial x^\mu}\ .$$

In the observable universe, the conformal symmetry is broken. Taking into account the violation of conformal symmetry, we set the scalar equal to $\beta = 1$, and, doing so, we obtain the effective Lagrangian L_{AB} to describe the interaction of the electromagnetic field A_μ and the analog of Weyl vector B_μ:

$$L_1 \supset -4L_{AB} = \delta^2 F_{\mu\nu}F^{\mu\nu} + E_{\mu\nu}E^{\mu\nu} - 2\varepsilon\delta F_{\mu\nu}E^{\mu\nu} - 2m_B^2 B_\mu B^\mu - 4J_A^\mu A_\mu - 4J_B^\mu B_\mu \ .$$

Here, $2m_B^2 = -\alpha$, J_A and J_B are electromagnetic and dilaton currents of the scalar field φ.

These equations hold for the strengths of the vectors A_μ and B_μ:

$$E^{\mu\nu}{}_{;\mu} + m_B^2 B^\nu = \varepsilon\delta m_B^2 F^{\mu\nu}{}_{;\mu} - \xi\left[\partial^\mu\left(\varphi\varphi^*\right) - 2B^\mu\left(\varphi\varphi^*\right)\right] ,$$

$$\delta^2 F^{\mu\nu}{}_{;\mu} = \varepsilon\delta m_B^2 E^{\mu\nu}{}_{;\mu} - \xi\left[i\varphi^*\partial^\mu\varphi - i\varphi\cdot\partial^\mu\varphi^* - 2A^\mu\left(\varphi\varphi^*\right)\right] \ .$$

Let's rename the variables: $\tilde{A}^\mu = \delta\cdot A^\mu$, $\varepsilon = \sin\chi_0$, $m_B^2 = \cos^2\chi_0\cdot m_{DF}^2$. Then

$$L_{AB} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}E_{\mu\nu}E^{\mu\nu} + \frac{\sin\chi_0}{2}\tilde{F}_{\mu\nu}E^{\mu\nu} + \frac{\cos^2\chi_0}{2}m_{DF}^2 B_\mu B^\mu + J_A^\mu\tilde{A}_\mu + J_B^\mu B_\mu \,.$$

If we neglect the dilaton current J_B, then the Lagrangian L_{AB} formally coincides with the Lagrangian from [4], which describes the interaction of photons $\tilde{A}^\mu$ and dark photons B^μ. Therefore, let's use the results of [4]. Basing on transformations

$$\{A^\mu, B^\mu\} \to \left\{\tilde{A}^\mu = \delta\cdot A^\mu, \tilde{B}^\mu = B^\mu\right\} \to \left\{\tilde{\tilde{A}}^\mu = \cos\chi_0\cdot\tilde{A}^\mu, \tilde{\tilde{B}}^\mu = B^\mu - \sin\chi_0\cdot\tilde{A}^\mu\right\},$$

we make a transition to the basis of interaction, in which oscillations of quanta of vector fields take place $\tilde{\tilde{A}}^\mu$ and $\tilde{\tilde{B}}^\mu$: $\tilde{\tilde{\gamma}}_A \leftrightarrow \tilde{\tilde{\gamma}}_B$.

There is a formula for the transition probability from [4]:

$$P(\tilde{\tilde{\gamma}}_A \to \tilde{\tilde{\gamma}}_B) = \sin^2\chi_0 \cdot \sin^2\left(\frac{m_{DF}^2 L_{AB}}{4\omega}\right),$$

where L_{AB} is the mixing length, ω is the angular frequency of photon $\tilde{\tilde{\gamma}}_A$. Thus, there is a transition of the quanta of the Weyl-gauge vector B^μ (as the analog of Weyl meson from [3]) into the quanta of the electromagnetic field A_μ. In this case, the analog of the dark photon is the vector B^μ , transforming according to the Weyl vector transformation rule, but not included in the affine connection (as in [2], in contrast to the Weyl meson from [3]).

The astrophysical consequences of the transition of ordinary photons to dark photons have been studied in various papers. In the recent publication [5], a variant was considered when dark photons, turning into ordinary ones, heat the interstellar gas to the observed temperature. The mass of dark photons, according to estimates in [5], is $m_{DF} \sim 8\cdot 10^{-14} eV$, the mixing parameter is $\varepsilon = \sin\chi_0 \sim 5\cdot 10^{-15}$. The probability P of the conversion $\tilde{\tilde{\gamma}}_B \to \tilde{\tilde{\gamma}}_A$ in a form convenient for comparison with observations is given in [5]:

$$P(\tilde{\tilde{\gamma}}_B \to \tilde{\tilde{\gamma}}_A) \cong \pi\varepsilon^2 \frac{m_{DF}^2 c^2}{\hbar^2} \left|\frac{d\ln m_\gamma^2}{dt}\right|_{t=t_R}^{-1}.$$

Here $m_\gamma(\vec{\mathrm{r}},\mathrm{t})$ is the effective plasma mass of an ordinary photon in interstellar gas, t_R is the time spent by the system $\tilde{\tilde{A}}^\mu$ and $\tilde{\tilde{B}}^\mu$ in resonance.

Thus, the existence of the interaction effect of the analogs of Weyl mesons and ordinary photons can be tested under astrophysical conditions.

About generalized Weyl transformations

For integrable Weyl geometry $\beta^2\breve{R} = \beta^2 R + 6\beta_\lambda\beta^\lambda$, where $\breve{R}$ is Weyl scalar curvature, R is scalar Riemann curvature, β is real positive Dirac scalar. The Weyl affine connection coefficients in the integrable case are defined as follows:

$$\breve{\Gamma}^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}\left(\breve{\partial}_\mu g_{\alpha\nu} + \breve{\partial}_\nu g_{\alpha\mu} - \breve{\partial}_\alpha g_{\mu\nu}\right) = \Gamma^\lambda_{\mu\nu} + \delta^\lambda_\mu \frac{\beta_\nu}{\beta} + \delta^\lambda_\nu \frac{\beta_\mu}{\beta} - g_{\mu\nu}\beta^\lambda$$

Let’s consider the Lagrangian:

$$L_w = \beta^2 R + 6\beta_\lambda\beta^\lambda + 2\lambda\beta^4 + \xi \cdot \frac{\beta^2}{\varphi\varphi^*} g^{\mu\nu}\left(\partial_\mu\varphi - (B_\mu + iA_\mu)\cdot\varphi\right)^* \cdot \left(\partial_\nu\varphi - (B_\nu + iA_\nu)\cdot\varphi\right)$$
$$+\delta^2\left(E^{\mu\nu}E_{\mu\nu} + F^{\mu\nu}F_{\mu\nu}\right)$$

Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $E_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$; λ, ξ and δ are parameters, φ is the complex scalar, A^μ is the real vector of electromagnetic potential, B^μ is the real Weyl-gauge vector. Under the Weyl transformations $B^\mu \to B^\mu - \frac{\beta^\mu}{\beta}$, under gauge transformations $A^\mu \to A^\mu - \alpha^\mu$. Here, β is an arbitrary real positive function, α is an arbitrary real function. Complex scalar $\varphi = \rho\cdot\exp(i\theta)$ transforms as: $\rho \to \rho\cdot\beta$, $\theta \to \theta + \alpha$.

Let's introduce ***complex Weyl-gauge vector*** $w^{\mu} = B^{\mu} + iA^{\mu}$. Let's introduce ***Weyl rotation*** in the complex plane of the complex Weyl-gauge vector $w^{\mu} = B^{\mu} + iA^{\mu}$:

$$w'^{\mu} = B'^{\mu} + iA'^{\mu} = \left(B^{\mu}\cos\chi - A^{\mu}\sin\chi\right) + i\left(A^{\mu}\cos\chi + B^{\mu}\sin\chi\right),$$

where χ is an arbitrary angle in the complex plane (B, A). We introduce the Weyl rotation of the logarithm of a complex scalar $\varphi = \rho\cdot\exp(i\theta)$, $\ln\varphi = \ln\rho + i\theta$:

$$\ln\varphi' = \ln\rho' + i\theta' = \left(\ln\rho\cdot\cos\chi - \theta\sin\chi\right) + i\left(\theta\cos\chi + \ln\rho\cdot\sin\chi\right)$$

We claim that the Lagrangian

$$L_w = \beta^2\breve{R} + 2\lambda\beta^4 + \xi\cdot\beta^2 g^{\mu\nu}\left(\partial_{\mu}\ln\varphi - w_{\mu}\right)^{*}\cdot\left(\partial_{\nu}\ln\varphi - w_{\mu}\right) + \delta^2\left(E^{\mu\nu}E_{\mu\nu} + F^{\mu\nu}F_{\mu\nu}\right)$$

is invariant under the Weyl rotation through the angle χ, and is invariant under the gauge transformation $A^{\mu} \to A^{\mu} - \alpha^{\mu}$, and is invariant under the Weyl transformation $B^{\mu} \to B^{\mu} - \dfrac{\beta^{\mu}}{\beta}$.

Note that when the local scale symmetry of the Lagrangian L_w is violated, one gauge invariance remains.

Some words about Stueckelberg Trick

Let's $\varphi = \exp(\psi + i\theta)$, $\psi = \ln|\varphi|$,

$$\xi \cdot \beta^2 g^{\mu\nu} \left(\partial_\mu \ln\varphi - w_\mu\right)^* \cdot \left(\partial_\nu \ln\varphi - w_\mu\right) = \xi \cdot \beta^2 g^{\mu\nu} \cdot \left[\left(\partial_\mu\psi - B_\mu\right)^2 + \left(\partial_\mu\theta - A_\mu\right)^2\right]$$

Here ψ and θ are two scalar Stueckelberg fields.

Under definite ***Weyl rotation:***

$$\xi \cdot \beta^2 g^{\mu\nu} \cdot \left[\left(\partial_\mu\psi - B_\mu\right)^2 + \left(\partial_\mu\theta - A_\mu\right)^2\right] \to \xi \cdot \beta^2 g^{\mu\nu} \cdot \left(\partial_\mu\psi' - B'_\mu\right)^2 ,$$

$$E^{\mu\nu}E_{\mu\nu} + F^{\mu\nu}F_{\mu\nu} = inv$$

Weyl meson B'_μ have mass now. Photon A'_μ have no mass.

What is Stueckelberg fields ?

Stueckelberg fields consist of a gauge vector field and the Stueckelberg scalar, and these two fields share the same mass m. In Stueckelberg mechanism Lorentz condition is not automatically satisfied just like in standard QED, but spurious scalar field is introduced so as to contribute to the commutation relation and make the Hamiltonian density positive definite.

Ozgur Akarsu, Metin Arik, Nihan Katirci. Inflation and late time acceleration designed by Stueckelberg massive photon. Found. Phys. 47 (2017) 769-796. ArXiv:1611.04545 [gr-qc].

In classical electromagnetism, encoded in the Maxwell theory, the Lagrangian density for the canonical electromagnetic field is given as follows:

$$\mathcal{L}_{\rm EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor and there is no mass term. It should be noted here that we use natural units with $\hbar = c = 1$ throughout this mini review. It had been believed by many that only massless vector field theories are gauge invariant. The massive spin-1 vector field is given by Proca Lagrangian density

$$\mathcal{L}_{\rm Proca} = -\frac{1}{4}F_{\mu\nu}(V)F^{\mu\nu}(V) + \frac{1}{2}m^2V^{\mu}V_{\mu}, \tag{2}$$

where V is the Proca field and m is its mass that breaks the gauge symmetry. Lorentz condition for the Proca field $\partial^{\mu}V_{\mu} = 0$ is automatically satisfied and the Hamiltonian density is positive definite. In commutation relations $1/m^2$ term leads to quadratic ultraviolet (UV) divergence, which can not be eliminated by renormalisation. Stueckelberg, on the other hand, achieved Lorentz covariance and even gauge invariance by introducing an auxiliary scalar field that contributes to the longitudinal polarisation of the massive photon of Proca theory in addition to the two transverse polarisations, known as helicities (original paper [22], see [23] for a detailed review). The so called Stueckelberg Lagrangian density is given as follows;

$$\mathcal{L}_{\rm Stu} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2\left(A_{\mu} - \frac{1}{m}\partial_{\mu}B\right)^2, \tag{3}$$

which is invariant under gauge transformation given by

$$A_{\mu} \to A_{\mu} + \partial_{\mu}\lambda \quad \text{and} \quad B \to B + m\lambda \tag{4}$$

Now we will say some final words about Weyl vector and it's use in Standard Model.

“Recently the discussion was reopened by proposals which foresee a Higgs-like role for the weak isospin extended scalar field of Weyl geometry, by introducing a second (real) scalar field σ transform□ing under rescaling with the same weight −1 as φ (Nishino/Rajpoot 2004, Nishino/Rajpoot 2007b). After the first year of taking data at the LHC, it may be the right time to reconsider the topic of a possible connection between electroweak symmetry “breaking” and gravity from a Weyl geometric perspective.” (Erhard Scholz , 2011)

Hitoshi Nishino and Subhash Rajpoot. Implication of Compensator Field and Local Scale Invariance in the Standard Model. Phys.Rev.D79:125025,2009. ArXiv:0906.4778 [hep-th]

Our total action invariant under (Diffeomorphisms) $\times SU(3) \times SU(2) \times U(1) \times \widetilde{U}(1)$ is [8]

$$
\begin{aligned}
I = \int d^4 x\, e \Big[& - \tfrac{1}{2}(\beta \Phi^\dagger \Phi + \zeta \sigma^2) \widetilde{R} - \tfrac{1}{4} g^{\mu\rho} g^{\nu\sigma} \Big\{ \mathrm{Tr}\, (W_{\mu\nu} W_{\rho\sigma}) + B_{\mu\nu} B_{\rho\sigma} + U_{\mu\nu} U_{\rho\sigma} \Big\} \\
& + \sum_{\substack{\mathrm{f}=q,l \\ \mathrm{g}=1,2,3}} \Big(\overline{\Psi}{}^{\mathrm{gf}}_{\mathrm{L}} \gamma^\mu D_\mu \Psi^{\mathrm{gf}}_{\mathrm{L}} + \sum_{i=1,2} \overline{\Psi}{}^{\mathrm{gf}}_{i\mathrm{R}} \gamma^\mu D_\mu \Psi^{\mathrm{gf}}_{i\mathrm{R}} \Big) + \sum_{\substack{\mathrm{f}=q,l \\ \mathrm{g},\mathrm{g}'=1,2,3 \\ i=1,2}} \Big(\mathbf{Y}^{\mathrm{f}}_{\mathrm{gg}'} \overline{\Psi}{}^{\mathrm{gf}}_{\mathrm{L}} \Phi \Psi^{\mathrm{g}'\mathrm{f}}_{i\mathrm{R}} + \mathbf{Y}'^{\mathrm{f}}_{\mathrm{gg}'} \overline{\Psi}{}^{\mathrm{gf}}_{\mathrm{L}} \widetilde{\Phi} \Psi^{\mathrm{g}'\mathrm{f}}_{i\mathrm{R}} \Big) + \mathrm{h.c.} \\
& + g^{\mu\nu} (D_\mu \Phi)(D_\nu \Phi^\dagger) + \tfrac{1}{2} g^{\mu\nu} (D_\mu \sigma)(D_\nu \sigma) - \lambda (\Phi^\dagger \Phi)^2 + \mu (\Phi^\dagger \Phi) \sigma^2 - \xi \sigma^4 \Big] ,
\end{aligned}
\tag{1.2}
$$

where $\gamma^\mu \equiv \gamma^m e_m{}^\mu$, and any $SU(3)$ color-related terms and indices are suppressed. The field strengths $W_{\mu\nu}$ and $B_{\mu\nu}$ are respectively those of W_μ and B_μ, while $U_{\mu\nu} \equiv \partial_\mu S_\nu - \partial_\nu S_\mu$. These field strengths are all invariant under $\widetilde{U}(1)$. The scale-invariant scalar curvature $\widetilde{R} \equiv g^{\mu\nu} \widetilde{R}_{\mu\nu}$ and the Ricci tensor $\widetilde{R}_{\mu\nu} \equiv \widetilde{R}_{\mu\rho\nu}{}^\rho$ are defined in terms of the scale-invariant Riemann tensor $\widetilde{R}_{\mu\nu\rho}{}^\sigma \equiv \partial_\mu \widetilde{\Gamma}_{\nu\rho}{}^\sigma - \partial_\nu \widetilde{\Gamma}_{\mu\rho}{}^\sigma - \widetilde{\Gamma}_{\mu\rho}{}^\tau \widetilde{\Gamma}_{\nu\tau}{}^\sigma + \widetilde{\Gamma}_{\nu\rho}{}^\tau \widetilde{\Gamma}_{\mu\tau}{}^\sigma$, where the scale-invariant affinity $\widetilde{\Gamma}$ is defined by $\widetilde{\Gamma}_{\mu\nu}{}^\rho \equiv (1/2) g^{\rho\sigma} (D_\mu g_{\nu\sigma} + D_\nu g_{\mu\sigma} - D_\sigma g_{\mu\nu})$ with $D_\mu g_{\rho\sigma} \equiv \partial_\mu g_{\rho\sigma} + 2 f S_\mu g_{\rho\sigma}$. The $\widetilde{\Phi}$ is $\widetilde{\Phi} \equiv i \sigma_2 \Phi^\dagger$ and the scale-covariant derivative D_μ is defined on each field by

$$
D_\mu \Psi^{\mathrm{gf}}_L = \Big(\partial_\mu + i g \tau \cdot W_\mu + \tfrac{i}{2} g' Y^{\mathrm{gf}}_{\mathrm{L}} B_\mu - \tfrac{1}{4} \widetilde{\omega}_\mu{}^{mn} \gamma_{mn} - \tfrac{3}{2} f S_\mu \Big) \Psi^{\mathrm{gf}}_L \ , \tag{1.3a}
$$

$$
D_\mu \Psi^{\mathrm{gf}}_{iR} = \Big(\partial_\mu + \tfrac{i}{2} g' Y^{\mathrm{gf}}_{i\mathrm{R}} B_\mu - \tfrac{1}{4} \widetilde{\omega}_\mu{}^{mn} \gamma_{mn} - \tfrac{3}{2} f S_\mu \Big) \Psi^{\mathrm{gf}}_{iR} \ , \tag{1.3b}
$$

$$
D_\mu \Phi = \Big(\partial_\mu + i g \tau \cdot W_\mu - \tfrac{i}{2} g' B_\mu - f S_\mu \Big) \Phi \ , \qquad D_\mu \sigma = (\partial_\mu - f S_\mu) \sigma \ , \tag{1.3c}
$$

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Thank you for your attention!