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On the recent polemics on Structure Function g_1 at small x

talk based on results obtained in collaboration with
M.Greco and S.I. Troyan

Consider lepton-hadron DIS.
The spin-dependent part of the hadronic tensor is parameterized by two structure functions:

$$W_{\mu\nu}{}^{spin} = i\, \varepsilon_{\mu\nu\lambda\rho}\, m_h\, (q_\lambda/Pq)\left[S_\rho\, g_1(x, Q^2) + (S_\rho - P_\rho)(Sq/q^2)\, g_2(x, Q^2)\right]$$

Structure Functions

where m_h , P and S are the hadron mass, momentum and spin;
q is the virtual photon momentum ($Q^2 = -q^2 > 0$). Both of the functions depend on Q^2 and $x = Q^2/2pq$, $0 < x < 1$.

The only theoretical instrument to calculate $g_1(x, Q^2)$ and $g_2(x, Q^2)$ is QCD
It involves integration over virtual parton momenta over the whole phaze space

However, QCD can be applied at large momenta only. The low momenta region is accounted for approximately through **QCD FACTORIZATION**

There are well-known the following kinds of QCD Factorization in the literature:

Collinear Factorization

Amati-Petronzio-Veneziano, Efremov-Ginzburg-Radyushkin, Libby-Sterman, Brodsky-Lepage, Collins-Soper-Sterman

K_T- Factorization/High-Energy Factorization

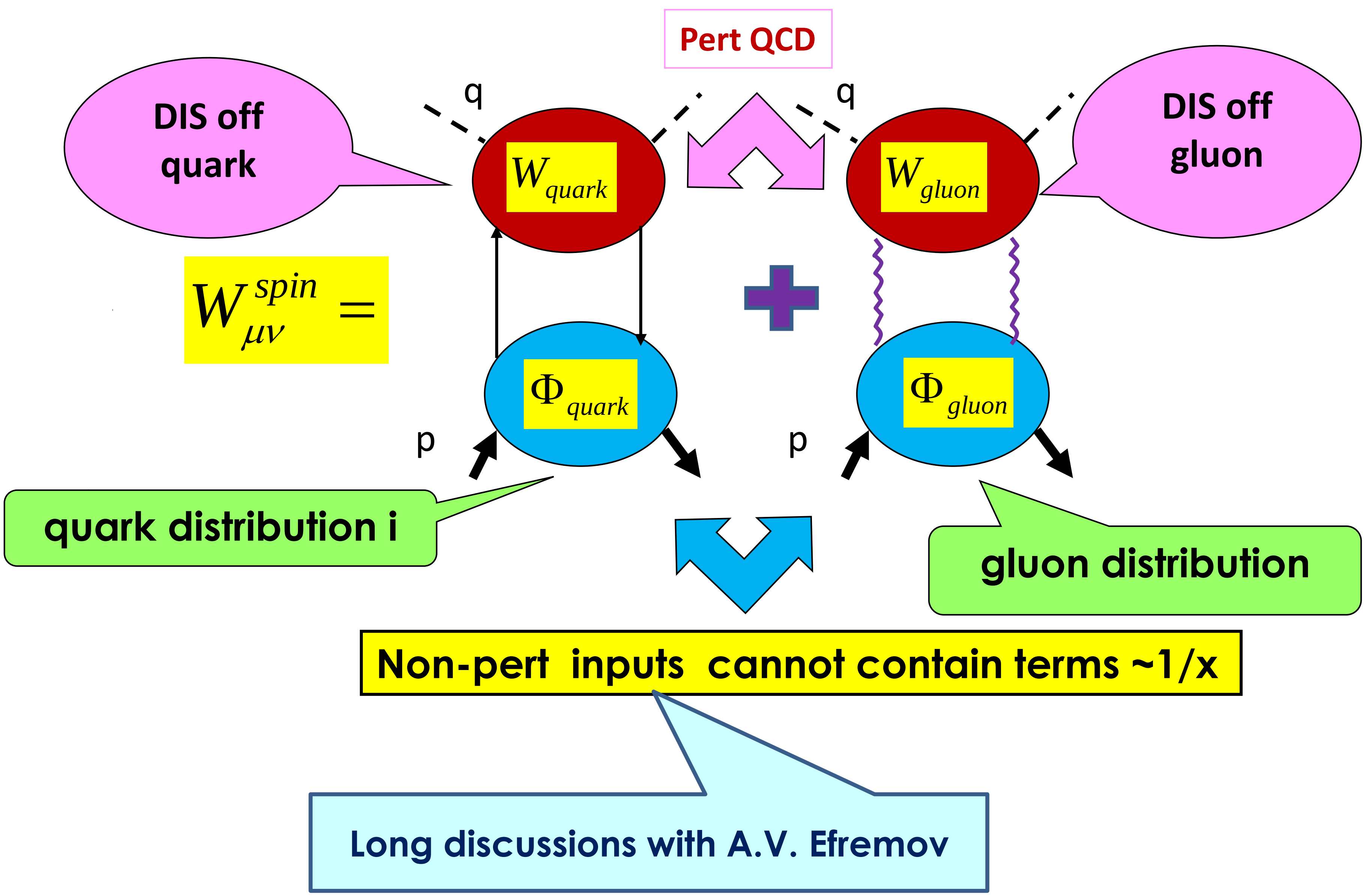
S. Catani - M. Ciafaloni – F. Hautmann

J.C. Collins, R.K. Ellis

These conventional forms of Factorization were introduced from different considerations and are used for different perturbative approaches

Recently a new, more general type of Factorization appeared:
Basic Factorization Step-by-step it can be reduced first to K_T and and then to Collinear Factorizations   Ermolaev-Greco-Troyan

QCD FACTORIZATION



$$g_{1,2} = g^{quark}{}_{1,2} \otimes \Phi_{quark} + g^{gluon}{}_{1,2} \otimes \Phi_{gluon}$$

Expression for g_1 look simpler in Mellin representation

Coefficient function

Anomalous dimension

Non-singlet:

$$g^{NS}{}_1(x, Q^2) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i}\, x^{-\omega}\, C_{NS}(\omega)\, e^{y\, h_{NS}}\, \Phi_{NS}(\omega)$$

Initial quark distribution

$$y = ln\left(Q^2/\mu^2\right)$$

Factorization scale

Expression for singlet g_1 is similar but more complicated singlet:

Initial quark distribution

$$g^S{}_1\left(x, Q^2\right) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i}\, x^{-\omega}\left(C^{(+)}\, e^{y\,\Omega_{(+)}} + C^{(-)}\, e^{y\,\Omega_{(+)}}\right)\ \Phi_q(\omega)$$

$$+\ \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i}\, x^{-\omega}\left(\widetilde{C}^{(+)}\, e^{y\,\Omega_{(+)}} + \widetilde{C}^{(-)}\, e^{y\,\Omega_{(+)}}\right)\ \Phi_g(\omega)$$

Initial gluon distribution

$\Omega_{(\pm)}(\omega)$ are made of anomalous dimensions

$$h_{(qq)}(\omega), h_{(qg)}(\omega), h_{(gq)}(\omega),.\ h_{(gg)}(\omega)$$

In the DGLAP framework Dokshitzer- Gribov- Lipatov- Altarelli - Parizi

$C^{(\pm)}(\omega), \widetilde{C}^{(\pm)}(\omega), \Omega_{(\pm)}(\omega)$ were calculated first in fixed orders in α_s

Later, both coefficient functions and anomalous dimensins for g_1 were calculated in double- logarithmic approximation:

Bartels – Ermolaev – Ryskin fixed α_s

Ermolaev – Greco – Troyan running α_s

There were obtained both explicit expressions for g_1 and the small-x asymptotics for both flavor singlet and non-singlet.

Double-Logarithmic Approximation

Example: elastic forward scattering amplitude:

$$M(s) = M^{Born}\left[1 + c_1\,\alpha_s\,ln^2 s + c_2\left(\alpha_s\,ln^2 s\right)^2 + c_3\left(\alpha_s\,ln^2 s\right)^3 + \ldots\right]$$

Structure function g_1 depends on two arguments, so DLA accounts for the terms

$$\sim {\alpha_s}^n\, ln\, x^{2n-1-k}\, ln^k\, Q^2 \qquad 0 \le k < n, \qquad n = 1, 2, \ldots.$$

In the ω -space

$$C_{NS} = 1 + \frac{c_1}{\omega^2} + \frac{c_2}{\omega^4} + \ldots \qquad h_{NS} = \frac{c_1}{\omega} + \frac{c_2}{\omega^3} + \ldots$$

DLA accounts for the contributions most singular at $\omega \to 0$

Applying the Saddle Point method to expressions for $g_1{}^{NS}$ and $g_1{}^{S}$ allowed us to obtain the small-x asymptotics

They proved to be of the Regge kind and their intercepts are the rightmost singularities :

Small-x asymptotics at x <<1 and $Q^2 >> \mu^2$

$\mu \approx 1\ \mathrm{GeV}$

Factorization scale

Using PMS

non-singlet

singlet

$$g_1{}^{NS} \sim x^{-\Delta_{NS}} \left(Q^2/\mu^2\right)^{\Delta_{NS}/2}$$

$$g_1{}^{S} \sim x^{-\Delta_{S}} \left(Q^2/\mu^2\right)^{\Delta_{S}/2}$$

intercepts

Asymptotic scaling

$$g_1{}^{NS} \sim \left(Q^2/x^2\right)^{\Delta_{NS}/2}$$

$$g_1{}^{S} \sim \left(Q^2/x^2\right)^{\Delta_{S}/2}$$

when α_s is kept fixed the intercept are:

$$\Delta_{NS} \approx \omega_+ \left[\frac{1 + \left(1 + 4/\left(N^2 - 1\right)\right)^{1/2}}{2}\right]^{1/2} \approx \boxed{\omega_+ \left(1 + 1/2\, N^2\right)^{1/2}}$$

$$\boxed{\omega_+ = \left(2\alpha_s C_F/\pi\right)^{1/2}}$$

$$\boxed{\Delta_S = 3.45\left(\alpha_s N/2\pi\right)^{1/2}}$$

Intercept of F_1^{NS}

Ermolaev- Manaenkov – Ryskin

Obtained by numerical calculations

$$C_F = \left(N^2 - 1\right)/2\,N, \qquad N = 3$$

WARNING: α_s is fixed at unknown scale Mostly, the DGLAP parametrization $\alpha_s = \alpha_s\left(Q^2\right)$ is used, which is incorrect at small x

When quark contributions to g_1 singlet are neglected, the intercept grows:

Pure gluon g_1 singlet intercept $\Delta_{gluon} = 3.66\,(\alpha_s N/2\pi)^{1/2}$

Represent $\Delta_{gluon} = z_h\,(\alpha_s N/2\pi)^{1/2}$ with $z_h = 3.66$

RUNNUNG COUPLING:

When α_s is running, the intercepts can be found numerically only:

$\boxed{\Delta_{NS} = 0.42}$ $\boxed{\Delta_S = 0.85}$

NB Pretty close to the value obtained by extrapolating HERA data to small x

Kochelev-Lipka-Nowak-Vento-Vinnikov

CRITICISM and ALTERNATIVE CALCULATIONS of SINGLET INTERCEPT

Interest to theoretical investigation of g_1 increased in 2015 when
Kovchegov-Pitonyak-Sievert 2015
investigated small-x asymptotics of helicity in DLA with
α_s in the ladder approximation. They confirmed our previous result on
Intercept non-singlet structure function F_1
Ermolaev-Manaenkov-Ryskin, 1995

Next year they included in consideration non-ladder graphs, obtaining
thereby intercept of g_1 and arrived at a huge
Disagreement with the result
Bartels- Ermolaev –Ryskin, 1996

They considered purely gluon DL contributions and represented their result on the intercept as follows:

$$\tilde{\Delta}_{gluon} = \tilde{z}_h \,(\alpha_s N/2\pi)^{1/2}$$

while our result is

$$\Delta_{gluon} = z_h \,(\alpha_s N/2\pi)^{1/2}$$

$\tilde{z}_h = 2.45$ vs $z_h = 3.66$

Strong discrepancy

Publishing such huge discrepancy provoked an extensive interest in the matter, so many authors contributed to this issue, namely

Kovchegov, Pitonyak, Sievert, Borden, Adamiak, Yossathom, Tawabutr, Santiago, Tarasov, Venugoplan, Chirilli, Gougoulic, Nayan Mani Nath, Jayanta Kumar Sarma, Zhou, ..

These authors also studied small- x evolution of helicity, using the JIMWLK -approach

Jalilian-Marian, Iancu, McLerran, Weigert,,Leonidov, Kovner

However, JIMWLK originally was designed for evolution of unpolarized objects , so
Kovchegov- Pitonyak - Sievert
generalized it to study the helicity evolution
and other authors also developed various modifications of JIMWLK trying to obtain most accurate estimates of z_h

This polemics continued till 2023

As a results of this polemics of 2016- 2023, the first estimate of 2016 (called KPS-evolution)
Kovchegov- Pitonyak - Sievert

$z_h = 2.45$

KPS 2016

was drastically corrected by
Kovchegov- Pitonyak - Sievert – Cougoulic- Tarasov- Tawabutr

when they constructed KSPTT evolution equation instead of KPS. Their estimate of 2023 is

$z_h = 3.66$

KPSCTT 2023

which coincides with BER result of 1996

However, recently accuracy of calculations in the framework of KPSCTT – evolution was increased, so same authors (e.g. Tawabutr) have concluded that there still remains a small disagreement

The newest estimate :

$$\tilde{z}_h = 3.661 \quad \text{vs} \quad z_h = 3.664$$

KPSCTT 2023

BER 1996

Obtained with modified JIMWLK equations

Obtained with InfraRed Evolution equations

Although the discrepancy is small, the authors of KPSCTT think that it requires further study

Comparison of KPSCTT and IREE approaches

KPSCTT

- Based on JIMWLK equation which is based on BFKL
- Intended to calculate small-x asymptotics of various objects with polarized partons
- Operates with fixed α_s only and because of that the obtained expressions for the intercepts contain α_s

IREE

- Based on evolution in Infra-Red cut off
- Intended to calculate various objects in DLA, unpolarized and polarized partons
- Operates with fixed and running α_s, so Intercepts do not contain α_s explicitly

ONSET of IREE

This method was invented by L.N. Lipatov .
It stems from the observation that the bremsstrahlung photon with minimal transverse momentum (the softest photon) can be factorized out of the radiative amplitudes with DL accuracy V.N. Gribov

Similarly, DL contributions of softest virtual quarks/gluons can be factorized

DL contributions of soft gluons are infrared (IR)-divergent. When quark masses are neglected, DL contributions from soft quarks also become IR-divergent.

In order to regulate them, one can introduce an IR cut-off μ
Lipatov suggested to introduce it in the transverse momentum space. It makes possible to use the factorization obtained by Gribov.

After factorizing the softest quarks and gluons, their transverse momenta act as a new IR cut-off, instead of μ , for integrating over momenta of other virtual partons.

Value of μ obeys the restriction $\mu \ll \Lambda_{QCD}$ in order to allow applying Perturbative QCD, otherwise it is arbitrary. This makes possible to evolve the objects under consideration with respect to μ

It is the reason why the method is named IREE. (M.Krawczyk)
The method proved to be effective and simple instrument for calculations in Double-Logarithmic Approximation (DLA), i.e. when contributions

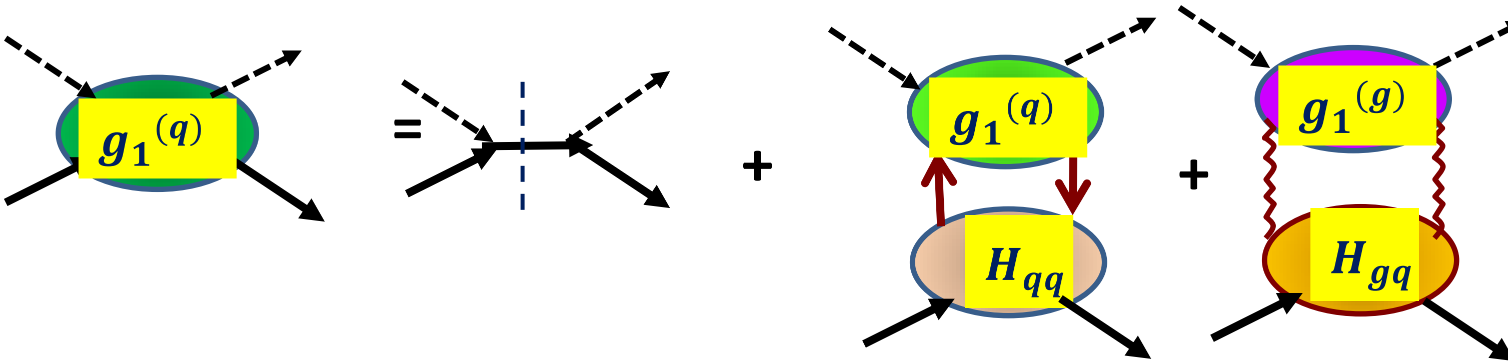
$$\sim {\alpha_s}^n \, ln \, x^{2n-1-k} \, ln^k \, Q^2 \qquad (n = 1, 2, \dots .)$$

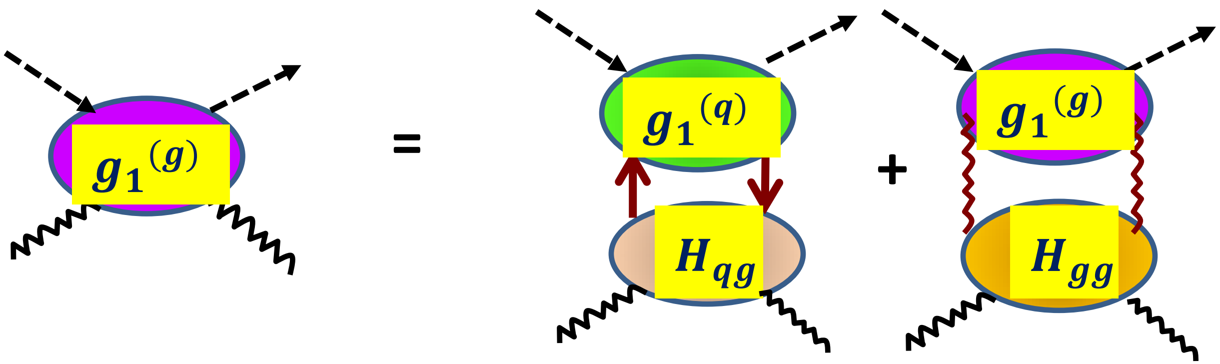
are accounted to all orders in α_s

At the beginning, IREE operated with fixed α_s
but later the running coupling effects were incorporated (Ermolaev-Greco-Troyan)

Now we demonstrate how to construct IREEs for perturbative components of g_1

Constructing IREEs for $g_1^{(q)}$ and $g_1^{(g)}$:





NB similar equations were obtained by Efremov and Ginzburg

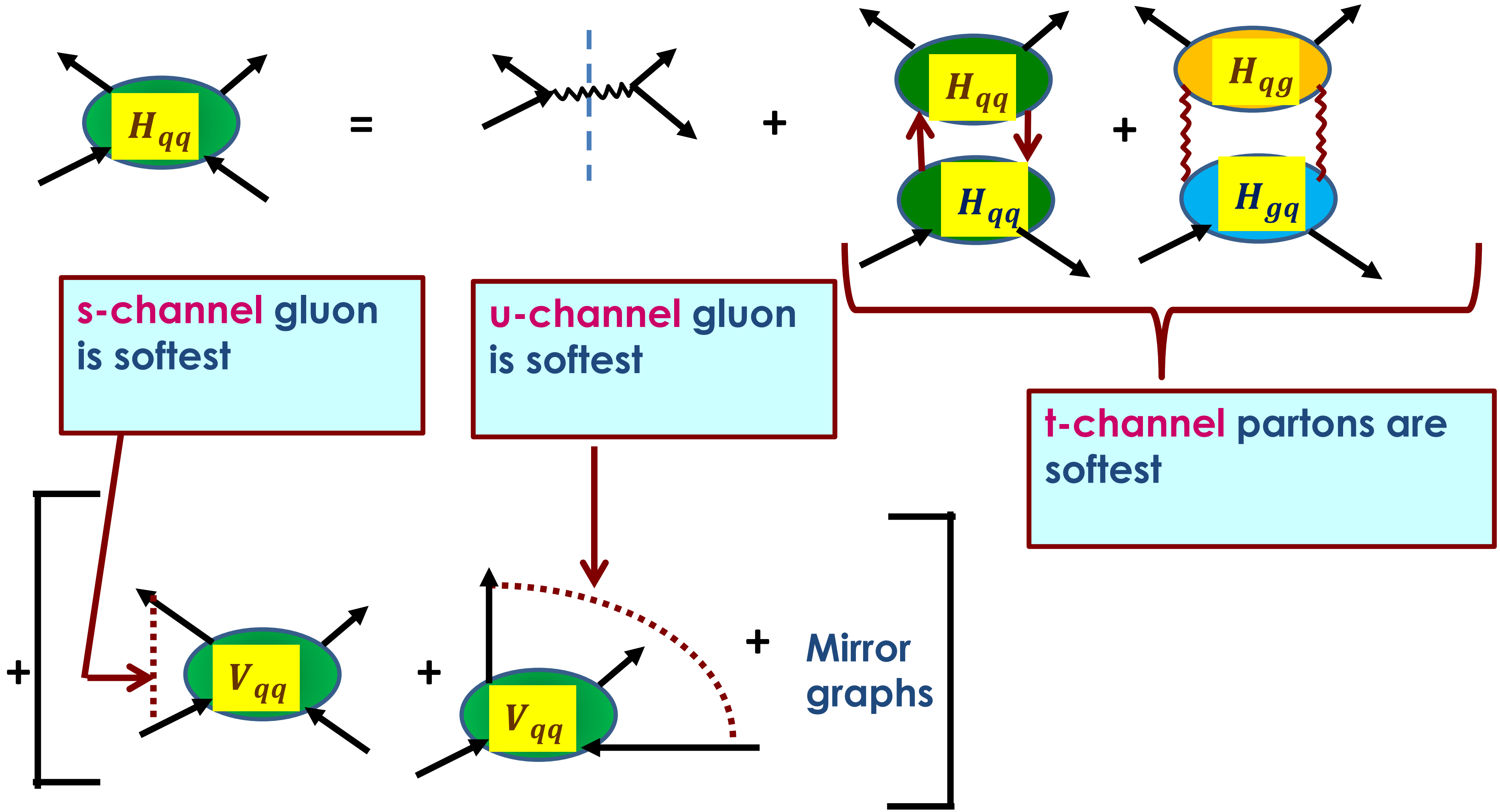
All intermediate t-channel partons have minimal transverse momenta and are called softest

Only two-parton intermediate states yield DL contributions

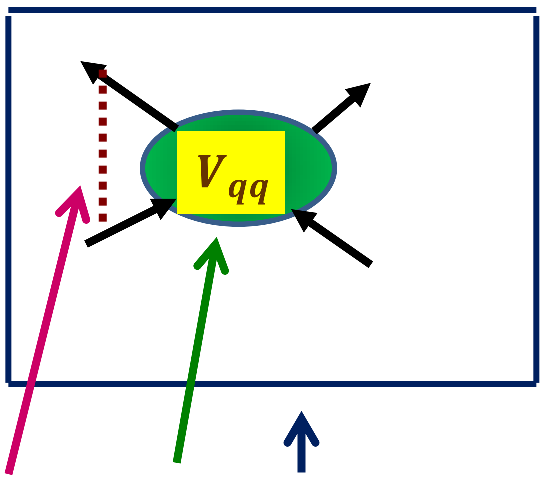
$g_1^{(q)}$ and $g_1^{(g)}$ are expressed through themselves and new parton-parton amplitudes H_{ij}

In turn, H_{ij} can also be obtained with constructing IREEs.

For instance, consider IREE for quark-quark amplitude H_{qq}

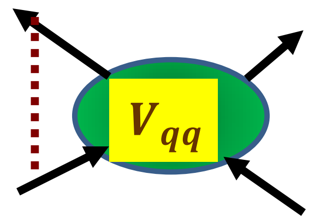


Object in frame is t-channel color singlet, so the blob is t-cannel octet

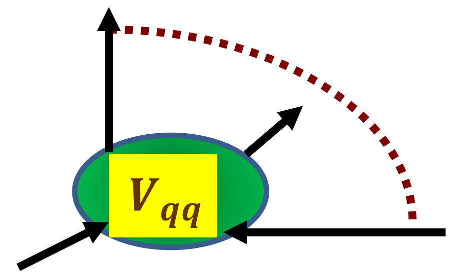


$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \ldots$$

These terms



and



have opposite signs and because of that they kill each other in the case of unpolarized structure functions

However, in the case of g_1 they have equal signs

Vanishing DL contributions from 2 ->2 non-ladder graphs with positive signature was first noticed in the QED context by Gorshkov-Lipatov-Nesterov

Singlet g_1 involves four octets: V_{qq} V_{qg} V_{gq} V_{gg}

NB The main technical problem of KPSCTT approach was including non-ladder graphs in consideration because its parent equation , JIMWLK was designed for studying unpolarized processes, where DL come from ladders only

In contrast, IREE approach accounts for non-ladder graphs through introducing octets, which is much simpler technically

For instance

$$V_{qq}\ (\omega) = \frac{\alpha_{sN}}{\pi} \frac{d}{d\,\omega}\, ln\left(e^{z^2} D_p(z)\right)$$ Kirschner-Lipatov

parabolic cylinder function

with

$z = \omega/\sqrt{\alpha_s\, N/2\,\pi}$ and $p = -1/2N^2$

For practical needs V_{qq} is often approximated by its Born value

$$V_{qq}\ (\omega) \approx -2\pi\alpha_s/N\,\omega$$

Comparison KPSCTT with IREE

KPSCTT main feature

BFKL
Total resummation of Leading Logs Solution presented through asymptotics

BK/JIMWLK/KPSCTT
Sum DL Solution is also presented through asymptotics

Absent in BFKL/KPSCTT

IREE

Total resummation of DL

Explicit expressions for amplitudes

Asymptotics are obtained with Saddle Point method

For implications, It is important to know what is applicability region for asymptotics

Applicability region of Regge asymptotics

Ermolaev-Greco-Troyan

Regge asymptotics are given by simple and elegant expressions.
However the applicability regions of the asymptotics are poorly known

Asymptotics

We introduce $R_{as}(x) = As(g_{1,})/g_1$
and numerically study its **x**-dependence at fixed Q^2

Asymptotics reliably represent $F_{1,2,L}$ when R_{as} is close to 1.
Numerical analysis yields

$x = 10^{-3}$ $R_{AS} \approx 0.5$

$x = 10^{-4}$ $R_{AS} \approx 0.7$

$x = 10^{-6}$ $R_{AS} \approx 0.9$

Appicability region for asymptotics

$x < x_0 = 10^{-6}$

Spin-dependent Pomeron

so, it is not Pomeron

Asymptotics of spin-dependent structure function g_1 is $x^{-\omega_0}$ with $\omega_0 = 0.86 < 1$ Ermolaev-Greco-Troyan

and the applicability region is $x \gg x_o = 10^{-6}$

Using asymptotics at $x > x_o$ means $x^{-a} \approx x_0^{-\Delta}$

For instance, choose $x = 10^{-4}$ $\Rightarrow$ $a \approx \Delta \frac{\ln x_0}{\ln x} = 0.86 \frac{\ln x_0}{\ln x}$

$$a = 0.86 \frac{6}{4} = 1.29$$

Spin-dependent Pomeron i.e. Illegal usage of asymptotics

CONCLUSIONS

Polemics on intercept of g_1 started in 2016 is now over.

It resulted in constructing KPSCTT evolution equation to replace JIMWLK

As a result, BER estimate of the intercept proved to be correct save small fraction

Ways to increase accuracy of calculating the intercept of g_1 within IREE approach

- Go beyond the Born approximation for the octets
- Account for single-logarithmic contributions