

T-odd angular coefficients for Drell-Yan process with Z neutral boson

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Efremov-90

- 1 Drell-Yan - theory part
- 2 Drell-Yan -from experiment

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Drell-Yan process

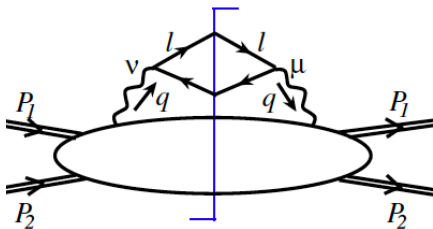


Figure: Diagrammatic representation of hadronic dilepton production via a virtual photon of four-momentum q .

Collins-Soper frame

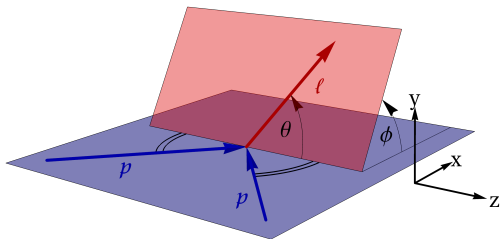


Figure: Definition of the polar and the azimuthal angles for the Drell-Yan process in the Collins-Soper frame.

Drell-Yan process $((\alpha_s)^0)$

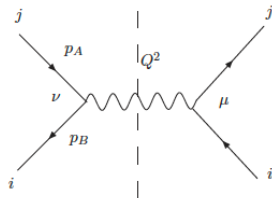


Figure: Born level

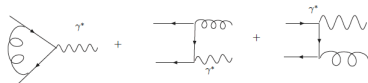
Drell-Yan process $((\alpha_s)^0)$

In general the hadronic tensor for the DY reaction can be written in terms of the set of nine structure functions W_i and with the use of the basis of orthogonal unit vectors

$$T^\mu = \mathbf{q}^\mu / \sqrt{Q^2} = (1, 0, 0, 0), \quad X^\mu = (0, 1, 0, 0), \quad Z^\mu = (0, 0, 0, 1), \\ Y^\mu = \epsilon^{\mu\nu\alpha\beta} T_\nu Z_\alpha X_\beta = (0, 0, 1, 0):$$

$$W^{\mu\nu} = (X^\mu X^\nu + Y^\mu Y^\nu) W_T \quad (1)$$

Drell-Yan process $(\alpha_s)^1$



and contributions from the gluon content of the hadrons,

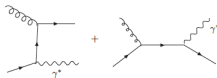


Figure: Diagram in parton level for Drell-Yan process $(\alpha_s)^1$

Drell-Yan process $(\alpha_s)^1$

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$$W^{\mu\nu} = (X^\mu X^\nu + Y^\mu Y^\nu) W_T + Z^\mu Z^\nu W_L \\ + (Y^\mu Y^\nu - X^\mu X^\nu) W_{\Delta\Delta} - (X^\mu Z^\nu + Z^\mu X^\nu) W_\Delta \quad (2)$$

Drell-Yan process $(\alpha_s)^1$ with Z

In general the hadronic tensor for the DY reaction can be written in terms of the set of nine structure functions W_i and with the use of the basis of orthogonal unit vectors

$$T^\mu = q^\mu / \sqrt{Q^2} = (1, 0, 0, 0), \quad X^\mu = (0, 1, 0, 0), \quad Z^\mu = (0, 0, 0, 1), \\ Y^\mu = \epsilon^{\mu\nu\alpha\beta} T_\nu Z_\alpha X_\beta = (0, 0, 1, 0):$$

with Z-boson

$$W^{\mu\nu} = (X^\mu X^\nu + Y^\mu Y^\nu)W_T + i(X^\mu Y^\nu - Y^\mu X^\nu)W_{T_P} + Z^\mu Z^\nu W_L \\ + (Y^\mu Y^\nu - X^\mu X^\nu)W_{\Delta\Delta} - (X^\mu Z^\nu + Z^\mu X^\nu)W_\Delta + i(Y^\mu Z^\nu - Z^\mu Y^\nu)W_{\nabla_P}, \quad (3)$$

Drell-Yan process with Z

In general the hadronic tensor for the DY reaction can be written in terms of the set of nine structure functions W_i and with the use of the basis of orthogonal unit vectors

$$T^\mu = \mathbf{q}^\mu / \sqrt{Q^2} = (1, 0, 0, 0), \quad X^\mu = (0, 1, 0, 0), \quad Z^\mu = (0, 0, 0, 1), \\ Y^\mu = \epsilon^{\mu\nu\alpha\beta} T_\nu Z_\alpha X_\beta = (0, 0, 1, 0):$$

full structure is:

$$\begin{aligned} W^{\mu\nu} &= (X^\mu X^\nu + Y^\mu Y^\nu)W_T + i(X^\mu Y^\nu - Y^\mu X^\nu)W_{T_P} + Z^\mu Z^\nu W_L \\ &+ (Y^\mu Y^\nu - X^\mu X^\nu)W_{\Delta\Delta} - (X^\mu Y^\nu + Y^\mu X^\nu)W_{\Delta\Delta_P} \\ &- (X^\mu Z^\nu + Z^\mu X^\nu)W_\Delta - (Y^\mu Z^\nu + Z^\mu Y^\nu)W_{\Delta_P} \\ &+ i(Z^\mu X^\nu - X^\mu Z^\nu)W_\nabla + i(Y^\mu Z^\nu - Z^\mu Y^\nu)W_{\nabla_P} \end{aligned} \quad (4)$$

Lepton angular distributions $dN/d\Omega$ are expanded in terms of hadronic structure functions as

$$\frac{dN}{d\Omega} = \frac{3}{16\pi(2W_T + W_L)} \left[g_T W_T + g_L W_L + g_\Delta W_\Delta + g_{\Delta\Delta} W_{\Delta\Delta} \right. \\ \left. + g_{T\rho} W_{T\rho} + g_{\nabla\rho} W_{\nabla\rho} + g_\nabla W_\nabla + g_{\Delta\Delta\rho} W_{\Delta\Delta\rho} + g_{\Delta\rho} W_{\Delta\rho} \right], \quad (5)$$

$$\frac{dN}{d\Omega} = \frac{3}{16\pi} \left(1 + \cos^2 \theta + \frac{A_0}{2} (1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi + \frac{A_2}{2} \sin^2 \theta \cos 2\phi \right. \\ \left. + A_3 \sin \theta \cos \phi + A_4 \cos \theta + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi \right) \quad (6)$$

and

$$\frac{dN}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left(1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right. \\ \left. + \tau \sin \theta \cos \phi + \eta \cos \theta + \xi \sin^2 \theta \sin 2\phi + \zeta \sin 2\theta \sin \phi + \chi \sin \theta \sin \phi \right). \quad (7)$$

Hadronic structure functions $W(x_1, x_2)$ relevant for the DY process with colliding of hadrons H_1 and H_2 are related to partonic-level structure functions $w^{ab}(x_1, x_2)$ in QCD by the collinear factorization formula

$$W(x_1, x_2) = \frac{1}{x_1 x_2} \sum_{a,b} \int_{x_1}^1 dz_1 \int_{x_2}^1 dz_2 w^{ab}(z_1, z_2) f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right), \quad (8)$$

Feynman diagram for T-odd Drell-Yan

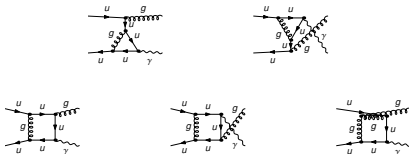


Figure: Diagrams describing NLO $u\bar{u} \rightarrow g\gamma$ subprocess at order $\mathcal{O}(\alpha_s^2)$.

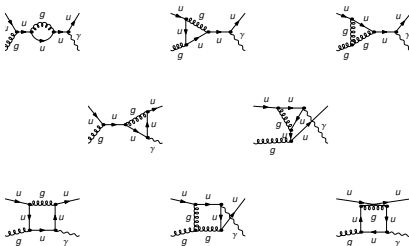


Figure: Diagrams describing NLO $u g \rightarrow u\gamma$ subprocess at order $\mathcal{O}(\alpha_s^2)$.

Calculation of T-odd helicity structures

In order to calculate the T -odd structure functions for convenience we use the orthogonal basis (P, R, K) [JHEP06(2021)066]:

$$\begin{aligned}P^\mu &= (p_1 + p_2)^\mu, \\R^\mu &= (p_1 - p_2)^\mu, \\K^\mu &= k_1^\mu - P^\mu \frac{P \cdot k_1}{P^2} - R^\mu \frac{R \cdot k_1}{R^2} = -q^\mu + P^\mu \frac{P \cdot q}{P^2} + R^\mu \frac{R \cdot q}{R^2}\end{aligned}\quad (9)$$

obeying the conditions

$$P^2 = -R^2 = \hat{s}, \quad K^2 = -\frac{\hat{u}\hat{t}}{\hat{s}}, \quad P \cdot R = P \cdot K = R \cdot K = 0. \quad (10)$$

Calculation of T-odd helicity structures

The (P, R, K) and (T, X, Y, Z) bases are related as:

$$\begin{aligned} X^\mu &= \frac{T^\mu \sqrt{1 + \rho^2}}{\rho} - \frac{P^\mu z_{12}^+ + R^\mu z_{12}^-}{2Q\rho\sqrt{1 + \rho^2}} \\ &= \frac{\rho(P^\mu z_{12}^+ + R^\mu z_{12}^-)}{2Q\sqrt{1 + \rho^2}} - \frac{K^\mu \sqrt{1 + \rho^2}}{Q\rho}, \\ Z^\mu &= \frac{P^\mu z_{12}^- + R^\mu z_{12}^+}{2Q\sqrt{1 + \rho^2}}, \\ Y^\mu &= -\epsilon^{\mu PRK} \frac{z_1 z_2}{Q^3 \rho(1 + \rho^2)} \end{aligned} \quad (11)$$

where $z_{12}^\pm = z_1 \pm z_2$, $Q = \sqrt{Q^2}$, and $\epsilon^{\mu PRK} = \epsilon^{\mu\nu\alpha\beta} P_\nu R_\alpha K_\beta$. Here $z_i = x_i/\xi_i$ are the fraction parameters with $p_i = \xi_i P_i$, factors $x_{1,2} = e^{\pm y} \sqrt{(Q^2 + Q_T^2)/s}$ are the fractions of the longitudinal gauge boson momentum at finite Q_T . We also define the factors $x_{1,2}^0 = e^{\pm y} Q/\sqrt{s}$ at $Q_T^2 = 0$.

Calculation of T-odd helicity structures

Partonic-level T-odd structure functions are fixed as

$$\begin{aligned}w_{\Delta\Delta P} &= -\frac{1}{2}(X^\mu Y^\nu + X^\nu Y^\mu) w_{\mu\nu} \\ &= -\frac{z_1 z_2}{4Q^4 \rho^2 (1 + \rho^2)^{3/2}} \left[\epsilon^{\mu PRK} \left(P^\nu z_{12}^+ + R^\nu z_{12}^- \right) + \epsilon^{\nu PRK} \left(P^\mu z_{12}^+ + R^\mu z_{12}^- \right) \right] w_{\mu\nu},\end{aligned}\quad (12)$$

$$\begin{aligned}w_{\Delta P} &= -\frac{1}{2}(Y^\mu Z^\nu + Y^\nu Z^\mu) w_{\mu\nu} \\ &= \frac{z_1 z_2}{4Q^4 \rho (1 + \rho^2)^{3/2}} \left[\epsilon^{\mu PRK} \left(P^\nu z_{12}^- + R^\nu z_{12}^+ \right) + \epsilon^{\nu PRK} \left(P^\mu z_{12}^- + R^\mu z_{12}^+ \right) \right] w_{\mu\nu},\end{aligned}\quad (13)$$

$$w_{\nabla} = \frac{i}{2}(X^\mu Z^\nu - X^\nu Z^\mu) w_{\mu\nu} = \frac{iz_1 z_2}{2Q^2 \rho (1 + \rho^2)} \left(P^\nu R^\mu - P^\mu R^\nu \right) w_{\mu\nu}. \quad (14)$$

Calculation of T-odd helicity structures

For $q\bar{q}$ annihilation we find

$$\begin{aligned}
 w_{\Delta\Delta_P}^{q\bar{q}} &= \frac{g_{q\bar{q};1}}{2} \sqrt{\frac{Q^2\hat{s}}{(Q^2-\hat{u})(Q^2-\hat{t})}} \left[-\frac{C_F}{2} \left(\frac{Q^2-\hat{t}}{Q^2-\hat{u}} + \frac{Q^2-\hat{u}}{Q^2-\hat{t}} \right) \right. \\
 &+ \left. C_1 \left(\frac{Q^2-\hat{t}}{\hat{t}} \left(1 - \frac{\hat{s}}{\hat{t}} \log \frac{Q^2-\hat{u}}{\hat{s}} \right) + \frac{Q^2-\hat{u}}{\hat{u}} \left(1 - \frac{\hat{s}}{\hat{u}} \log \frac{Q^2-\hat{t}}{\hat{s}} \right) \right) \right], \quad (15)
 \end{aligned}$$

$$\begin{aligned}
 w_{\Delta_P}^{q\bar{q}} &= \frac{g_{q\bar{q};1}}{2} \frac{Q^2\hat{s}}{\sqrt{(Q^2-\hat{u})(Q^2-\hat{t})}\hat{u}\hat{t}} \left[C_F \left(\frac{Q^2-\hat{t}}{Q^2-\hat{u}} - \frac{Q^2-\hat{u}}{Q^2-\hat{t}} \right) \right. \\
 &+ \left. C_1 \left(\frac{Q^2-\hat{u}}{\hat{u}} \log \frac{Q^2-\hat{t}}{\hat{s}} - \frac{Q^2-\hat{t}}{\hat{t}} \log \frac{Q^2-\hat{u}}{\hat{s}} \right) \right], \quad (16)
 \end{aligned}$$

$$\begin{aligned}
 w_{\nabla}^{q\bar{q}} &= g_{q\bar{q};2} \sqrt{\frac{Q^2\hat{s}}{\hat{u}\hat{t}}} \left[\frac{C_F}{2} \frac{(2Q^2\hat{s} + \hat{u}\hat{t})(Q^2 + \hat{s})(\hat{u} - \hat{t})}{(Q^2 - \hat{u})^2(Q^2 - \hat{t})^2} \right. \\
 &+ \left. C_1 \left(-\frac{Q^2(\hat{u} - \hat{t})}{(Q^2 - \hat{u})(Q^2 - \hat{t})} + \frac{\hat{s}}{\hat{u}} \log \frac{Q^2 - \hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \log \frac{Q^2 - \hat{u}}{\hat{s}} \right) \right]. \quad (17)
 \end{aligned}$$

$$w_{\Delta\Delta\rho}^{q\bar{q}} = -\frac{g_{q\bar{q};1}}{4z_1z_2} \frac{1}{\sqrt{1+\rho^2}} \left[(C_F - C_1)(z_1^2 + z_2^2) + C_1(z_1^2 F_1(z_2) + z_2^2 F_1(z_1)) \right], \quad (18)$$

$$w_{\Delta\rho}^{q\bar{q}} = -\frac{g_{q\bar{q};1}}{2z_1z_2} \frac{1}{\rho\sqrt{1+\rho^2}} \left[(C_F - C_1)(z_1^2 - z_2^2) + C_1(z_1^2 F_2(z_2) - z_2^2 F_2(z_1)) \right], \quad (19)$$

$$w_{\nabla}^{q\bar{q}} = -\frac{g_{q\bar{q};2}}{z_1z_2} \frac{1}{\rho} \left[\left(C_F \left(1 - \frac{\rho^2}{2(1+\rho^2)} \right) - C_1 \right) (z_1^2 - z_2^2) + C_1(z_1 F_2(z_2) - z_2 F_2(z_1)) \right] \quad (20)$$

Here, the functions F_1 and F_2 are defined as:

$$F_1(z) = \frac{1+z}{1-z} + \frac{2z \log(z)}{(1-z)^2} = 2 \sum_{N=1}^{\infty} \frac{(1-z)^N}{(N+1)(N+2)} = \mathcal{O}(1-z)$$

$$, \quad F_2(z) = 1 + \frac{z \log(z)}{1-z} = \frac{1-z}{2} (1 + F_1(z)) = \sum_{N=1}^{\infty} \frac{(1-z)^N}{N(N+1)} = \mathcal{O}(1-z). \quad (21)$$

We have full agreement with results obtained in Refs. [Phys.Rev.Lett.52 Kaoru Hagiwara, Ken-ichi Hikasa, Naoyuki Kai and in Nucl.Phys.B 387 (1992) 3—85 E. Mirkes.]

From the phase space

$$S(2\pi)^4 \delta(p_1 + p_2 - q - p_4) \frac{d^3 p_4}{(2\pi)^3 2E_4} = 2\pi S \delta(s + t + u - Q^2) \quad (22)$$

In the calculation of the phase space factor corresponding to the one-parton production we get the well-known delta function (see, e.g., in Ref. [D. Boer and P. J. Mulders PhysRevD.57.5780]) relevant for the DY process

$$\delta\left((1 - z_1)(1 - z_2) - \frac{\rho^2}{1 + \rho^2} z_1 z_2\right) \varphi(z_1, z_2). \quad (23)$$

From Gelfand-Shilov:

$$\delta(xy - c) = -2\delta(x, y) \ln c + \left(\frac{\delta(y)}{x} + \frac{\delta(x)}{y} \right) + o(c).$$

$$\delta(G - c) = \delta(G) - c\delta'(G) + \frac{c^2}{2}\delta''(G) + \dots + \frac{(-1)^k}{k!}\delta^{(k)}(G) + \dots$$

ρ^2 – expansion

In $Q_T Q \rightarrow 0$ limit we can use famous expansion for delta-function which comes from phase space factor

$$\begin{aligned} \frac{S}{z_1 z_2} \delta(\hat{s} + \hat{t} + \hat{u} - Q^2) \Rightarrow \frac{1}{x_1 x_2} \left[\frac{\delta(1 - z_2)}{(1 - z_1)_+} + \frac{\delta(1 - z_1)}{(1 - z_2)_+} \right. \\ \left. + \delta(1 - z_1) \delta(1 - z_2) \ln \frac{Q^2}{Q_1^2} \right]. \end{aligned} \quad (23)$$

The standard definition of “+” distribution is

$$\int_x^1 dz \frac{f(z)}{(1 - z)_+} = \int_x^1 dz \frac{f(z) - f(1)}{(1 - z)} + f(1) \ln(1 - x).$$

$$\begin{aligned}
 W_{\Delta\Delta\rho}^{\text{LO};q\bar{q}}(x_1^0, x_2^0) &= -\frac{g_{q\bar{q};1}}{4x_1^0x_2^0} (C_F - C_1) (2 \log \rho^2 + 3) q_1(x_1^0) \bar{q}_2(x_2^0) \\
 &- \frac{g_{q\bar{q};1}}{4x_1^0x_2^0} \frac{C_F - C_1}{C_F} \left[\int_{x_2^0}^1 \frac{dz_2}{z_2} P_{qq}(z_2) q_1(x_1^0) \bar{q}_2\left(\frac{x_2^0}{z_2}\right) + \int_{x_1^0}^1 \frac{dz_1}{z_1} P_{qq}(z_1) q_1\left(\frac{x_1^0}{z_1}\right) \bar{q}_2(x_2^0) \right] \\
 &- \frac{g_{q\bar{q};1}}{4x_1^0x_2^0} C_1 \left[\int_{x_2^0}^1 \frac{dz_2}{z_2} f_1(z_2) q_1(x_1^0) \bar{q}_2\left(\frac{x_2^0}{z_2}\right) + \int_{x_1^0}^1 \frac{dz_1}{z_1} f_1(z_1) q_1\left(\frac{x_1^0}{z_1}\right) \bar{q}_2(x_2^0) \right], \tag{24}
 \end{aligned}$$

$$\begin{aligned}
 W_{\nabla}^{\text{LO};q\bar{q}}(x_1^0, x_2^0) &= 2 \frac{g_{EW;1}}{g_{EW;2}} W_{\Delta\rho}^{\text{LO};q\bar{q}}(x_1^0, x_2^0) \\
 &= -\frac{g_{q\bar{q};1}}{\rho x_1^0 x_2^0} \frac{C_F - C_1}{C_F} \left[\int_{x_2^0}^1 \frac{dz_2}{z_2} \tilde{P}_{qq}(z_2) q_1(x_1^0) \bar{q}_2\left(\frac{x_2^0}{z_2}\right) - \int_{x_1^0}^1 \frac{dz_1}{z_1} \tilde{P}_{qq}(z_1) q_1\left(\frac{x_1^0}{z_1}\right) \bar{q}_2(x_2^0) \right] \\
 &- \frac{g_{q\bar{q};1}}{\rho x_1^0 x_2^0} C_1 \left[\int_{x_2^0}^1 \frac{dz_2}{z_2} f_2(z_2) q_1(x_1^0) \bar{q}_2\left(\frac{x_2^0}{z_2}\right) - \int_{x_1^0}^1 \frac{dz_1}{z_1} f_2(z_1) q_1\left(\frac{x_1^0}{z_1}\right) \bar{q}_2(x_2^0) \right], \tag{25}
 \end{aligned}$$

ρ^2 – expansion

$$W_{\Delta\Delta\rho}^{\text{LO};qg}(x_1^0, x_2^0) = -\frac{g_{qg;1}}{4x_1^0 x_2^0} \int_{x_2^0}^1 \frac{dz_2}{z_2} P'_{qg}(z_2) q(x_1^0) g\left(\frac{x_2^0}{z_2}\right), \quad (26)$$

$$W_{\nabla}^{\text{LO};qg}(x_1^0, x_2^0) = 2 \frac{g_{EW;1}}{g_{EW;2}} W_{\Delta\rho}^{\text{LO};qg}(x_1^0, x_2^0) = \frac{g_{qg;1}}{\rho x_1^0 x_2^0} \int_{x_2^0}^1 \frac{dz_2}{z_2} P''_{qg}(z_2) q(x_1^0) g\left(\frac{x_2^0}{z_2}\right), \quad (27)$$

where

$$\begin{aligned} P_{qq}(z) &= C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right], \\ \tilde{P}_{qq}(z) &= C_F(1+z), \\ P'_{qg}(z, \rho^2) &= C_F(1+2z) + C_1 z (2 \log \rho^2 - z), \\ P''_{qg}(z, \rho^2) &= C_F(1-z) + C_1 z (\log \rho^2 (1-z) + 1 - 2z). \end{aligned} \quad (28)$$

Here

$$f_i(z) = \frac{F_i(z)}{1-z} \quad (29)$$

Sample frame title

For W_L and $W_{\Delta\Delta}$ we have Lam-Tung relation. This relation is work in LO and violate at high ρ^2 in NNLO (α_s^2) order. [Nucl.Phys.B 387 (1992) 3—85 E. Mirkes.; JHEP 11 (2017) 003 R. Gauld and etc.]

For T-odd structure helicity hadron fuctions of Drell-Yan process, we obtain new relation which exist at low ρ^2

Relation for T-odd

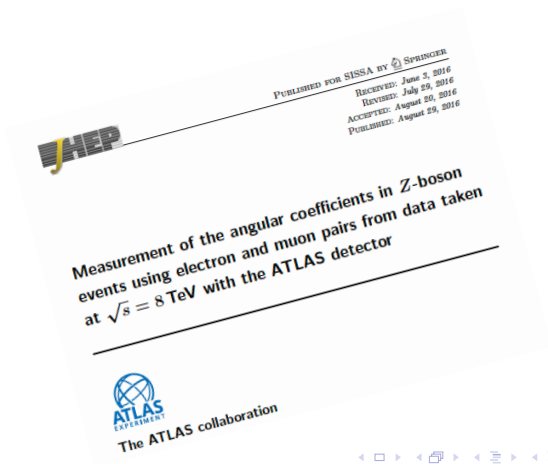
$$W_{\nabla}^{\text{LO};ab}(x_1^0, x_2^0) = 2 \frac{g_{\text{EW};1}}{g_{\text{EW};2}} W_{\Delta_P}^{\text{LO};ab}(x_1^0, x_2^0) \quad (30)$$

at small $\rho^2 = Q_T^2/Q^2$.

- 1 Drell-Yan - theory part
- 2 Drell-Yan -from experiment

Experiment measurements

Experimental measurements of T-odd $A_{5,6,7}$ angular coefficients were done in 2016.



y^Z -binned A_5			
p_T^Z range [GeV]	$0 < y^Z < 1$	$1 < y^Z < 2$	$2 < y^Z < 3.5$
0.0-2.5	$-0.002 \pm 0.005 \pm 0.003$	$0.000 \pm 0.007 \pm 0.004$	$-0.030 \pm 0.072 \pm 0.025$
2.5-5.0	$-0.003 \pm 0.002 \pm 0.001$	$-0.002 \pm 0.003 \pm 0.002$	$0.012 \pm 0.026 \pm 0.009$
5.0-8.0	$-0.003 \pm 0.002 \pm 0.001$	$-0.001 \pm 0.002 \pm 0.001$	$0.013 \pm 0.015 \pm 0.005$
8.0-11.4	$-0.002 \pm 0.002 \pm 0.001$	$0.000 \pm 0.002 \pm 0.001$	$0.006 \pm 0.013 \pm 0.005$
11.4-14.9	$0.000 \pm 0.002 \pm 0.001$	$0.003 \pm 0.002 \pm 0.001$	$0.004 \pm 0.013 \pm 0.005$
14.9-18.5	$0.000 \pm 0.002 \pm 0.001$	$0.004 \pm 0.002 \pm 0.001$	$0.000 \pm 0.014 \pm 0.005$
18.5-22.0	$0.002 \pm 0.002 \pm 0.001$	$0.005 \pm 0.002 \pm 0.001$	$0.004 \pm 0.016 \pm 0.005$
22.0-25.5	$0.003 \pm 0.002 \pm 0.001$	$0.005 \pm 0.002 \pm 0.001$	$0.013 \pm 0.018 \pm 0.006$
25.5-29.0	$0.003 \pm 0.002 \pm 0.001$	$0.004 \pm 0.003 \pm 0.001$	$0.012 \pm 0.020 \pm 0.006$
29.0-32.6	$0.004 \pm 0.002 \pm 0.001$	$0.003 \pm 0.003 \pm 0.001$	$0.025 \pm 0.022 \pm 0.007$
32.6-36.4	$0.004 \pm 0.002 \pm 0.001$	$0.002 \pm 0.003 \pm 0.001$	$0.026 \pm 0.023 \pm 0.008$
36.4-40.4	$0.003 \pm 0.003 \pm 0.001$	$0.001 \pm 0.003 \pm 0.002$	$0.041 \pm 0.025 \pm 0.008$
40.4-44.9	$0.002 \pm 0.003 \pm 0.002$	$0.001 \pm 0.004 \pm 0.002$	$0.030 \pm 0.025 \pm 0.008$
44.9-50.2	$0.002 \pm 0.003 \pm 0.002$	$0.002 \pm 0.004 \pm 0.002$	$0.025 \pm 0.023 \pm 0.009$
50.2-56.4	$0.001 \pm 0.003 \pm 0.002$	$0.004 \pm 0.004 \pm 0.002$	$0.002 \pm 0.025 \pm 0.010$
56.4-63.9	$0.001 \pm 0.003 \pm 0.002$	$0.006 \pm 0.004 \pm 0.002$	$-0.014 \pm 0.026 \pm 0.011$
63.9-73.4	$0.001 \pm 0.004 \pm 0.002$	$0.009 \pm 0.005 \pm 0.002$	$-0.010 \pm 0.028 \pm 0.012$
73.4-85.4	$0.003 \pm 0.005 \pm 0.002$	$0.011 \pm 0.005 \pm 0.003$	$-0.052 \pm 0.032 \pm 0.014$
85.4-105	$0.006 \pm 0.005 \pm 0.003$	$0.010 \pm 0.006 \pm 0.003$	$0.005 \pm 0.049 \pm 0.026$
105-132	$0.011 \pm 0.006 \pm 0.003$	$0.006 \pm 0.007 \pm 0.004$	
132-173	$0.018 \pm 0.008 \pm 0.004$	$-0.004 \pm 0.010 \pm 0.005$	
173-253	$0.030 \pm 0.014 \pm 0.007$	$-0.023 \pm 0.017 \pm 0.008$	
253-600	$0.045 \pm 0.025 \pm 0.012$	$-0.055 \pm 0.031 \pm 0.014$	

Table 19. The angular coefficient $A_5 \pm \delta_{\text{stat}} \pm \delta_{\text{sys}}$ in bins of y^Z .

	$p_T^Z = 5 - 8 \text{ GeV}$		$p_T^Z = 22 - 25.5 \text{ GeV}$		$p_T^Z = 132 - 173 \text{ GeV}$	
	NLO	NNLO	NLO	NNLO	NLO	NNLO
A_0	$0.0115^{+0.0006}_{-0.0003}$	$0.0150^{+0.0006}_{-0.0008}$	$0.1583^{+0.0008}_{-0.0009}$	$0.1577^{+0.0041}_{-0.0018}$	$0.8655^{+0.0008}_{-0.0006}$	$0.8697^{+0.0017}_{-0.0023}$
A_2	$0.0113^{+0.0004}_{-0.0004}$	$0.0060^{+0.0010}_{-0.0017}$	$0.1588^{+0.0014}_{-0.0009}$	$0.1161^{+0.0092}_{-0.0028}$	$0.8632^{+0.0013}_{-0.0009}$	$0.8012^{+0.0073}_{-0.0215}$
$A_0 - A_2$	$0.0002^{+0.0007}_{-0.0005}$	$0.0090^{+0.0014}_{-0.0013}$	$-0.0005^{+0.0016}_{-0.0012}$	$0.0416^{+0.0036}_{-0.0067}$	$0.0023^{+0.0015}_{-0.0011}$	$0.0685^{+0.0200}_{-0.0082}$
A_1	$0.0052^{+0.0004}_{-0.0003}$	$0.0074^{+0.0020}_{-0.0008}$	$0.0301^{+0.0013}_{-0.0013}$	$0.0405^{+0.0014}_{-0.0038}$	$0.0600^{+0.0013}_{-0.0015}$	$0.0611^{+0.0018}_{-0.0023}$
A_3	$0.0004^{+0.0002}_{-0.0001}$	$0.0012^{+0.0003}_{-0.0006}$	$0.0066^{+0.0003}_{-0.0005}$	$0.0070^{+0.0017}_{-0.0020}$	$0.0545^{+0.0003}_{-0.0016}$	$0.0584^{+0.0018}_{-0.0047}$
A_4	$0.0729^{+0.0023}_{-0.0006}$	$0.0757^{+0.0021}_{-0.0025}$	$0.0659^{+0.0019}_{-0.0003}$	$0.0672^{+0.0018}_{-0.0050}$	$0.0253^{+0.0007}_{-0.0002}$	$0.0247^{+0.0024}_{-0.0018}$
A_5	$0.0001^{+0.0002}_{-0.0002}$	$0.0001^{+0.0007}_{-0.0007}$	< 0.0001	$0.0011^{+0.0013}_{-0.0030}$	$-0.0004^{+0.0005}_{-0.0005}$	$0.0044^{+0.0042}_{-0.0026}$
A_6	$-0.0002^{+0.0002}_{-0.0003}$	$0.0013^{+0.0006}_{-0.0005}$	$0.0004^{+0.0006}_{-0.0004}$	$0.0017^{+0.0043}_{-0.0015}$	$0.0003^{+0.0003}_{-0.0006}$	$0.0028^{+0.0017}_{-0.0018}$
A_7	< 0.0001	$0.0014^{+0.0007}_{-0.0004}$	$0.0002^{+0.0003}_{-0.0007}$	$0.0024^{+0.0013}_{-0.0013}$	$0.0003^{+0.0004}_{-0.0007}$	$0.0048^{+0.0027}_{-0.0012}$

Table 1. Summary of predictions from DYNNLO at NLO and NNLO for A_0 , A_2 , $A_0 - A_2$, A_1 , A_3 , A_4 , A_5 , A_6 , and A_7 at low (5–8 GeV), mid (22–25.5 GeV), and high (132–173 GeV) p_T^Z for the y^Z -integrated configuration. The uncertainty represents the sum of statistical and systematic uncertainties.

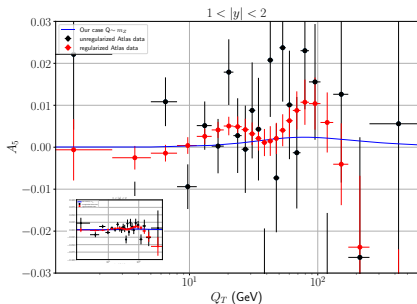


Figure: Comparison of analytical results for the A_5 coefficient at $Q \sim m_Z$ with ATLAS data [JHEP 08 (2016) 159]. Black and red experimental points denote data with statistical error and regularized points [JHEP 08 (2016) 159], respectively. Results for rapidity $1 < |y| < 2$. The scale of the y-axis is chosen to enhance visibility of the small predicted values of $A_{5,6,7}$. Some data points fall outside this range. Full set of data in small scale is shown in the left lower corner of Figure.

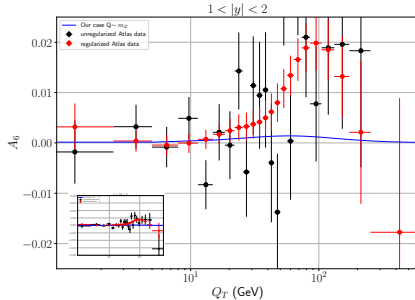


Figure: Comparison of analytical results for the A_6 angular coefficient at $Q \sim m_Z$ with ATLAS data [JHEP 08 (2016) 159] for rapidity $1 < |y| < 2$. Black and red experimental points denote data with statistical error and regularized points [JHEP 08 (2016) 159], respectively.

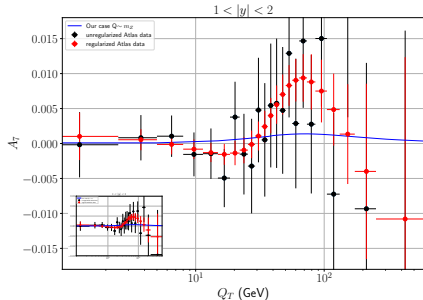


Figure: Comparison of analytical results for the A_7 coefficient at $Q \sim m_Z$ with ATLAS data [JHEP 08 (2016) 159]. Black and red experimental points denote data with statistical error and regularized points [JHEP 08 (2016) 159], respectively.

- T-odd helicity structure function for Drell-Yan process was obtained. In limit of small $\rho^2 = Q_T^2/Q^2$, we have new identity for W_{∇} and W_{Δ_P} .
- We need additional experimental analysis of T-odd structures for Drell-Yan process with Z and W weak boson creation.