Improved Muon-Capture Calculations in Light and Heavy Nuclei

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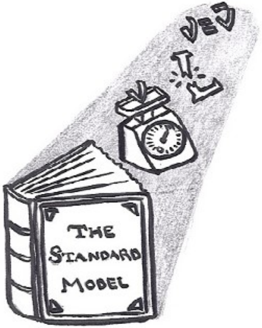
OMC4DBD Collaboration Meeting, 26/04/2022

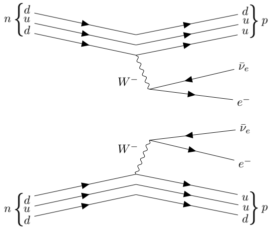
Introduction

Motivation

- Current knowledge on particles and interactions between them is based on the Standard Model (SM) of particle physics
- According to the SM, neutrinos are extremely weakly interacting, massless fermions
- Yet we know from solar neutrino experiments that neutrinos must have a non-zero mass
  - But what is the absolute mass scale?
  - What else is there beyond the SM?
- *Observing neutrinoless double-beta decay would provide answers!*



Two-Neutrino Double-Beta ($2\nu\beta\beta$) Decay



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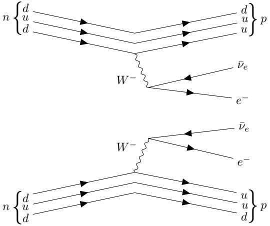
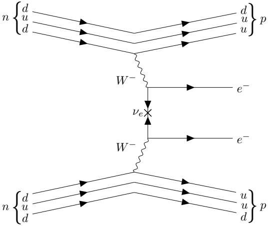


$$^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z+2}\mathrm{Y}_{N-2} + 2e^{-} + 2\bar{\nu}_e$$

- May happen, when β-decay is not energetically allowed
- Allowed by the Standard Model
- Measured in ≈ 10 isotopes
  - Half-lives of the order 10^{20} years or longer

Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay



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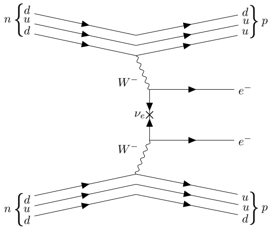

$$^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z+2}\mathrm{Y}_{N-2} + 2e^{-}$$

- Requires that the neutrino is a Majorana particle
- Violates the lepton-number conservation law by two
- $\frac{1}{t_{1/2}^{(0\nu)}} \propto |\langle m_\nu \rangle|^2$

Half-life of $0\nu\beta\beta$ Decay

$$\boxed{\frac{1}{t_{1/2}^{0\nu}} = g_{\mathrm{A}}^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2}$$

[1] Agostini *et al.*, arXiv:2202.01787 (2022)

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- Axial-vector coupling ($g_{\mathrm{A}}^{\mathrm{free}} \approx 1.27$)

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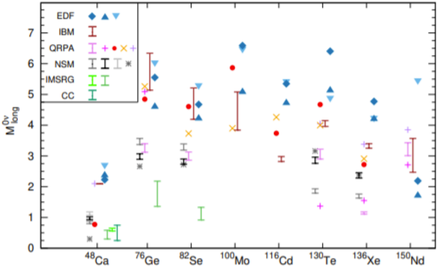

Matrix elements of $0\nu\beta\beta$ decays [1]

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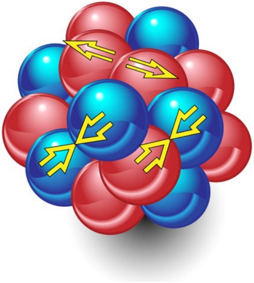
Nuclear Many-Body Methods

- Ab initio methods (CC, **IMSRG**, ...)



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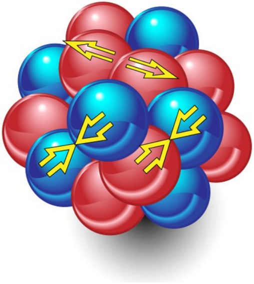

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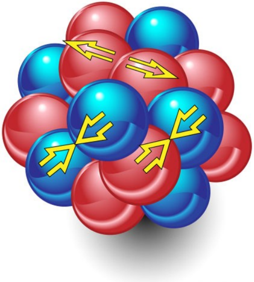

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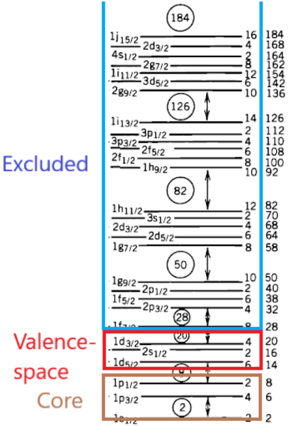

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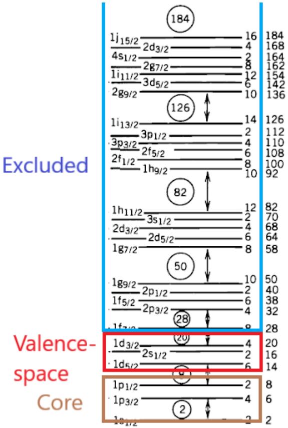

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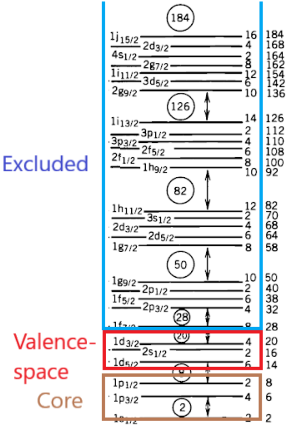

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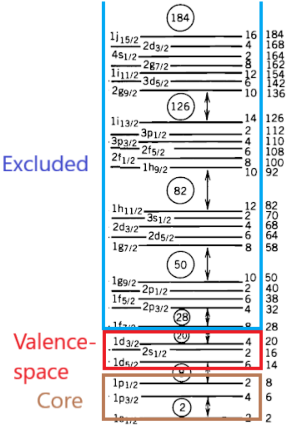

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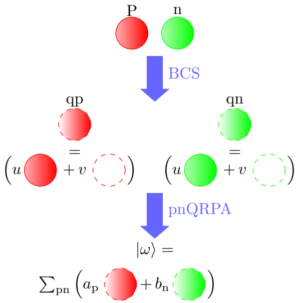

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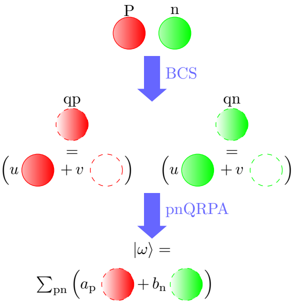

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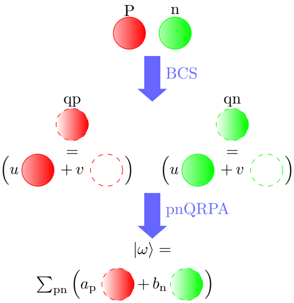

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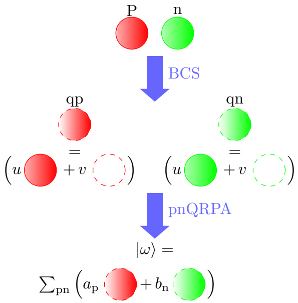

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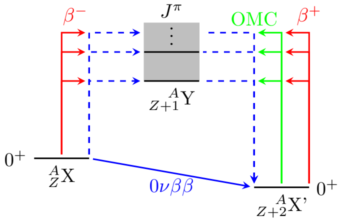
- ...

Ordinary Muon Capture as a Probe of $0\nu\beta\beta$ Decay

Advantages of OMC as a Probe of $0\nu\beta\beta$ Decay

- OMC leads to transitions to all J^π states up to high energies



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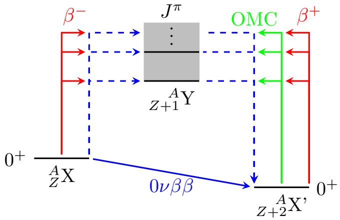

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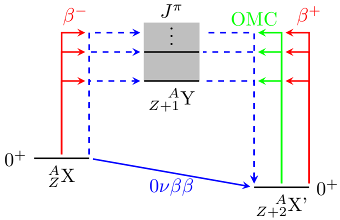
- Left-hand-side leg probed by charge-exchange reactions

$$\boxed{{}^{a}_{z}a + {}^{A}_{Z}X \rightarrow {}^{a}_{z-1}b + {}^{A}_{Z+1}Y,}$$

where (a, b) can be (p, n), $({}^{3}\mathrm{He}, t)$, ...



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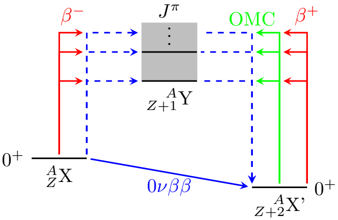
- Ordinary muon capture (OMC)

$$\boxed{\mu^{-} + {}^{A}_{Z}X \rightarrow \nu_\mu + {}_{Z-1}^{A}Y}$$

can probe the right-hand side



Advantages of OMC as a Probe of $0\nu\beta\beta$ Decay

- Both OMC and $0\nu\beta\beta$ decay involve couplings $g_{\rm A}$ and $g_{\rm p}$:

$$\boxed{W^{(OMC)} \propto |g_{\rm A} M_{\rm A} + g_{\rm V} M_{\rm V} + g_{\rm P} M_{\rm P}|^2}$$

$$\boxed{M^{0\nu} = M^{0\nu}_{\rm GT}(g_{\rm A}, g_{\rm P}, g_{\rm M}) - \left(\frac{g_{\rm V}}{g_{\rm A}}\right)^2 M^{0\nu}_{\rm F}(g_{\rm V}) + M^{0\nu}_{\rm T}(g_{\rm A}, g_{\rm P}, g_{\rm M})\ ,}$$

$$\boxed{[t^{0\nu}_{1/2}]^{-1} = g_{\rm A}^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2}$$

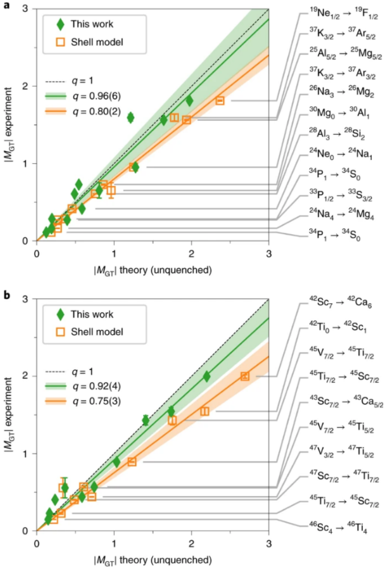
- ...so if
  - we know the involved nuclear structure precisely enough, and
  - OMC rates to individual nuclear states can be measured

...we can probe $g_{\rm A}$ and $g_{\rm p}$ on the relevant momentum-exchange regime for $0\nu\beta\beta$ decay

Muon Capture on Light Nuclei

g_A Quenching at High Momentum Exchange?

- Recently, first *ab initio* solution to g_A quenching puzzle was proposed for β-decay [2]
- How about g_A quenching at high momentum transfer $q \approx 100$ MeV/c?
  - OMC could provide a hint!



[2] P. Gysbers *et al.*, *Nature Phys.* **15**, 428 (2019)

Ingredients 1: Particle Physics

Muon-Capture Theory

- Interaction Hamiltonian $\rightarrow$ capture rate:

$$W(J_i \rightarrow J_f) = \frac{2J_f + 1}{2J_i + 1}\left(1 - \frac{q}{m_\mu + AM}\right) q^2 \sum_{\kappa u} |g_{\mathrm{V}} M_{\mathrm{V}} + g_{\mathrm{A}} M_{\mathrm{A}} + g_{\mathrm{P}} M_{\mathrm{P}}|^2$$

PHYSICAL REVIEW VOLUME 118, NUMBER 2 APRIL 15, 1960

Theory of Allowed and Forbidden Transitions in Muon Capture Reactions*

MASATO MORITA
Columbia University, New York, New York

AND

AKIHIKO FUJII†
Brookhaven National Laboratory, Upton, Long Island, New York

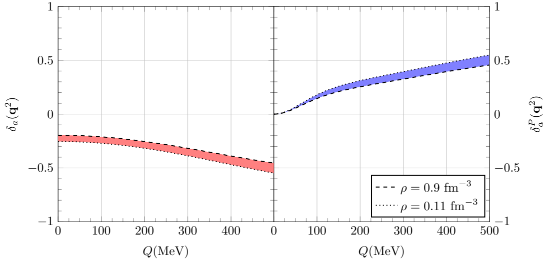
(Received November 9, 1959)

Ingredients 1: Particle Physics

NEW!! Hadronic Two-Body Currents

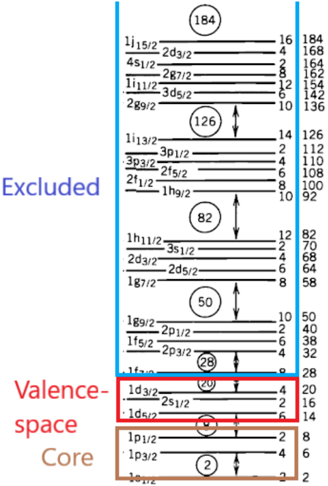
- Correct the couplings by effective one-body currents [3]

$$g_{\rm A} \rightarrow (1+\delta_a(\mathbf{q}^2))g_{\rm A} \text{ and } g_{\rm P} \rightarrow \left(1-\frac{q^2+m_\pi^2}{q^2}\delta_a^P(\mathbf{q}^2)\right)g_{\rm P}$$



[3] Hoferichter *et al.*, *Phys. Rev. C* **102**, 074018 (2020)

- Hamiltonian based on the chiral EFT with EM 1.8/2.0 interaction
- Valence-space Hamiltonian and OMC operators decoupled from complimentary space with a unitary transformation
    - Operators can be made consistent with the Hamiltonian!



Ingredients 3: Atomic Physics

Bound-Muon Wave Function

- Solve the Dirac equations for the muon:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}r}G_{-1} + \frac{1}{r}G_{-1} = \frac{1}{\hbar c}(mc^2 - E + V(r))F_{-1} \\ \frac{\mathrm{d}}{\mathrm{d}r}F_{-1} - \frac{1}{r}F_{-1} = \frac{1}{\hbar c}(mc^2 + E - V(r))G_{-1} \end{cases}$$

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- For a **point-like**

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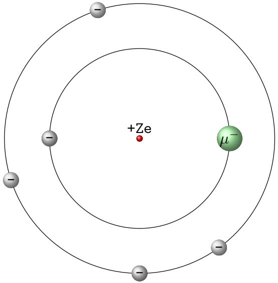

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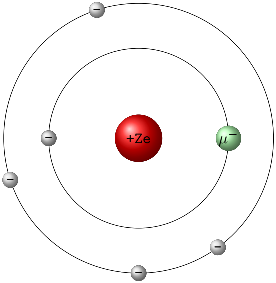

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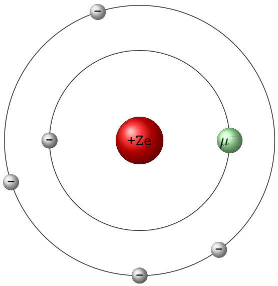
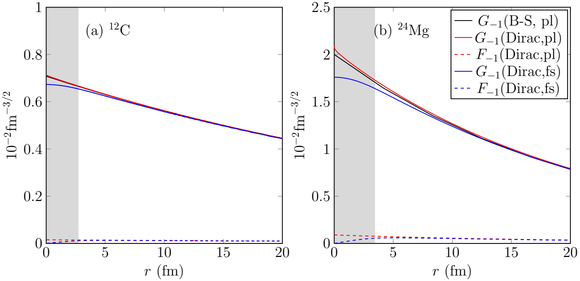
- Solve the Dirac equations for the muon:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}r} G_{-1} + \frac{1}{r} G_{-1} = \frac{1}{\hbar c}(mc^2 - E + V(r))F_{-1} \\ \frac{\mathrm{d}}{\mathrm{d}r} F_{-1} - \frac{1}{r} F_{-1} = \frac{1}{\hbar c}(mc^2 + E - V(r))G_{-1} \end{cases}$$

- For a **point-like** or **finite-size** nucleus





Test case: Capture Rates on Low-Lying States in ^{12}C

Nuclear Shell Model + Two-Body Currents + Realistic Muon Wave Function

- Nuclear shell-model calculation in *p*-shell with chiral two-body currents and realistic bound-muon wave functions
- Quite good agreement with experiment

J_i^π	E_{exp} (MeV)	Rate (10^3 1/s)			
		Exp.		NSM	
		Measday[4]	Double Chooz[5]	1bc	1bc+2bc
1^+_{gs}	0	6.04 ± 0.35	$5.68^{+0.14}_{-0.23}$	6.48	4.56–4.86
2^+_1	0.953	0.21 ± 0.1	$0.31^{+0.09}_{-0.07}$	0.42	0.32–0.34
2^+_2	3.759	-	$0.026^{+0.015}_{-0.011}$	0.011	0.009–0.009

[4] Measday, *Phys. Rep.* **354**,243 (2001)

[5] Abe *et al.*, *Phys. Rev. C* **93**,054608 (2016)

Results: Capture Rates on Low-Lying States in ^{24}Na

J_i^{π}	E_{exp} (MeV)	Rate (10^3 1/s)				
		Exp.[6]	NSM		IMSRG	
			1bc	1bc+2bc	1bc	1bc+2bc
1_1^+	0.472	(21.0 ± 6.6)	4.0	3.0	22.3	15.2
1_2^+	1.347	17.5 ± 2.3	32.7	21.3	7.7	4.9
Sum(1^+)		38.5 ± 8.9	36.7	24.5	30.0	20.0
2_1^+	0.563	17.5 ± 2.1	1.0	0.7	0.5	0.3
2_2^+	1.341	3.4 ± 0.5	3.1	2.5	1.0	0.9
Sum(2^+)		20.9 ± 2.6	4.1	3.2	1.5	1.2

[LJ, T. Miyagi, S.R. Stroberg, J.D. Holt, J. Kotila and J. Suhonen, arXiv:2111.12992]

[6] P. Gorringe *et al.*, Phys. Rev. C **60**, 055501 (1999)

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- Generally, IMSRG gives smaller capture rates
- 1^+ states mixed
- Agreement with experiment could be better

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Improved Muon-Capture Framework

Improvements to Morita-Fujii Formalism

- Morita-Fujii[7] (MF) and Walecka[8] formalisms for OMC combined with pnQRPA tend to give different capture rates

$\boxed{g_{\mathrm{A}}^{\mathrm{eff}}(\mathrm{Jok.})^{9} \approx 0.6}$ $\boxed{g_{\mathrm{A}}^{\mathrm{eff}}(\check{\mathrm{S}}\mathrm{im.})^{10} \approx 1.27}$ $\boxed{g_{\mathrm{A}}^{\mathrm{eff}}(\mathrm{Cic.})^{11} \approx 1.0}$

- Not straigthforward to compare the two formalisms
- Something has to be done (ongoing work w/ E. Ydrefors and J. Suhonen)
  1. Check the assumptions made in the MF formalism
  2. Introduce 'Walecka-like' multipole operators into MF formalism $\rightarrow$ compare (first test case: $^{100}\mathrm{Mo}$)

[7] Morita and Fujii, *Phys. Rev.* **118**, 606 (1960)
[8] Walecka, *Muon Physics II* p.113 (Academic Press, New York) (1975)
[9] LJ, Suhonen, *Phys. Rev. C* **100**, 014619 (2019)
[10] Šimkovic *et al.*, *Phys. Rev. C* **102**, 034301 (2020)
[11] Ciccarelli *et al.*, *Phys. Rev. C* **102**, 034306 (2020)

Summary

- By studying OMC we can shed light on the unknown effective value of g_{A}
- In order to probe the effective value of g_{P} we would need to have data on capture rates to individual states
- First *ab initio* muon-capture studies performed
- Next: improved muon-capture theory

Thank you!