Theory estimates for muon capture in light and heavy nuclei

Lotta Jokiniemi MONUMENT Collaboration meeting 24/05/2023









Introduction

Ab initio studies on muon capture in light nuclei

Phenomenological study on ¹³⁶Ba

Summary and Outlook



Ordinary Muon Capture (OMC)

$$\mu^- +^A_Z \mathcal{X}(J_i^{\pi_i}) \to \nu_\mu +^A_{Z-1} \mathcal{Y}(J_f^{\pi_f})$$

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Ordinary = non-radiative

$$\begin{pmatrix} \text{Radiative muon capture (RMC):} \\ \mu^{-} +^{A}_{Z} X(J_{i}^{\pi_{i}}) \rightarrow \nu_{\mu} +^{A}_{Z-1} Y(J_{f}^{\pi_{f}}) + \boldsymbol{\gamma} \end{pmatrix}$$





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• Weak interaction process with momentum transfer $q \approx 100 \text{ MeV}/c^2$

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Same currents involved (vector, magnetic, axial-vector and psedoscalar) \rightarrow Similar to $0\nu\beta\beta$ decay!

Muon-Capture Theory

• Interaction Hamiltonian \rightarrow capture rate:

$$W(J_i \to J_f) = \frac{2J_f + 1}{2J_i + 1} \left(1 - \frac{q}{m_\mu + AM} \right) q^2 \sum_{\kappa u} |g_V M_V(\kappa, u) + g_M M_M(...) + g_A M_A(...) + g_P M_P(...)|^2$$

PHYSICAL REVIEW

VOLUME 118, NUMBER 2

APRIL 15, 1960

Theory of Allowed and Forbidden Transitions in Muon Capture Reactions*

MASATO MORITA Columbia University, New York, New York

AND

AKIHIKO FUJII† Brookhaven National Laboratory, Upton, Long Island, New York (Received November 9, 1959)

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PHYSICAL REVIEW VOLUME 118, NUMBER 2 APRIL 15, 1960 Theory of Allowed and Forbidden Transitions in Muon Capture Reactions* MASATO MORITA Columbia University, New York, New York AND Акцико Fujut Delle Brookhaven National Laboratory, Upton, Long Island, New York (Received November 9, 1959) ISCOV

Use realistic bound-muon wave functions

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- Use realistic bound-muon wave functions
- Add the effect of two-body currents

Nuclear matrix elements

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- Operator (muon capture)



Axial-Vector Quenching

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Märkisch et al., Phys. Rev. Lett. 122, 242501 (2019)

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$$\log ft = \log(f_0 t_{1/2}[\mathbf{s}]) = \log\left(\frac{\kappa}{B_{\mathrm{F}} + B_{\mathrm{GT}}}\right)$$

$$B_{\rm F} = \frac{g_{\rm V}^2}{2J_i + 1} |(J_f^{\pi_f}||\sum_a \tau_a^-||J_i^{\pi_i})|^2 ,$$

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≈ TRIUMF q_A Quenching at High Momentum Exchange?

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P. Gysbers et al., Nature Phys. 15, 428 (2019)



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 - OMC could provide a hint!



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- Solution: adding two-body currents and missing correlations
- ► How about g_A quenching at high momentum transfer ≈ 100 MeV/c?
 - OMC could provide a hint!
- ► In principle, one could also access the pseudoscalar coupling *g*_P



Gysbers et al., Nature Phys. 15, 428 (2019)





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TRIUMF First principles or *ab initio* nuclear theory



Figure courtesy of P. Navrátil



First principles or *ab initio* nuclear theory – what is done at present



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Figure courtesy of P. Navrátil

No-Core Shell Model (NCSM)

 OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis



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No-Core Shell Model (NCSM)

- OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis
 - ► HO basis truncated with N_{max}
- Quasiexact nuclear many-body method
- Restricted to nuclei with $oldsymbol{A} \lesssim 20$
- \rightarrow OMC on 6 Li, 12 C and 16 O



Capture Rates to the Ground State of ⁶He





*****TRIUMF Capture Rates to the Ground State of ⁶He

- NCSM in keeping with experiment
- The rates can be compared with other ab initio calculations

King et al., Phys. Rev. C 105, L042501 (2022)



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Capture Rates to the Ground State of ¹²B

Interaction dependence



work in progress

Capture Rates to the Ground State of ¹²B

- Interaction dependence
- Converge slow (clustering effects?)



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Hayes et al., Phys. Rev. Lett. 91, 012502 (2003)



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*** TRIUMF** Capture Rates to the Ground State of ¹²B

- Interaction dependence
- Converge slow (clustering effects?)
- The results can be compared against earlier NCSM ones

Hayes et al., Phys. Rev. Lett. 91, 012502 (2003)

► 3-body forces essential to reproduce the measured rate



 ${}^{12}C(0^+_{\sigma s}) + \mu^- \rightarrow {}^{12}B(1^+_{\sigma s}) + \nu_{\mu}$



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RIUMF Capture Rates to Low-Lying States in ¹²B ${}^{12}C(0^+_{gs}) + \mu^- \rightarrow {}^{12}B(2^+_2) + \nu_{\mu}$ ${}^{12}C(0^+_{os}) + \mu^- \rightarrow {}^{12}B(2^+_1) + \nu_{\mu}$ 0.5Abe 2016 Measday 2001 - NN-N⁴LO+3N_{1n1} 40 Abe 2016 - NN-N⁴LO+3N^{*}_{1n1} - NN-N⁴LO+3N_{lnl} 0.4- NN-N³LO^{*}+3N_{1n1} - NN-N³LO^{*}+3N_{lnl} 30 $Rate(10^3/s)$ 0.3Rate(1/s)200.2100.1- 1b 1b + 2b- 1b + 2b0 0 ∞ 12 ∞ 10 12 $\mathbf{2}$ 6 8 10 0 2 $\mathbf{4}$ 6 8 0

 $N_{\rm max}$

LJ, Navrátil, Kotila, Kravvaris, work in progress

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LJ, Navrátil, Kotila, Kravvaris, work in progress

Capture Rates to Low-Lying States in ^{16}N



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LJ, Navrátil, Kotila, Kravvaris, work in progress

Capture Rates to Low-Lying States in $^{16}\mathrm{N}$



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 Color gradient: increasing N_{max} (3,5,7 for ¹²C and 2,4,6 for ¹⁶O)



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- \blacktriangleright Rates obtained summing over ~ 50 final states of each parity
 - Can be improved
- Summing up the rates up to ~ 20 MeV, we capture ~ 85% of the total rate in both ¹²B and ¹⁶N



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Total Muon-Capture Rates

Experiment:

$$\mu^- + {}^{100} \text{ Mo} \to \nu_\mu + {}^{100} \text{ Nb}$$



Hashim et al., Phys. Rev. C 97, 014617 (2018)

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Missing potentially important contribution from high energies

Total Muon-Capture Rates

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- ► Calls for phenomenology:

see 0_i^+ 136 Xe $\beta\beta$ 0_f^+ OMC 136 Ba 0_f^+ Atomic number

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- ► Calls for phenomenology:
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- Solution: (phenomenological) nuclear shell model and proton-neutron QRPA



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Excitation energies in 136 Cs ($J \le 5$)

 The shell-model and pnQRPA energies are surprisingly similar



Excitation energies in 136 Cs ($J \le 5$)

 The shell-model and pnQRPA energies are surprisingly similar

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 Agreement with experiment gets much better with the new measurement

B. M. Rebeiro et al., arXiv:2301.11371 (2023)



Muon capture rates to low-lying states in $^{136}\mbox{Cs}$

• Summing up the rates to states with $E_X < 1$ MeV:

P. Gimeno, LJ, J. Kotila, M. Ramalho, J. Suhonen, 10.20944/preprints202304.0899.v1 (submitted to Universe)

	Rate (1b)($10^{3}1/s$)	Rate (1b+2b)($10^{3}1/s$)	Rate (1b+2b) / Total rate
NSM	248	150 - 174	1.4 - 1.5%
pnQRPA	1103	592 - 807	5-7%

- ▶ pnQRPA gives ≈4 times larger rates than NSM
 - ▶ With experimental data, we will know which one is (more) correct
 - May hint which model is more reliable for the $0\nu\beta\beta$ decay of ¹³⁶Xe!
- ► Similar study ongoing for OMC on ^{128,130}Xe
- ► In pnQRPA, can be extended to strength functions!





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- Ab initio muon-capture studies could shed light on g_A quenching at finite momentum exchange regime relevant for 0νββ decay
- No-core shell-model describes well partial muon-capture rates in light nuclei ⁶He, ¹²B and ¹⁶N
- Phenomenological methods still needed for heavy/difficult systems







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Outlook

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- ► The "brute force" method cannot reach the total muon-capture rates → use the Lanczos strength-function method, instead



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- ► The "brute force" method cannot reach the total muon-capture rates → use the Lanczos strength-function method, instead
- Study the effect of exact two-body currents and/or continuum on the OMC rates

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Thank you Merci



Excitation Energies of 12 **B**

		$E_{ m exc.}$ (MeV)			
J_i^{π}	Interaction	$N_{\rm max} = 4$	$N_{\rm max} = 6$	$N_{\max} = 8(IT)$	Exp.
1_{1}^{+}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	0.0 0.135	0.0 0.000	0.0 0.000	0.0
2_{1}^{+}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	0.251 0.000	0.465 0.027	0.538 0.097	0.953
0_{1}^{+}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	2.073 3.306	1.831 2.909	1.713 2.761	2.723
2^{+}_{2}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	3.816 4.919	3.490 4.463	3.344 4.281	3.760
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Excitation Energies of ¹⁶N

		$E_{ m exc.}$ (MeV)			
J_i^{π}	Interaction	$N_{\rm max} = 4$	$N_{\rm max} = 6$	$N_{\max} = 8(IT)$	Exp.
2_{1}^{-}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	0.154 0.214	0.087 0.146	0.064 0.133	0.0
0_{1}^{-}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	2.245 2.807	1.487 2.065	1.010 1.606	0.120
3_{1}^{-}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	0.000 0.000	0.000 0.000	0.000 0.000	0.298
1_{1}^{-}	NN(N ⁴ LO)-3NInI NN(N ⁴ LO)-3NInIE7	2.561 2.985	1.833 2.310	1.363 1.869	0.397

Discovery, accelerated