Critical effects on the evolution of Quark Gluon Plasma

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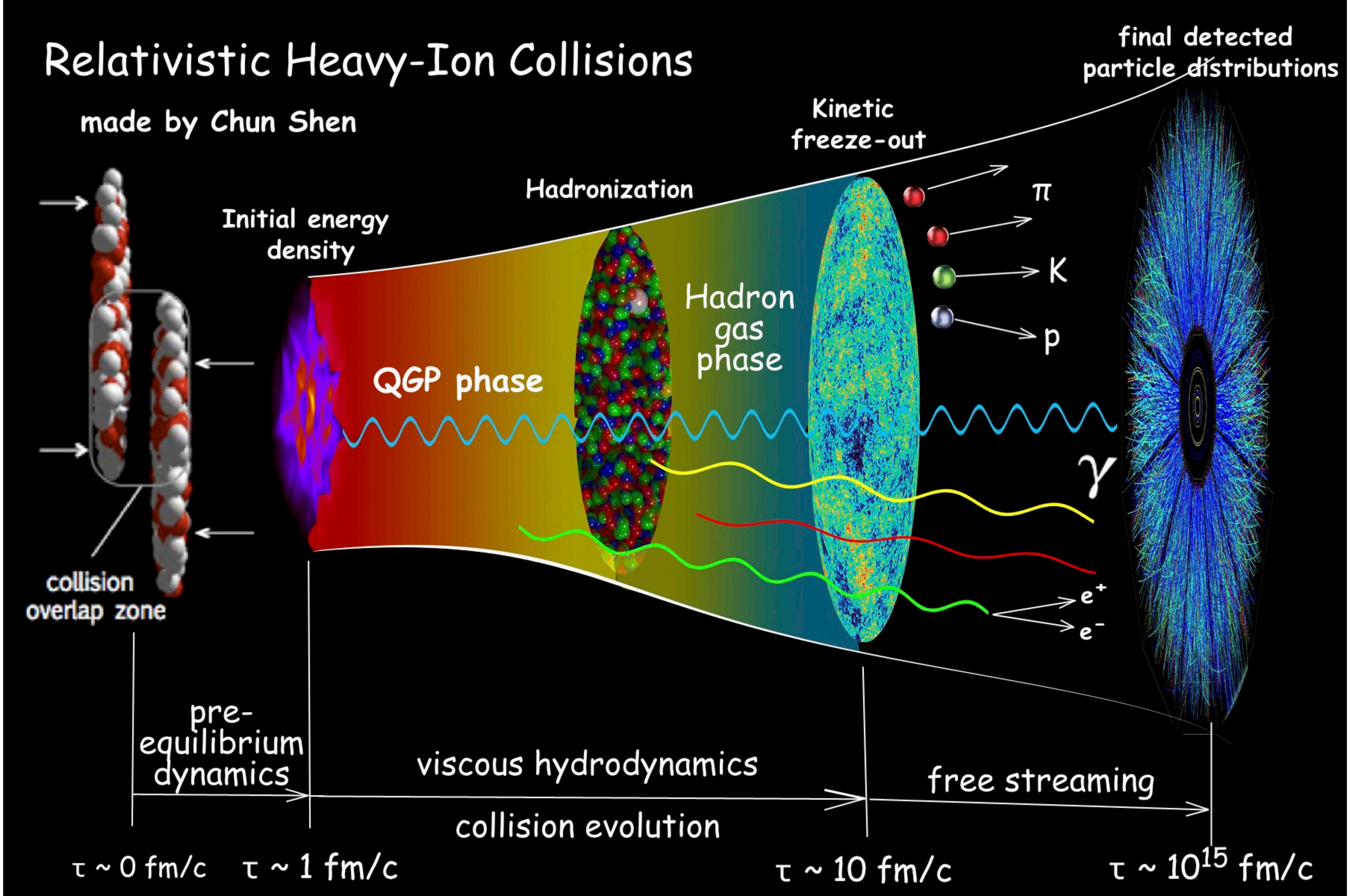
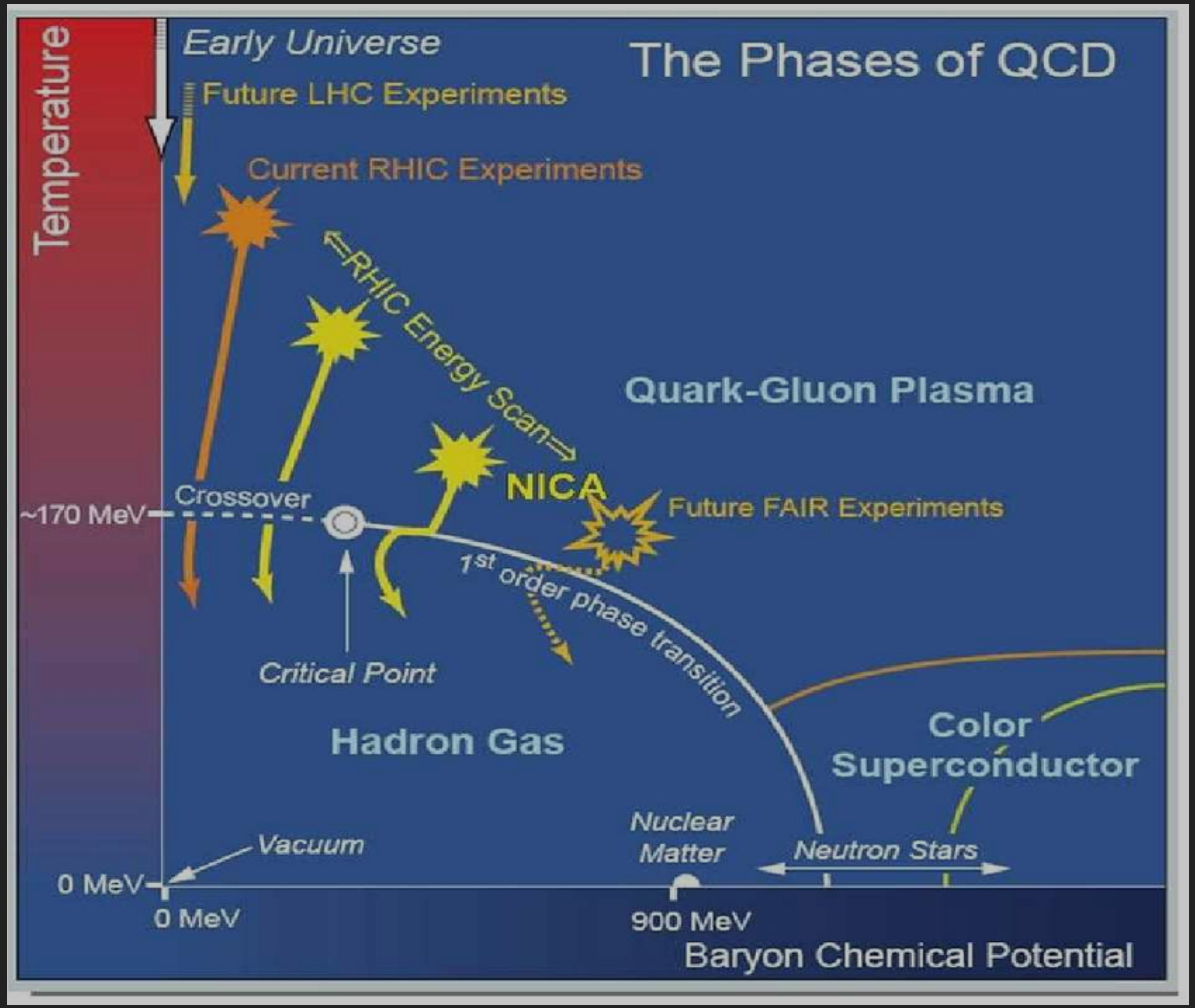
OUTLINE

- Motivation
- Hydro equations and the EOS with a critical point
- Propagation of perturbations in the QGP
  - Linear perturbations
  - Nonlinear perturbations
- Results & Discussion
- Summary & conclusion


Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
Hadron gas phase
QGP phase
π
K
p
γ
e+
e−
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

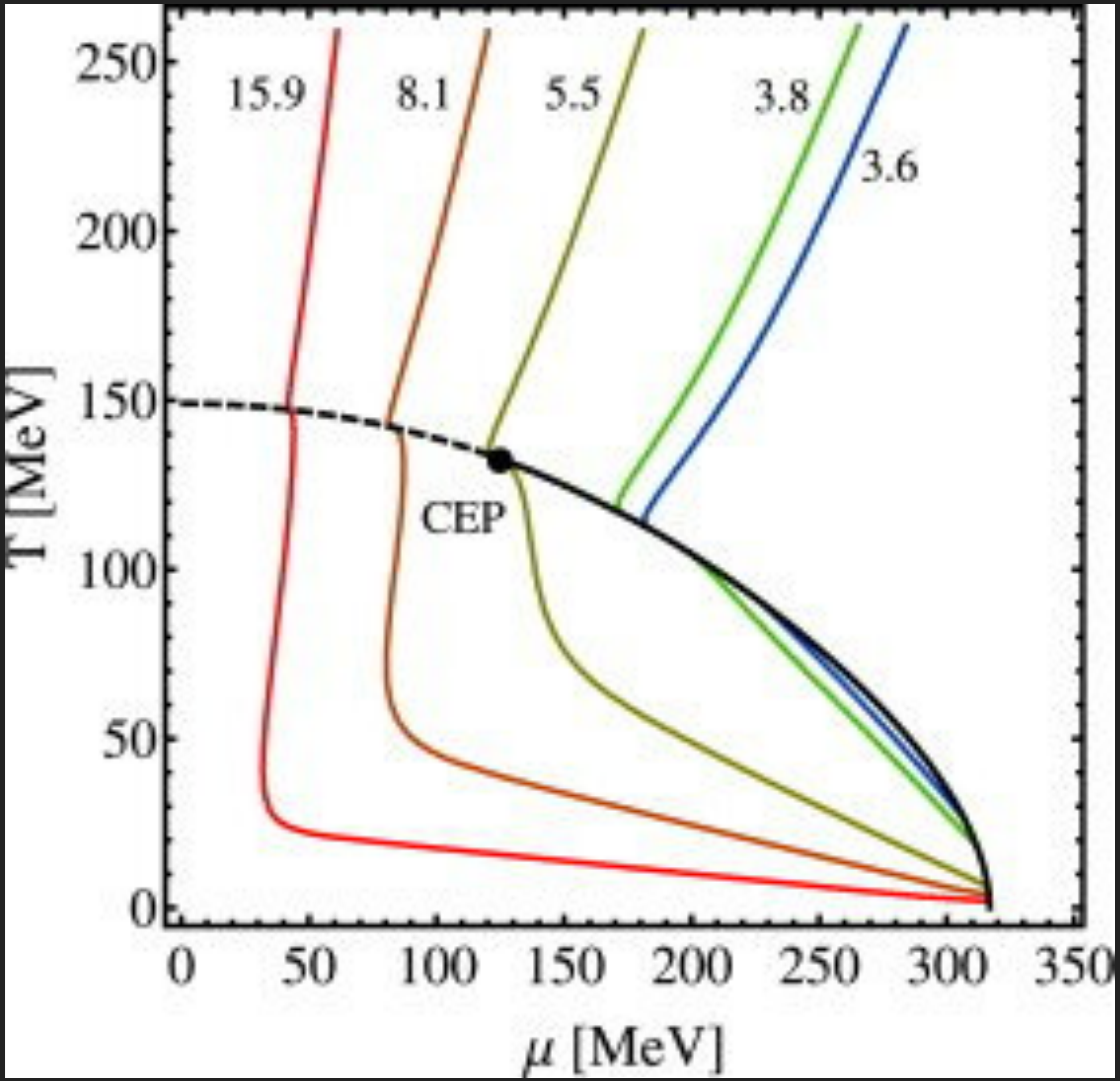
MOTIVATION



- An anticipated critical end point (CEP) at the end of the first-order phase transition line at finite μ_B.
- Location of the CEP is predicted by effective models, depends on the parameters of the models.
- Ab-initio lattice calculations are not valid at finite μ_B .
- Relevant for RHIC-BES program, GSI-FAIR, NICA energies.

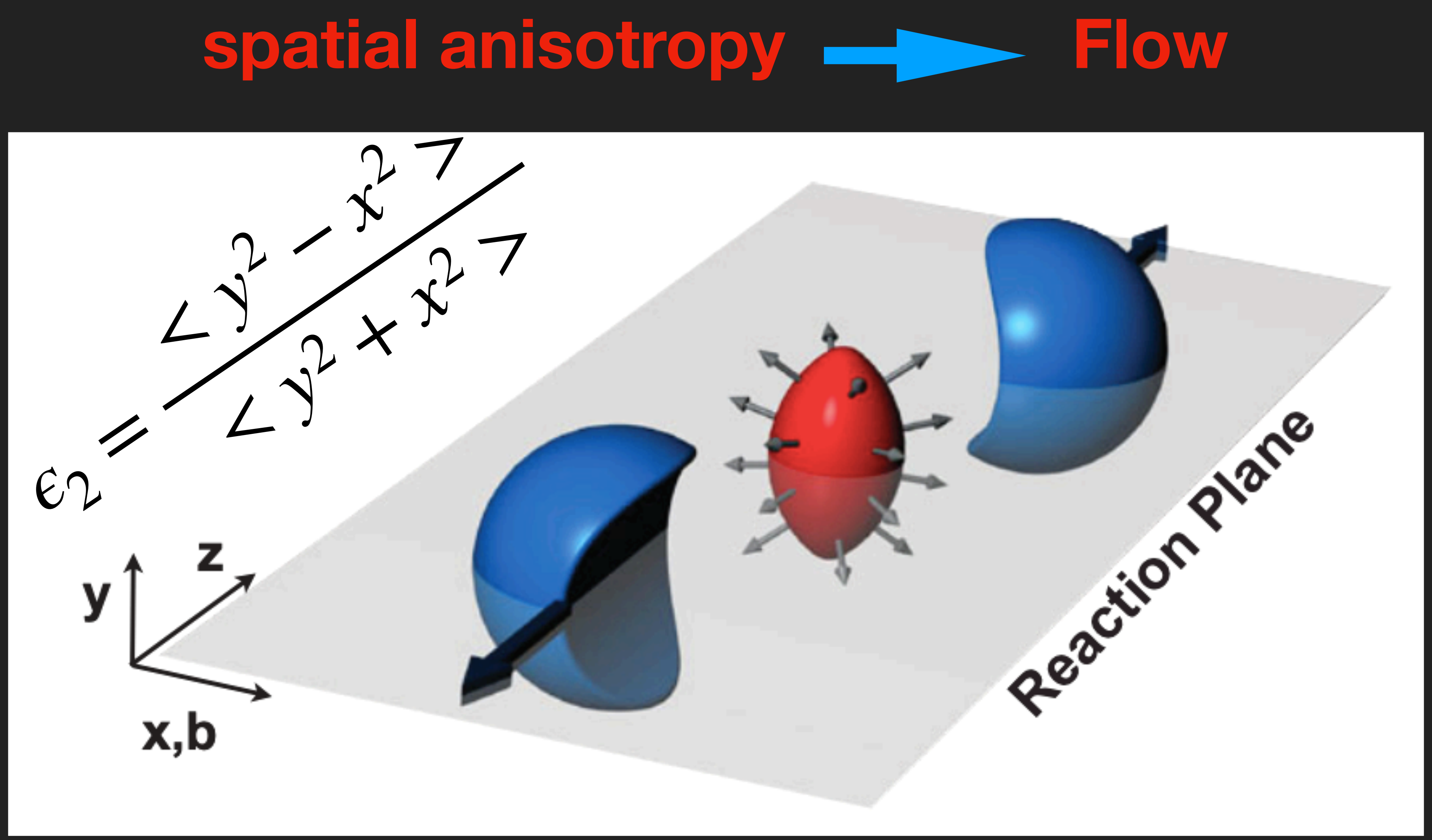
WHY PERTURBATIONS?

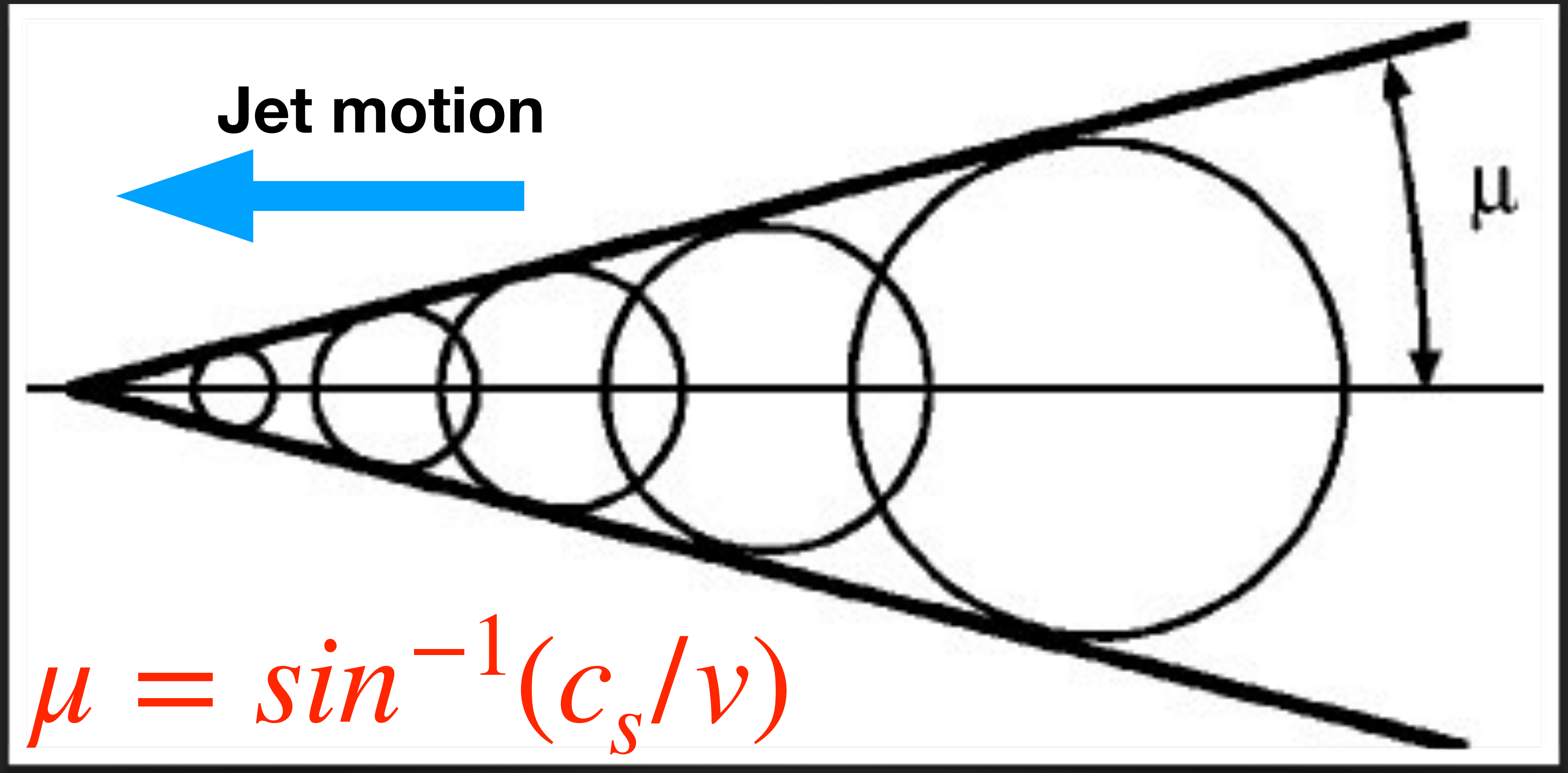
- Medium response is recorded to study the propagation of perturbations.
- Response could be imprinted in the hadron spectra.
- Isentropic trajectories near the CEP and away from the CEP will get influenced separately.
- Trajectories could provide information on the collision energies.
- Separating the collision energies by identifying hadron spectra.
- Dealing with small (linear) as well as large (nonlinear) perturbations.



OBSERVABLES

- Promising observables: Flow harmonics are attributed to the hydrodynamic response of QGP to the initial geometry.
- Formation of Mach cone due to shock wave propagation.
- Aim to observe the effects of the CEP on such observables.

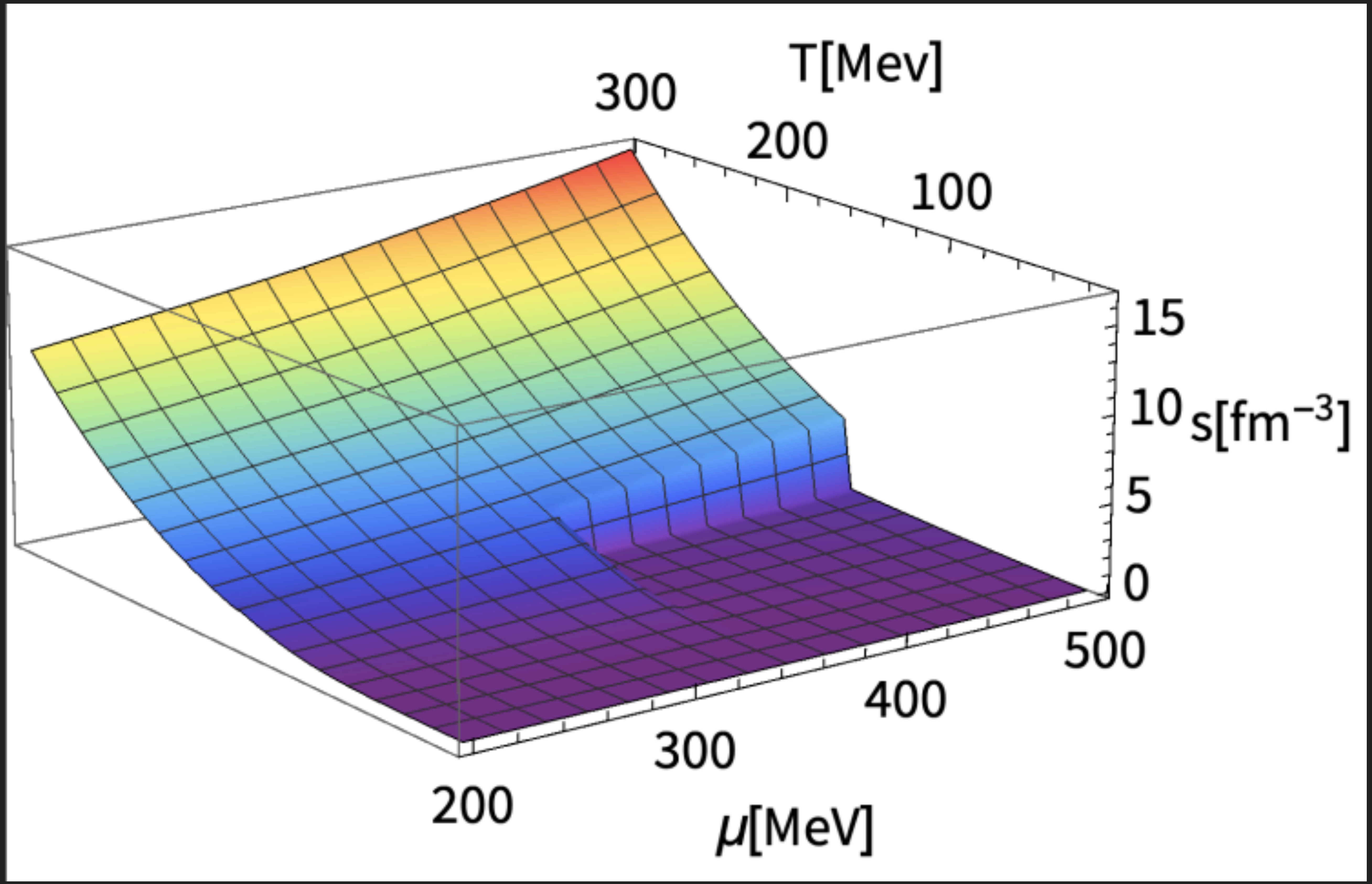




EOS IN HYDRO EQUATIONS

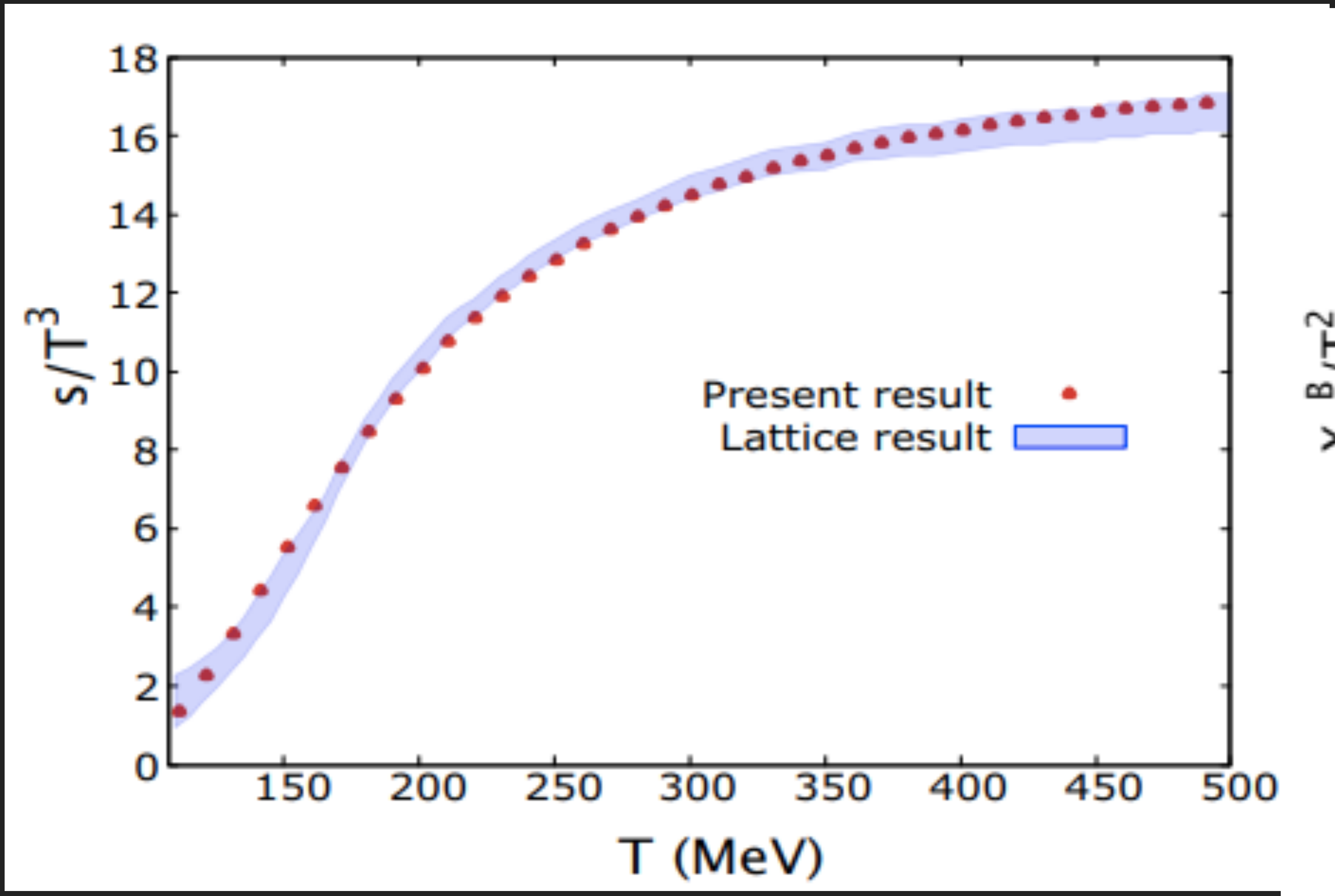
- Müller-Israel-Stewart theory for describing the fluid.
- An equation of state (EoS) to close the hydro equations.
- Used an EoS embedding a critical point: constructed on the basis of universality hypothesis.
- QCD critical point belongs to the same universality class as of the critical point of Ising model.

Entropy density $(T_c, \mu_c) = (154, 367) MeV$

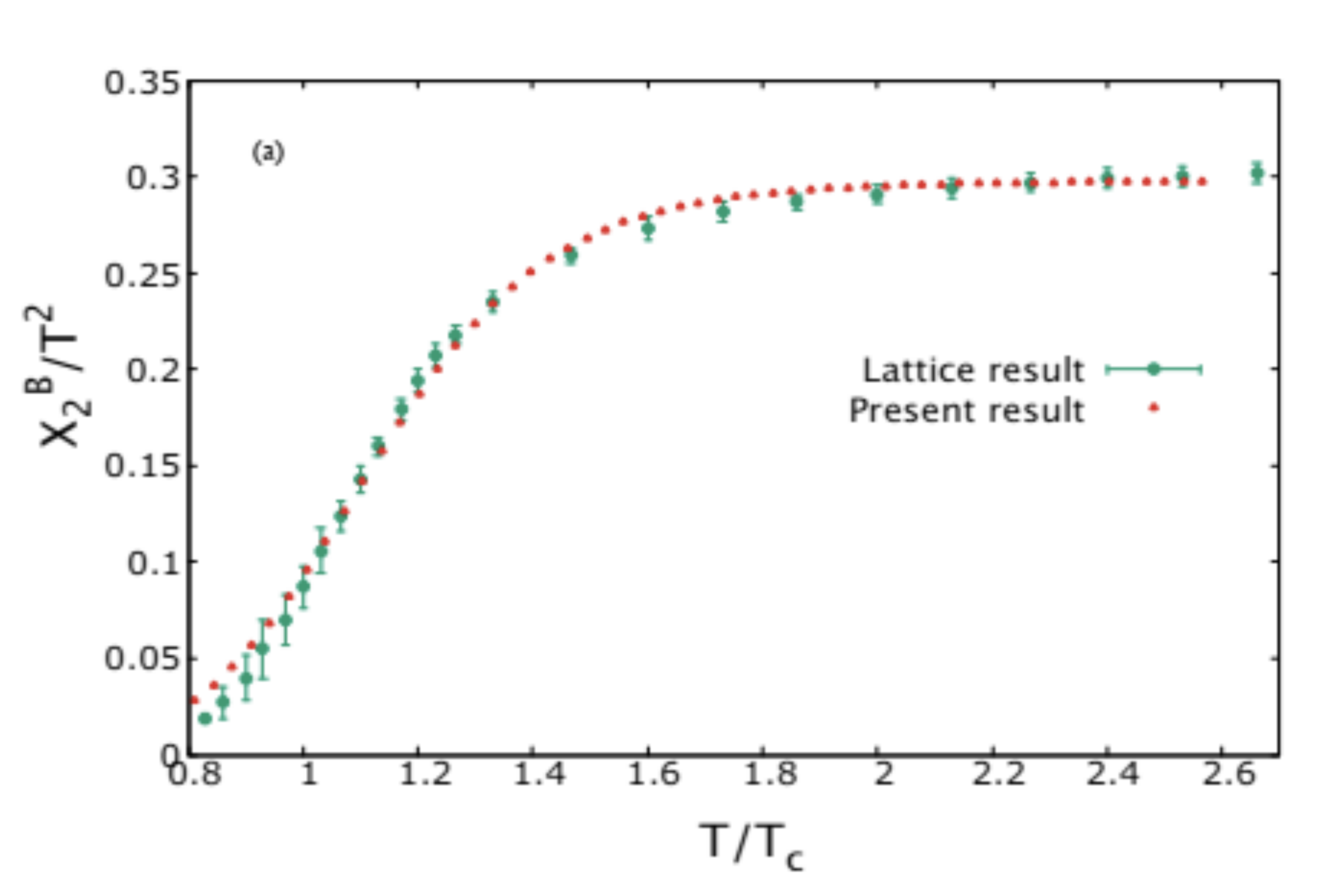


❖ **To check the reliability of the EoS, we compare our results with the available lattice results**

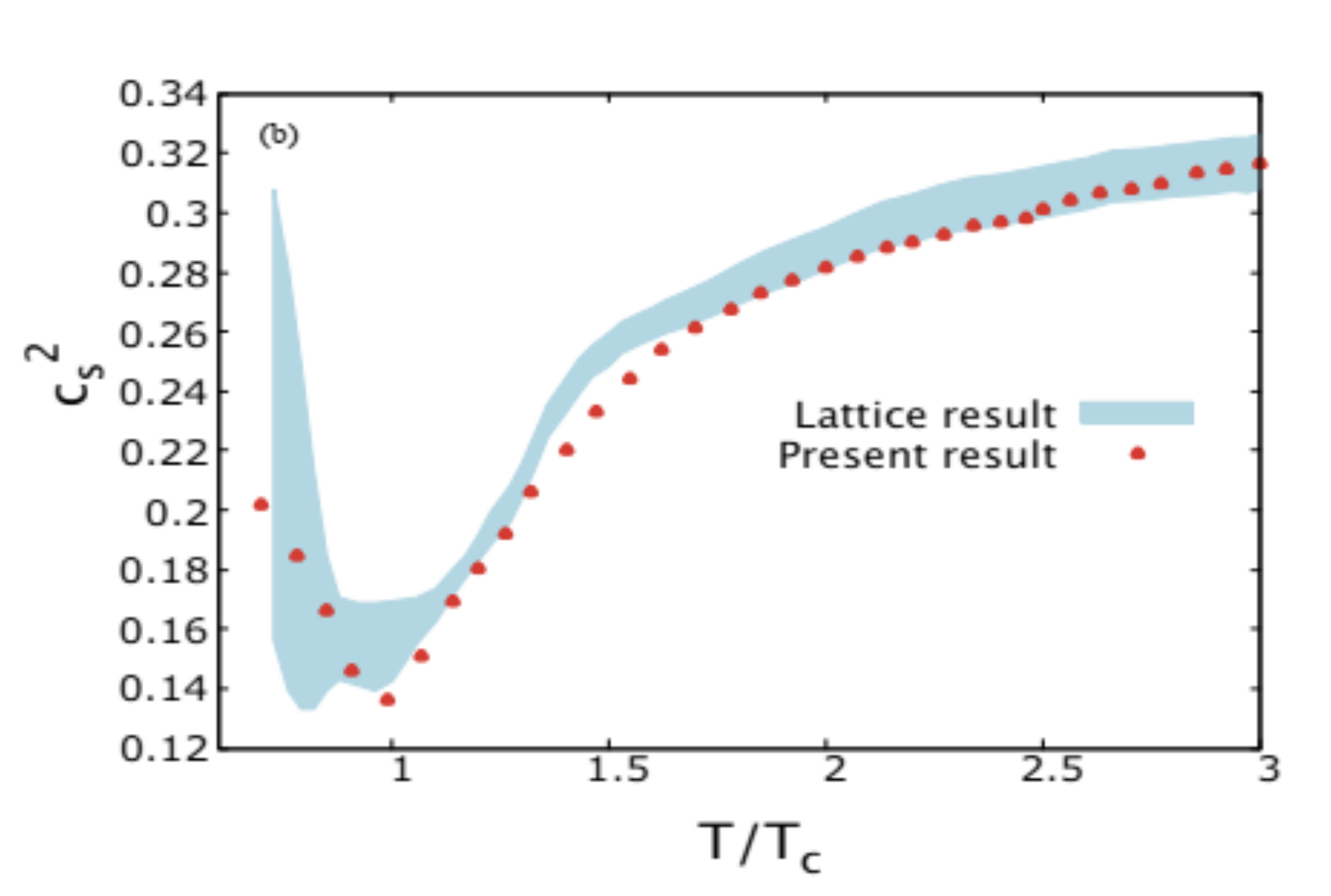
S. Borsanyi et al, JHEP (2012), PLB (2014)



Entropy density



Baryon number susceptibility ~ $\left(\frac{\partial^2 P}{\partial \mu^2}\right)$



Speed of sound~ $\left(\frac{\partial P}{\partial \epsilon}\right)$

LINEAR PERTURBATIONS

- As strategy, a space-time dependent perturbation, $e^{i(kx-\omega t)}$ is placed into the fluid via hydro variables.

  S. Weinberg, Astrophys. J. 168, 175 (1971)

- Separated out the real part and the imaginary part from dispersion relation.

- Fate of a wave is dictated by magnitude of the real and the imaginary part of the dispersion relation.

- A threshold wavelength is calculated via k_{th} by putting $\left|\frac{\omega_{\Re e}}{\omega_{\mathcal{I}m}}\right| = 1$ Liao and Koch, PRC (2010)

$$k_{th} = \sqrt{\frac{a_0 d_0^2}{b_0 c_0^2}} \left[1 - \frac{1}{2}\left(\frac{d_1 c_2^2}{b_0 d_0^2} + \frac{a_1 d_0}{a_0 c_0} - \frac{a_1 c_2^2 d_1}{a_0 b_0 c_0 d_0} + \frac{b_1^2 c_2^3 d_1}{a_0 b_0 d_0^3} - \frac{a_1 d_0}{a_0^2 b_0}\right)\right] \left[1 + \frac{1}{2}\left(\frac{b_1 d_0^2}{b_0 c_0^2} + \frac{c_2 d_0^2}{c_0^3} - \frac{a_1 d_0}{a_0 b_0^2} + \frac{a_1 b_1 d_0^3}{a_0 b_0^3 c_0^2} - \frac{a_1 c_2 d_0^3}{b_0^3 c_0^3}\right)\right]$$

First-order correction

Second-order correction

FATE OF THE PERTURBATION

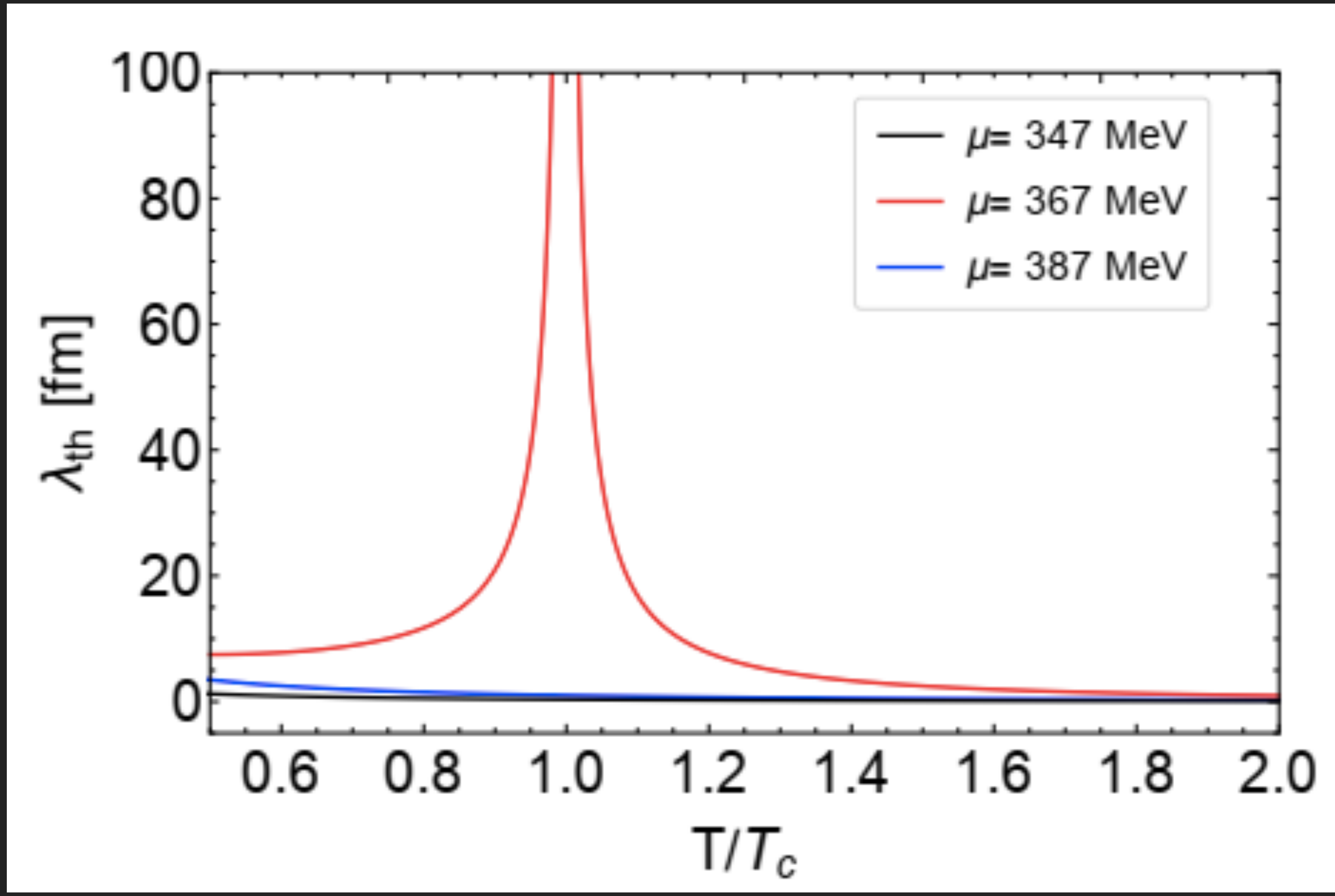
- A wave with wavelength $\lambda \geq \lambda_{th}$, can only propagate and rest will dissipate in the medium.

  Staig and Shuryak, PRC (2011)

- Viscous horizon is a length scale $(R_v) \sim \lambda_{th}$: separates the wavelengths of the sound which are and are not dissipated by the viscosity effects.

- R_v → highest order of surviving flow harmonics $(n_v) \sim 1/R_v$

- At the CEP, we found λ_{th} to be very large, implies survival of no wave.



FATE OF A PERTURBATION

- As consequences, Mach cone formation is prevented.

- The viscous horizon scale is also divergent, means no wave can survive from viscous damping.

- All the possible flow harmonics collapse at the CEP.

PROCEEDINGS OF SCIENCE

Collapse of Flow: Probing the Order of the Phase Transition

Horst Stöcker*

H. Stoecker, arXiv:0710.5089

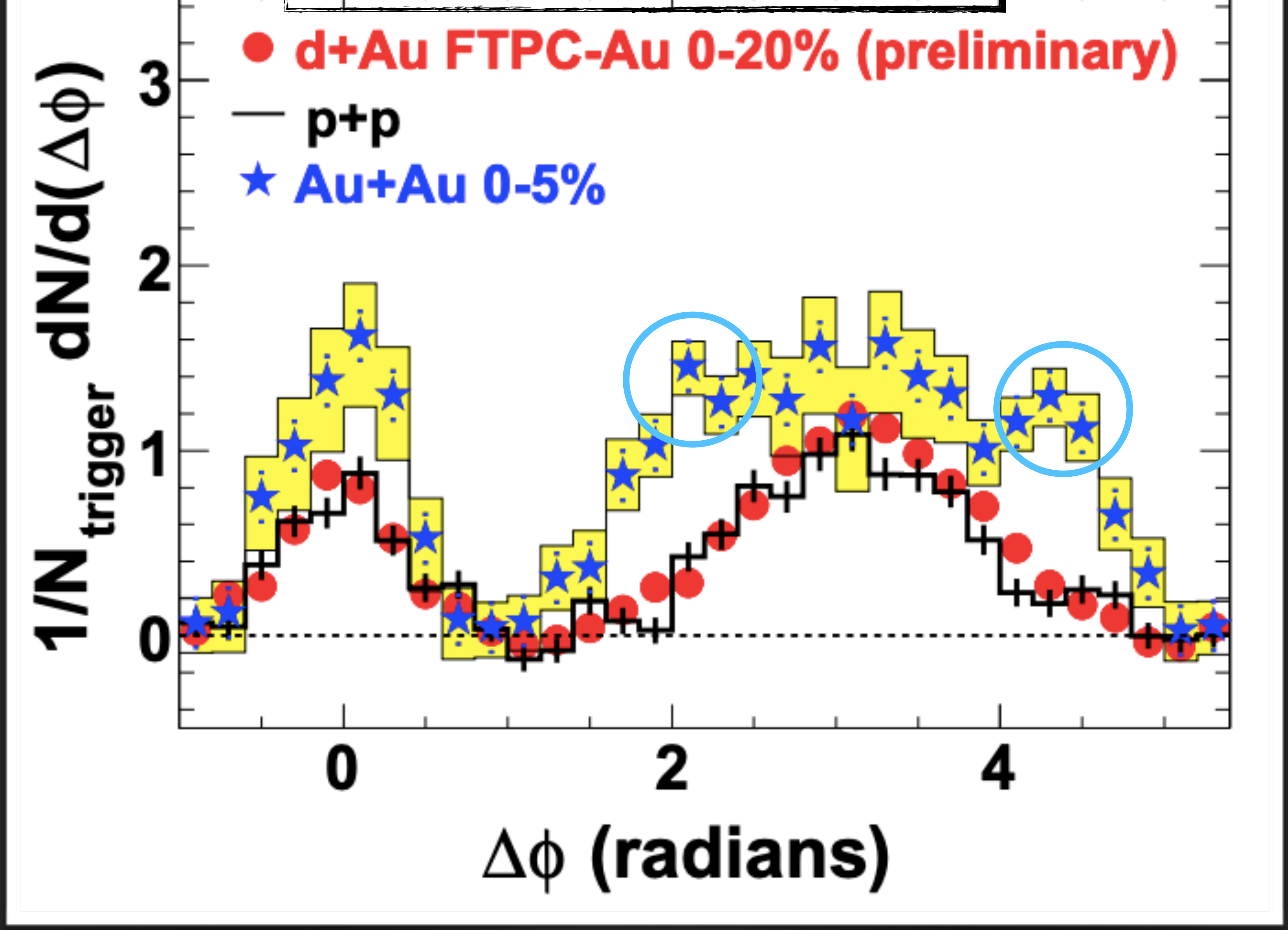
- Ideally fully collapsed harmonics may be the signature of the CEP, but in experiments from the formation of the QGP to hadron is superposition of all the T, μ states, one expects suppression.

NONLINEAR PERTURBATIONS

- Propagation of jets in QGP creates disturbances~ 1 GeV/ fm^3.
- Linear analysis is not valid, as they are nonlinear perturbations.
- The nonlinear perturbations are treated as nonlinear waves.

G. Sarwar, ***MH****, M. Rahaman , A. Bhattacharyya and J. Alam,* ***PLB (2021)***

STAR Collaboration, NPA(2006)



- Double hump structure at away side jet in particle correlation at $\Delta\phi = \pi \pm 1.2rad$

STAR collaboration, PRL (2005)

Formation of Mach cone

- Mach cone formation is prevented in linear analysis.
- We aim to observe if such phenomena occurs in nonlinear waves too.

EOM: NONLINEAR PERTURBATIONS

- **Hydro equations from MIS theory**
- **Adopted Reductive Perturbative Method (RPM) technique:**

Stretched coordinate

$$X = \frac{\sigma^{1/2}}{L}(x - c_s t), \quad , \quad Y = \frac{\sigma^{3/2}}{L}(c_s t)$$

Fogaça et. al., PRC (2010)

$$\hat{\epsilon} = \frac{\epsilon}{\epsilon_0} = 1 + \sigma\epsilon_1 + \sigma^2\epsilon_2 + \sigma^3\epsilon_3 + \ldots$$

- **Different order of σ results in Breaking wave, Burger's, KdV equations.**

Initial profiles

$$\hat{\epsilon}_1 = A_1\left[\text{sech}\frac{(x - x_0)}{B_1}\right]^2$$

$$\hat{\epsilon}_2 = A_2\left[\text{sech}\frac{(x - x_0)}{B_2}\right]^2$$

$$\hat{\epsilon}_1 = \sigma\,\epsilon_1,\ \hat{\epsilon}_2 = \sigma^2\,\epsilon_2$$

EOM for First-order perturbation

$$\frac{\partial\hat{\epsilon}_1}{\partial t} + [1 + \frac{3}{2}q_1\hat{\epsilon}_1]c_s\frac{\partial\hat{\epsilon}_1}{\partial x} + [\frac{1}{2(\epsilon_0 + p_0)}(\zeta + \frac{4}{3}\eta) - \kappa q_2]\frac{\partial^2\hat{\epsilon}_1}{\partial x^2} = 0$$

EOM for Second-order perturbation

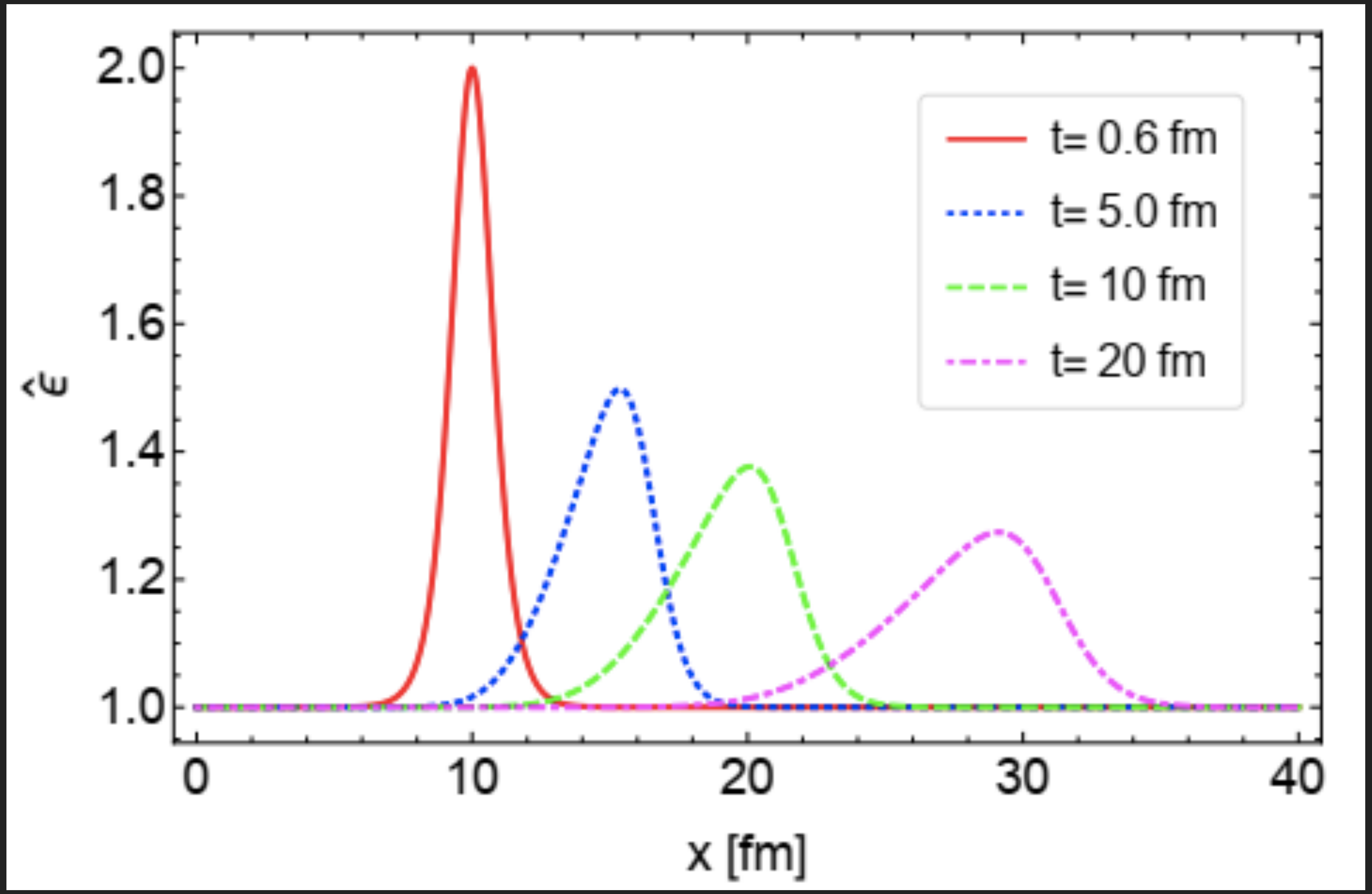
$$\frac{\partial\hat{\epsilon}_2}{\partial t} + \mathcal{S}_1\frac{\partial\hat{\epsilon}_2}{\partial x} + \mathcal{S}_2\frac{\partial\hat{\epsilon}_1}{\partial x} + \mathcal{S}_3(\frac{\partial\hat{\epsilon}_1}{\partial x})^2 + \mathcal{S}_4\frac{\partial^2\hat{\epsilon}_1}{\partial x^2} + \mathcal{S}_5\frac{1}{\hat{\epsilon}_1}\frac{\partial\hat{\epsilon}_1}{\partial x}\frac{\partial^2\hat{\epsilon}_1}{\partial x^2} + \mathcal{S}_6\frac{\partial^3\hat{\epsilon}_1}{\partial x^3} + \mathcal{S}_7\frac{\partial^2\hat{\epsilon}_2}{\partial x^2} = 0$$

FATE OF NONLINEAR PERTURBATIONS

AWAY FROM CRITICAL POINT

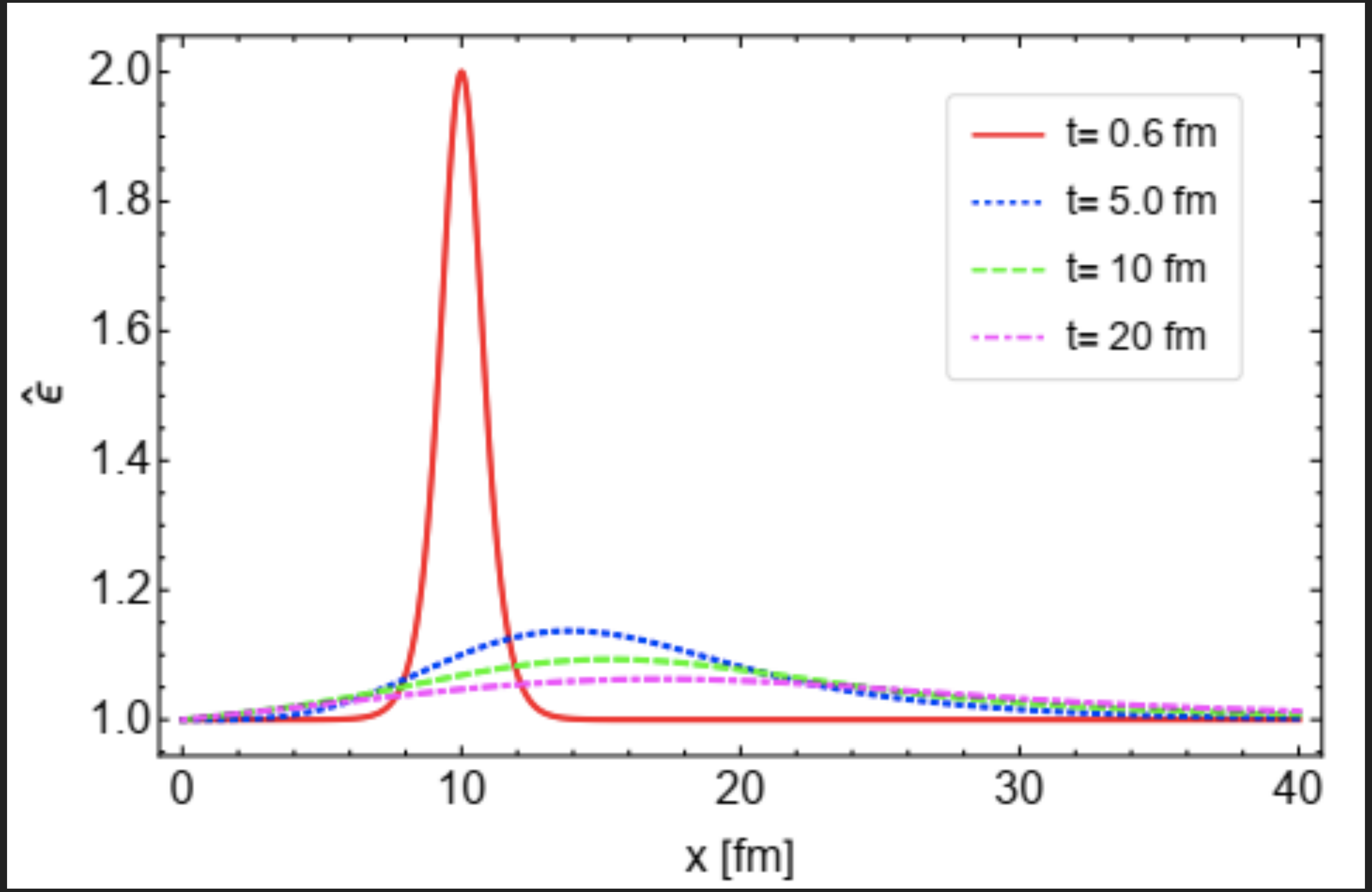
$\hat{\epsilon} = 1 + \hat{\epsilon}_1 + \hat{\epsilon}_2$

NEAR CRITICAL POINT



- Viscous coefficients η, ζ, κ are present.
- Waves are attenuated, but not dissipated completely.

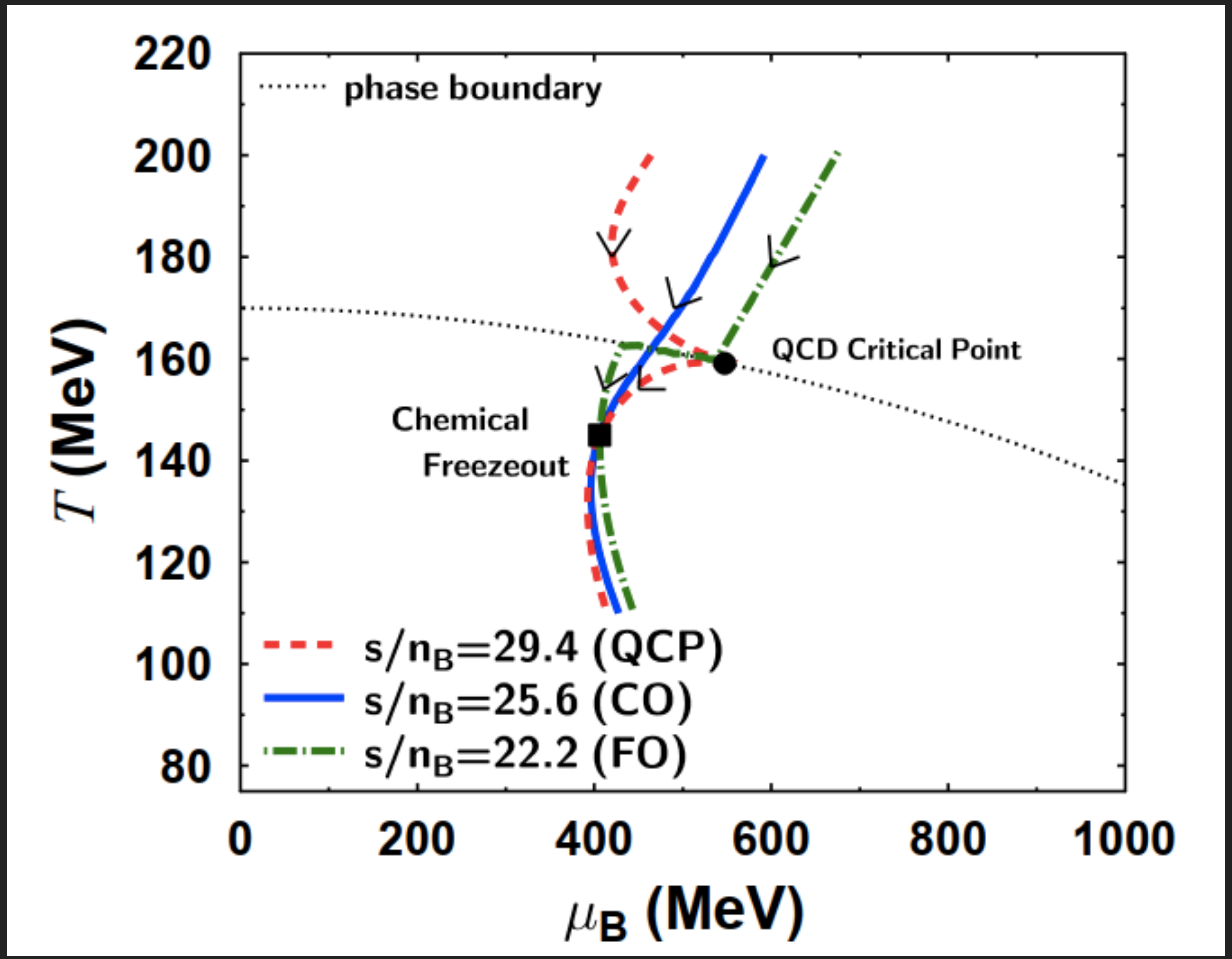
- Viscous coefficients η, ζ, κ are present.
- Waves dissipate rapidly in presence of the critical point.



CRITICAL EFFECTS

- Like the linear disturbances, the nonlinear disturbances too created by jets are highly suppressed.
- The Mach cone can not form, thus it will affect the particle correlation.
- Flow harmonics will be suppressed.
- The trajectory of the system in the QCD phase diagram will be event-dependent.

Nonaka et. al, NPA (2009)



In event-by-event analysis, isentropic trajectories form a few events may go through the CEP and the others away from the CEP, cause large event-by-event fluctuation of flow harmonics.

SUMMARY

- **Studied linear perturbations by imparting a perturbation $e^{i(kx-\omega t)}$ through hydro variables.**
- **Critical effects are observed through an EoS containing the critical point.**
- **λ_{th} is calculated via dispersion relation, and it is found to be diverging near the critical point, implying forbiddance of propagation of perturbations.**
- **Consequently, the speed of sound in the QGP vanishes, insinuates the formation of Mach cone.**
- **Viscous horizon scale also diverges and we find no harmonics to survive when the system hits the critical point.**

SUMMARY

- Along with the linear perturbations, we have also studied the nonlinear perturbations in the fluid.
- The nonlinear waves survives the damping, when the system is away from the CEP, but highly dissipated near the CEP.
- As a result, the Mach cone effect will disappear in the particle correlation of jets.

- We argue that the disappearance of Mach cone effects in particle correlation and the enhancement of fluctuations in flow harmonics in the event-by-event analysis may be considered as signals of the critical point.

Thank you

BACKUP: MIS THEORY

General frame →

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \Pi)\Delta^{\mu\nu} + h^\mu u^\nu + h^\nu u^\mu + \pi^{\mu\nu}$$

$$N^\mu = nu^\mu + \nu^\mu = nu^\mu - nq^\mu/(\epsilon + p)$$

Landau-Lifshitz frame/Energy frame: $h^\mu = 0,\ \nu^\mu \neq 0$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N^\mu = nu^\mu + \nu^\mu = nu^\mu - nq^\mu/(\epsilon + p)$$

$$\Pi = -\frac{1}{3}\zeta(\partial_\mu u^\mu + \beta_0 D\Pi - \alpha_0 \partial_\mu q^\mu)$$

$$\pi^{\mu\nu} = -2\eta\Delta^{\mu\nu\alpha\beta}\left[\partial_\alpha u_\beta + \beta_2 D\pi_{\alpha\beta} - \alpha_1 \partial_\alpha q_\beta\right]$$

$$q^\mu = \kappa T \Delta^{\mu\nu}[\frac{nT}{\epsilon + P}(\partial_\nu \alpha) - \beta_1 D q_\nu + \alpha_0 \partial_\nu \Pi + \alpha_1 \partial_\lambda \pi^\lambda_\nu]$$

Eckart frame/Particle frame: $h^\mu \neq 0,\ \nu^\mu = 0$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \Pi)\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu}$$

$$N^\mu = nu^\mu$$

$$\Pi = -\frac{1}{3}\zeta(\partial_\mu u^\mu + \beta_0 D\Pi - \tilde{\alpha}_0 \partial_\mu q^\mu)$$

$$\pi^{\mu\nu} = -2\eta\Delta^{\mu\nu\alpha\beta}\left[\partial_\alpha u_\beta + \beta_2 D\pi_{\alpha\beta} - \tilde{\alpha}_1 \partial_\alpha q_\beta\right]$$

$$q^\mu = -\kappa T \Delta^{\mu\nu}[\frac{1}{T}(\partial_\nu T) - \tilde{\beta}_1 D q_\nu + \alpha_0 \partial_\nu \Pi + \tilde{\alpha}_1 \partial_\lambda \pi^\lambda_\nu]$$

$$\tilde{\alpha}_0 - \alpha_0 = \tilde{\alpha}_1 - \alpha_1 = -(\tilde{\beta}_1 - \beta_0) = -[(\epsilon + p)]^{-1}$$

- **The EOM is obtained by the conservation equations:** $\partial_\mu T^{\mu\nu} = 0; \partial_\mu N^\mu = 0$

- **Constructed on the basis of universality hypothesis: says that CEP of the QCD belongs to the same universality class of 3D Ising model.**

- **The universality hypothesis permits the linear mapping between the CEP of the Ising model and the CEP of the QCD phase diagram as:**

$$r = \frac{\mu - \mu_c}{\Delta\mu_c}, \ h = \frac{T - T_c}{\Delta T_c}$$

- **Magnetization, $M(r, h)$, plays as an order parameter in Ising model and that is mapped as critical entropy density $s_c(T, \mu)$ in QCD**

$$s_c(T, \mu) = \frac{M(r, h)}{\Delta T_c} = M\left(\frac{\mu - \mu_c}{\Delta\mu_c}, \frac{T - T_c}{\Delta T_c}\right)\frac{1}{\Delta T_c}$$

- **A dimensionless entropy density is calculated as:**

$$S_c = \left[D\sqrt{\Delta T_c^2 + \Delta\mu_c^2}\right] s_c$$

- **The entropy density of s_Q and s_H is connected by using S_c as switching function as:**

$$s(T,\mu) = \frac{1}{2}[1 - \tanh S_c(T,\mu)]s_Q(T,\mu) + \frac{1}{2}[1 + \tanh S_c(T,\mu)]s_H(T,\mu)$$

- **Once entropy density is known, other thermodynamic variables can be calculated as:**

$$n(T,\mu) = \int_0^T \frac{\partial s(T',\mu)}{\partial\mu} dT' \qquad p(T,\mu) = \int_0^T s(T',\mu)dT' \qquad \epsilon(T,\mu) = Ts(T,\mu) - p(T,\mu) + \mu n$$

- **To get the first order phase boundary, the discontinuity in the entropy density along the transition line**

$$\left|\frac{\partial T_c(\mu)}{\partial\mu}\right| \left[s(T_c(\mu)+\Delta,\mu) - s(T_c(\mu)-\Delta,\mu)\right]$$

DISPERSION RELATION

- **The EOMs for different components of perturbation are obtained by the conservation equations.**

Dispersion relation

Real part

$$\omega_{\rm Re} = \sqrt{\frac{a_0}{b_0}}\left[k - \frac{1}{2}\frac{a_1}{a_0}k^2 + \left(\frac{1}{2}\frac{a_2}{a_0} - \frac{1}{8}a_1a_0^2 + \frac{b_1}{b_0}\right)k^3 + \left(\frac{1}{4}\frac{a_1a_2}{a_0^2} + \frac{1}{16}a_1^2a_0^3 - \frac{1}{2}\frac{a_1b_1}{a_0b_0}\right)k^4\right].$$

$$a_0 = 9h\left[\left(\frac{\partial P}{\partial T}\right)_n + \alpha_1 n\left\{\left(\frac{\partial \epsilon}{\partial n}\right)_T\left(\frac{\partial P}{\partial T}\right)_n - \left(\frac{\partial \epsilon}{\partial T}\right)_n\left(\frac{\partial P}{\partial n}\right)_T\right\}\right],$$

$$a_1 = \frac{9\alpha\beta_1 n^2T^2\kappa^2}{h} + 12\alpha_1\eta\kappa nT\left[\alpha + \frac{\alpha n}{h}\left(\frac{\partial \epsilon}{\partial n}\right)_T - \frac{T}{h}\left(\frac{\partial P}{\partial T}\right)_n + \frac{T}{h}\left(\frac{\partial \epsilon}{\partial T}\right)_n\right],$$

$$a_2 = \frac{9\beta_1\kappa^2n^2T}{h}\left[\left(\frac{\partial n}{\partial T}\right)_\mu\left(\frac{\partial \mu}{\partial n}\right)_T + \left(\frac{\partial P}{\partial T}\right)_n\left(\frac{\partial P}{\partial n}\right)_T\right] + \frac{12\alpha_1\eta\kappa nT}{h}\left[\left(\frac{\partial P}{\partial T}\right)_n + n\left(\frac{\partial n}{\partial T}\right)_\mu\left(\frac{\partial \mu}{\partial n}\right)_T + \frac{n}{h}\left(\frac{\partial P}{\partial T}\right)_n\left(\frac{\partial \epsilon}{\partial n}\right)_T\right],$$

$$b_0 = 9h\left(\frac{\partial \epsilon}{\partial T}\right)_n,$$

$$b_1 = 24\beta_2\eta^2\left(\frac{\partial \epsilon}{\partial T}\right)_n + \frac{9\beta_1\kappa^2n^2}{h}\left[T\left(\frac{\partial \mu}{\partial n}\right)_T\left(\frac{\partial n}{\partial T}\right)_\mu - T^2\left(\frac{\partial \epsilon}{\partial n}\right)_T\left(\frac{\partial \mu}{\partial n}\right)_n + \alpha\left(\frac{\partial \epsilon}{\partial n}\right)_T\right]$$

Imaginary part

$$\omega_{\rm Im} = -\frac{c_0}{d_0}\left[k^2 - \frac{c_1}{c_0}k^3 - \left(\frac{d_1}{d_0} + \frac{c_2}{c_0}\right)k^4\right]$$

$$c_0 = 2\eta h^2\left(\frac{\partial \epsilon}{\partial T}\right)_n - 3hn^2\kappa T\beta_1\left[\alpha\kappa\left(\frac{\partial \epsilon}{\partial n}\right)_T + h\left(\frac{\partial n}{\partial T}\right)_\mu\left(\frac{\partial \mu}{\partial n}\right)_T + \frac{\alpha_1}{\beta_1}\left(\frac{\partial P}{\partial T}\right)_n\left(\frac{\partial \epsilon}{\partial n}\right)_T\right]$$

$$c_1 = 2\alpha\beta_1\eta n^2T\left(T\kappa^2 + 4h\eta\frac{\beta - 2}{\beta_1}\right),$$

$$c_2 = 8\beta_2\eta\kappa n^2T\left[\left(\frac{\partial \epsilon}{\partial T}\right)_n - \left(\frac{\partial P}{\partial n}\right)_T^2\left(\frac{\partial \epsilon}{\partial n}\right)_T\right],$$

$$d_0 = 3h^3\left(\frac{\partial \epsilon}{\partial T}\right)_n,$$

$$d_1 = 3h\beta_1n^2\kappa\left[\alpha\kappa\left(\frac{\partial \epsilon}{\partial n}\right)_T + 4T^2\kappa\left(\frac{\partial \epsilon}{\partial T}\right)_n\left(\frac{\partial \mu}{\partial n}\right)_T^2 - T\left(\frac{\partial n}{\partial T}\right)_n\left(\frac{\partial \epsilon}{\partial n}\right)_T\right].$$