Investigating a non-extensive QCD medium in extreme conditions

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Outline

- **Magnetic field in Heavy Ion Collisions and its impact on the QCD medium**
- **Effective model treatment, namely the NJL model**
- **Nonextensive statistics and its relevance**
- **Results**
- **Conclusion**

Production of a strong magnetic field in HICs

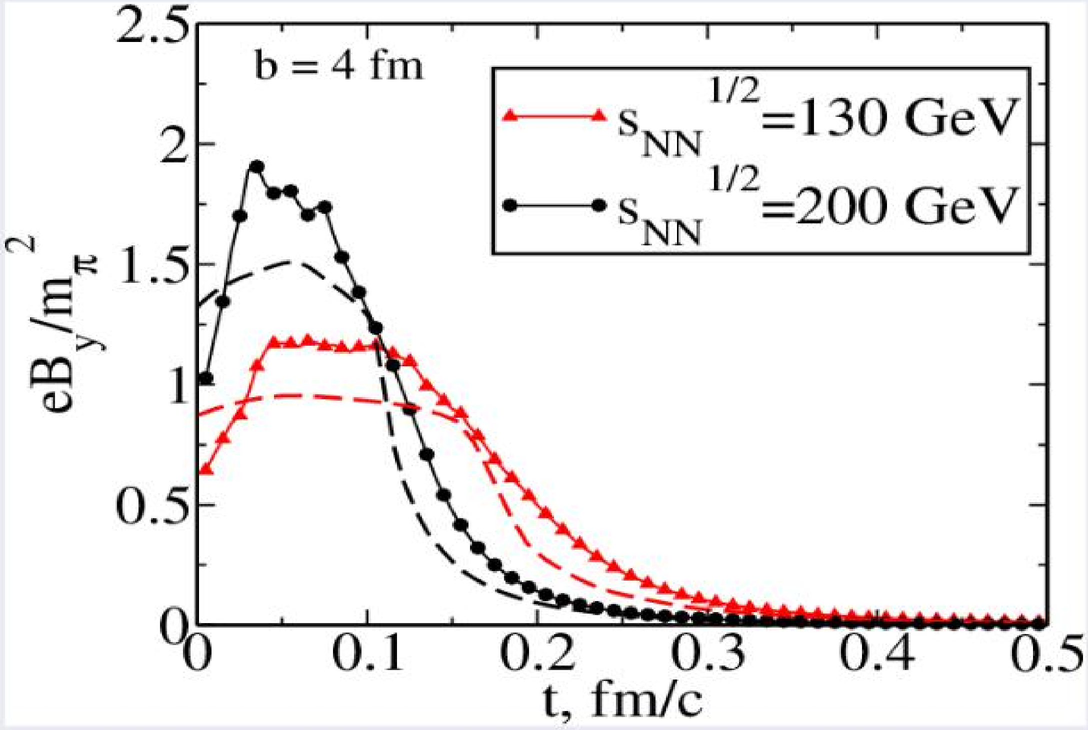
- A very strong magnetic field ($\approx m_\pi^2$ at RHIC and $\approx 10\,m_\pi^2$ at LHC) is generated in the direction perpendicular to the reaction plane, due to the relative motion of the ions themselves.

  ($m_\pi^2 = 1.96 \times 10^{-2}$ GeV$^2 \approx 10^{18}$ Gauss)

- A comparison with other terrestrial strengths: Earth $\approx 10^{-18}\ m_\pi^2$, usual laboratory $\approx 10^{-13}\ m_\pi^2$, max.

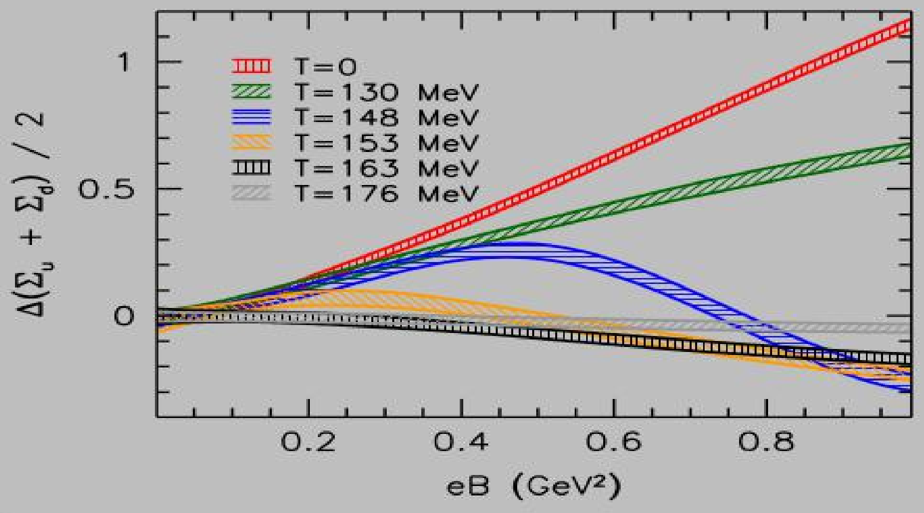
- A magnetar: $\approx 10^-5 - 10^-3\ m_\pi^2$.

[V. Skokov et al, IJMP,2009]

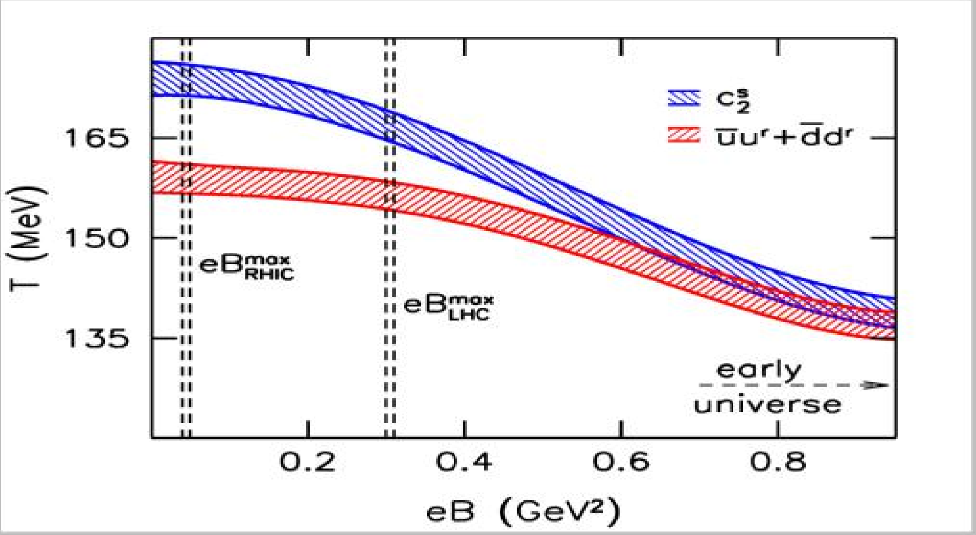


Phase diagram in presence of eB

- We observe magnetic catalysis (MC) and inverse magnetic catalysis (IMC) in presence of a magnetic field. [G. S. Bali et al., Phys. Rev. D 86, 071502]



- The PD in presence of eB looks as: [G. S. Bali et al., JHEP 02 (2012) 044]



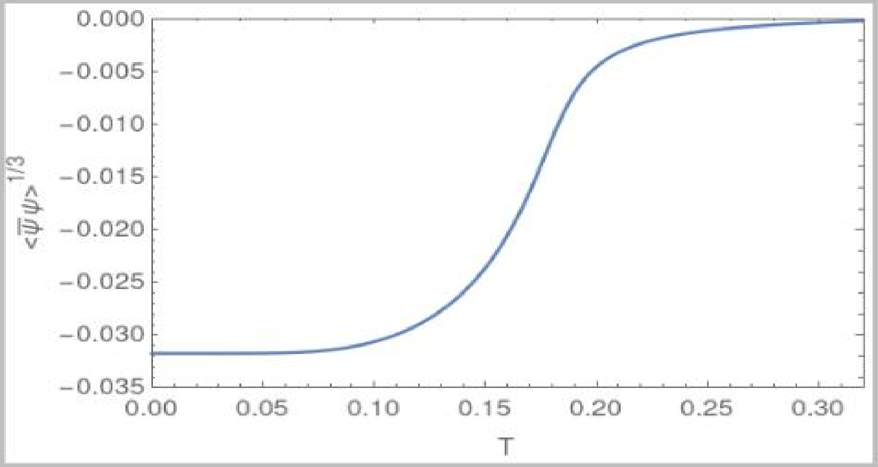
- **A word of caution** about the IMC effect.

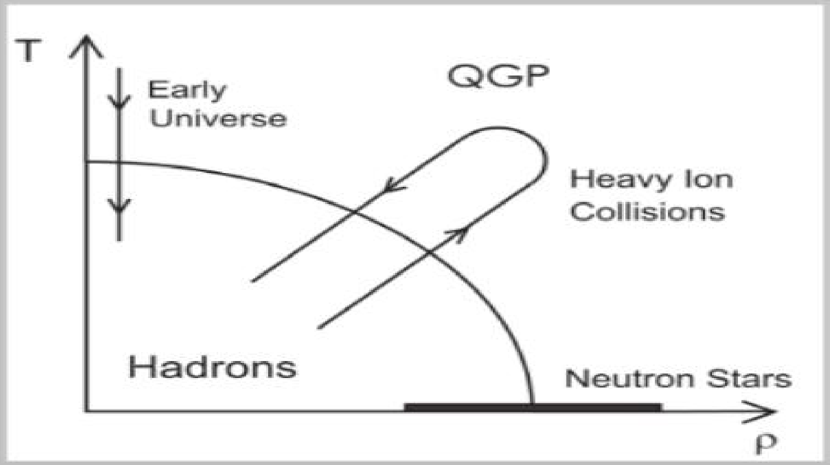
NJL, a classic example of QCD model

- Nambu–Jona-Lasinio (NJL) model is one such candidate, which is often used as an effective QCD model. [Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345(1961); 124, 246(1961)]

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \quad (1)$$

- It mainly incorporates the chiral symmetry and the chiral condensate, breaks the chiral symmetry spontaneously. This symmetry is restored at high temperature.





A 3f NJL model in the prsence of a magnetic field

- We work with a 3-flavour NJL model:

$$\mathcal{L}_{\rm NJL}^{B} = \bar{\psi}(i\not{D} - m_0)\psi + \mathcal{L}_1 + \mathcal{L}_2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad \text{with}$$

$$\mathcal{L}_1 = \frac{G_S}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2] \text{ and}$$

$$\mathcal{L}_2 = -G_D\{\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\}.$$

[D. P. Menezes et al, Phys. Rev. C 79, 035807 (2009)]

- Because of the magnetic field there will be two important modifications:

$$E_f(B) = [M_f^2 + p_z^2 + (2l + 1 - s)|q_f|B]^{1/2}$$

$$\int \frac{d^3p}{(2\pi)^3} \to \frac{|q_f|B}{2\pi}\sum_{l=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}$$

- To obatain the thermodynamic potential we need to linearise the Lagrangian.

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$$\Omega_{\rm NJL}^{B}(T,\mu) = G_S \sum_{f=u,d,s} \sigma_f^2 - 4G_D\sigma_u\sigma_d\sigma_s + \sum_{f=u,d,s}\left(\Omega_{\rm vac}^f + \Omega_{\rm mag}^f + \Omega_{\rm med}^f\right) + \frac{B^2}{2}$$

$$\Omega_{\rm vac}^f = -2N_c \int_\Lambda \frac{d^3p}{(2\pi)^3} E_p^f$$

$$\Omega_{\rm mag}^f = -\frac{N_c}{2\pi^2}\sum_f (|q_f|B)^2\left(\zeta'(-1,x_f) + \frac{x_f^2}{4} - \frac{1}{2}(x_f^2 - x_f)\ln x_f\right) \quad \text{and}$$

$$\Omega_{\rm med}^f = -\frac{N_c}{2\pi}T\sum_{f,l,s}|q_f|B\int_{-\infty}^{\infty}\frac{dp_z}{(2\pi)}\left[\ln\left(1+e^{-(E_f(B)-\mu)/T}\right) + \ln\left(1+e^{-(E_f(B)+\mu)/T}\right)\right]$$

Nonextensive statistics

- In 1988 Constantino Tsallis entertained the possibility of ensembles where the entropy took a nonadditive form involving a parameter q. [C. Tsallis, J. Stat. Phys. 52, 479 (1988)]

- As compared to the exponentials in traditional extensive statistics, the nonextensive statistics lead to power laws.

- It has been applied to many areas of the natural and social sciences.
  [C. Tsallis, Introduction to Nonextensive Statistical Mechanics, Springer, 2008]

- In the analysis of the data, Tsallis distribution has gained prominence with very good fits to the transverse momentum distributions made both at RHIC and LHC.
  [J. Cleymans & D. Worku, EPJA 48, 160 (2012)]

Nonextensive statistics

- Mathematically, the nonextensive statistics or the so-called Tsallis statistics can be obtained by replacing

$$\exp(-\frac{E}{T}) \to \exp_q(-\frac{E}{T}) \quad \text{where}$$

$$\exp_q(x) \equiv \begin{cases} \left(1 + (q-1)x\right)^{1/(q-1)}, & \text{if } x > 0 \\ \left(1 + (1-q)x\right)^{1/(1-q)}, & \text{if } x \leq 0. \end{cases}$$

- It reduces to the usual distribution in the limit q $\rightarrow$ 1.

$$\lim_{q\to 1} f_T^B(E) = f^B(E),$$

$$\lim_{q\to 1} f_T^{FD}(E) = f^{FD}(E),$$

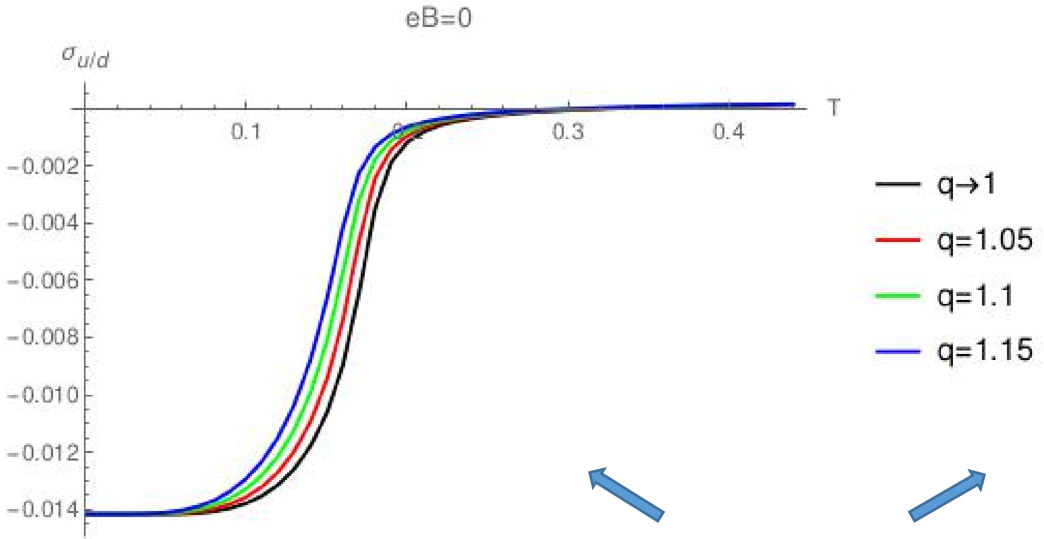
$$\lim_{q\to 1} f_T^{BE}(E) = f^{BE}(E).$$

- Derivation of the Tsallis distribution based on the Boltzmann equation can be found: [T.S. Biró & G. Purcsel, Phys. Rev. Lett. 95, 162302 (2005)]

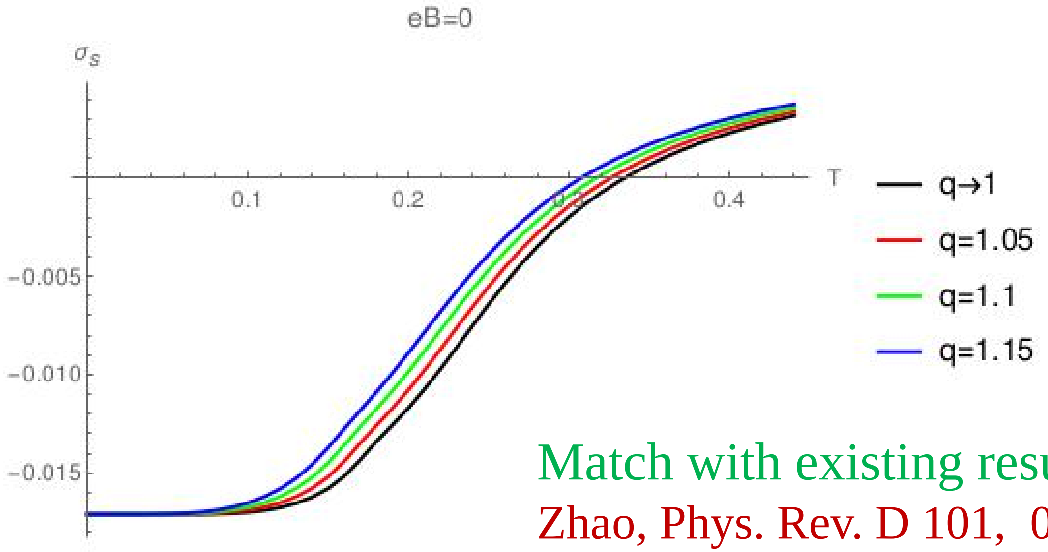
- Typical values of q parameter is close to unity and should not deviate from 1 by more than 20 percentage in HICs. [ALICE Collaboration, Eur. Phys. J. C71 1655 (2011)]

Results

Condensates and effective masses (eB=0)



up/down

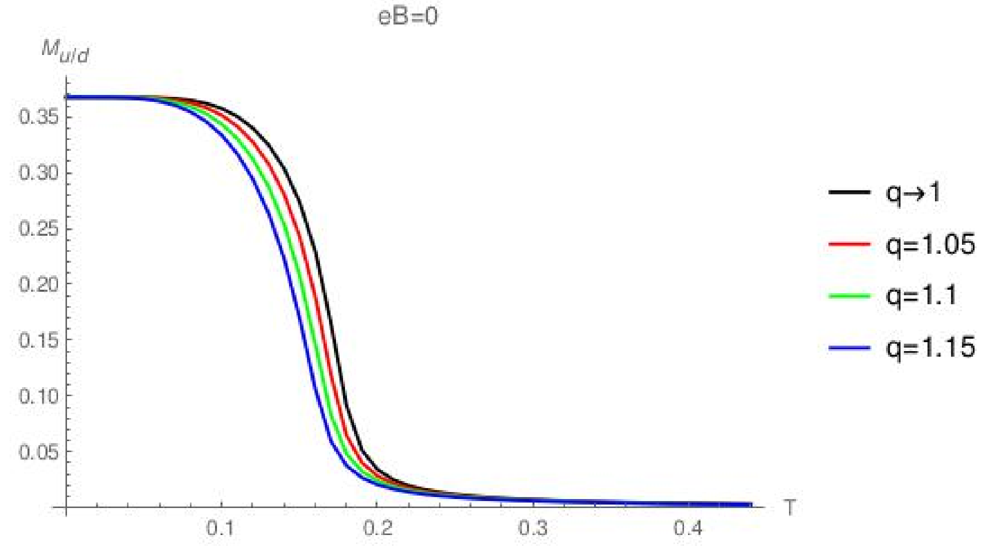


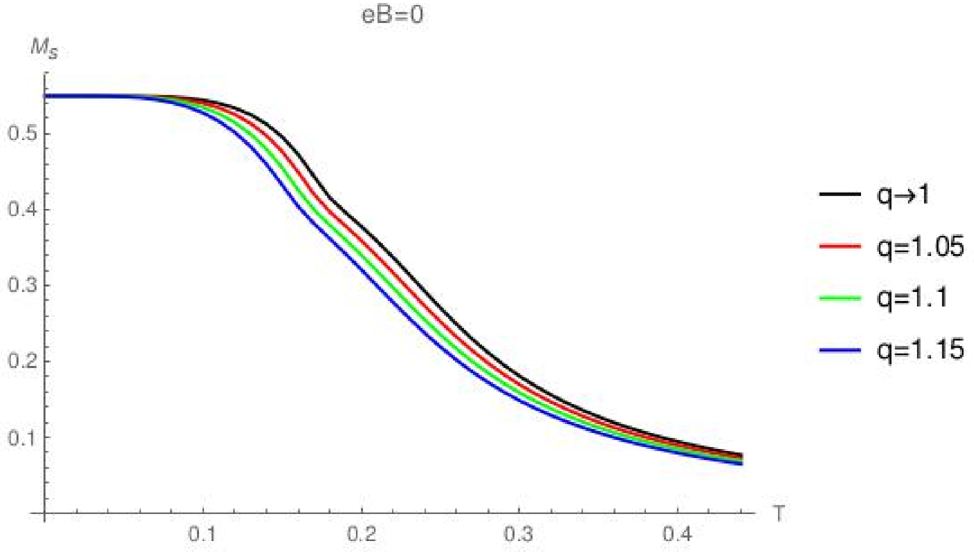
strange

Condensates

Match with existing results [Y. P. Zhao, Phys. Rev. D 101, 096006 (2020)]

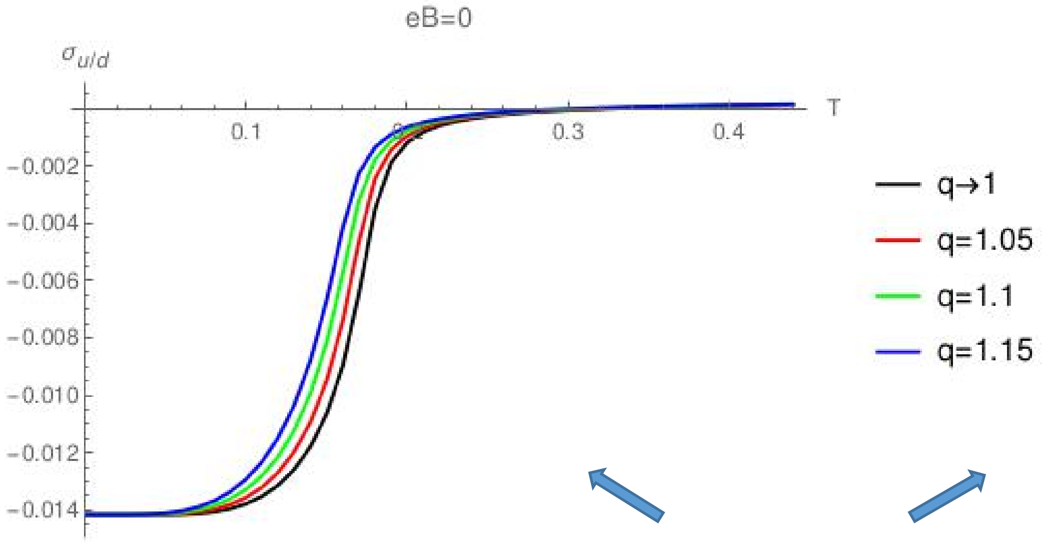
$$M_u = m_u - 2G_S\sigma_u + 2G_D\sigma_d\sigma_s$$
$$M_d = m_d - 2G_S\sigma_d + 2G_D\sigma_u\sigma_s$$
$$M_s = m_s - 2G_S\sigma_s + 2G_D\sigma_u\sigma_d$$

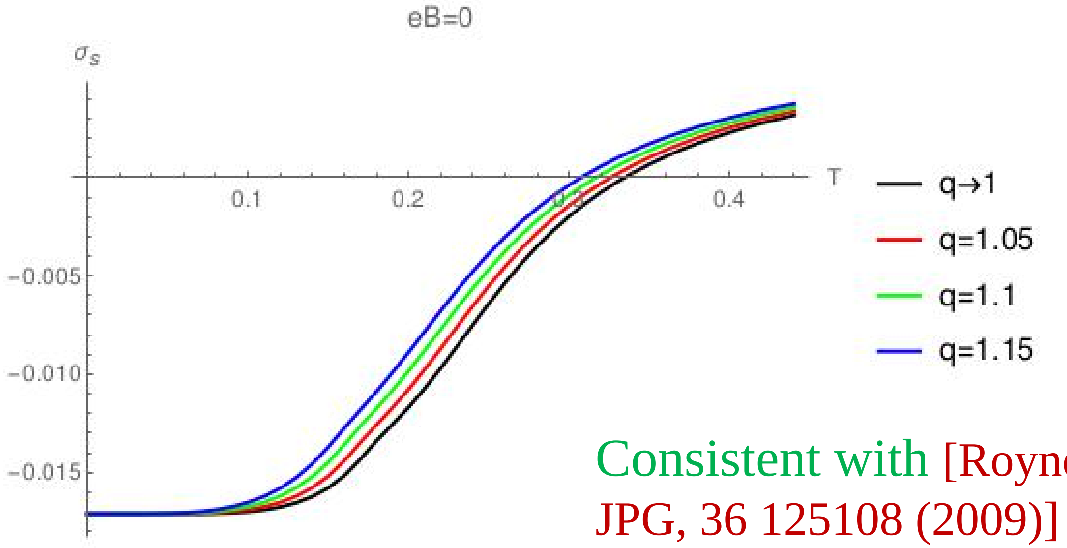




Effective masses

Chiral transition temperature (eB=0)



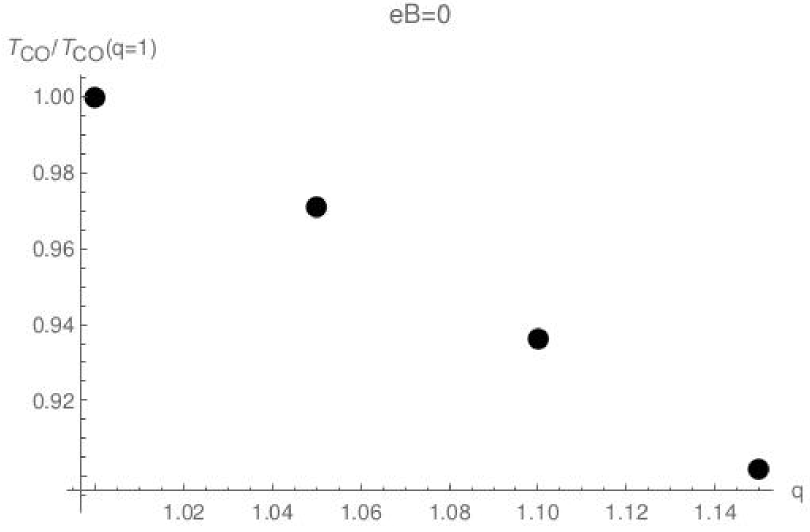


Condensates

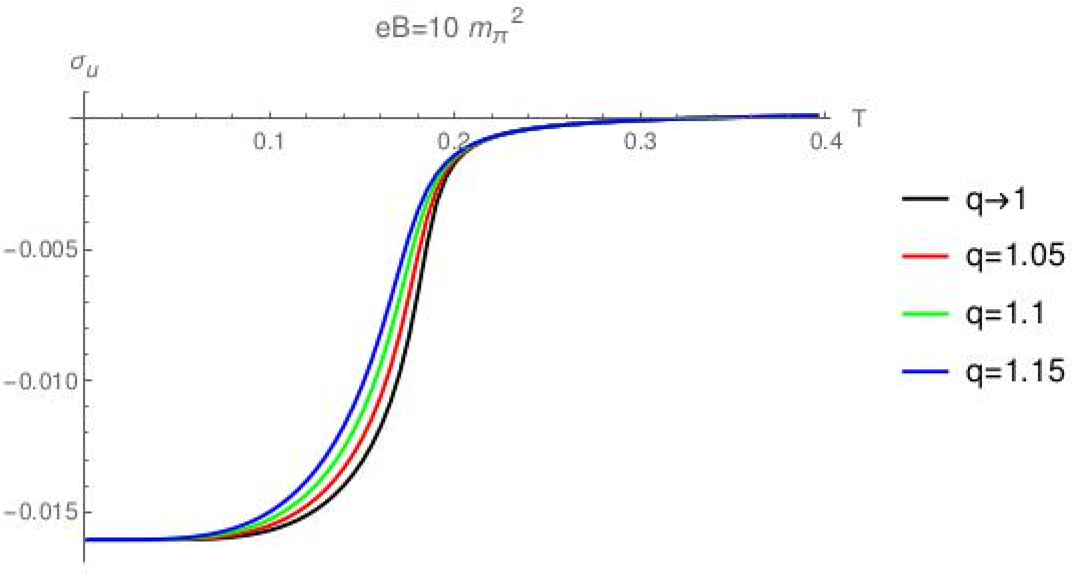
Consistent with [Roynek & Wilk, JPG, 36 125108 (2009)]

Chiral transition temperature calculated from the inflection points → It decreases.

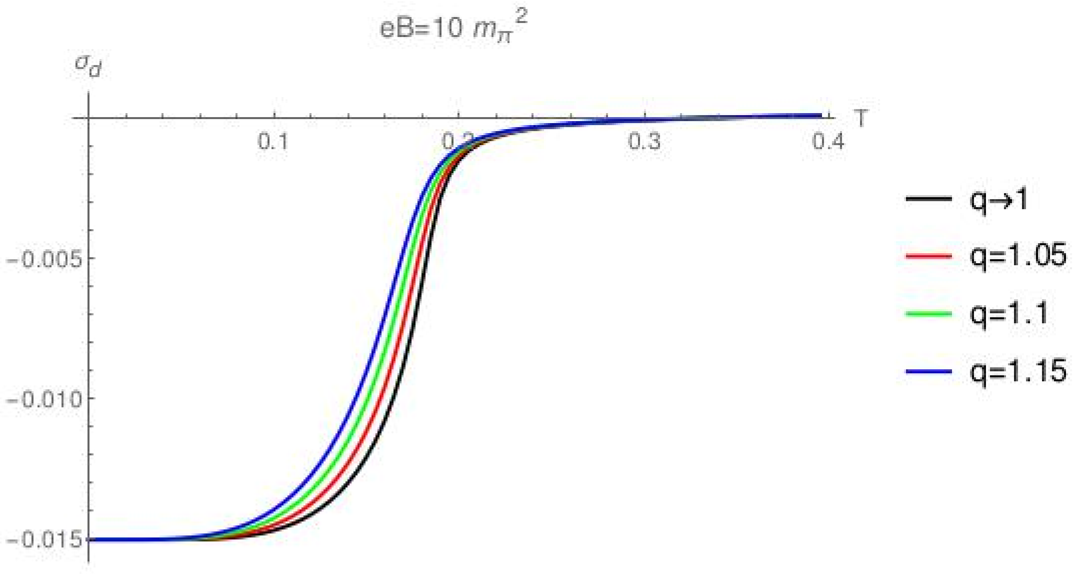
q	1	1.05	1.10	1.15
T_{CO} (MeV)	173	168	162	156



Chiral condensates (eB≠0)



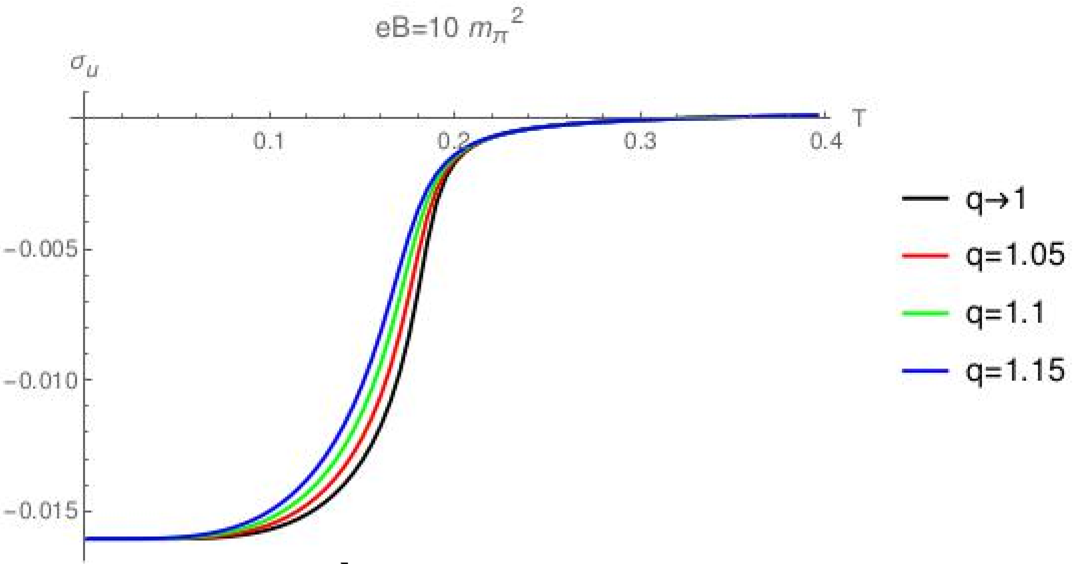
up quark



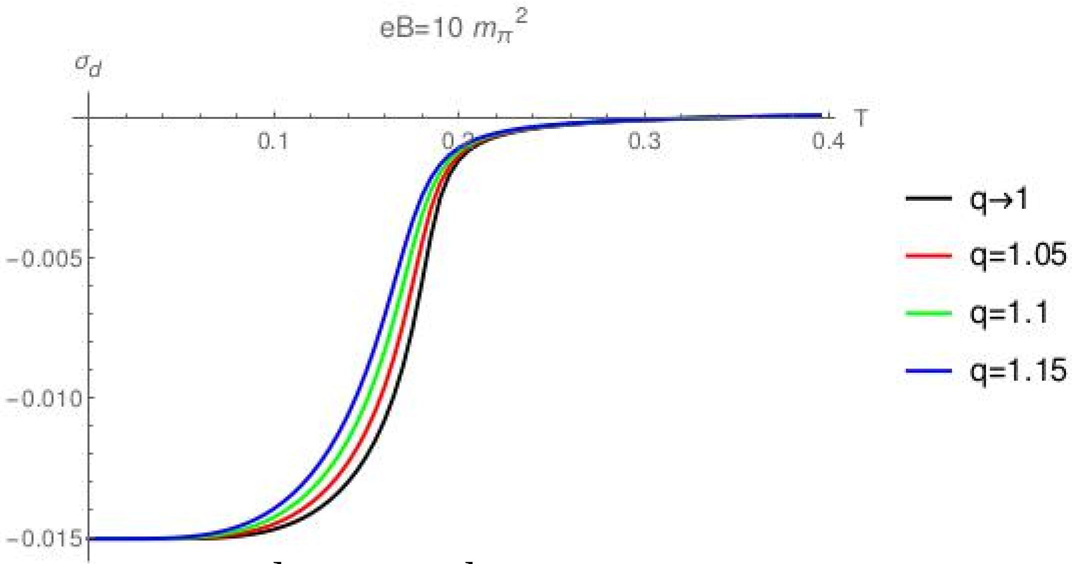
down quark

- In presence of a magnetic field up an down quarks split.
- The chiral condensates decrease with increasing q, particularly around the transition region.
- The effects on the effective masses are obvious.

Chiral transition temperature (eB≠0)



up quark

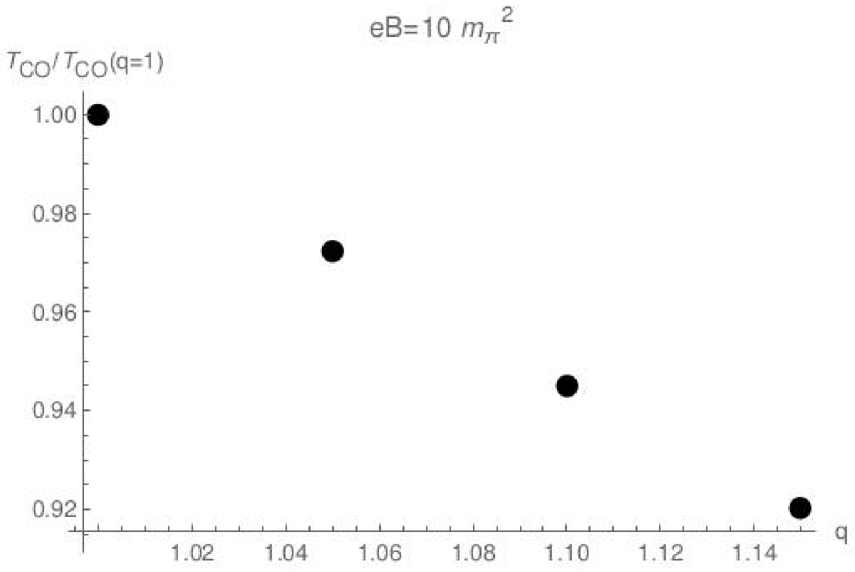


down quark

q	1	1.05	1.10	1.15
$T_{\rm CO}$ (MeV)	181.5	176.5	171.5	167

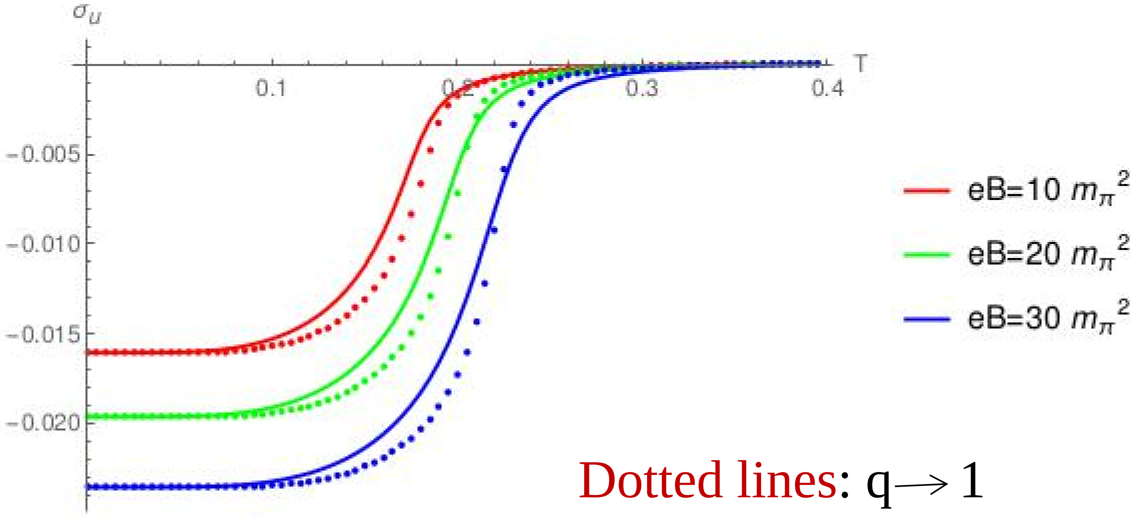
Table: Chiral transition tempearture

→

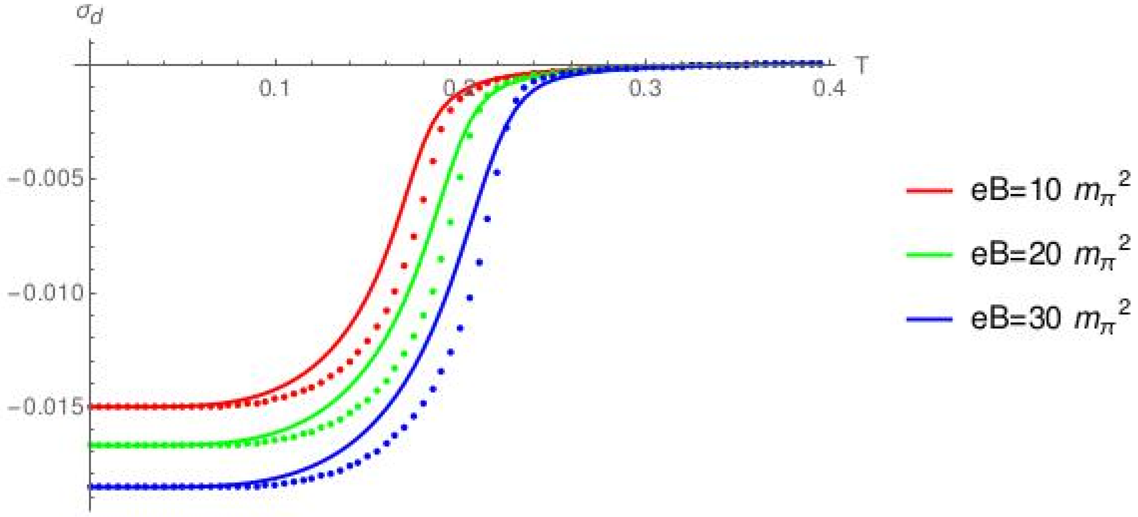


- There is a slight enhancement in the percentage decrease in the presence of a magnetic field.

Chiral condensates comparison (eB≠0)



up quark

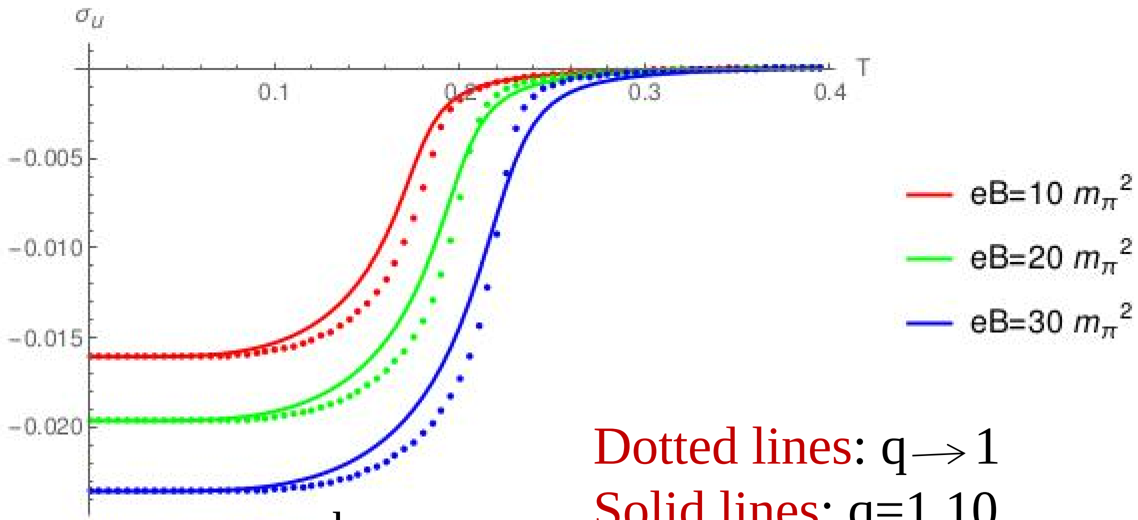


down quark

Dotted lines: $q \rightarrow 1$
Solid lines: $q=1.10$

- Magnetic field splits up an down quarks.
- Presence of magnetic catalysis for both cases.
- There is an overall decrease in the condensates in the nonextensive scenarios for most of the eB values.

Chiral transition temperature comparison (eB≠0)



up quark

Dotted lines: $q \to 1$
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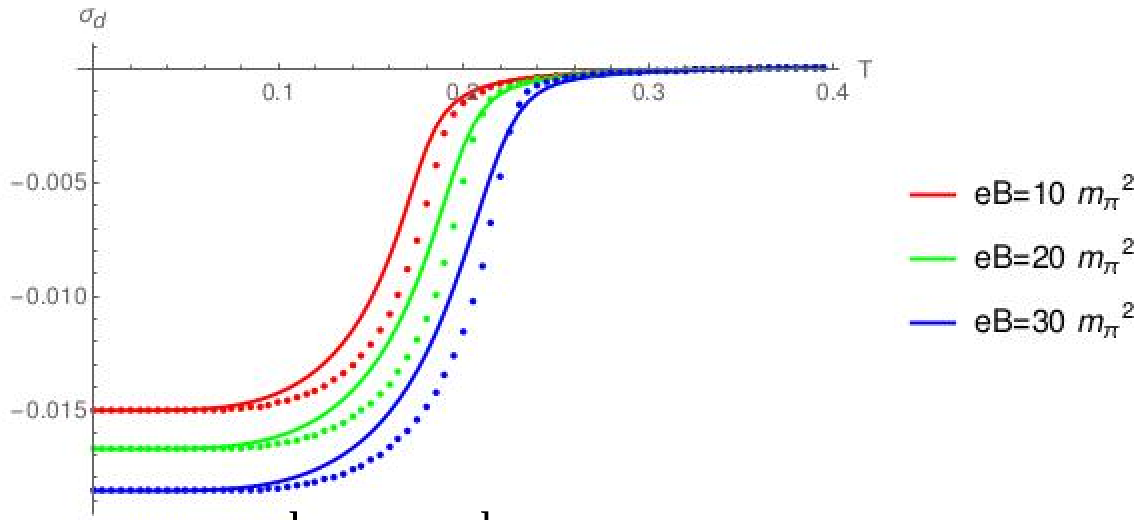
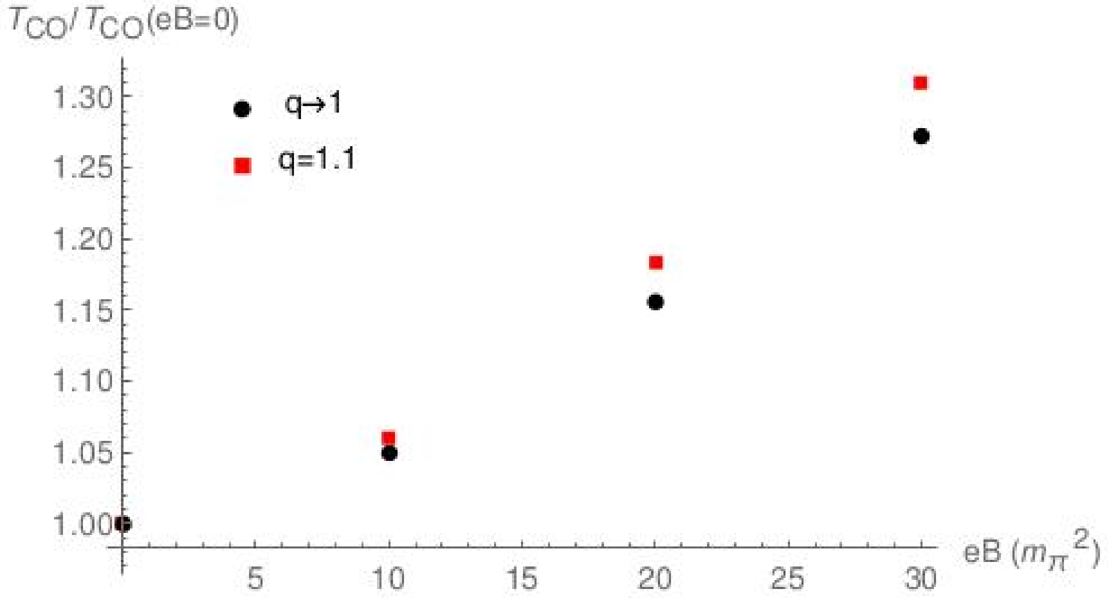


down quark

$eB\ (m_\pi^2)$		0	10	20	30
$T_{\rm CO}$ (MeV)	$q \to 1$:	173	181.5	200	220
	$q = 1.1$:	162	171.5	191.5	212

Table: Chiral transition tempearture



- There is an enhancement in the scaled transition temperatures for the nonextensive scenarios.

Upshots

- Chiral transition has been investigated for a nonextensive medium using NJL model.

- The transition temperature decreases with increasing q values, i.e., the nonextensivity of the medium.

- This features remains unaltered in the presence of a magnetic field for a constant coupling.

- It will be interesting to investigate the chiral transition with running coupling constant: ongoing project.

Thank You