Effect of Coriolis force on Electrical Conductivity: A Non-Relativistic Description

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Introduction

- In off-central or peripheral HIC, a huge magnetic field as well as angular velocity can be created [2, 3].

  F. Becattini, F. Piccinini, and J. Rizzo Phys. Rev. C 77, 024906
  $\implies \sqrt{s_{NN}} = 200 GeV$ *and* $L_0 = 5 \times 10^5 \hbar$ at an impact parameter b = 5 fm.

  Even 0.01% angular momentum(ang. mom.) transfer $\implies 10^3 \hbar$ ang. mom. of QGP

- This implies QGP must have a high average vorticity.

  Question:

- Can the transport coefficients of QGP be modified in the presence of rotation?

  In this presentation, we will answer this question in a non-relativistic setup.

Formalism

Non-Relativistic Frame Transformation:

Operator connection from inertial to rotating frame:

$$\left(\frac{d}{dt}\right) \equiv \left(\frac{d}{dt}\right)' + \vec{\Omega} \times \ . \tag{1}$$

Using the Eq. (1) one gets the following important identity:

$$\vec{a} = \vec{a}' + 2(\vec{\Omega} \times \vec{v}') + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}') + \dot{\vec{\Omega}} \times \vec{r}' \ , \tag{2}$$

$$\text{Coriolis force} = -2m(\vec{\Omega} \times \vec{v}') \ ,$$

$$\text{Centrifugal force} = -m\,\vec{\Omega} \times (\vec{\Omega} \times \vec{r}') \ ,$$

$$\text{Euler force} = -m\,(\dot{\vec{\Omega}} \times \vec{r}') \ .$$

Our set-up:

A non-relativistic single-component fluid rotating with angular velocity $\vec{\Omega} = const$.

$\vec{\Omega} = const$. makes Euler force zero.

- In our analysis, we will ignore the centrifugal force as if it were not there(added in the backup slides).

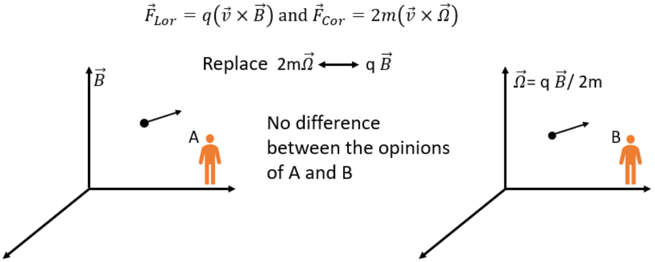

Figure: Motion under finite magnetic fields and rotation

Formalism(contd...)

- To see the statistical effects $\implies$ Boltzmann Transport Equation(BTE)
- Out of eqbm. $\implies$ transport coefficients of the medium
- Navier-Stokes Theory $\implies$ Dissipative flows = Transport coefficients $\times$ Driving force
- Our Interest: Charge current : $J_i = \sigma_{ij} \tilde{E}_j$
- Kinetic theory definition of j_i :

$$j_i = q\, g \int \frac{d^3\vec{p}}{(2\pi)^3} v_i \, \delta f \, , \tag{3}$$

- Assumption: Total distribution (f) = local eqbm. distribution (f_0) + deviation δf,

$$f^0 = \frac{1}{\exp\left\{\frac{E-\mu(\vec{r},t)-\vec{u}(\vec{r},t)\cdot\vec{p}}{T(\vec{r},t)}\right\} + 1} \, , \tag{4}$$

- f obeys the Boltzmann Transport Equation(BTE).

Formalism(contd...)

- BTE in Relaxation Time Approximation (RTA) :

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} + \vec{F}_{\text{ext}}\cdot\frac{\partial f}{\partial \vec{p}} = -\frac{\delta f}{\tau_c} \,. \tag{5}$$

- External forces in different scenarios are different.

- Our BTE will be:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} + [q\vec{\tilde{E}} + 2m(\vec{v}\times\vec{\Omega}]\cdot\frac{\partial f}{\partial \vec{p}} = -\frac{\delta f}{\tau_c} \,. \tag{6}$$

- We ignore terms: $\frac{\partial u_i}{\partial x_j}, \frac{\partial T}{\partial x_i}, \frac{\partial \mu}{\partial x_i}$

  Eq (6) will become,

$$[q\vec{\tilde{E}} + 2m(\vec{v}\times\vec{\Omega})]\cdot\frac{\partial f^0 + \delta f}{\partial \vec{p}} = -\frac{\delta f}{\tau_c} \,. \tag{7}$$

- In Eq. (7) keeping terms of order δf :

$$\implies \frac{\partial f^0}{\partial E}\vec{v}\cdot(q\vec{\tilde{E}}) + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial \delta f}{\partial \vec{p}} = -\frac{\delta f}{\tau_c} \,. \tag{8}$$

Formalism(contd...)

Solving Eq. (8) for δf

- Ansatz: $\delta f = -\vec{p}\cdot\vec{F}\left(\frac{\partial f^0}{\partial E}\right)$

  $\vec{F} = \alpha\hat{e} + \beta\hat{\omega} + \gamma(\hat{e}\times\hat{\omega})$

  $\vec{\tilde{E}} = \tilde{E}\hat{e},\ \vec{\Omega} = \Omega\hat{\omega}$

  α, β, γ are unknown constants

- Substituting δf in Eq. (8):

$$\left(\frac{q\tilde{E}}{m} - 2\Omega\gamma\right)\hat{e} + 2\gamma\Omega(\hat{\omega}\cdot\hat{e})\hat{\omega} + 2\alpha\Omega(\hat{e}\times\hat{\omega}) = \frac{\alpha}{\tau_c}\hat{e} + \frac{\beta}{\tau_c}\hat{\omega} + \frac{\gamma}{\tau_c}(\hat{e}\times\hat{\omega})\ . \tag{9}$$

- Following linear eq.s to solve:

$$\frac{q\tilde{E}}{m} - \frac{\gamma}{\tau_\Omega} = \frac{\alpha}{\tau_c}\ , \qquad \frac{\gamma}{\tau_\Omega}(\hat{\omega}.\hat{e}) = \frac{\beta}{\tau_c}\ , \qquad \frac{\alpha}{\tau_\Omega} = \frac{\gamma}{\tau_c}\ , \tag{10}$$

$\tau_\Omega \equiv \frac{1}{2\Omega}$.

Formallism(contd...)

Eq. 10 can be simplified as:

$$\alpha = \frac{\tau_c\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}, \qquad \gamma = \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}, \qquad \beta = \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)^2(\hat{\omega}.\hat{e})\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}. \tag{11}$$

- See two time scales τ_c and τ_Ω (more about this later).

The Eqs. (11) can be used to get the expression of δf as:

$$\begin{aligned}
\delta f &= -\vec{p}\cdot\vec{F}\frac{\partial f^0}{\partial E} \\
&= -\frac{\partial f^0}{\partial E} m\vec{v}\cdot(\alpha\hat{e} + \beta\hat{\omega} + \gamma(\hat{e}\times\hat{\omega})) \\
&= -\frac{\partial f^0}{\partial E}\left[\frac{q\tau_c\tilde{E}}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\hat{e}\cdot\vec{v} + \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)^2(\hat{\omega}\cdot\hat{e})q\tilde{E}}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\hat{\omega}\cdot\vec{v} + \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)q\tilde{E}}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)}(\hat{e}\times\hat{\omega})\cdot\vec{v}\right] \\
&= -\frac{\partial f^0}{\partial E}\left(\frac{q\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\right)\left[\delta_{jl} + \left(\frac{\tau_c}{\tau_\Omega}\right)^2\omega_j\omega_l + \left(\frac{\tau_c}{\tau_\Omega}\right)\epsilon_{ljk}\omega_k\right]\tilde{E}_j v_l\,.
\end{aligned} \tag{12}$$

Current density:

$$j_i = qg \int \frac{d^3\vec{p}}{(2\pi)^3} v_i \frac{\partial f^0}{\partial E} \left(\frac{q\tau_c}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} \right) \left[\delta_{jl} + \left(\frac{\tau_c}{\tau_\Omega}\right)^2 \omega_j \omega_l + \left(\frac{\tau_c}{\tau_\Omega}\right) \epsilon_{ljk} \omega_k \right] \tilde{E}_j v_l \,. \tag{13}$$

Final expression:

$$j_i = -q^2 g \int \frac{d^3p}{(2\pi)^3} \frac{\partial f^0}{\partial E} \frac{v^2}{3} \left(\frac{\tau_c}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} \right) \left[\delta_{ij} + \left(\frac{\tau_c}{\tau_\Omega}\right) \epsilon_{ijk} \omega_k \left(\frac{\tau_c}{\tau_\Omega}\right)^2 \omega_j \omega_i \right] \tilde{E}_j \,. \tag{14}$$

- Comparing Eq.(14) with $j_i = \sigma_{ij} \tilde{E}_j$

$$\sigma_{ij} = -gq^2 \int \frac{d^3p}{(2\pi)^3} \frac{\partial f^0}{\partial E} \frac{v^2}{3} \left(\frac{\tau_c}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} \right) \left[\delta_{ij} + \left(\frac{\tau_c}{\tau_\Omega}\right) \epsilon_{ijk} \omega_k \left(\frac{\tau_c}{\tau_\Omega}\right)^2 \omega_j \omega_i \right] \,. \tag{15}$$

We can re-express Eq. (15) as follows:

$$\sigma_{ij} = \sigma_0 \delta_{ij} + \sigma_1 \epsilon_{ijk} \omega_k + \sigma_2 \omega_i \omega_j \,, \text{with } \sigma_n = \frac{gq^2}{T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_c \left(\frac{\tau_c}{\tau_\Omega}\right)^n}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} \times \frac{v^2}{3} f^0 (1 - f^0) \,. \tag{16}$$

$\sigma_0, \sigma_1, \sigma_2$ are scalars that make up the conductivity tensor.

Formalism(contd...)

- σ_n in terms of fermi functions: $\sigma_n = \frac{\sqrt{m\pi}gq^2}{2\pi^2\sqrt{2}} \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)^n}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{\frac{3}{2}} f_{3/2}(A)$, $A = e^{\mu/T}$.
- fermi functions: $f_j(A) = \frac{1}{\Gamma(j)} \int_0^\infty \frac{x^{j-1}dx}{A^{-1}e^x+1}$
- Let's define the following:

$$\begin{aligned}
\sigma_{||} = \sigma_0 &= \frac{g\sqrt{m}q^2}{(2\pi)^{3/2}} \tau_c \, T^{\frac{3}{2}} f_{3/2}(A) \, , \\
\sigma_\perp = \sigma_0 + \sigma_2 &= \frac{g\sqrt{m}q^2}{(2\pi)^{3/2}} \frac{\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{\frac{3}{2}} f_{3/2}(A) \, , \\
\sigma_\times = \sigma_1 &= \frac{g\sqrt{m}q^2}{(2\pi)^{3/2}} \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{\frac{3}{2}} f_{3/2}(A) \, .
\end{aligned} \tag{17}$$

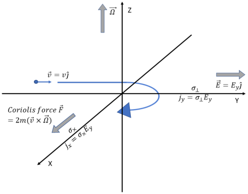

Figure: perpendicular conduction

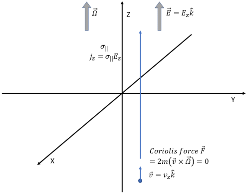

Figure: parallel conduction

For $\vec{\Omega} = \Omega\hat{k}$:

$$[\sigma] = \begin{pmatrix} \sigma_0 & \sigma_1 & 0 \\ -\sigma_1 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 + \sigma_2 \end{pmatrix} .$$

$\sigma_0 \equiv \sigma_\perp = \sigma_{yy} = \sigma_{xx}$, $\sigma_0 + \sigma_2 \equiv \sigma_{||} = \sigma_{zz}$, and $\sigma_1 \equiv \sigma_\times = \sigma_{xy}$

Non-Zero Coriolis force in XY-plane $\Longrightarrow$ $j_x = \sigma_{xy}E_y = \sigma_\times E_y$ $j_y = \sigma_{yy}E_y = \sigma_\perp E_y$

No Coriolis force in Z-direction $\Longrightarrow$ $j_z = \sigma_{zz}E_z = \sigma_{||}E_z$

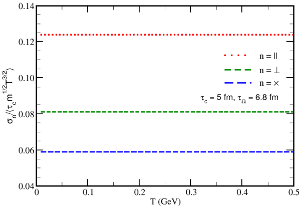

Figure: $\sigma_{normalized}$ vs T, $\mu = 0$

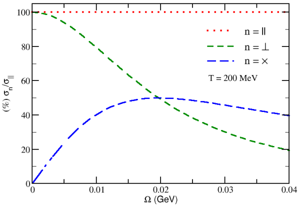

Figure: $\sigma/\sigma_{||}$ vs Ω, $\mu = 0$

For $\mu \to 0$

$$\sigma_{||} = 0.29g \,\frac{\sqrt{m}q^2}{(2\pi)^{3/2}}\, \tau_c \, T^{\frac{3}{2}} \zeta(3/2) \, ,$$
$$\sigma_{\perp} = 0.29g \,\frac{\sqrt{m}q^2}{(2\pi)^{3/2}}\, \frac{\tau_c}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{\frac{3}{2}} \zeta(3/2) \, ,$$
$$\sigma_{\times} = 0.29g \,\frac{\sqrt{m}q^2}{(2\pi)^{3/2}}\, \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{\frac{3}{2}} \zeta(3/2) \, , \tag{18}$$

$$\sigma_{zz} = \sigma_{||} \propto \tau_c \, ,$$
$$\sigma_{xx} = \sigma_{yy} = \sigma_{\perp} \propto \tau_{\perp} = \frac{\tau_c}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} \, ,$$
$$\sigma_{xy} = \sigma_{\times} \propto \tau_{\times} = \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} \, ,$$
$$\sigma_{\perp}(\Omega \to 0 \; or \; \tau_\Omega \to \infty) = \sigma_{||} \, ,$$
$$\sigma_{\times}(\Omega \to 0 \; or \; \tau_\Omega \to \infty) = 0 \, .$$

Summary

- We explored the equivalence nature of the Lorentz force and the Coriolis force in transport phenomena.
- The microscopic quantity- the deviation from the equilibrium distribution, is guessed with three unknown constants.
- The deviation is found by substituting it in the relaxation time approximated Boltzmann equation with the Coriolis force term.
- Then, it is used to obtain the microscopic expression of current density and is compared with the macroscopic definition of current density to get the conductivity tensor.

References

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Thank you!

All the oraganizers, the collaborators and the audiences.

Backups

Inclusion of Centrifugal Force: To see the effect of Centrifugal Force on the current, one may add the centrifugal force as a force term in the LHS of Eq. (7) as follows:

$$
\begin{aligned}
\left[q\vec{\tilde{E}} + 2m(\vec{v}\times\vec{\Omega}) - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r})\right]\cdot\frac{\partial f}{\partial\vec{p}} &= -\frac{\delta f}{\tau_c}\,,\\
\left[q\vec{\tilde{E}} - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r})\right]\cdot\frac{\partial f^0}{\partial\vec{p}} + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial f^0+\delta f}{\partial\vec{p}} &= -\frac{\delta f}{\tau_c}\,,\\
\left[q\vec{\tilde{E}} - m((\vec{\Omega}\cdot\vec{r})\vec{\Omega} - \Omega^2\vec{r})\right]\cdot\frac{\partial f^0}{\partial\vec{p}} + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial f^0+\delta f}{\partial\vec{p}} &= -\frac{\delta f}{\tau_c}\,,\\
\left[q\vec{\tilde{E}} - m((\vec{\Omega}\cdot\vec{r})\vec{\Omega} - \Omega^2\vec{r})\right]\cdot\frac{\partial f^0}{\partial E}(\vec{v}-\vec{u}) + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial f^0}{\partial E}(\vec{v}-\vec{u}) + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial\delta f}{\partial\vec{p}} &= -\frac{\delta f}{\tau_c}\,.
\end{aligned}
\tag{19}
$$

Since conductivity (in general, any transport coefficient) is independent of the fluid velocity, we will put $\vec{u}=0$ in the Eq. (19) to get:

$$
\left[q\vec{\tilde{E}} - m((\vec{\Omega}\cdot\vec{r})\vec{\Omega} - \Omega^2\vec{r})\right]\cdot\frac{\partial f^0}{\partial E}\vec{v} + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial f^0}{\partial E}\vec{v} + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial\delta f}{\partial\vec{p}} = -\frac{\delta f}{\tau_c}\,,
$$

$$
\left[q\vec{\tilde{E}} - m((\vec{\Omega}\cdot\vec{r})\vec{\Omega} - \Omega^2\vec{r})\right]\cdot\frac{\partial f^0}{\partial E}\vec{v} + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial\delta f}{\partial\vec{p}} = -\frac{\delta f}{\tau_c}\,. \tag{20}
$$

Backups

Let us assume that, $\delta f = -\vec{p} \cdot \vec{F}(\frac{\partial f^0}{\partial E})$ with $\vec{F} = \alpha\hat{e} + \beta\hat{\omega} + \gamma(\hat{e} \times \hat{\omega})$, where $\vec{\tilde{E}} = \tilde{E}\hat{e}$, $\vec{\Omega} = \Omega\hat{\omega}$, and α, β, γ are unknown constants. The Eq.(20) with the substitution of δf becomes:

$$[\frac{q\vec{\tilde{E}}}{m} - ((\vec{\Omega} \cdot \vec{r})\vec{\Omega} - \Omega^2\vec{r}) + 2(\vec{F} \times \vec{\Omega})] \cdot \frac{\partial f^0}{\partial E}\vec{v} = \frac{1}{\tau_c}\vec{F} \cdot \left(\vec{v}\frac{\partial f^0}{\partial E}\right) . \tag{21}$$

We have three linearly independent vectors, i.e, $\hat{e}$, $\hat{\omega}$, and $\hat{\omega} \times \hat{e}$. One may, without loss of generality, assume that $\hat{\omega} = \hat{k}$, $\frac{\hat{e}\times\hat{\omega}}{|\hat{e}\times\hat{\omega}|} = \hat{j}$, and $\frac{(\hat{e}\times\hat{\omega})\times\hat{\omega}}{|\hat{e}\times\hat{\omega}|} = \frac{\hat{\omega}\times(\hat{\omega}\times\hat{e})}{|\hat{e}\times\hat{\omega}|} = \hat{i}$, where $\hat{i}, \hat{j}$, and $\hat{k}$ are the unit vectors along X, Y, and Z axes, respectively. Now we will express the expression of centrifugal force in terms of the linear combination of $\hat{e}$, $\hat{\omega}$, and $\hat{e} \times \hat{\omega}$.

$$\begin{aligned}
\text{Centrifugal acceleration} &= \Omega^2(\vec{r} - (\hat{\omega} \cdot \vec{r})\hat{\omega}) , \\
&= \Omega^2(x\hat{i} + y\hat{j}) , \\
&= \Omega^2 \left[x\frac{\hat{\omega} \times (\hat{\omega} \times \hat{e})}{|\hat{e} \times \hat{\omega}|} + y\frac{\hat{e} \times \hat{\omega}}{|\hat{e} \times \hat{\omega}|}\right] , \\
&= \frac{1}{4\tau_\Omega^2}\left[-\frac{x}{|\hat{e} \times \hat{\omega}|}\hat{e} + \frac{x(\hat{\omega} \cdot \hat{e})}{|\hat{e} \times \hat{\omega}|}\hat{\omega} + \frac{y}{|\hat{e} \times \hat{\omega}|}\hat{e} \times \hat{\omega}\right] .
\end{aligned} \tag{22}$$

Backups

Further one notices $\vec{F} \times \vec{\Omega} = -\gamma\Omega\hat{e} + \gamma\Omega(\hat{\omega}\cdot\hat{e})\hat{\omega} + \alpha\Omega(\hat{e}\times\hat{\omega})$. By equating the coefficients of the linearly independent basis vectors from both sides of Eq. (21), we get the following three equations:

$$\frac{q\tilde{E}}{m} - \frac{\gamma}{\tau_\Omega} - \frac{x}{4\tau_\Omega^2|\hat{e}\times\hat{\omega}|} = \frac{\alpha}{\tau_c}\,, \quad \frac{\gamma}{\tau_\Omega}(\hat{\omega}.\hat{e}) + \frac{x(\hat{\omega}\cdot\hat{e})}{4\tau_\Omega^2|\hat{e}\times\hat{\omega}|} = \frac{\beta}{\tau_c}\,, \quad \frac{\alpha}{\tau_\Omega} + \frac{y}{4\tau_\Omega^2|\hat{e}\times\hat{\omega}|} = \frac{\gamma}{\tau_c}\,. \tag{23}$$

The above equations can be simplified to give:

$$\begin{aligned}
\alpha &= \frac{\tau_c\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} - \frac{\frac{\tau_c}{\tau_\Omega}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|\left(1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2\right)}\left(x+\frac{\tau_c}{\tau_\Omega}y\right)\,,\\
\gamma &= \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} + \frac{\frac{\tau_c}{\tau_\Omega}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|\left(1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2\right)}\left(-x\frac{\tau_c}{\tau_\Omega}+y\right)\,,\\
\beta &= \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)^2(\hat{\omega}.\hat{e})\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} + \frac{\frac{\tau_c}{\tau_\Omega}\hat{\omega}.\hat{e}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|\left(1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2\right)}\left(x+\frac{\tau_c}{\tau_\Omega}y\right)\,.
\end{aligned} \tag{24}$$

Backups

$$
\begin{aligned}
\delta f &= -\vec{p}\cdot\vec{F}\frac{\partial f^0}{\partial E} \\
&= -\frac{\partial f^0}{\partial E} m\vec{v}\cdot\left(\alpha\hat{e}+\beta\hat{\omega}+\gamma(\hat{e}\times\hat{\omega})\right) , \\
&= -\frac{\partial f^0}{\partial E} m\vec{v}\cdot\Bigg[\Big[\frac{\tau_c\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} - \frac{\frac{\tau_c}{\tau_\Omega}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|\left(1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2\right)}(x+\frac{\tau_c}{\tau_\Omega}y)\Big]\hat{e} \\
&\quad +\Big[\frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)^2(\hat{\omega}.\hat{e})\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} + \frac{\frac{\tau_c}{\tau_\Omega}\hat{\omega}.\hat{e}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|\left(1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2\right)}(x+\frac{\tau_c}{\tau_\Omega}y)\Big]\hat{\omega} \\
&\quad +\Big[\frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)\left(\frac{q\tilde{E}}{m}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} + \frac{\frac{\tau_c}{\tau_\Omega}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|\left(1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2\right)}(-x\frac{\tau_c}{\tau_\Omega}+y)\Big](\hat{e}\times\hat{\omega})\Bigg] ,
\end{aligned}
\tag{25}
$$

Backups

Finally, we have,

$$\delta f = -\frac{\partial f^0}{\partial E}\left(\frac{q\tau_c}{1+(\frac{\tau_c}{\tau_\Omega})^2}\right)\left[\delta_{jl}+\left(\frac{\tau_c}{\tau_\Omega}\right)^2\omega_j\omega_l+\left(\frac{\tau_c}{\tau_\Omega}\right)\epsilon_{ljk}\omega_k\right]\tilde{E}_j v_l - m\frac{\partial f^0}{\partial E}\frac{\frac{\tau_c}{\tau_\Omega}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|(1+(\frac{\tau_c}{\tau_\Omega})^2)}$$

$$\left[\text{-}\delta_{jl}\left(x+\left(\frac{\tau_c}{\tau_\Omega}\right)y\right)+\omega_j\omega_l\left(x+\left(\frac{\tau_c}{\tau_\Omega}\right)y\right)+\epsilon_{ljk}\omega_k\left(-x\left(\frac{\tau_c}{\tau_\Omega}\right)+y\right)\right]e_j v_l \; (26)$$

The second term of RHS of Eq. (25) gives a current that is not proportional to the electric field. Hence, it will not contribute to the electrical conductivity components. We may call it a Centrifugal Current($\vec{j_c}$):

$$j_{c_i} = -mqg\int\frac{d^3p}{(2\pi)^3}\frac{\partial f^0}{\partial E}\frac{v^2}{3}\frac{\frac{\tau_c}{\tau_\Omega}}{4\tau_\Omega|\hat{e}\times\hat{\omega}|(1+(\frac{\tau_c}{\tau_\Omega})^2)}$$

$$\left[\text{-}\delta_{ij}\left(x+\left(\frac{\tau_c}{\tau_\Omega}\right)y\right)+\omega_j\omega_i\left(x+\left(\frac{\tau_c}{\tau_\Omega}\right)y\right)+\epsilon_{ijk}\omega_k\left(-x\left(\frac{\tau_c}{\tau_\Omega}\right)+y\right)\right]e_j \; .(27)$$