Twist-3 TMD factorization using Gaussian ansatz in the LFQDM

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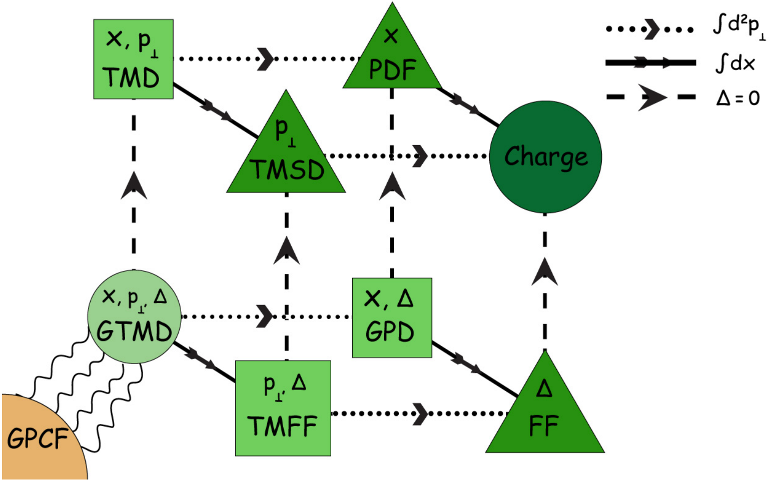
Outline

Introduction



- *S. Sharma and H. Dahiya, arXiv:2310.03592 (2023)*

Outline

Light-Front Quark-Diquark Model I

- In this model the proton is described as an aggregate of an active quark and a diquark spectator of definite mass.

- The proton has spin-flavor $SU(4)$ structure and it has been expressed as a made up of isoscalar-scalar diquark singlet $|u\,S^0\rangle$, isoscalar-vector diquark $|u\,A^0\rangle$ and isovector-vector diquark $|d\,A^1\rangle$ states as [2, 3]

$$|P;\pm\rangle = C_S|u\,S^0\rangle^{\pm} + C_V|u\,A^0\rangle^{\pm} + C_{VV}|d\,A^1\rangle^{\pm}.$$

  Here, the scalar and vector diquark has been denoted by S and A respectively. Their isospin has been represented by the superscripts on them.

- The light-cone convention $z^{\pm} = z^0 \pm z^3$ has been used.

Light-Front Quark-Diquark Model II

- The momentum of the proton (P), struck quark (p) and diquark (P_X) are

$$
\begin{aligned}
P &\equiv \Big(P^+, \frac{M^2}{P^+}, \mathbf{0}\Big),\\
p &\equiv \Big(xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\Big),\\
P_X &\equiv \Big((1-x)P^+, P_X^-, -\mathbf{p}_\perp\Big).
\end{aligned}
$$

- The Fock-state expansion in the case of two particle for $J^z = \pm 1/2$ for the scalar diquark can be expressed as

$$
\begin{aligned}
|u\,S\rangle^\pm = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\Big[\psi_+^{\pm(\nu)}(x,\mathbf{p}_\perp)\Big| +\frac{1}{2}\; s; xP^+, \mathbf{p}_\perp\Big\rangle\\
+ \psi_-^{\pm(\nu)}(x,\mathbf{p}_\perp)\Big| -\frac{1}{2}\; s; xP^+, \mathbf{p}_\perp\Big\rangle\Big],
\end{aligned}
$$

where, flavour index is $\nu = u, d$.

Light-Front Quark-Diquark Model III

- $|\lambda_q\, \lambda_S\,; xP^+, \mathbf{p}_\perp\rangle$ represents the state of two particle having helicity of struck quark as λ_q and helicity of a scalar diquark as λ_S.
- The LFWFs for the scalar diquark are expressed as [4]

$$\psi_{+}^{+(\nu)}(x,\mathbf{p}_\perp) = N_S\ \varphi_1^{(\nu)}(x,\mathbf{p}_\perp),$$
$$\psi_{-}^{+(\nu)}(x,\mathbf{p}_\perp) = N_S\Big(-\frac{p^1+ip^2}{xM}\Big)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),$$
$$\psi_{+}^{-(\nu)}(x,\mathbf{p}_\perp) = N_S\Big(\frac{p^1-ip^2}{xM}\Big)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),$$
$$\psi_{-}^{-(\nu)}(x,\mathbf{p}_\perp) = N_S\ \varphi_1^{(\nu)}(x,\mathbf{p}_\perp).$$

Here $\varphi_i^{(\nu)}(x,\mathbf{p}_\perp)$ are LFWFs and N_S is the normalization constant.

Light-Front Quark-Diquark Model IV

- Similarly, Fock-state expansion in the case of two particle for the vector diquark is given as [5]

$$
\begin{aligned}
|\nu\, A\rangle^{\pm} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \Bigg[& \psi^{\pm(\nu)}_{++}(x,\mathbf{p}_\perp) \Big| +\frac{1}{2}\;\; +1; xP^+, \mathbf{p}_\perp \Big\rangle \\
& + \psi^{\pm(\nu)}_{-+}(x,\mathbf{p}_\perp) \Big| -\frac{1}{2}\;\; +1; xP^+, \mathbf{p}_\perp \Big\rangle + \psi^{\pm(\nu)}_{+0}(x,\mathbf{p}_\perp) \Big| +\frac{1}{2}\;\; 0; xP^+, \mathbf{p}_\perp \Big\rangle \\
& + \psi^{\pm(\nu)}_{-0}(x,\mathbf{p}_\perp) \Big| -\frac{1}{2}\;\; 0; xP^+, \mathbf{p}_\perp \Big\rangle + \psi^{\pm(\nu)}_{+-}(x,\mathbf{p}_\perp) \Big| +\frac{1}{2}\;\; -1; xP^+, \mathbf{p}_\perp \Big\rangle \\
& + \psi^{\pm(\nu)}_{--}(x,\mathbf{p}_\perp) \Big| -\frac{1}{2}\;\; -1; xP^+, \mathbf{p}_\perp \Big\rangle \Bigg].
\end{aligned}
$$

Here $|\lambda_q\; \lambda_D; xP^+, \mathbf{p}_\perp\rangle$ is the state of two-particle with helicity of quark being $\lambda_q = \pm\frac{1}{2}$ and helicity of vector diquark being $\lambda_D = \pm 1, 0$ (triplet).

Light-Front Quark-Diquark Model V

- The LFWFs for the vector diquark for the case when $J^z = +1/2$ are given as

$$\psi^{+(\nu)}_{+\,+}(x, \mathbf{p}_\perp) = N_1^{(\nu)} \sqrt{\frac{2}{3}}\Big(\frac{p^1 - ip^2}{xM}\Big)\varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{-\,+}(x, \mathbf{p}_\perp) = N_1^{(\nu)} \sqrt{\frac{2}{3}}\varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{+\,0}(x, \mathbf{p}_\perp) = -N_0^{(\nu)} \sqrt{\frac{1}{3}}\varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{-\,0}(x, \mathbf{p}_\perp) = N_0^{(\nu)} \sqrt{\frac{1}{3}}\Big(\frac{p^1 + ip^2}{xM}\Big)\varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{+\,-}(x, \mathbf{p}_\perp) = 0,$$

$$\psi^{+(\nu)}_{-\,-}(x, \mathbf{p}_\perp) = 0,$$

Light-Front Quark-Diquark Model VI

- The LFWFs for the vector diquark for the case when $J^z = -1/2$ are given as

$$\psi_{+\,+}^{-(\nu)}(x, \mathbf{p}_\perp) = 0,$$
$$\psi_{-\,+}^{-(\nu)}(x, \mathbf{p}_\perp) = 0,$$
$$\psi_{+\,0}^{-(\nu)}(x, \mathbf{p}_\perp) = N_0^{(\nu)} \sqrt{\frac{1}{3}} \Big(\frac{p^1 - ip^2}{xM} \Big) \varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi_{-\,0}^{-(\nu)}(x, \mathbf{p}_\perp) = N_0^{(\nu)} \sqrt{\frac{1}{3}} \varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi_{+\,-}^{-(\nu)}(x, \mathbf{p}_\perp) = -N_1^{(\nu)} \sqrt{\frac{2}{3}} \varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi_{-\,-}^{-(\nu)}(x, \mathbf{p}_\perp) = N_1^{(\nu)} \sqrt{\frac{2}{3}} \Big(\frac{p^1 + ip^2}{xM} \Big) \varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$

where N_0, N_1 are the normalization constants.

Light-Front Quark-Diquark Model VII

- Generic ansatz of LFWFs $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ is being adopted from the soft-wall AdS/QCD prediction [6, 7] and the parameters $a_i^\nu,\ b_i^\nu$ and δ^ν are established as [8]

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\, x^{a_i^\nu}(1-x)^{b_i^\nu} \exp\left[-\delta^\nu \frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

Outline

1. Introduction
2. Light-Front Quark-Diquark Model
3. **Input Parameters**
4. TMD Correlator and Parameterization
5. Result Analysis
6. Summary

Input Parameters I

- The parameters a_i^ν and b_i^ν, have been fitted at model scale $\mu_0 = 0.313$ GeV using the Dirac and Pauli data of form factors. [9, 10, 11].

ν	a_1^ν	b_1^ν	a_2^ν	b_2^ν	δ^ν
u	0.280	0.1716	0.84	0.2284	1.0
d	0.5850	0.7000	0.9434	0.64	1.0

Table 1: Values of model parameters corresponding to up and down quarks.

ν	N_S	N_0^ν	N_1^ν
u	2.0191	3.2050	0.9895
d	2.0191	5.9423	1.1616

Table 2: Values of normalization constants N_i^2 corresponding to both up and down quarks.

Input Parameters II

- The AdS/QCD scale parameter κ is chosen to be 0.4 GeV [12].
- Constituent quark mass (m) and the proton mass (M) are taken to be 0.055 GeV and 0.938 GeV sequentially.
- The coefficients C_i of scalar and vector diquarks are given as

$$\begin{aligned} C_S^2 &= 1.3872, \\ C_V^2 &= 0.6128, \\ C_{VV}^2 &= 1. \end{aligned}$$

Outline

TMD Correlator

TMD Correlator

- The unintegrated quark-quark correlator in the light-front formalism for SIDIS is defined as [13]

$$\Phi^{\nu[\Gamma]}_{[\Lambda^{N_i}\Lambda^{N_f}]}(x, \mathbf{p}_\perp; S) = \frac{1}{2}\int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ip.z} \langle P; \Lambda^{N_f} | \overline{\psi}^\nu(0) \Gamma \mathcal{W}_{[0,z]} \psi^\nu(z) | P; \Lambda^{N_i} \rangle \Big|_{z^+=0}.$$

- $|P; \Lambda^{N_i}\rangle$ and $|P; \Lambda^{N_f}\rangle$ are the initial and final states of the proton having momentum P with helicities Λ^{N_i} and Λ^{N_f}, respectively.
- The momentum of the proton (P), struck quark (p) and diquark (P_X) are

$$\begin{aligned} P &\equiv \left(P^+, \frac{M^2}{P^+}, \mathbf{0}\right), \\ p &\equiv \left(xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\right), \\ P_X &\equiv \left((1-x)P^+, P_X^-, -\mathbf{p}_\perp\right). \end{aligned}$$

- The value of Wilson line $\mathcal{W}_{[0,z]}$ is chosen to be 1.

TMD Parameterization for proton at twist-3

$$\Phi^{[1]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[e^{\nu}\left(x,\mathbf{p}_\perp^2\right) - \frac{i\sigma^{i+}p_T^i}{P^+}e_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i})$$

$$\Phi^{[\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[-\frac{i\sigma^{i+}\gamma_5 k_T^i}{P^+}e_T^{\nu}\left(x,\mathbf{p}_\perp^2\right) - i\sigma^{+-}\gamma_5 e_L^{\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i}),$$

$$\Phi^{[\gamma^j]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[\frac{p_T^j}{M}f^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{M\,i\sigma^{j+}}{P^+}f_T^{\perp'\nu}\left(x,\mathbf{p}_\perp^2\right)\right.$$
$$\left. + \frac{p_T^j\,i\sigma^{k+}p_T^k}{M\,P^+}f_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{i\sigma^{ij}p_T^i}{M}f_L^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i}),$$

$$\Phi^{[\gamma^j\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[+\frac{i\varepsilon_T^{ij}p_T^i}{M}g^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}g_T^{\prime\nu}\left(x,\mathbf{p}_\perp^2\right)\right.$$
$$\left. + \frac{p_T^j\,i\sigma^{k+}\gamma_5 p_T^k}{M\,P^+}g_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\,i\sigma^{+-}\gamma_5}{M}g_L^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i}),$$

$$\Phi^{[i\sigma^{ij}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = -\frac{i\varepsilon_T^{ij}}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\Big[-h^{\nu}\left(x,\mathbf{p}_\perp^2\right)+\frac{i\sigma^{k+}p_T^k}{P^+}h_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\Big]u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\Big[+\frac{i\sigma^{i+}\gamma_5 p_T^i}{P^+}h_T^{\nu}\left(x,\mathbf{p}_\perp^2\right)+i\sigma^{+-}\gamma_5 h_L^{\nu}\left(x,\mathbf{p}_\perp^2\right)\Big]u(P,\Lambda^{N_i})\,.$$

Outline

Explicit Expressions of TMDs

For proton, the twist-3 TMDs can be given as

$$xe^{\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 + C_A^2\Big(\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\Big)\Big)\frac{m}{M}\Big[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big],$$

$$xf^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 + C_A^2\Big(\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\Big)\Big)\Big[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big],$$

$$xg_L^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{32\pi^3}\Big(C_S^2 N_s^2 + C_A^2\Big(-\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\Big)\Big)\Big[|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2 - \frac{2m}{xM}|\varphi_1^\nu||\varphi_2^\nu|\Big],$$

$$xg_T^{\prime\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\Big)\frac{m}{M}\Big[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big],$$

$$xg_T^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{8\pi^3}\Big(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\Big)\Big[\frac{1}{x}|\varphi_1^\nu||\varphi_2^\nu| - \frac{m}{x^2M}|\varphi_2^\nu|^2\Big],$$

S-Wave P-Wave D-Wave

Explicit Expressions of TMDs

$$xh_L^\nu(x,\mathbf{p_\perp^2}) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 + C_A^2\Big(-\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\Big)\Big)\frac{1}{M}\Big[m\Big(|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big) + \frac{2p_\perp^2}{xM}|\varphi_1^\nu||\varphi_2^\nu|\Big],$$

$$xh_T^\nu(x,\mathbf{p_\perp^2}) = \frac{1}{8\pi^3}\Big(-C_S^2 N_s^2 + \frac{1}{3}C_A^2|N_0^\nu|^2\Big)\Big[|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2 - \frac{2m}{xM}|\varphi_1^\nu||\varphi_2^\nu|\Big],$$

$$xh_T^{\perp\nu}(x,\mathbf{p_\perp^2}) = \frac{1}{16\pi^3}\Big(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\Big)\Big[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\Big].$$

- ***S. Sharma, N. Kumar and H. Dahiya, Nucl. Phys. B (2023)***

Equation of Motion

$$xe^{q}(x,\mathbf{p}_\perp^2) = x\tilde{e}^{q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} f_1^{q}(x,\mathbf{p}_\perp^2),$$

$$xf^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{f}^{\perp q}(x,\mathbf{p}_\perp^2) + f_1^{q}(x,\mathbf{p}_\perp^2),$$

$$xg_L^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_L^{\perp q}(x,\mathbf{p}_\perp^2) + g_1^{q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} h_{1L}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xg_T^{\prime q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_T^{\prime q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} h_1^{q}(x,\mathbf{p}_\perp^2) - \frac{m}{M}\frac{\vec{p}_T^{\,2}}{2M^2} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xg_T^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_T^{\perp q}(x,\mathbf{p}_\perp^2) + g_{1T}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xh_L^{q}(x,\mathbf{p}_\perp^2) = x\tilde{h}_L^{q}(x,\mathbf{p}_\perp^2) - \frac{\vec{p}_T^{\,2}}{M^2} h_{1L}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} g_1^{q}(x,\mathbf{p}_\perp^2),$$

$$xh_T^{q}(x,\mathbf{p}_\perp^2) = x\tilde{h}_T^{q}(x,\mathbf{p}_\perp^2) - h_1^{q}(x,\mathbf{p}_\perp^2) - \frac{\vec{p}_T^{\,2}}{2M^2} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} g_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xh_T^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{h}_T^{\perp q}(x,\mathbf{p}_\perp^2) + h_1^{q}(x,\mathbf{p}_\perp^2) - \frac{\vec{p}_T^{\,2}}{2M^2} h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2).$$

Gaussian Ansatz

- TMD from Guassian Ansatz, average transverse momentum and Gaussian transverse dependence ratio is defined as

$$\Upsilon^{\nu}_{Gauss}(x, \mathbf{p}_{\perp}^2) = \frac{\Upsilon^{\nu}(x)}{\pi\langle \mathbf{p}_{\perp}^2(\Upsilon)\rangle^{\nu}}\; e^{\frac{-\mathbf{p}_{\perp}^2}{\langle \mathbf{p}_{\perp}^2(\Upsilon)\rangle^{\nu}}},$$

$$\langle \mathbf{p}_{\perp}^r(\Upsilon)\rangle^{\nu} = \frac{\int dx \int d^2p_{\perp} p_{\perp}^r \Upsilon^{\nu}(x, \mathbf{p}_{\perp}^2)}{\int dx \int d^2p_{\perp} \Upsilon^{\nu}(x, \mathbf{p}_{\perp}^2)},$$

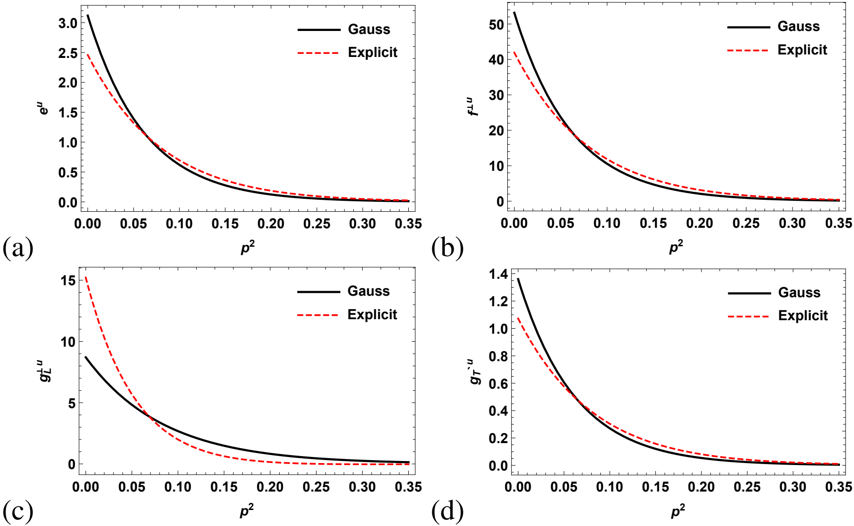
$$R_G(\Upsilon)^{\nu} = \frac{2}{\sqrt{\pi}} \frac{\langle \mathbf{p}_{\perp}^1(\Upsilon)\rangle^{\nu}}{\langle \mathbf{p}_{\perp}^2(\Upsilon)\rangle^{\nu\,1/2}}.$$

-

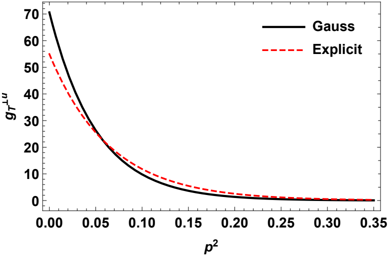
TMD Υ	e^u	$f^{\perp u}$	h_L^u	$g_L^{\perp u}$	$g_T^{\perp u}$	$g_T^{\prime u}$	$h_T^{\perp u}$	h_T^u
$\langle p_{\perp}\rangle^u$	0.22	0.22	0.28	0.28	0.20	0.22	0.22	0.28
$\langle p_{\perp}^2\rangle^u$	0.06	0.06	0.09	0.09	0.05	0.06	0.06	0.09
$R_G(\Upsilon)^{\nu}$	1.02	1.02	1.04	1.09	1.00	1.02	1.02	1.09

- S. Sharma, N. Kumar and H. Dahiya, Nucl. Phys. B (2023)

TMD vs $p_\perp^2$ at $x = 0.3$



TMD vs $p_\perp^2$ *at* $x = 0.3$



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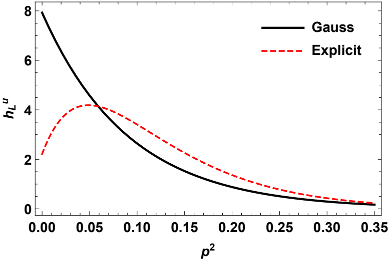

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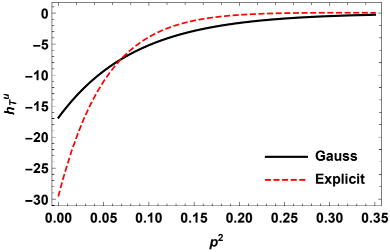

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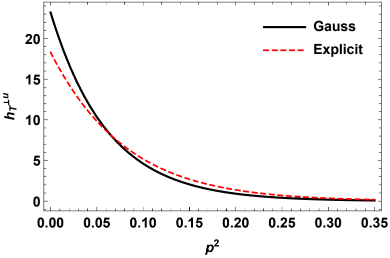

(h)

Outline

Summary

- For u quark the TMDs with Gaussian transverse dependence ratio R^{ν}_{G} less than 1.04 is successfully demonstrated by Gaussian factorization.
- There is no direct connection between the TMD associated waves being S, P and D with applicability of Gaussian Ansatz.
- No direct connection is found between the equation of motion being quadratic in transverse momenta and the validity of Gaussian Ansatz.
- This approach successfully describes a vast body of data at leading twist [8] and is useful for estimating the outcome of experimental measurements at higher twist.
- Factorization of collinear and transverse momentum dependence is certainly violated in full TMD evolution.

Thank you!

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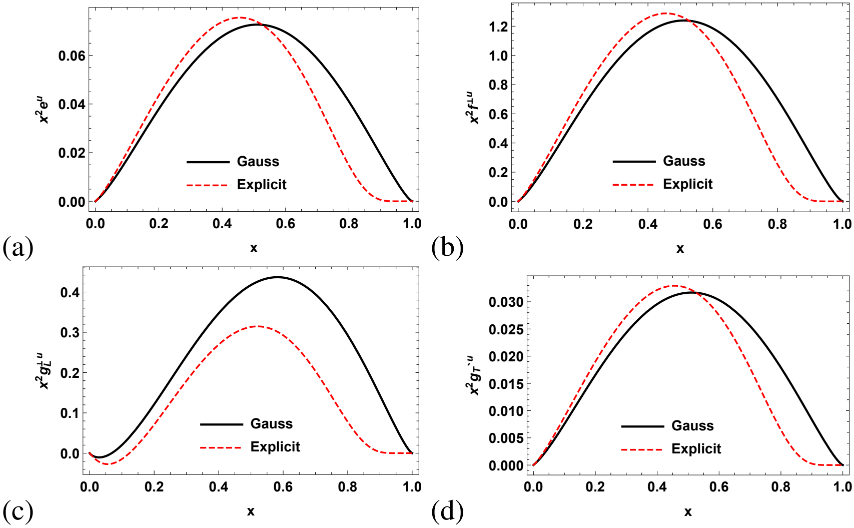
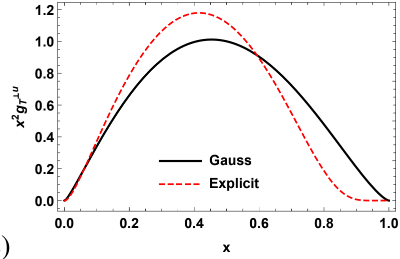
$TMD\ vs\ x\ at\ p_\perp^2 = 0.1$



TMD vs x at $p_\perp^2 = 0.1$



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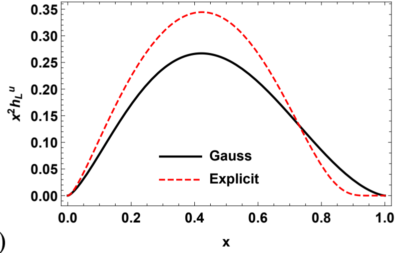
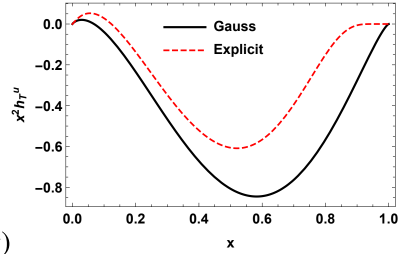
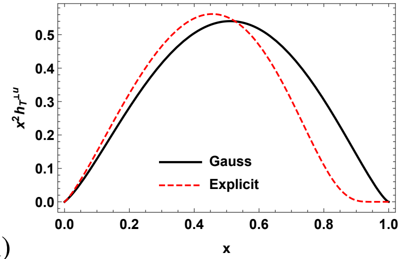


(f)



(g)



(h)