QCD mesonic screening masses using Gribov quantization

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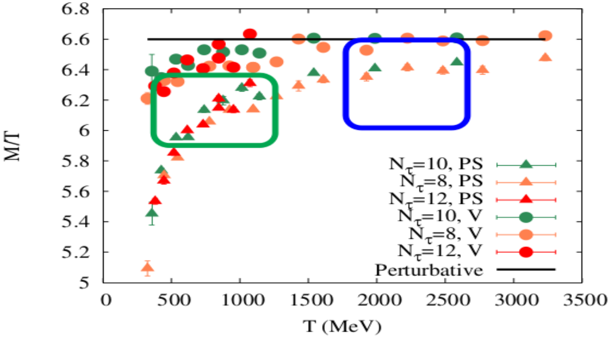
Introduction

- At finite temperature, Lorentz symmetry is broken $\Rightarrow$ temporal and spatial directions are, in general, unrelated.
- Correlation function in the time direction is used to define spectral functions, which gives information about the plasma's real-time properties, such as particle production rate.
- Correlation functions in spatial direction give info about
  (1) At which length scale are thermal fluctuation correlated ?
  (2) At which length scale are external charges screened ?
- These "static" observables are physical and eminently suited to measurements in lattice calculation results.
- Different operator gives different correlation lengths depending on their discrete and continuous global symmetry properties.

Continued

- Screening mass can show us how perturbative the medium is.
- Vector-like excitations can reach the perturbative estimate more quickly than pseudo-scalar excitation
- Perturbative result: $M/T = 2\pi + \frac{g^2 C_F}{2\pi}(\frac{1}{2} + E_0)$ (HotQCD 19)



Detailed setup

- Correlator in momentum space:

$$C_{\mathbf{q}}\left[O^a, O^b\right] \equiv \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3 x e^{i\mathbf{q}\cdot\mathbf{x}} \left\langle O^a(\tau, \mathbf{x}) O^b(0, \mathbf{0}) \right\rangle \quad (1)$$

- In configuration space, correlation function in z direction:

$$C_z\left[O^a, O^b\right] = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^2 \mathrm{x}_\perp \left\langle O^a\left(\tau, \mathrm{x}_\perp, z\right) O^b(0, \mathbf{0}, 0) \right\rangle \quad (2)$$

- In the limit of $z \to \infty$

$$C_z\left[O^a, O^b\right] \sim \mathrm{e}^{-2\omega_0 z} = \mathrm{e}^{-mz},$$

where $\omega_n = 2\pi T(n + \frac{1}{2})$, $\zeta^{-1} = 2\pi T = m \to$ Screening mass
- We are interested in the correlation lengths ζ of mesonic observables in which operators have the form : $\mathcal{O} = \bar{\psi}\Gamma F^a \psi$, where

$$\Gamma = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\},$$

Next-to-leading order for flavour non-singlet correlators

- A large number of higher order graphs that need to be considered.

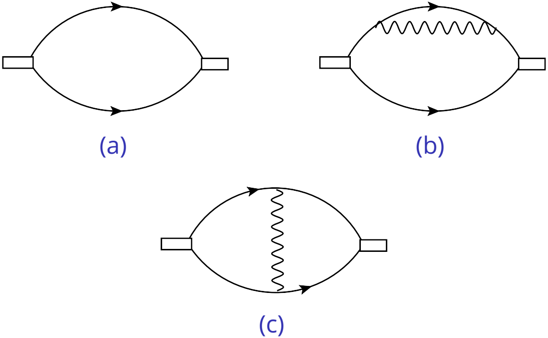


Figure: The diagrams which contribute to meson correlation function: (a) free theory correlator (b) quark self-energy graph (c) interaction of quark and anti-quark through gluon exchange.

Effective theory

- A convenient way of resummation of all the diagrams is offered by the effective field theory, namely NRQCD.
- Correlation lengths can be seen as (2+1)-dimensional bound states of heavy particles of mass "p_0", which is much larger than infrared scale gT, g^2T.
- Correlation function in leading order dominates only at zero Matsubara mode.

$$\mathcal{L}_E^{\psi} = \bar{\psi}\left[i\gamma_0 p_0 - ig\gamma_0 A_0 + \gamma_k D_k + \gamma_3 D_3\right]\psi \tag{3}$$

- The "diagonalized" on-shell effective lagrangian for two independent light modes with a non-relativistic structure:

$$\mathcal{L}_E^{\psi} \approx i\chi^{\dagger}\left[p_0 - gA_0 + D_3 - \frac{1}{2p_0}\left(D_k^2 + \frac{g}{4i}\left[\sigma_k, \sigma_l\right]F_{kl}\right)\right]\chi + \\ + i\phi^{\dagger}\left[p_0 - gA_0 - D_3 - \frac{1}{2p_0}\left(D_k^2 + \frac{g}{4i}\left[\sigma_k, \sigma_l\right]F_{kl}\right)\right]\phi + \mathcal{O}\left(\frac{1}{p_0^2}\right) \tag{4}$$

Continued....

- Free propagators in the effective theory are:

$$\langle \chi_u(p)\chi_v^*(q)\rangle = \delta_{uv}(2\pi)^3\delta^{(3)}(p-q)\frac{-i}{M+ip_3+p_\perp^2/(2p_0)}$$
$$\langle \phi_u(p)\phi_v^*(q)\rangle = \delta_{uv}(2\pi)^3\delta^{(3)}(p-q)\frac{-i}{M-ip_3+p_\perp^2/(2p_0)} \quad (5)$$

- Since we are interested in results up to $\mathcal{O}(g^2)T$, we can ignore the spatial gauge fields.

$$\mathcal{L}_E^{\psi} = i\chi^\dagger\left(M - g_{\mathrm{E}}A_0 + D_t - \frac{\nabla_\perp^2}{2p_0}\right)\chi + i\phi^\dagger\left(M - g_{\mathrm{E}}A_0 - D_t - \frac{\nabla_\perp^2}{2p_0}\right)\phi \quad (6)$$

- To be consistent at $\mathcal{O}(g^2T)$, we should replace ω_0 of the tree-level effective Lagrangian by a matching coefficient $M = \omega_0 + \mathcal{O}(g^2T)$ that needs to be determined.

Matching conditions from QCD to NRQCD, with Gribov

- Gribov Propagator:

$$D^{\mu\nu}(P) = \left[\delta^{\mu\nu} - (1-\xi)\frac{P^\mu P^\nu}{P^2}\right]\frac{P^2}{P^4+\gamma_G^4} \tag{7}$$

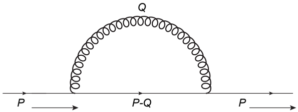


Figure: Quark self-energy correction to one-loop

$$\begin{aligned} S^{-1}(P) &= i\not{P} - ig^2 C_F \sum_Q \frac{\gamma_\mu(\not{P}-\not{Q})\gamma_\mu}{(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4}\right)_b + ig^2 \\ &\times \quad C_F \sum_Q \frac{\not{Q}(\not{P}-\not{Q})\not{Q}}{Q^2(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4} - \frac{\xi\, Q^2}{Q^4+\gamma_G^4}\right)_b \end{aligned} \tag{8}$$

Continued...

- In order to study the zeroes of $S^{-1}(P)$, one can consider $\left[\gamma_0 S^{-1}(P)\right]_{11}$
  $\left[\gamma_0 S^{-1}(P)\right]_{11} \propto \sum\!\!\!\!\!\!\int_Q \frac{2(q_0+iq_3)}{(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4}\right)_b\Big|_{p_0=-ip_3}$
- The only integral required to find the $\mathcal{O}(g^2)$ correction is

$$\sum\!\!\!\!\!\!\!\int_Q \frac{2\left(q_0+iq_3\right)}{(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4}\right)_b\Bigg|_{p_0=-ip_3} = I_1+I_2 \tag{9}$$

where,

$$I_1 = \frac{-1}{p_0}\int_0^\infty \frac{q^2\,dq}{(2\pi)^2}\left[\frac{n^+}{E_+}+\frac{n^-}{E_-}\right]. \tag{10}$$

$$I_2 = \frac{1}{p_0}\sum\!\!\!\!\!\!\!\int_Q \frac{Q^2}{(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4}\right)_b\Bigg|_{p^2=0} = \frac{1}{p_0}\left[\frac{-T^2}{24}+X\right] \tag{11}$$

where X is given by

$$X = \frac{\gamma_G^4}{T^2}\int \frac{q^2 dq}{(2\pi)^2}\frac{\sin\theta d\theta}{8EE_+E_-}\left[\left\{\frac{\tilde{n}+n^-}{i\pi - E + E_-} - \frac{\tilde{n}+n^+}{i\pi - E + E_+}\right\}\right.$$
$$\left. + \left\{\frac{\tilde{n}+n^+}{i\pi + E - E_+} - \frac{\tilde{n}+n^-}{i\pi + E - E_-}\right\}\right]\frac{1}{E_+ - E_-} \quad (12)$$

- Here $\tilde{n}$ is the Fermi-Dirac (F.D) distribution function with energy $E = q - p_3(1+\cos\theta)$.

$$p_3 \approx i\left[p_0 - g^2 C_F(I_1 + I_2)\right] \quad (13)$$

- The pole location is simply $p_3 = iM$ on $NRQCD_3$ side.

$$M = p_0 - g^2 C_F(I_1 + I_2) \quad (14)$$

Solution for the screening states

- Correlators we have considered are

$$C_z\left[O^a, O^b\right] \sim \int \mathrm{d}^2 x_\perp \left\langle O^a\left(x_\perp, z\right) O^b\left(\mathbf{0}_\perp, 0\right)\right\rangle, \tag{15}$$

- We focus on the correlators which decay more slowly that is, which are represented by operators of the type $\phi^\dagger \chi + \chi^\dagger \phi$

- The EOM obeyed by the Green's function (at large z) is of the form

$$(\partial_z - H)G(z) = C\,\delta(z) \tag{16}$$

- We define the correlation function now as

$$C(r, z) \equiv \int \mathrm{d}^2 \boldsymbol{R}\left\langle \phi^*\left(\boldsymbol{R}+\frac{r}{2}, z\right) \chi\left(\boldsymbol{R}-\frac{r}{2}, z\right) \chi^*(\mathbf{0}, 0) \phi(\mathbf{0}, 0)\right\rangle. \tag{17}$$

Quark anti-quark potential in effective theory

- The tree-level contribution reads as

$$\left[\partial_z + 2M - \frac{1}{p_0}\nabla^2_{\boldsymbol{r}}\right] C^{(0)}(r,z) \propto \delta(z)\delta^{(2)}(r). \tag{18}$$

- Similarly, the 1-loop contribution can be written as

$$\left[\partial_z + 2M - \frac{1}{p_0}\nabla^2_{\boldsymbol{r}}\right] C^{(1)}(\boldsymbol{r},z) = -g^2_{\mathrm{E}} C_F \mathcal{K}\left(\frac{1}{zp_0}, \frac{\nabla_{\boldsymbol{r}}}{p_0}, \frac{\gamma_G^4}{p_0^4}, rp_0\right) C^{(0)}(r,z), \tag{19}$$

where the kernel $\mathcal{K}$ is dimensionless.

- The 1-loop static potential obtained from the above kernel is,

$$V(r) = g^2_{\mathrm{E}} \frac{C_F}{2\pi}\left[\ln\frac{\gamma_{\mathrm{G}} r}{2} + \gamma_E - K_0\left(\gamma_{\mathrm{G}} r\right)\right]. \tag{20}$$

- Combining now tree level and 1-loop contribution, we get

$$\left[\partial_z + 2M - \frac{1}{p_0}\nabla^2_{\boldsymbol{r}} + V(r)\right] C(r,z) \propto \delta(z)\delta^{(2}(r), \tag{21}$$

Continued....

- This potential determines the coefficient of the exponential fall-off, $\xi^{-1} \equiv m$, through

$$\left[2M - \frac{\nabla_r^2}{p_0} + V(r)\right]\Psi_0 = m\Psi_0 \tag{22}$$

- In order to find the solution numerically, we rescale

$$r \equiv \frac{\hat{r}}{m_{\mathrm{E}}}, \quad m - 2M \equiv g_{\mathrm{E}}^2 \frac{C_F}{2\pi}\hat{E}_0, \quad \rho = \frac{p_0 g_{\mathrm{E}}^2 C_F}{2\pi{\gamma_{\mathrm{G}}}^2}$$

$$\left[-\left(\frac{\mathrm{d}^2}{\mathrm{d}\hat{r}^2} + \frac{1}{\hat{r}}\frac{\mathrm{d}}{\mathrm{d}\hat{r}}\right) + \rho\left(\ln\frac{\hat{r}}{2} + \gamma_E - K_0(\hat{r}) - \hat{E}_0\right)\right]\Psi_0 = 0, \tag{23}$$

where

$$\Psi_0(\hat{r}) \approx \Psi_0(0)\left[1 + \frac{1}{2}\rho\hat{r}^2\left(\ln\frac{\hat{r}}{2} + \gamma_E - 1 - \frac{1}{2}\hat{E}_0\right)\right]. \tag{24}$$

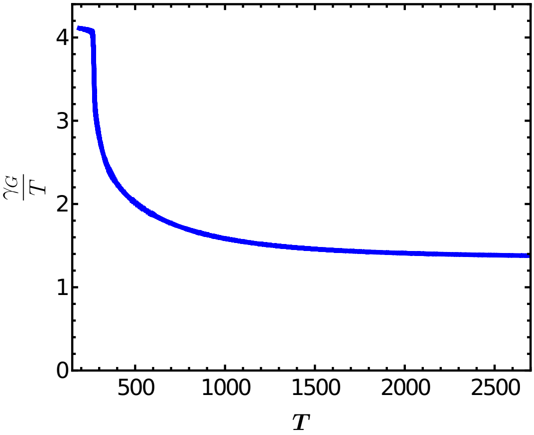


Figure: Temperature dependence of scaled Gribov mass parameter obtained using lattice (thermodynamics) data.

Results and Discussion

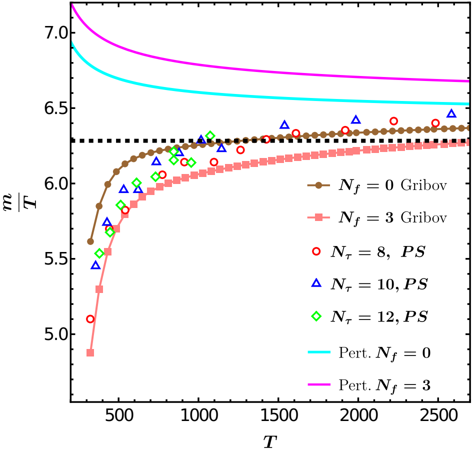


Figure: The temperature dependence of the scaled screening mass. The dashed line represents the free theory result from $m/T = 2\pi$.

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Thank you for your attention.