Influence of relativistic rotation on QCD properties

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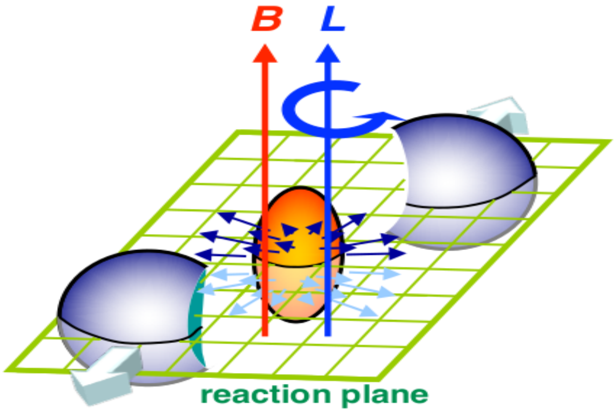
Outline:

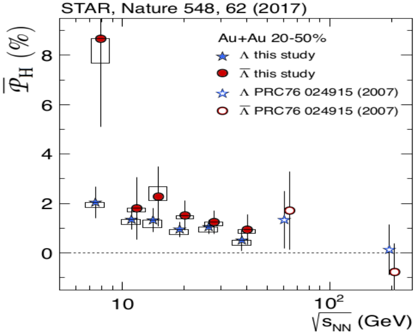
- Introduction
- Critical temperatures
- Moment of inertia of GP
- Inhomogeneous phase transitions in GP
- Conclusion

Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

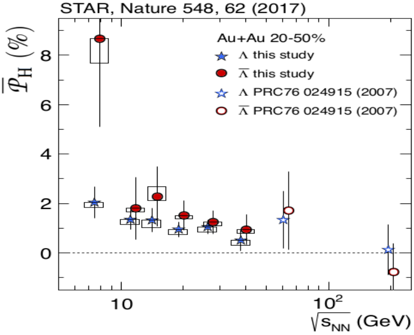
Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar{\Lambda}})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 10^{22} s^{-1}$)
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions



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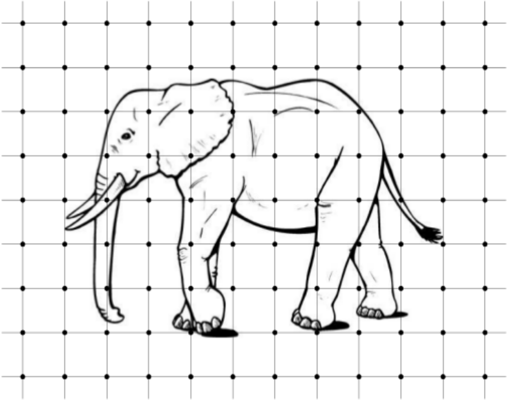
How relativistic rotation influences QCD?

Lattice QCD



Lattice simulation

- Allows to study strongly interacting systems
- Based on the first principles of quantum field theory
- Powerful due to modern supercomputers and algorithms

Lattice simulation of QCD

- We study QCD in thermodynamical equilibrium
- The system is in the finite volume
- Calculation of the partition function

$$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s...} \det \left(\hat{D}_i(U) + m_i\right)$$

- Monte Carlo calculation of the integral
- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: g^2 and masses of quarks

Study of rotating QGP

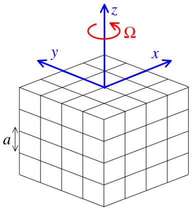
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\, \exp\left[-\beta \hat{H}\right] = \int DA \;\exp\left[-S_G\right]$$

- Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2} \int d^4x \,\sqrt{g_E}\, g_E^{\mu\nu} g_E^{\alpha\beta} F_{\mu\alpha}^{(a)} F_{\nu\beta(a)}$$

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \mathrm{Tr}\Big[(1-r^2\Omega^2) F_{xy}^a F_{xy}^a + (1-y^2\Omega^2) F_{xz}^a F_{xz}^a +$$

$$+(1-x^2\Omega^2) F_{yz}^a F_{yz}^a + + F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz} F_{zy}\Big]$$

Details of the simulations

- *Ehrenfest–Tolman effect:* **In gravitational field the temperature is not constant in space at thermal equilibrium**

$$T(r)\sqrt{g_{00}} = const = 1/\beta$$

$$T(r)\sqrt{1 - r^2\Omega^2} = 1/\beta$$

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- Rotation effectively heats the system from the rotation axis to the boundaries $T(r) > T(r = 0)$
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- We use the designation $T = T(r = 0) = 1/\beta$

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$

- *The dependence on boundary conditions is the property of all approaches*
- *One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening*

Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \mathrm{Tr}\big[(1 - r^2\Omega^2)F^a_{xy}F^a_{xy} + (1 - y^2\Omega^2)F^a_{xz}F^a_{xz} +$$

$$+(1 - x^2\Omega^2)F^a_{yz}F^a_{yz} + +F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} -$$

$$-2iy\Omega(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) + 2ix\Omega(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2 F_{xz}F_{zy}\big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω

Details of the simulations: critical temperatures

Confinement/deconfinement phase transition

- Polyakov line

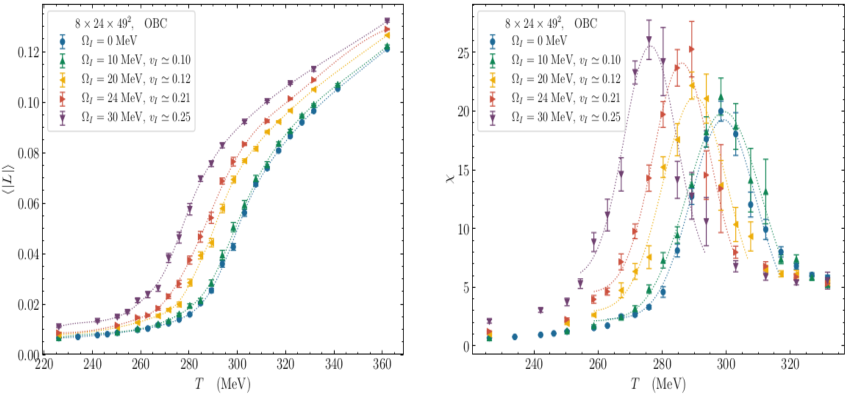
$$L = \left\langle \mathrm{Tr}\mathcal{T}\exp\left[ig\int_{[0,\beta]} A_4\, dx^4\right]\right\rangle$$

- Susceptibility of the Polyakov line

$$\chi = N_s^2 N_z \left(\langle|L|^2\rangle - \langle|L|\rangle^2\right)$$

- T_c is determined from Gaussian fit of the $\chi(T)$

Results of the calculation (Neumann b.c.)



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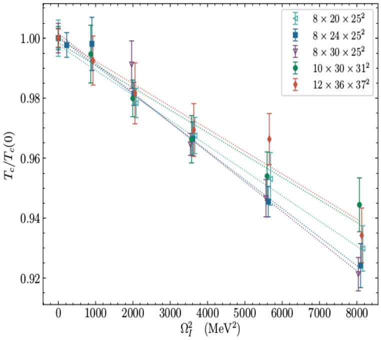
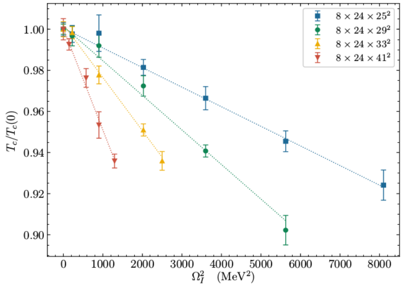


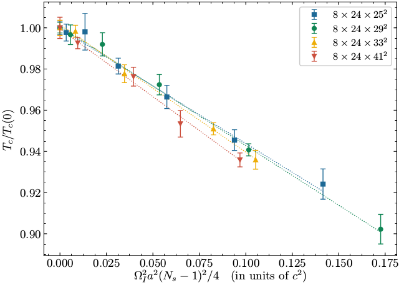
- The results can be well described by the formula ($C_2 > 0$)

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

- **The critical temperature rises with angular velocity**
- The results weakly depend on lattice spacing and the volume in z-direction

Dependence on the transverse size



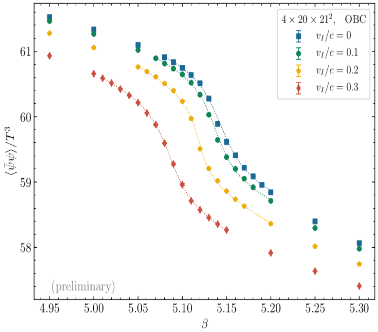


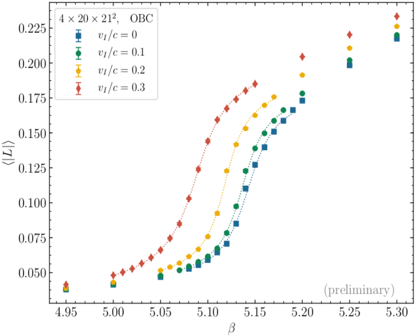
- The results can be well described by the formula

$$\frac{T_c(\Omega)}{T_c(0)} = 1 - B_2 v_I^2, \quad v_I = \Omega_I (N_s - 1) a/2, \quad C_2 = B_2 (N_s - 1)^2 a^2/4$$

- **Periodic b.c.:** $B_2 \sim 1.3$
- **Dirichlet b.c.:** $B_2 \sim 0.5$
- **Neumann b.c.:** $B_2 \sim 0.7$
- **Good variable is $v = \Omega R$, rather than Ω**

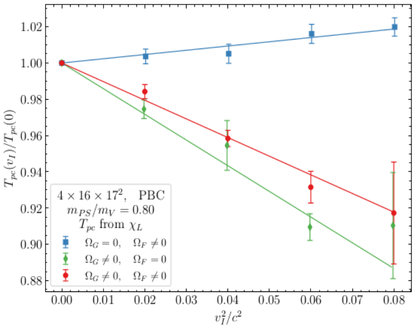
Simulation with fermions

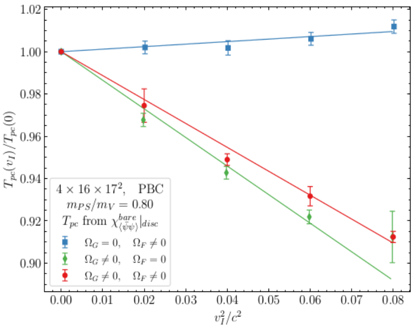




- Lattice simulation with Wilson fermions
- Critical couplings of both transitions coincide
- Critical temperatures are increased

Simulation with fermions





- QCD action: $S = S_f(\Omega_F) + S_g(\Omega_G)$
- One can introduce velocities for gluons Ω_G and fermions Ω_F
- $\Omega_F \neq 0, \Omega_G = 0$ decreases critical temperatures
- $\Omega_F = 0, \Omega_G \neq 0$ increases critical temperatures
- $\Omega_G = \Omega_F \neq 0$ pull system to opposite directions but gluons win

EoS of rotating gluodynamics

- Free energy of rotating QGP

$$F(T,R,\Omega) = F_0(T,R) + C_2\Omega^2 + ...$$

- The moment of inertia

$$C_2 = -\frac{1}{2} I_0(T,R), \quad I_0(T,\Omega) = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_{T,\Omega\to 0}$$

- Instead of $I_0(T,R)$ we calculate $K_2 = -\frac{I_0(T,R)}{F_0(T,R)R^2}$
- Sign of K_2 concides with the sign of $I_0(T,R)$

EoS of rotating gluodynamics

- Classical moment of inertia

$$I_0(R) = \int_V d^3x x_\perp^2 \rho_0(x_\perp)$$

- Related to the trace of EMT $T^\mu_\mu = \rho_0(x_\perp)c^2$
- Generation of mass scale in QCD and scale anomaly

$$T^\mu_\mu \sim \langle G^2 \rangle \sim \langle H^2 + E^2 \rangle$$

- In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- *One could anticipate:* $\rho_0 \sim \langle H^2 + E^2 \rangle$?

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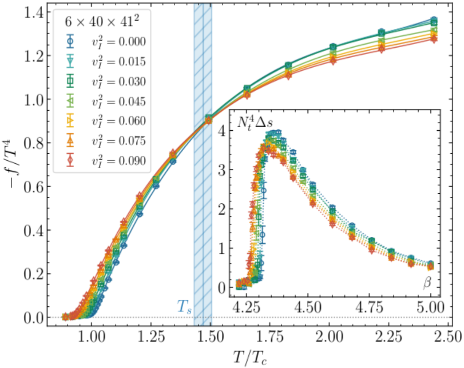
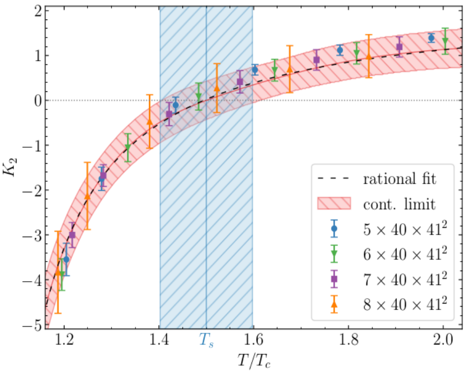
- In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- *One could anticipate:* $\rho_0 \sim \langle H^2 + E^2 \rangle$?
- $I_0 = I_{fluct} + I_{cond}$ *valid for QCD!*
  $I_{fluct} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2$
  $I_{cond} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle$

Calculation of free energy on the lattice

- $F = -T \log Z$ impossible to calculate on the lattice
- $\frac{\partial F}{\partial \beta} \sim \langle \Delta s(\beta) \rangle = s(\beta)_T - s(\beta)_{T=0}, \quad \beta = \frac{6}{g^2}$
- $\frac{F(T)}{T^4} \sim \int_{\beta_0}^{\beta_1} d\beta' \langle \Delta s(\beta') \rangle$

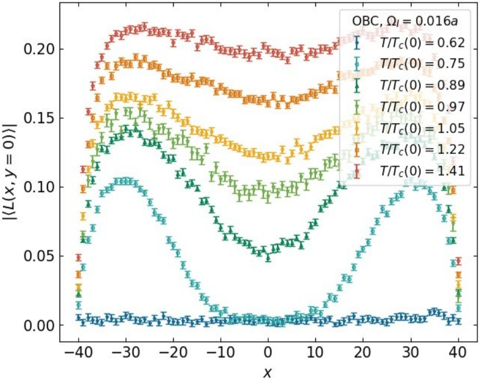


Moment of inertia of gluon plasma



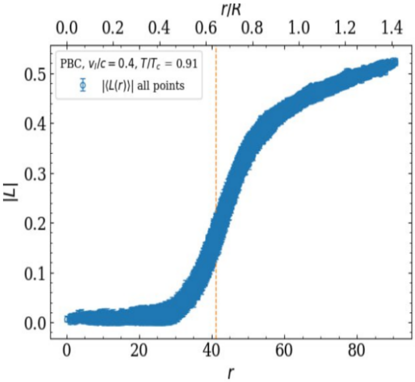
- $I(T, R) = -F_0(T, R) K_2 R^2$
- $I < 0$ for $T < 1.5T_c$ and $I > 0$ for $T > 1.5T_c$
- Negative moment of inertia indicates a thermodynamic instability of rigid rotation
- The region of $I < 0$ is related to magnetic condensate and the scale anomaly
- We believe that the same is true for QCD

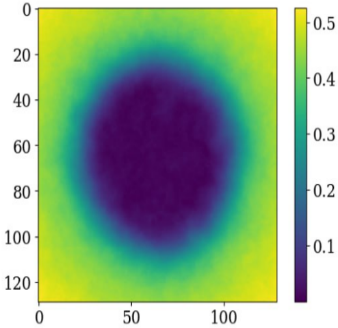
Polyakov loop in rotating QGP(preliminary!)



- Weak dependence at large temperatures
- No dependence at low teperatures
- Strong inhomogeneity of Polyakov loop close to $\sim T_c$

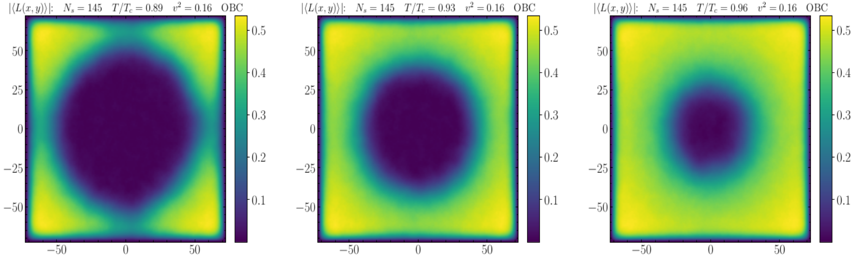
Inhomogeneous phase transitions in QGP





- Confinement in the center and deconfinement close to boundary
- Such configurations can be found close to T_c
- Vortex?

Inhomogeneous phase transitions in QGP



- As temperature is encreased, vortex penetrates closer to center
- Deconfinement appears on the boundaries and captures all volume

Conclusion

- Lattice study of rotating gluodynamics and QCD have been carried out
- Critical temperatures rise with rotation
- We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- Negative moment of inertia indicates a thermodynamic instability of rigid rotation
- We observed inhomogeneous phase transitions in GP
- We believe that all observed effects remain in QCD

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THANK YOU!