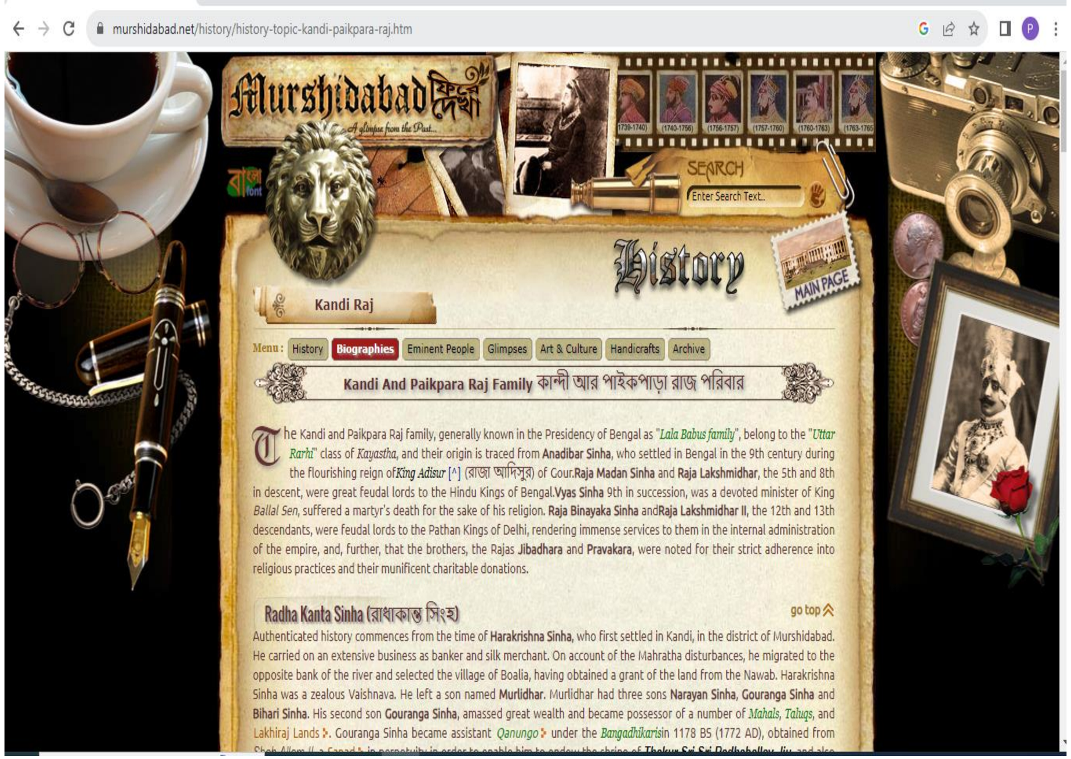
Professor Bikash Sinha –
The Scientist, Educator, Author, Inspiring Leader and Institute Builder

(June 16, 1945 - August 11, 2023)


murshidabad.net/history/history-topic-kandi-paikpara-raj.htm

Murshidabad দর্শন
A glimpse from the Past...

History

Kandi Raj

Menu : History | Biographies | Eminent People | Glimpses | Art & Culture | Handicrafts | Archive

Kandi And Paikpara Raj Family কান্দী আর পাইকপাড়া রাজ পরিবার

The Kandi and Paikpara Raj family, generally known in the Presidency of Bengal as "*Lala Babus family*", belong to the "*Uttar Rarhi*" class of *Kayastha*, and their origin is traced from **Anadibar Sinha**, who settled in Bengal in the 9th century during the flourishing reign of*King Adisur* [^] (রাজা আদিসুর) of Gour.**Raja Madan Sinha** and **Raja Lakshmidhar**, the 5th and 8th in descent, were great feudal lords to the Hindu Kings of Bengal.**Vyas Sinha** 9th in succession, was a devoted minister of King *Ballal Sen*, suffered a martyr's death for the sake of his religion. **Raja Binayaka Sinha** and**Raja Lakshmidhar II**, the 12th and 13th descendants, were feudal lords to the Pathan Kings of Delhi, rendering immense services to them in the internal administration of the empire, and, further, that the brothers, the Rajas **Jibadhara** and **Pravakara**, were noted for their strict adherence into religious practices and their munificent charitable donations.

Radha Kanta Sinha (রাধাকান্ত সিংহ) go top

Authenticated history commences from the time of **Harakrishna Sinha**, who first settled in Kandi, in the district of Murshidabad. He carried on an extensive business as banker and silk merchant. On account of the Mahratha disturbances, he migrated to the opposite bank of the river and selected the village of Boalia, having obtained a grant of the land from the Nawab. Harakrishna Sinha was a zealous Vaishnava. He left a son named **Murlidhar**. Murlidhar had three sons **Narayan Sinha**, **Gouranga Sinha** and **Bihari Sinha**. His second son **Gouranga Sinha**, amassed great wealth and became possessor of a number of *Mahals*, *Talugs*, and Lakhiraj Lands. Gouranga Sinha became assistant *Qanungo* under the *Bangadhikaris*in 1178 BS (1772 AD), obtained from


The World at His Feet
"I salute thee"

Prof Bikash Sinha was born in Kandi Paikpara Raj Family on June 16th, 1945.

Bikash Chandra Sinha (বিকাশ চন্দ্র সিংহ)

go top

Bikash Chandra Sinha hailed from the famous Paikpara Raj family, born on 16th June 1945. He is an eminent Indian physicist, active in the fields of nuclear physics and high energy physics. He was graduated from the Presidency College at Calcutta with honours in Physics (first class) in the year 1964 and soon thereafter, left India for higher studies at Christ's College, Cambridge and joined Tripos, a three years Degree course in Science at Cambridge. He obtained his Tripos from Cambridge University in natural science in 1967 and a Ph.D. from London University in 1970 and a D.Sc. in 1981. Dr. Sinha lived in England for about 12 years, teaching and researching in the Rutherford High Energy Physics Laboratory and Kings College, London. He was Member of the Scientific Advisory Committee to the Cabinet, Government of India 1997/1999 and Chairman of the International Radiation Physics in 1997 and the Chairman of the National Committee for International Union for Pure and Applied Physics in 1998. Dr. Sinha worked in the Bhaba Atomic Research Centre at Bombay from 1976 till 1984 and then he returned to Calcutta. He became the Director at Variable Energy Cyclotron Centre under the Department of Atomic Energy, Government of India, Salt Lake City, in the year 1987 and concurrently, became the Director of Saha Institute of Nuclear Physics in 1992; recipient of a number of awards and honours, including Padma Shri in 2001, the R.D. Birla Award for Excellence in Physics in 2002 and the Padma Bhushan Award in 2010. He is father of a daughter named **Tania** born on 19th August, 1974, and a son named **Amartya** born on July 7,1979 at Bombay.

Education

- School: Scottish Church Collegiate School, Kolkata
- B.Sc. (H) in Physics from Presidency College, Kolkata
- B.A. Natural Sciences (Physics Tripos) from Cambridge University – 1967
- M.A. Natural Sciences (Physics Tripos) from Cambridge University – 1968
- Ph.D. King's College, London University, 1970
- D.Sc. London University, 1981

Cambridge, Senate House, after graduation,
NO SMOKING please

- Senior Research Fellow at King's College, London: 1970-1976

On the graduation day, Cambridge University,
Senate house, 1967

- On Dr Raja Ramanna's invitation, Dr Sinha returned permanently to India and joined Bhabha Atomic Research Centre (BARC), Mumbai in 1976.

Dr. Raja Ramanna, at home in Kolkata
after Beethoven and Liszt, a little relaxation

- Scientific Officer (1976-1983) at Nuclear Physics Division, BARC .
- Moved to Variable Energy Cyclotron Centre, Kolkata in 1983
- Assumed charge of -
  - Director, Variable Energy Cyclotron Centre: 1987-2009,
  - Director, Saha Institute of Nuclear Physics: 1992-2009,
  - Vice Chancellor, West Bengal University of Technology: Feb. – Dec. 2003.

Research: on Low/Intermediate Energy Nuclear Physics at King's College (1970-1976); Bhabha Atomic Research Centre (1976-1983).

1983 -1984: Transition period (low energy nuclear physics to Quark Gluon Plasma (QGP) and relativistic heavy ion collisions.

Research: on QGP (1983-2022) and Hellium Exploration.

He spent 52 years (1970-2022) of his life on basic research.

Volume 31B, number 5 PHYSICS LETTERS 2 March 1970

Nuclear optical potential (1970)

THE NEUTRON EXCESS DISTRIBUTION IN ^{208}Pb

D. C. SINHA and V. R. W. EDWARDS
Wheatstone Laboratory, King's College, Strand, London UK

Received 12 December 1969

It is shown that adding a derivative Saxon Woods term to the real part of the optical potential to represent the symmetry potential arising from the excess neutrons greatly improves the fits to the elastic scattering of 30.3, 40.0 and 61.4 MeV protons from ^{208}Pb. The depth and geometry of the best fit derivative term are found to be consistent with:- (i) the cross sections for the ^{208}Pb(p,n)^{208}Bi reaction at 30 and 50 MeV; (ii) the Coulomb displacement energy of ^{208}Pb and (iii) the strength of the isospin dependent component of the effective two nucleon force.

Volume 128B, number 1,2 PHYSICS LETTERS 18 August 1983

Electromagnetic signal of QGP (1983)

UNIVERSAL SIGNALS OF A QUARK–GLUON PLASMA

Bikash SINHA
Nuclear Physics Division, Bhabha Atomic Research Centre, Bombay 400 085, India

Received 27 April 1983

It is shown that the ratio of the production rates ($\gamma/\mu^+\mu^-$) and ($\pi^{0,\pm}/\mu^+\mu^-$) from a quark–gluon plasma is independent of the space–time evolution of the plasma fireball and thus universal signals of the quark phase.

Last publication (2022) by Prof Sinha

Published: 12 May 2022

Hawking Radiation from Relics of the QCD Phase Transition—Strange Quark Nuggets, Primordial Black Holes, and White Holes

Bikash Sinha

Physics of Particles and Nuclei **53**, 159–166 (2022) | Cite this article

Low/Intermediate Energy Nuclear Physics (1970 - early years of 1980's)

Transition to Quark Gluon Plasma research (early years of 1980's)

His main interest was –

- **Signals of QGP mainly electromagnetic probes (photons and lepton pairs),**
- **Evolution of QGP in space and time,**
- **Relativistic hydrodynamics,**
- **Application of Non-Equilibrium Statistical Mechanics to understand the thermalization in QGP,**
- **QCD phase transition in early universe, etc.**
- **Effects of geological activity on the abundance of He and other gases emanating from hot spring.**

Publications of Professor Bikash Sinha -

- **Participated/Delivered lectures in Conference / Symposium / Workshops, etc. ~ 300**
- **Publications of general interest ~ 100.**
- **More than 200 publications in theory.**
- **Total publication: more than 400 (including international collaborations).**

Prof. Bikash Sinha as an Institute Builder

Architect of several large scale International Collaborations:

- WA98 (CERN, Geneva), Super Proton Synchrotron
- ALICE (CERN, Geneva), Large Hadron Collider
- STAR (BNL, USA), Relativistic Heavy Ion Collider
- FAIR (GSI, Germany), Facility for Antiproton and Ion Research

Jammu University, Punjab University, Rajasthan, University, Guwahati University, IIT Bombay, IIT Indore, NISER, SINP, IOP, Bose Institute, …… participated in these collaborations.

Dr Sinha has made Kolkata "The City of Cyclotron" of India.
There was one cyclotron operational when took charge of Director, VECC.
He added Super Conducting Cyclotron & Medical Cyclotron at VECC and FRENA at SINP.

He set up modern laboratories at hot springs of Bakreswar (West Bengal) and Tantloi (Jharkhand) to study He abundance in the gas emanating from these hot springs. He found correlation between change in the abundance of He gas with the geological activities (earthquake, etc.).

- Recipient of many awards:
  - S N Bose centenary award 1994,
  - DAE Raja Ramanna prize 2001,
  - Rais Ahmed Memorial Lecture Award,
  - Padma Shri (2001),
  - Padma Bhusan (2010),

Greeted by the President of India K. R. Narayanan by "Padmashree" award.

  - Humbolt Research Award of the Alexander von Humbolt Foundation (2005).

- Member of several Indian and Foreign academies.

- Recipient of Honoris Causa, D.Sc. from several Indian and Foreign Institutions.

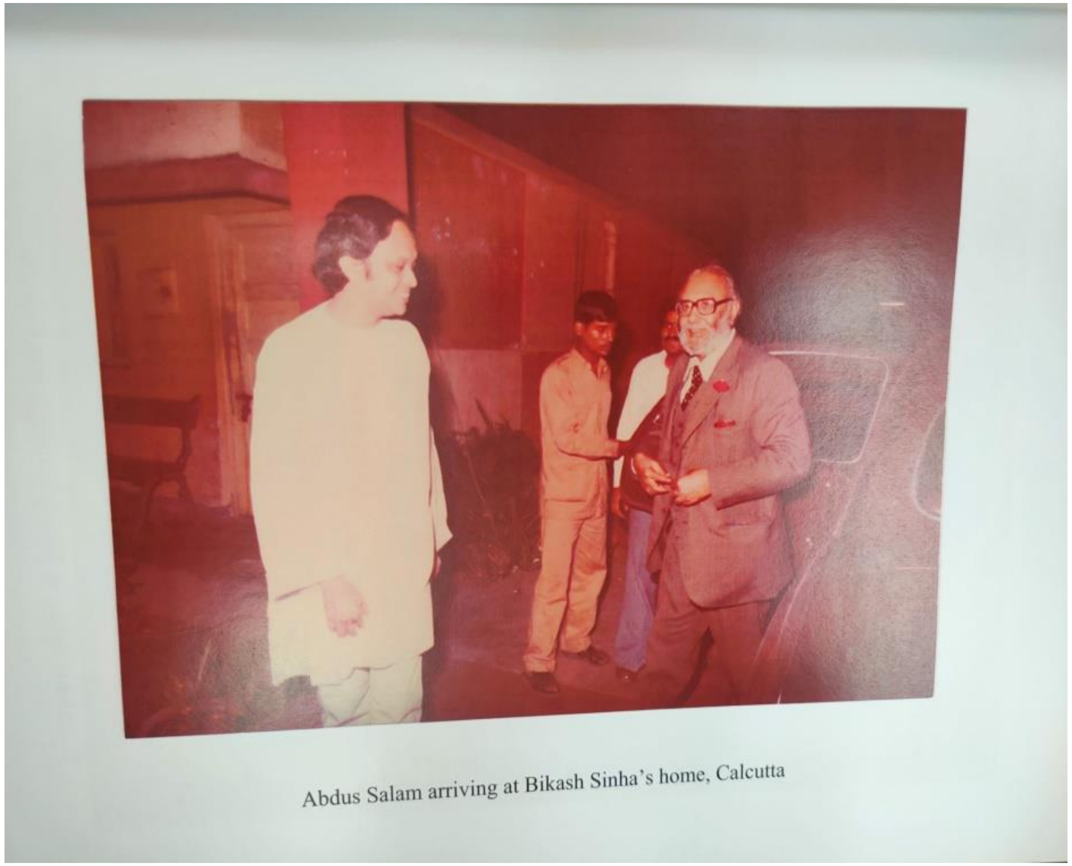
- **Member of Governing Council/Executive Council/Senate of about a dozen of Institutions.**

- **Member of Governing Council/Executive Council/Senate of several academic Institutes.**

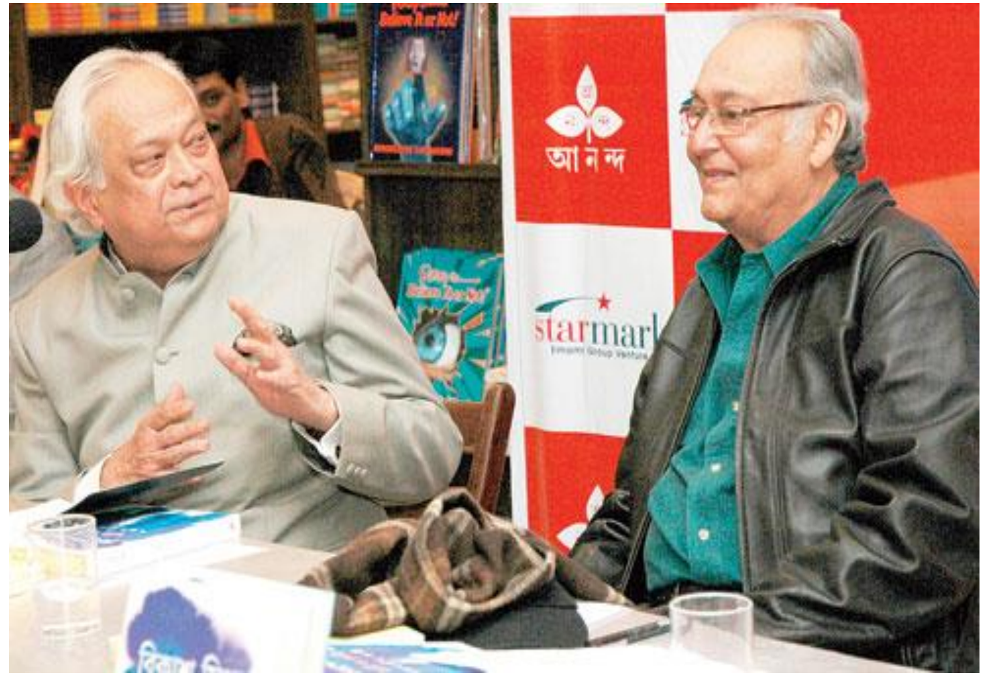
- **Member of:**

  - **Scientific Advisory Committee of the Prime Minister of India, New Delhi (2005-2013);**
  - **Scientific Advisory Committee of the Cabinet, Government of India, New Delhi (2003-2006).**

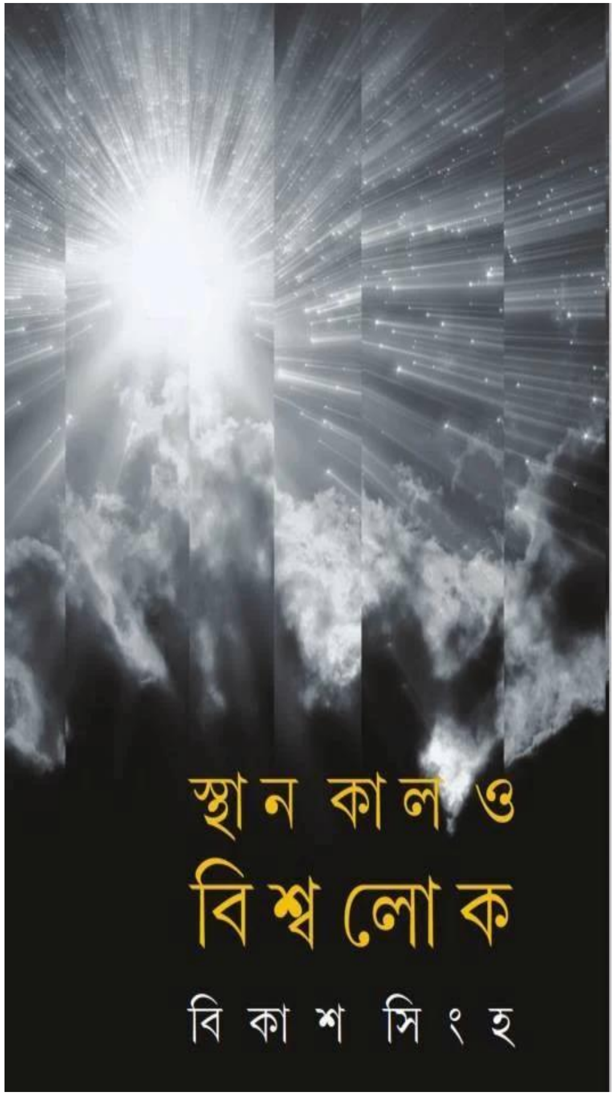
President of – International Radiation Physics Society, etc.



Abdus Salam arriving at Bikash Sinha's home, Calcutta

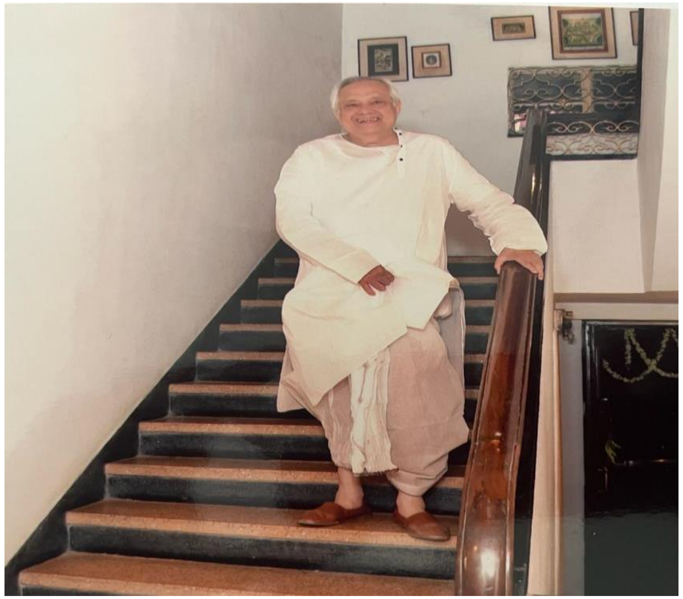


At home with science and culture


স্থান কাল ও
বিশ্বলোক
বিকাশ সিংহ


বিকাশ সিংহ
সৃষ্টি ও কৃষ্টি
বন্ধনহীন গ্রন্থি

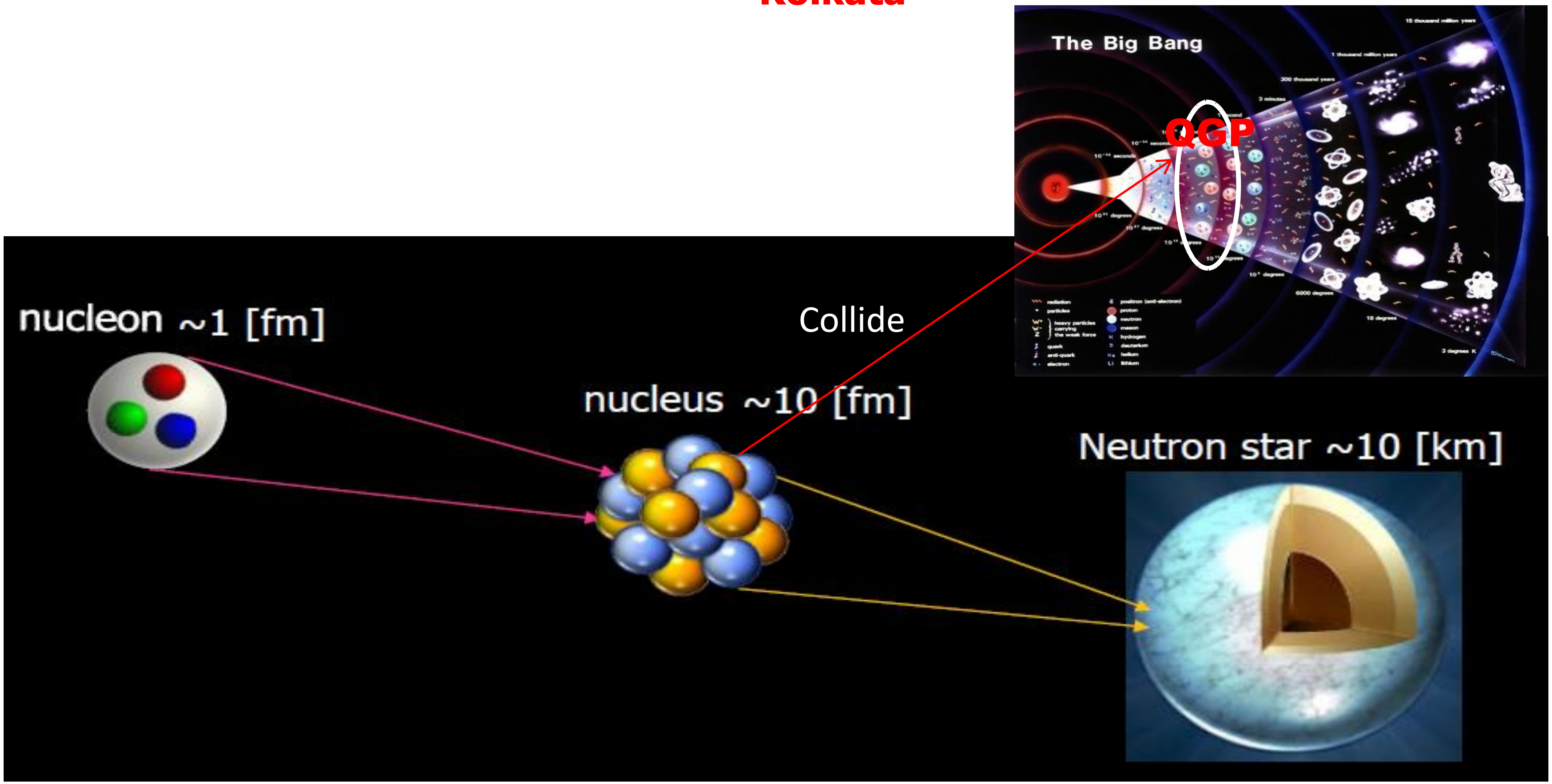
In different moods



Our respectful homage

Spin polarization as a signal of critical point

Jan-e Alam
Variable Energy Cyclotron Centre
Kolkata



India-JINR workshop, JINR, Dubna, Russia. *October 16-18, 2023.*

Plan

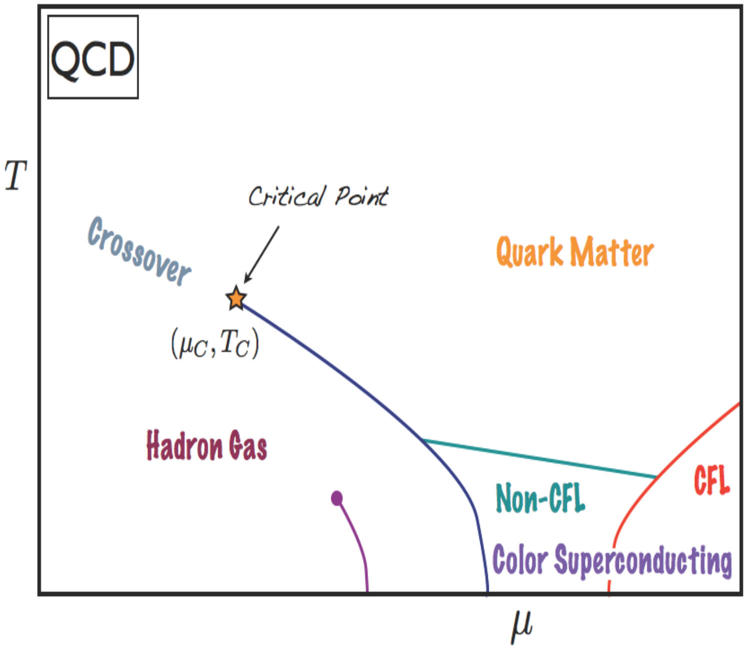
- **Motivation**
- **Solving relativistic viscous hydrodynamics with critical point**
- **Results**
- **Summary**

Based on:

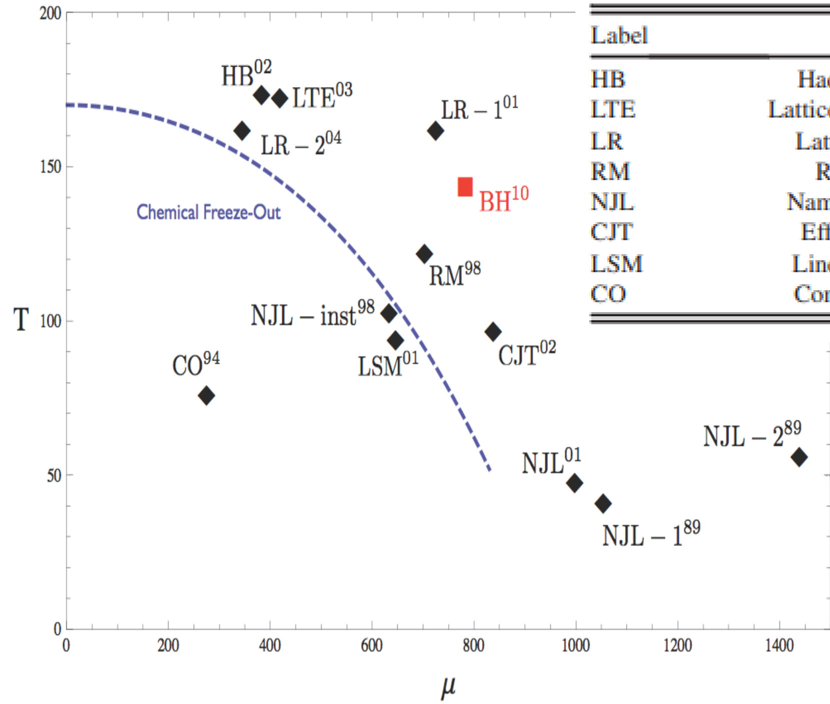
1. **Sushant K Singh & J.A., "Suppression of spin polarization as an indicator of QCD critical point", Eur. Phys. J. C 83, (2023) 585.**
2. **Sushant K. Singh & J. A. , "Effects of the QCD critical point on the spectra and flow coefficients of hadrons", Phys. Rev. D 107, (2023) 074042.**

Motivation

The phase diagram of QCD



Where is the critical point?

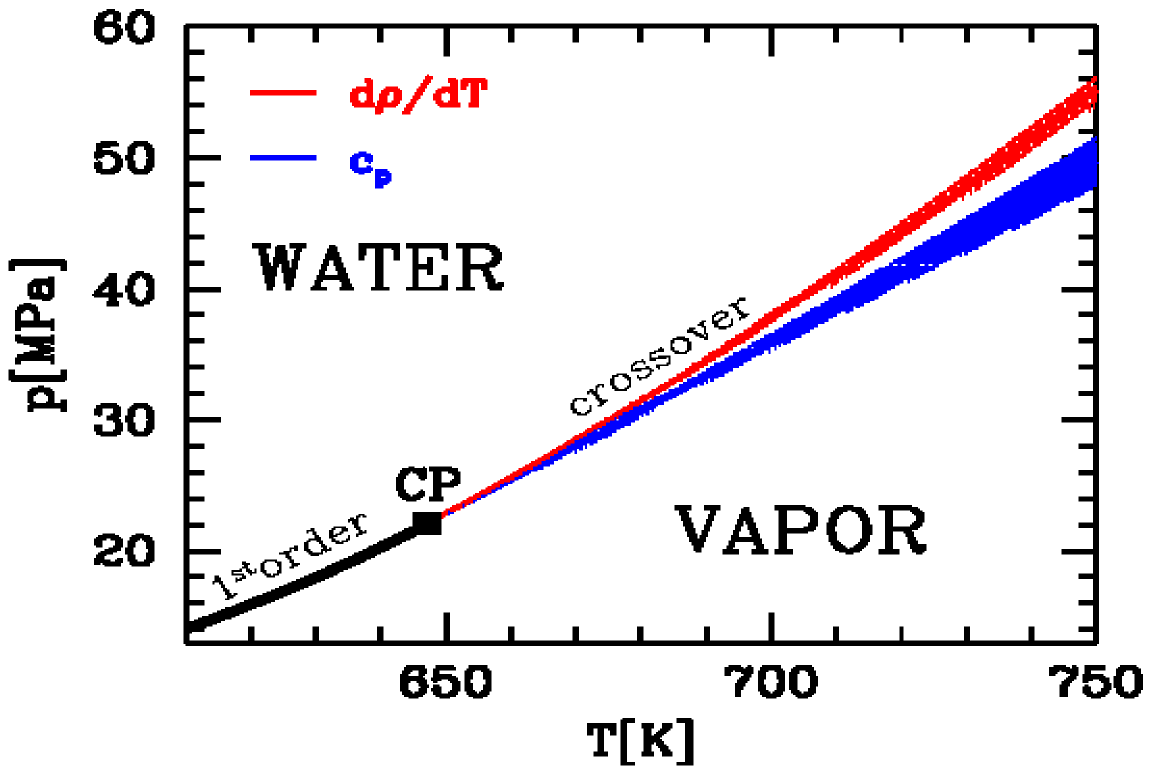


Label	Method	Reference
HB	Hadronic bootstrap	[33]
LTE	Lattice Taylor expansion	[34]
LR	Lattice reweighting	[35,36]
RM	Random matrix	[37]
NJL	Nambu–Jona-Lasinio	[38–40]
CJT	Effective potential	[41]
LSM	Linear sigma model	[39]
CO	Composite operator	[42,43]

DeWolfe et al., PRD 83 (2011) 086005

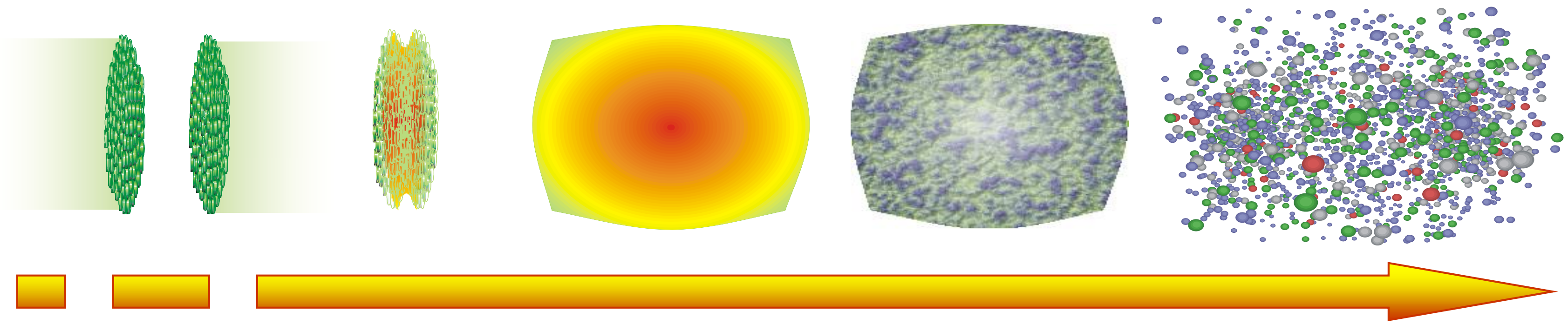
- **The location of critical point in the QCD phase diagram is not precisely known from the first principle calculations.**

The water-vapour phase diagram

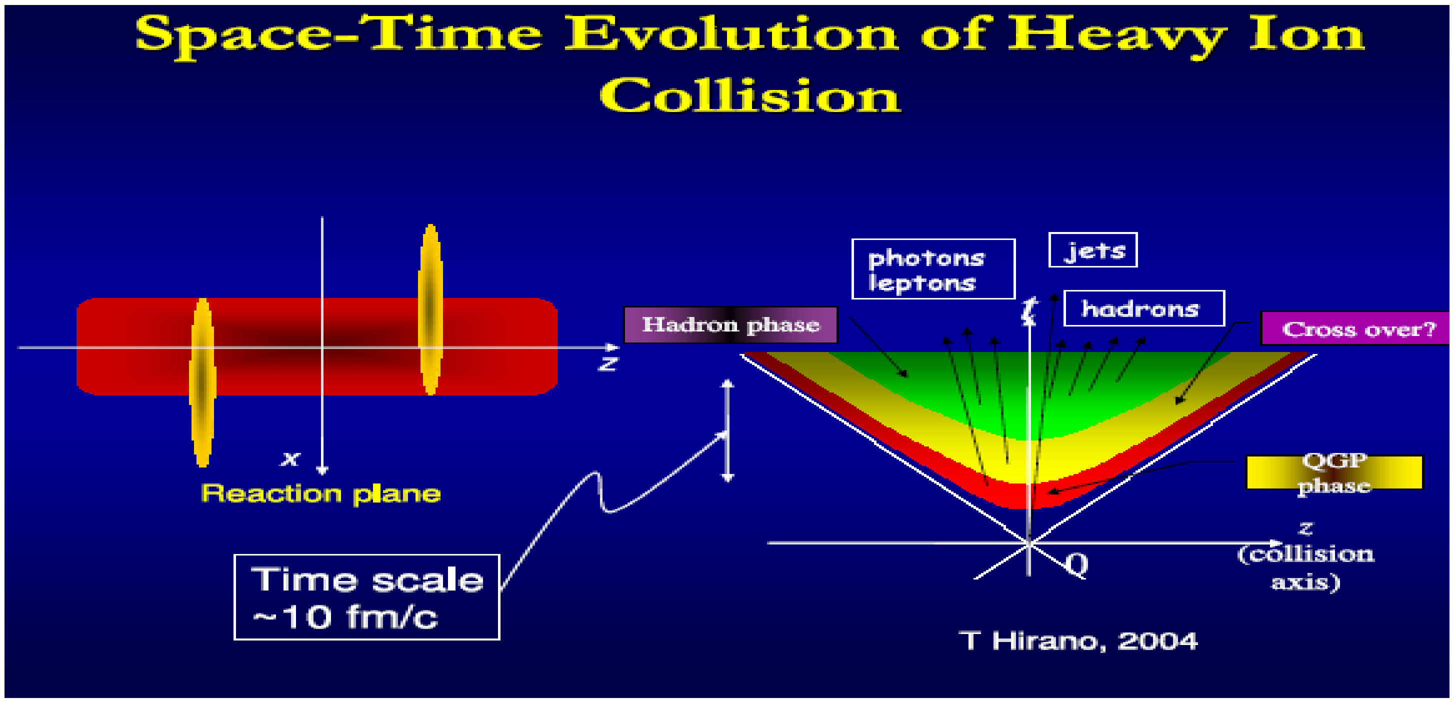


Fodor & Katz, arXiv:0908.341 [hep-ph]

Nuclear Collisions at Relativistic Energies



Time



Space time evolution of quark gluon plasma

Relativistic viscous hydrodynamics - a tool to describe the space-time evolution of matter produced in nuclear collision at relativistic energies

Israel-Stewart hydrodynamics is used here:

Hydrodynamic equations

$$D_\mu T^{\mu\nu} = 0$$

$$D_\mu N_B^\mu = 0$$

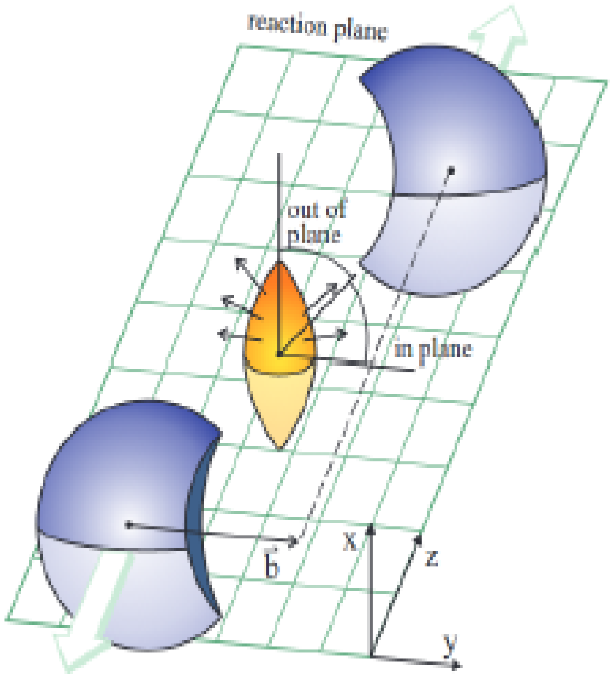
$$\Delta^{\mu\nu}_{\alpha\beta} u^\gamma D_\gamma \pi^{\alpha\beta} = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu} D_\gamma u^\gamma$$

$$u^\gamma D_\gamma \Pi = -\frac{\Pi - \Pi_{NS}}{\tau_\Pi} - \frac{4}{3}\Pi D_\gamma u^\gamma$$

Inputs:
Initial conditions (energy density, net baryon density and velocity)
Equation of State,
Transport coefficinets,
Freeze-out condition

$T^{\mu\nu}$: energy momentum tensor,
N_B^μ :baryon current
$\pi/\pi^{\mu\nu}$: bulk/stress tensors
NS: for Navier-Stokes limit

Large OAM in non-central heavy-ion collision



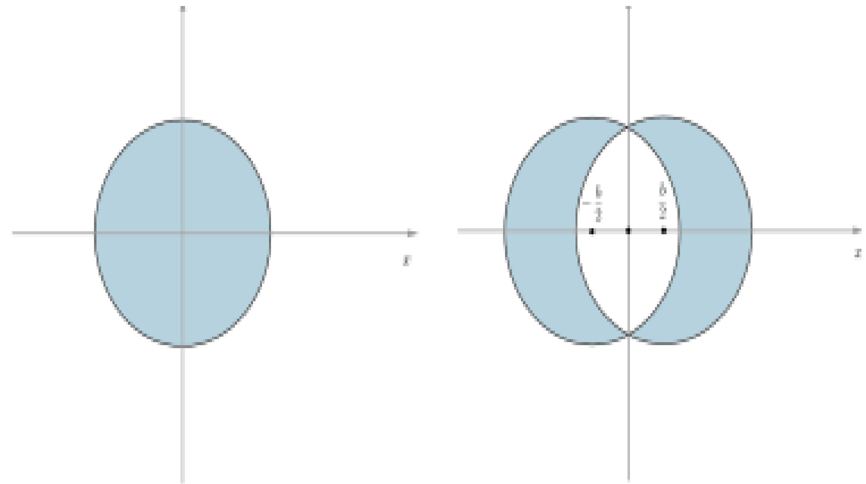
arXiv:0910.4114

- Nuclei carry a large orbital angular momentum (OAM), $L_0 = pb \simeq A\sqrt{s_{NN}}b/2$.
- e.g. for $\sqrt{s_{NN}} = 200$ GeV and $b = 5$ fm, $L_0 \sim 5 \times 10^5$.
- A fraction of L_0 is transferred to QGP fireball.

What is the fraction of the initial angular momentum deposited in the fireball?

To estimate the fraction, define thickness function (number of nucleons per unit transverse area) as

$$T(x,y) = \int dz\, \rho(x,y,z) \quad , \qquad \rho \rightarrow \text{Nuclear density distribution}$$



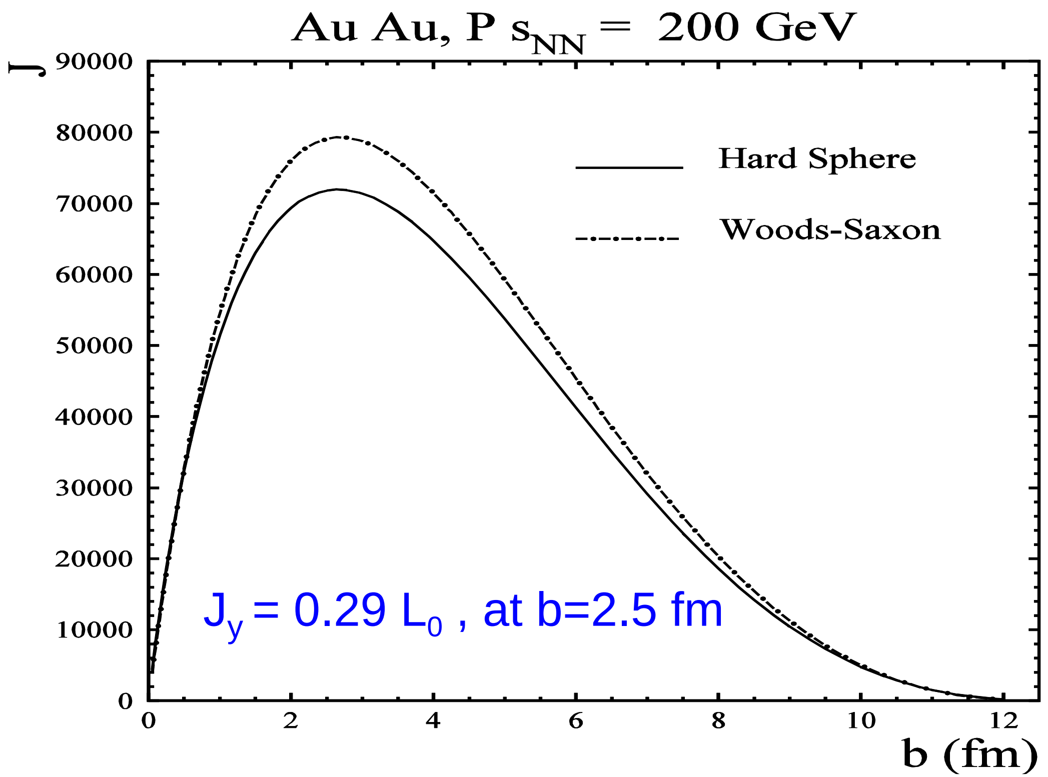
T(x,y) T(x+b/2,y) T(x-b/2,y)

$$\frac{dP}{dxdy} = \left[T(x-b/2,y) - T(x+b/2,y)\right]\frac{\sqrt{s_{NN}}}{2}$$

Becattini et al., PRC 77 (2008) 024906

The initial angular momentum of the fireball is:

$$J_y \sim \int dx \int dy\, x \frac{dP}{dxdy} = \int dx \int dy\, x\,[T(x-b/2,y) - T(x+b/2,y)]\,\frac{\sqrt{s_{NN}}}{2}$$



Becattini et al., PRC 77 (2008) 024906

Vorticity in nature (diameter):

Superfluid vortices (~angstrom), Vortex in the tail of aircraft (1-2 meter, Dust Devils (1-10 meter), Tornadoes (10-500 meter), Hurricanes (100-2000 km), Jupiter's Red Spot (25,000 km) , Spiral Glaxies (light years).

QGP (size ~ 5 fm), velocity ~ 0.1 c, vorticity O(10^{21} sec^{-1}) (small size and high velocity). Tornado vorticity ~ O(10^{-1} sec^{-1})

FreeImages & ScienceAlert

Evolution of vorticity

$$\frac{\partial \vec{\omega}}{\partial t} = \left(\vec{\omega} \cdot \vec{\nabla}\right) \vec{v} - \left(\vec{v} \cdot \vec{\nabla}\right) \vec{\omega} - \theta \vec{\omega} + \frac{1}{\rho^2} \vec{\nabla} \rho \times \vec{\nabla} p - \frac{1}{\rho^2} \left(\zeta + \frac{1}{3} \eta\right) \vec{\nabla} \rho \times \vec{\nabla} \theta - \frac{\eta}{\rho^2} \vec{\nabla} \rho \times \nabla^2 \vec{v} + \frac{\eta}{\rho} \nabla^2 \vec{\omega}.$$

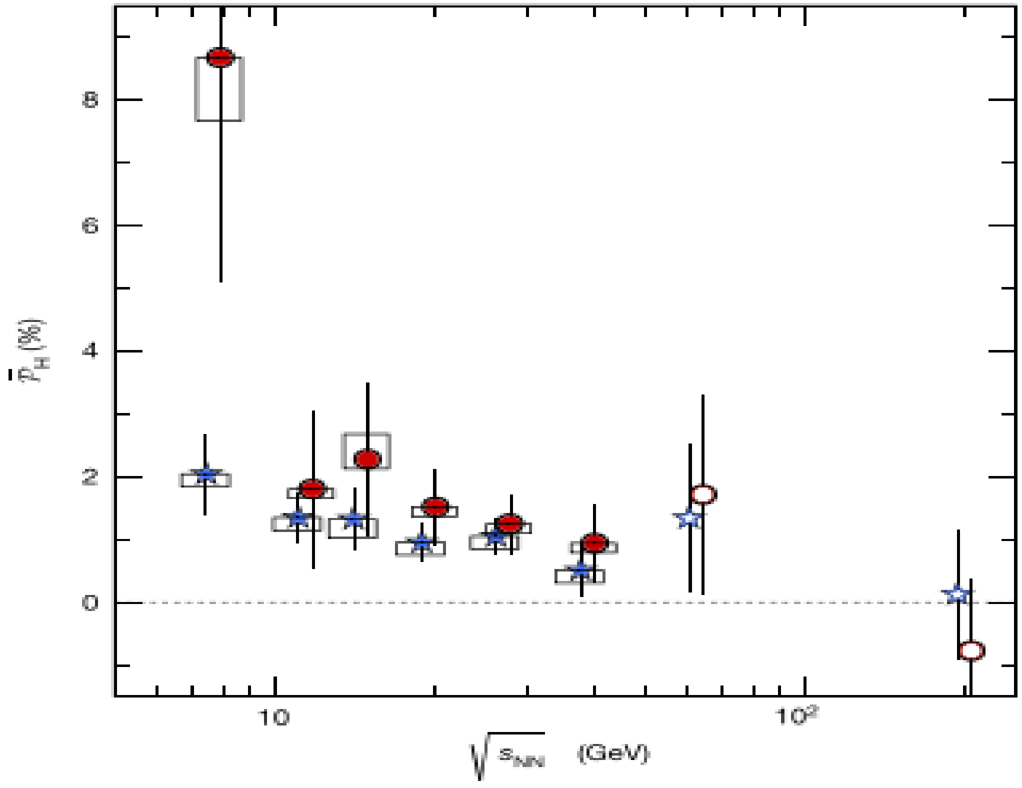
where $\theta = \nabla \cdot \vec{v}.$

LETTER

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*



Polarization of Λ-hyperon for Au+Au collisions measured by STAR collaboration at RHIC-BNL.

Solving relativistic hydrodynamics

Initial energy density and baryon density profiles -

Use optical Glauber model + symmetric rapidity profile +local energy momentum conservation as in Shen et al., PRC 102 (2020) 014909.

The local collision energy and net longitudinal momentum at a point in the transverse plane are

$$E(x,y) = [n_A(x,y) + n_B(x,y)]\, m_N \cosh(y_{\text{beam}}) = M(x,y) \cosh(y_{CM})$$

$$P_z(x,y) = [n_A(x,y) - n_B(x,y)]\, m_N \sinh(y_{\text{beam}}) = M(x,y) \sinh(y_{CM})$$

n_A & n_B are obtained by integrating the nuclear density over z. E=energy/area, and P_z is the longitudinal momentum/area.

where

$$M(x,y) = m_N\sqrt{n_A^2 + n_B^2 + 2n_A n_B \cosh(y_{\mathrm{beam}})}$$

$$y_{\mathrm{CM}} = \tanh^{-1}\left[\frac{n_A - n_B}{n_A + n_B}\cosh(y_{\mathrm{beam}})\right]$$

We must have

$$\int dxdy\ E(x,y) = \int d\Sigma_\mu T^{\mu t}$$

$$\int dxdy\ P_z(x,y) = \int d\Sigma_\mu T^{\mu z}$$

Connection of the geometry of the collisions with the fluid dynamics through energy momentum tensor.

Assuming $u^\tau = 1$ and $u^x = u^y = u^{\eta_s} = 0$, we have $T^{\tau\tau} = \varepsilon(x,y,\eta_s)$ and $T^{\tau\eta} = 0$, so that

$$M(x,y) = \int \tau_0 d\eta_s\ \varepsilon(x,y,\eta_s)\cosh(\eta_s - y_{\mathrm{CM}})$$

$$0 = \int \tau_0 d\eta_s\ \varepsilon(x,y,\eta_s)\sinh(\eta_s - y_{\mathrm{CM}})$$

Shen et al., PRC 102 (2020) 014909.

Further assume:

$$\varepsilon(x,y,\eta_s) = \mathcal{N}_e(x,y)\exp\left[-\frac{(|\eta_s - y_{\rm CM}| - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s - y_{\rm CM}| - \eta_0)\right]$$

we get

$$\mathcal{N}_e(x,y) = \frac{M(x,y)}{2\sinh(\eta_0) + \sqrt{\frac{\pi}{2}}\sigma_\eta e^{\sigma_\eta^2/2} C_\eta}$$

with

$$C_\eta = e^{\eta_0}\mathrm{erfc}\left(-\sqrt{\frac{1}{2}}\sigma_\eta\right) + e^{-\eta_0}\mathrm{erfc}\left(\sqrt{\frac{1}{2}}\sigma_\eta\right)$$

Initial net baryon density is taken as

$$n_B(x,y,\eta_s;\tau_0) = \mathcal{N}_B\left[g_A(\eta_s)n_A(x,y) + g_B(\eta_s)n_B(x,y)\right]$$

where $\mathcal{N}_B$ is fixed by the condition

$$\int \tau_0\, dx\, dy\, d\eta_s\; n_B(x,y,\eta_s;\tau_0) = N_{\rm part}$$

Parameters are fixed by reproducing data on rapidity distribution of elliptic flow and charged particle multiplicities for different centralities [Shen et al., PRC 102 (2020) 014909].

Equation of State

- The pressure at non-zero T and μ_B can be obtained through a Taylor series expansion about $\mu_B = 0$ as follows

$$P_{QCD}(\mu_B, T) = T^4 \sum_n c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

- The presence of CP makes some of the coefficients diverge

$$T^4 c_n(T) \to T^4 c_n^{\text{Non-Ising}}(T) + T_c^4 c_n^{\text{Ising}}(T)$$

Equivalently,

$$P_{QCD}(\mu_B, T) = P^{\text{reg}}(\mu_B, T) + P^{\text{crit}}(\mu_B, T)$$

- Choose and adjust P^{reg} such that $P_{QCD}(0, T) = P^{\text{LAT}}(T)$.

Equation of State (contd.)

- Obtain P^{crit} by mapping to 3D-Ising model. The mapping is done as follows:

Parotto et al., PRC 101 (2020) 034901

r: reduced temperature
h: magnetic field

$$\frac{T - T_C}{T_C} = w\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$

$$\frac{\mu_B - \mu_{BC}}{T_C} = w\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

- The Ising pressure in the critical region is given by

$$P_{\texttt{Ising}}(R,\theta) = h_0 M_0 R^{2-\alpha}\left[\theta\tilde{h}(\theta) - g(\theta)\right],$$

where

$$h = h_0 R^{\beta\delta}\tilde{h}(\theta) \quad , \quad r = R(1-\theta^2)$$

and

$$\tilde{h}(\theta) = \theta(1+a\theta^2+b\theta^4) \quad , \quad g(\theta) = c_0+c_1(1-\theta^2)+c_2(1-\theta^2)^2+c_3(1-\theta^2)$$

Equation of State (contd.)

- The Ising coefficients are obtained from $P_{\mathtt{Ising}}$ as follows:

$$c_n^{\mathrm{Ising}}(T) = \frac{1}{n!} T^n \left. \frac{\partial^n P^{\mathrm{Ising}}}{\partial \mu_B^n} \right|_{\mu_B=0}$$

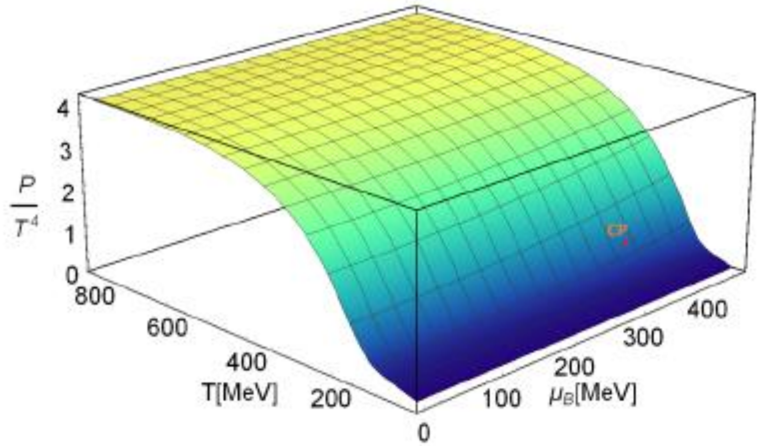
- The "Non-Ising" coefficients are obtained as follows:

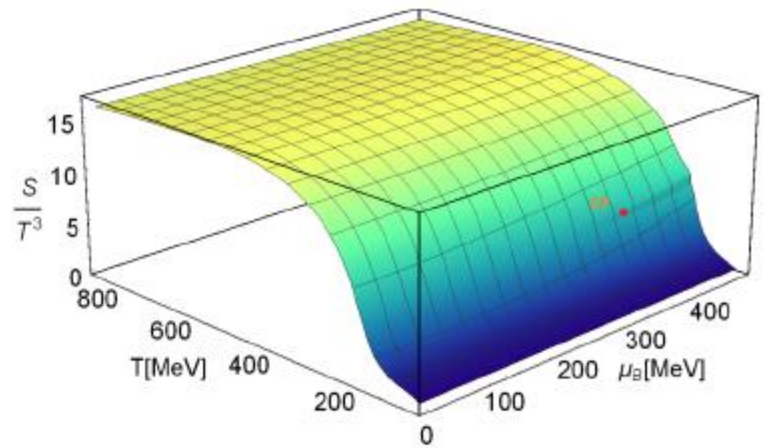
$$T^4 c_n^{\mathrm{LAT}}(T) = T^4 c_n^{\mathrm{Non\text{-}Ising}}(T) + T_c^4 c_n^{\mathrm{Ising}}(T).$$

- The QCD pressure is then

$$P_{\mathrm{QCD}}(T, \mu_B) = T^4 \sum_n c_{2n}^{\mathrm{Non\text{-}Ising}}(T) \left(\frac{\mu_B}{T}\right)^{2n} + T_c^4 P^{\mathrm{Ising}}(R(T, \mu_B), \theta(T, \mu_B))$$

Equation of State (contd.)





For hydrodynamics we require $p \equiv p(\varepsilon, n_B)$. Discretize $\varepsilon - n_B$ plane:

$$\Delta\varepsilon\,(\mathrm{GeV/fm^3}) = \begin{cases} 0.002 & \text{if} \quad 0.001 \le \varepsilon < 1.001, \\ 0.02 & \text{if} \quad 1.001 \le \varepsilon < 11.001, \\ 0.1 & \text{if} \quad 11.001 \le \varepsilon < 61.001, \\ 0.5 & \text{if} \quad 61.001 \le \varepsilon < 101.001. \end{cases}$$

$$\Delta n_B\,(\mathrm{fm^{-3}}) = \begin{cases} 0.0005 & \text{if} \quad 0 \le n_B < 0.15, \\ 0.001 & \text{if} \quad 0.15 \le n_B < 0.3, \\ 0.01 & \text{if} \quad 0.3 \le n_B < 1, \\ 0.025 & \text{if} \quad 1 \le n_B < 5. \end{cases}$$

Equilibrium Correlation Length, ξ

$$\xi^2 = \frac{1}{H_0}\left(\frac{\partial M(r,h)}{\partial h}\right)_r$$

Assume $H_0 = 1$. M is parameterized in terms of R and θ as

$$M(R,\theta) = M_0 R^\beta \theta$$

We have

$$\left(\frac{\partial M}{\partial h}\right)_r = \left(\frac{\partial M}{\partial R}\right)_\theta \left(\frac{\partial R}{\partial h}\right)_r + \left(\frac{\partial M}{\partial \theta}\right)_R \left(\frac{\partial \theta}{\partial h}\right)_r$$

so that ξ is given by

$$\xi^2 = \frac{M_0}{h_0}\frac{R^{\beta(1-\delta)}}{2\beta\delta\theta\tilde{h}(\theta) + (1-\theta^2)\tilde{h}'(\theta)}\left[1 + (2\beta - 1)\theta^2\right]$$

Transport coefficients

Outside the critical region *i.e.* for $\xi < \xi_0$, we have

$$\eta_0(\mu_B, T) = 0.08\left(\frac{\varepsilon + p}{T}\right) \quad , \quad \zeta_0(\mu_B, T) = 15\eta_0(\mu_B, T)\left(\frac{1}{3} - c_s^2\right)^2$$

Near the critical point, the transport coefficients diverge as

$$\zeta \sim \xi^3 \quad , \quad \eta \sim \xi^{0.05}$$

Monnai et al. PRC 95 (2017)034902

The critical behavior of transport coefficients can be modeled for $\xi > \xi_0$ as

$$\zeta = \zeta_0\left(\frac{\xi}{\xi_0}\right)^3 \quad , \quad \eta = \eta_0\left(\frac{\xi}{\xi_0}\right)^{0.05}$$

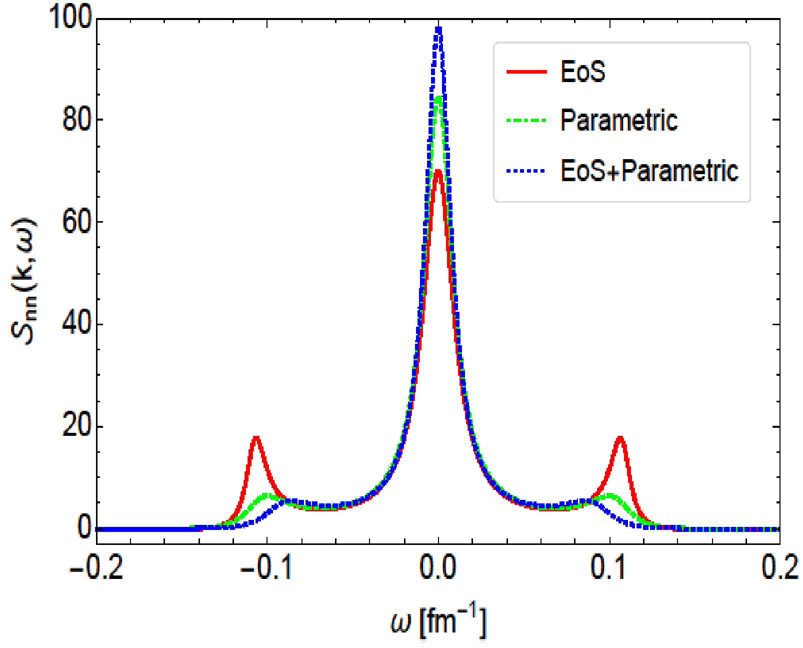
Similarly for τ_π and τ_Π, i.e.

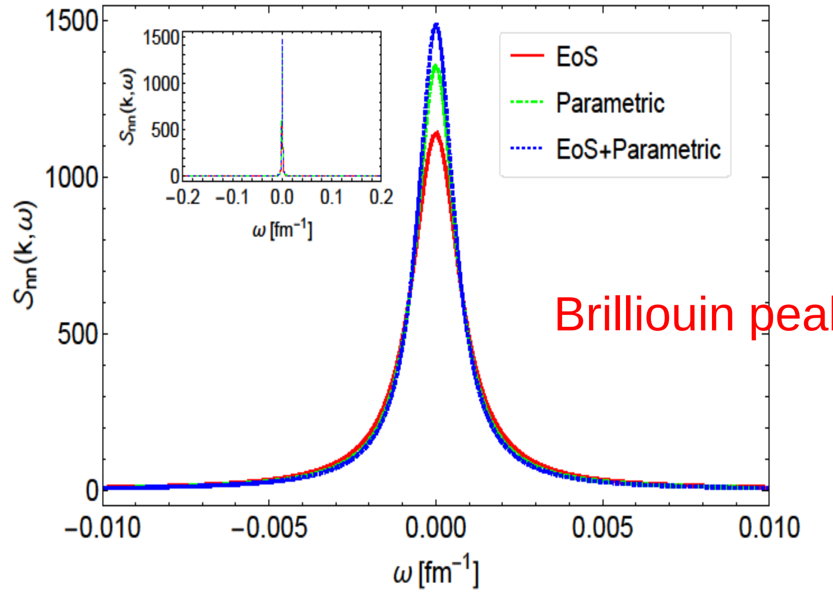
$$\tau_\Pi = \tau_\Pi^0\left(\frac{\xi}{\xi_0}\right)^3 \quad , \quad \tau_\pi = \tau_\pi^0\left(\frac{\xi}{\xi_0}\right)^{0.05}$$

ξ_0 is a parameter for deciding the boundary of the critical region. We choose $\xi_0 = 1.75$ fm.

The correlation in density fluctuation: $S'_{nn}(k,\omega) = \langle \delta n(k,\omega)\delta n(k,0) \rangle$

The dynamic structure factor is given by: $S_{nn}(k,\omega) = \dfrac{S'_{nn}(k,\omega)}{\langle \delta n(k,0)\delta n(k,0) \rangle}$





Brillioin peaks vanish

Dynamical spectral structure of density fluctuation away from the QCD critical point.

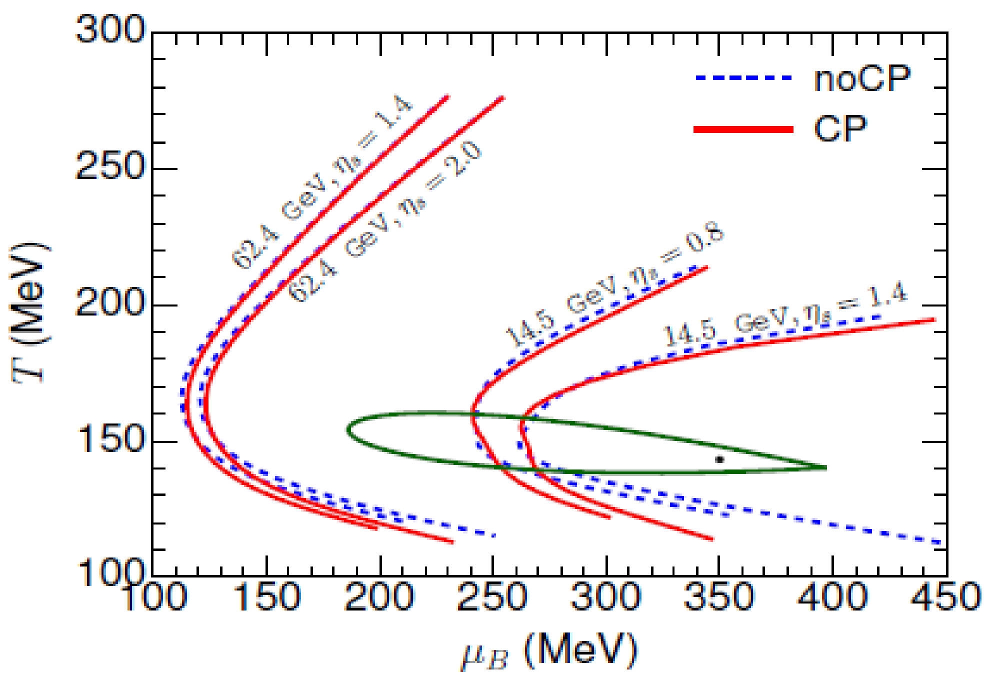
Near the QCD critical point

Stopping criterion

- Constant energy density, $\varepsilon = 0.3$ GeV/fm^3. Close to transition line.
- The surface is found using the CORNELIUS code.
- The surface is input to the UrQMD.
- The spin polarization analysis is done on this surface

Results

Hydrodynamic trajectories in phase diagram



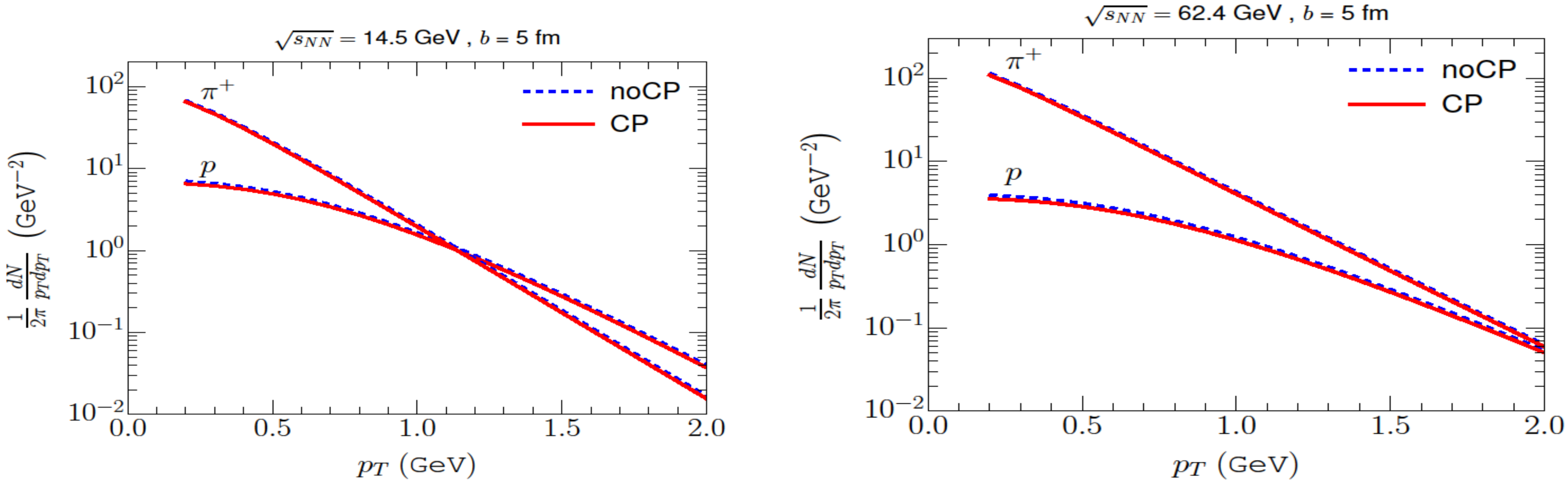
SKS and J. Alam, arXiv:2205.14469

The effect of QCD critical point (CP) on the transverse momentum spectra of pion and proton is found to be insignificant.

The momentum spectrum of the particle 'i' at the freeze-out surface can be estimated by using the Cooper-Frye formula:

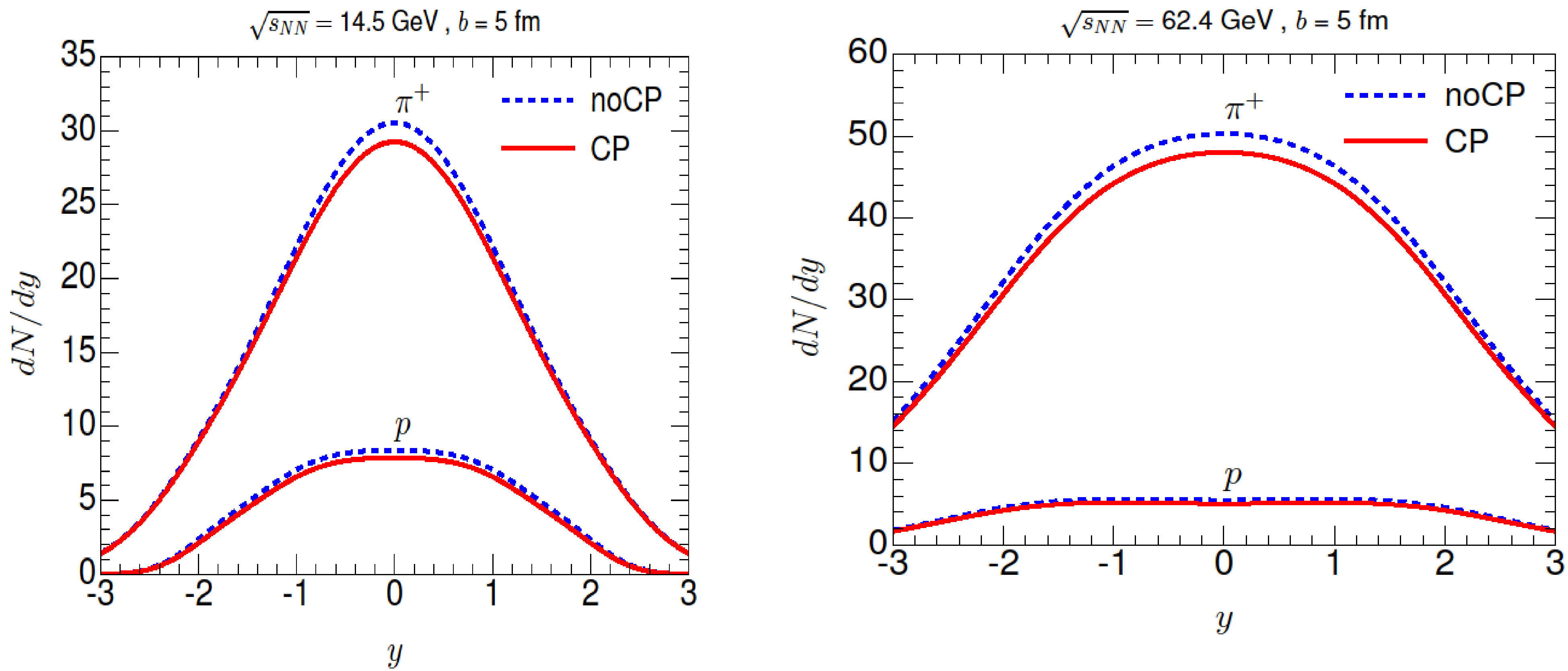
$$E\frac{dN_i}{d^3p} = \int_\Sigma (d\Sigma.p)\, f_i(x,p)$$

The distribution function contains the information of the hydrodynamic evolution through temperature, chemical potential and flow velocity.



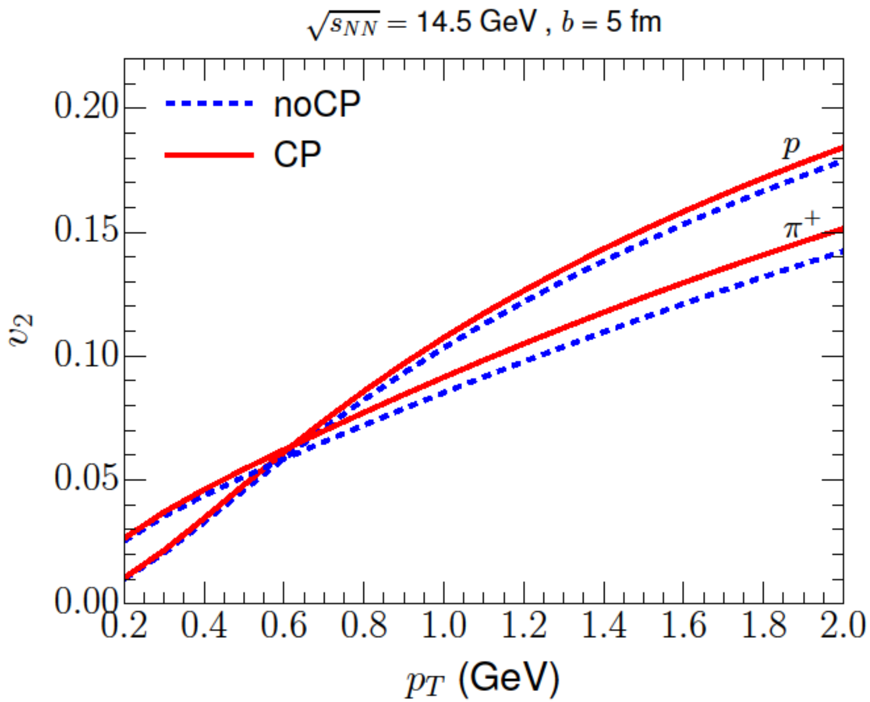
Insensitive to CP

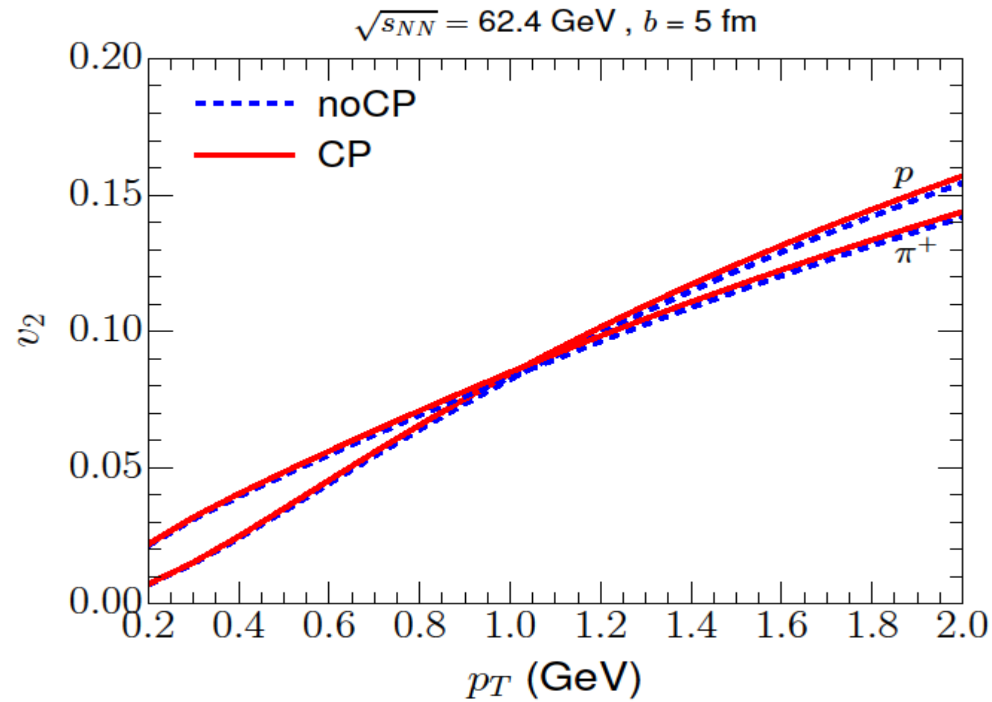
The rapidity distribution with and without QCD critical point



Insensitive to CP

The elliptic flow with and without QCD critical point





Insensitive to CP

Suppression of the vorticity in the presence of CP

The thermal vorticity is defined as:

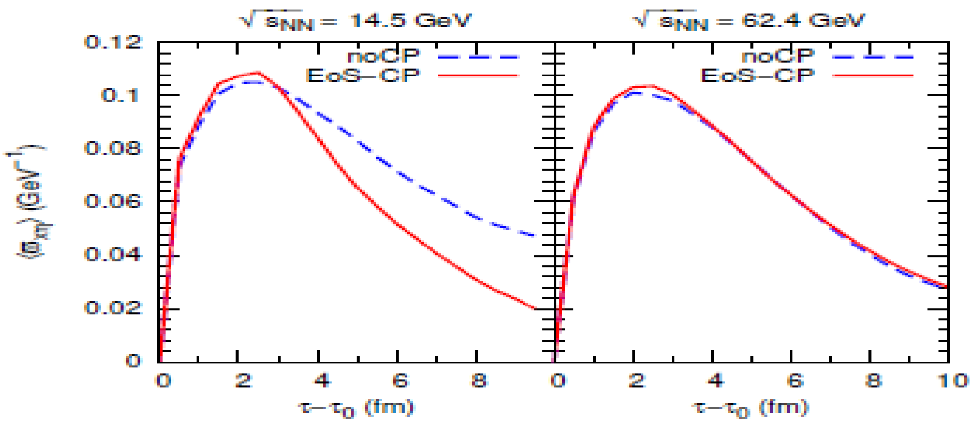
$$\varpi_{\mu\nu} = \frac{1}{2}\left[\partial_\nu \beta_\mu - \partial_\mu \beta_\nu\right] \qquad \beta_\mu = u_\mu / T.$$

In the non-relativistic limit the vorticity is defined as the curl of the velocity, $\vec{\omega} = \nabla \times \vec{v}$

The evolution of the vorticity is governed by the following equation:

$$\frac{\partial \vec{\omega}}{\partial t} = \left(\vec{\omega}\cdot\vec{\nabla}\right)\vec{v} - \left(\vec{v}\cdot\vec{\nabla}\right)\vec{\omega} - \theta\vec{\omega} + \frac{1}{\rho^2}\vec{\nabla}\rho \times \vec{\nabla}p$$
$$-\frac{1}{\rho^2}\left(\zeta + \frac{1}{3}\eta\right)\vec{\nabla}\rho \times \vec{\nabla}\theta - \frac{\eta}{\rho^2}\vec{\nabla}\rho \times \nabla^2\vec{v} + \frac{\eta}{\rho}\nabla^2\vec{\omega}. \quad \text{where} \quad \theta = \nabla\cdot\vec{v}.$$

Several competing mechanisms determine the evolution of the vorticity.



Suppression of vorticity due to QCD CP.

Results for zero initial angular momentum

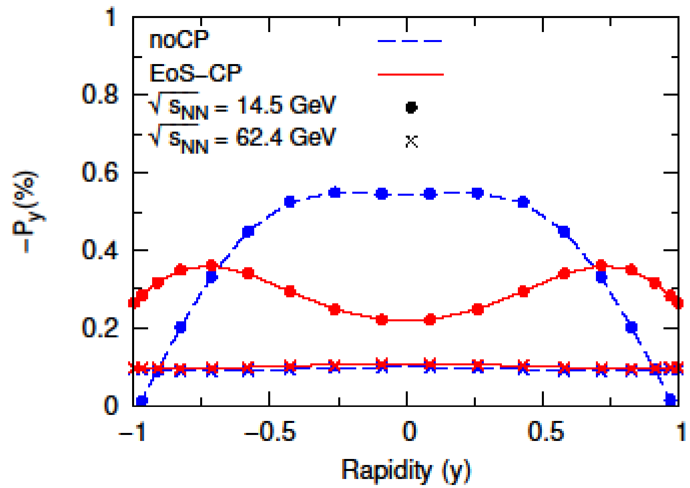
Polarization vector for spin ½ particle:

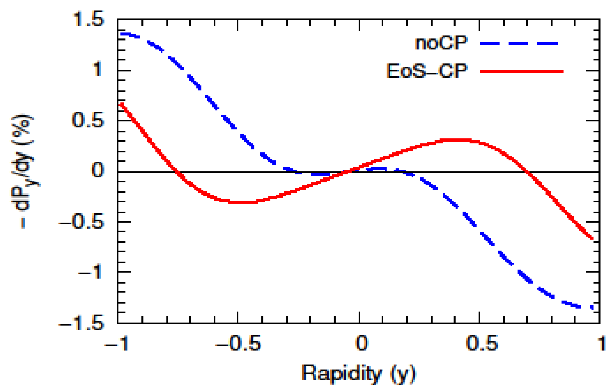
$$P_\mu(x,p) = -\frac{1}{8m}\epsilon_{\mu\rho\sigma\tau}(1-n_F)\varpi^{\rho\sigma}p^\tau$$

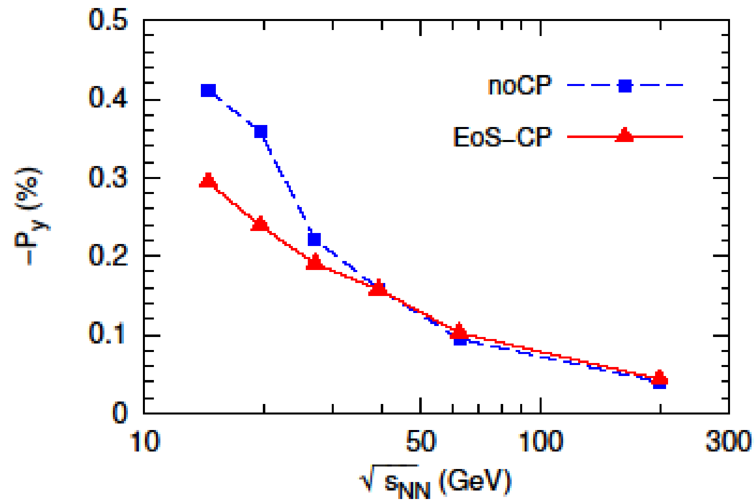
where $\varpi^{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma)$ with $\beta_\rho = \frac{u_\rho}{T}$

Cooper-Frye formula for particles with spin

$$P_\mu(p) = \frac{\int_\Sigma (d\Sigma.p)\, P_\mu(x,p)\, n_F(x,p)}{\int_\Sigma (d\Sigma.p)\, n_F(x,p)}$$







$$\vec{S}^*(x,p) \propto \frac{\gamma}{T^2}\vec{v}\times\nabla T + \frac{1}{T}(\vec{\omega}-(\vec{\omega}\cdot\vec{v})\vec{v}) + \frac{1}{T}\gamma\vec{A}\times\vec{v},$$

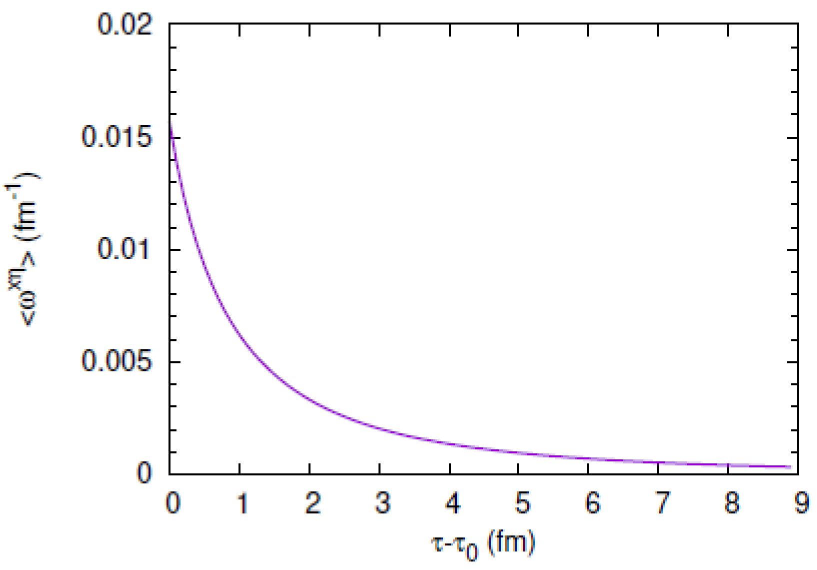
Non-zero initial angular momentum

The IC model of C. Shen *et al.* has been generalized to include non-zero initial vorticity in S. Ryu *et al.* PRC 104, 054908 (2021). The initial energy-momentum current is assumed to have the following form:

$$T^{\tau\tau} = \varepsilon(x, y, \eta_s)\cosh(y_L) \quad , \quad T^{\tau\eta_s} = \frac{1}{\tau_0}\varepsilon(x, y, \eta_s)\sinh(y_L)$$

where

$$y_L = f\ y_{\mathtt{CM}} \qquad , \qquad f \in [0, 1]$$

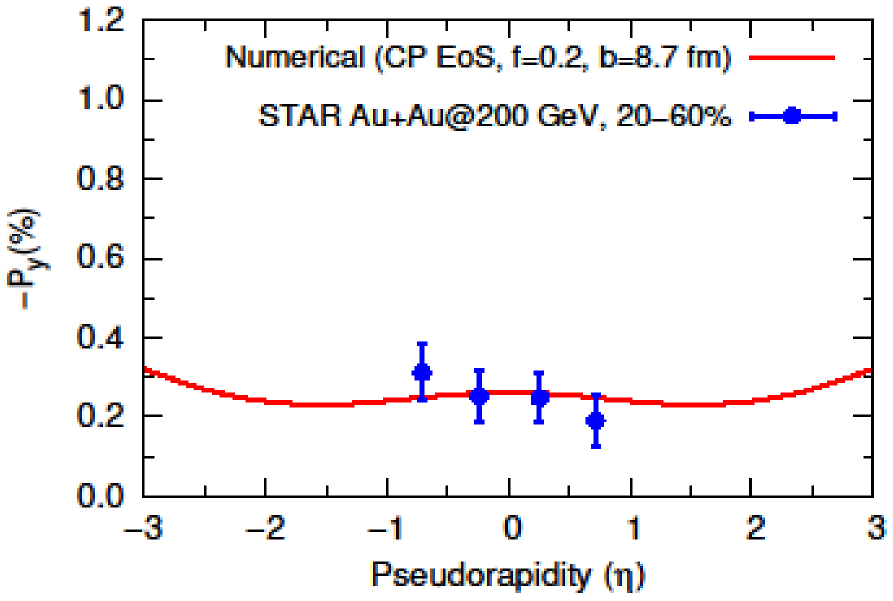


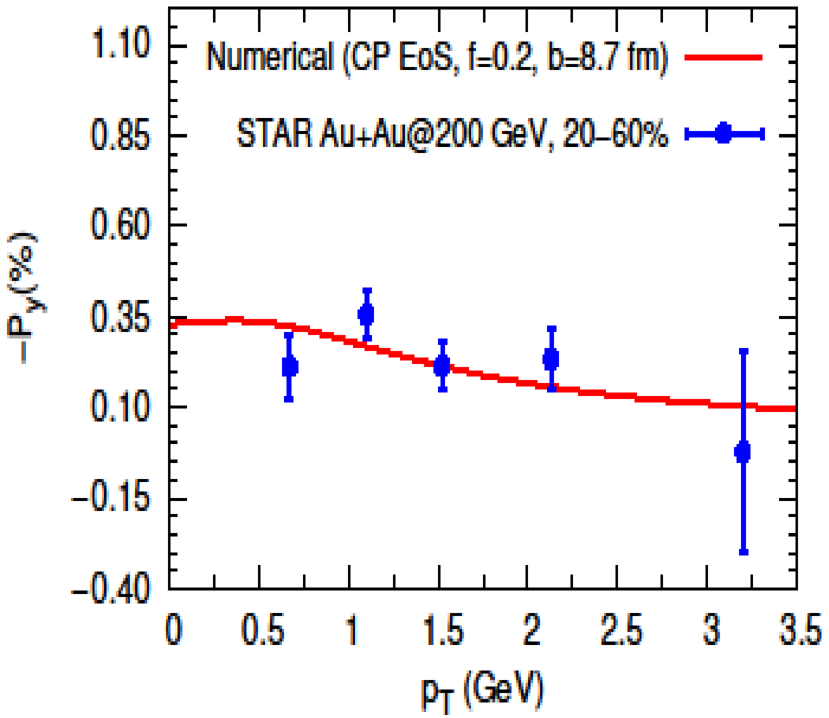
Polarization vector for spin ½ particle: [Results for non-zero initial angular momentum]

$$P_\mu(x,p) = -\frac{1}{8m}\epsilon_{\mu\rho\sigma\tau}(1-n_F)\varpi^{\rho\sigma}p^\tau \quad \text{where} \quad \varpi^{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma) \quad \text{with} \quad \beta_\rho = \frac{u_\rho}{T}$$

Cooper-Frye formula for particles with spin

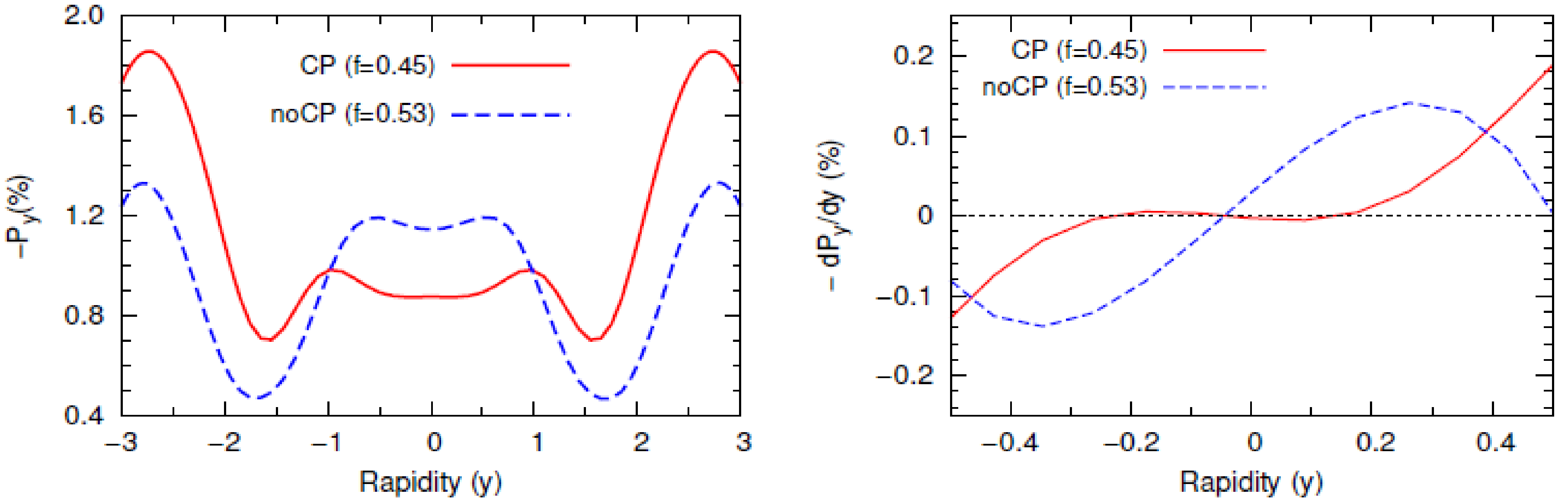
$$P_\mu(p) = \frac{\int_\Sigma (d\Sigma.p)\, P_\mu(x,p) n_F(x,p)}{\int_\Sigma (d\Sigma.p)\, n_F(x,p)}$$





Results for non-zero initial angular momentum

Au+Au collisions at $\sqrt{s_{NN}} = 14.5$ GeV with $b = 5.6$ fm



SKS and J. Alam, EPJC (2023) 83:585

- We also find that the other bulk observables like elliptic flow, p_T-spectra etc. are not much affected due to the CP. SKS and J. Alam, PRD 107, 074042 (2023).

Summary:

- The effects of the critical point on the spin polarization of Λ-hyperon has been studied by using relativistic viscous hydrodynamics.
- The other hadronic observables like elliptic flow, p_T spectra etc. are not affected much near the critical point (at most changed by 8%).
- It is found that the rapidity dependence of the spin polarization changes significantly (more than 25% quantitative change but there are qualitative in the slope also) as the critical point is approached.
- The study suggests that the spin polarization can be used as an indicator of the critical point.

THANK YOU

The QCD critical point drastically changes the rapidity distribution of spin polarization (y-component) of Lambda hyperon. The suppression of P_y can be used to detect the critical point.

Polarization vector for spin ½ particle:

$$P_\mu(x,p) = -\frac{1}{8m}\epsilon_{\mu\rho\sigma\tau}(1-n_F)\varpi^{\rho\sigma}p^\tau$$

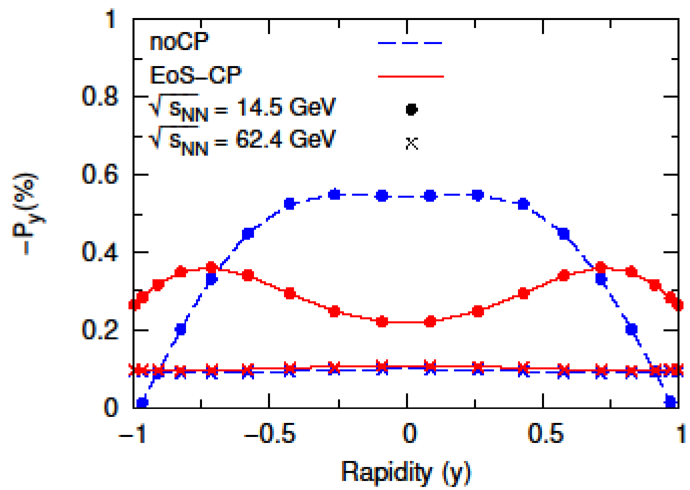
where

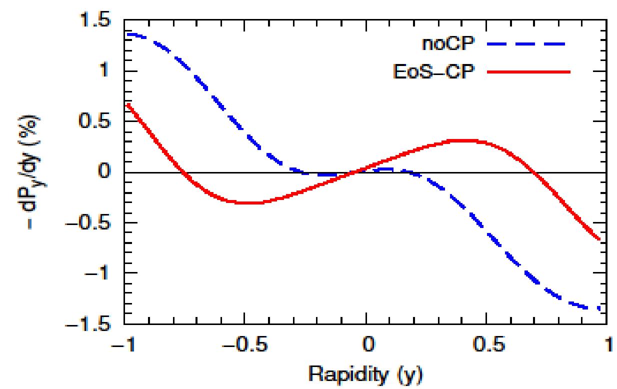
$$\varpi^{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma) \qquad \text{with} \qquad \beta_\rho = \frac{u_\rho}{T}$$

Cooper-Frye formula for particles with spin

Space-time integrated mean polarization vector:

$$P_\mu(p) = \frac{\int_\Sigma (d\Sigma.p)\, P_\mu(x,p) n_F(x,p)}{\int_\Sigma (d\Sigma.p)\, n_F(x,p)}$$





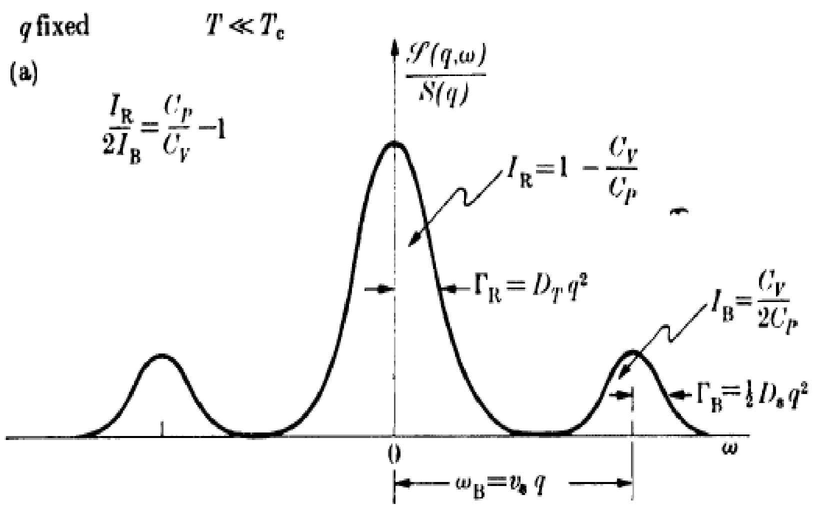
[Taken from Introduction to phase transition and critical phenomena by H. Eugene Stanley, Clarendon Press, Oxford, 1971.]

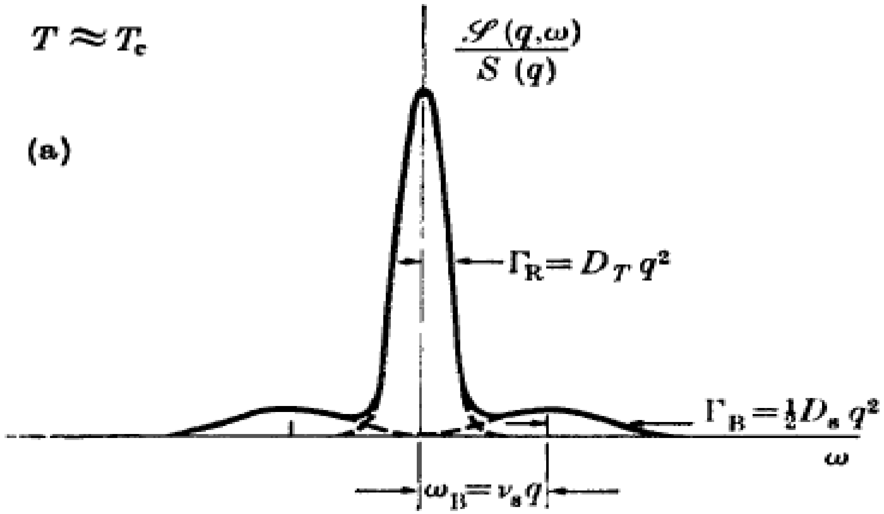
Space-time correlation in density fluctuation:

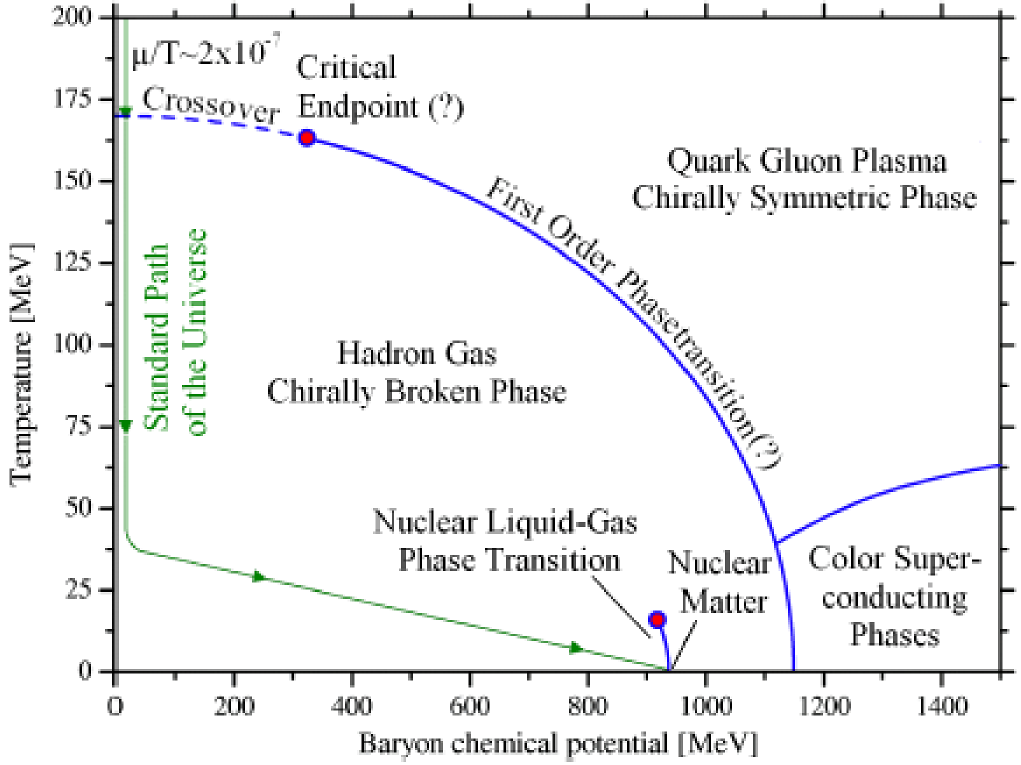
$$\delta n(\mathbf{r}, t) \equiv n(\mathbf{r}, t) - \langle n \rangle \qquad \mathscr{G}_{nn}(\mathbf{r}, t) \equiv \langle \delta n(\mathbf{r}, t)\, \delta n(0, 0) \rangle$$

The dynamic structure factor is given by

$$\mathscr{S}_{nn}(\mathbf{q}, \omega)/S_{nn}(\mathbf{q}) = \left(1 - \frac{C_V}{C_P}\right) \frac{2D_T q^2}{\omega^2 + (D_T q^2)^2} + \frac{C_V}{C_P} \left\{ \frac{\frac{1}{2} D_s q^2}{(\omega - v_s q)^2 + (\frac{1}{2} D_s q^2)^2} + \frac{\frac{1}{2} D_s q^2}{(\omega + v_s q)^2 + (\frac{1}{2} D_s q^2)^2} \right\}$$



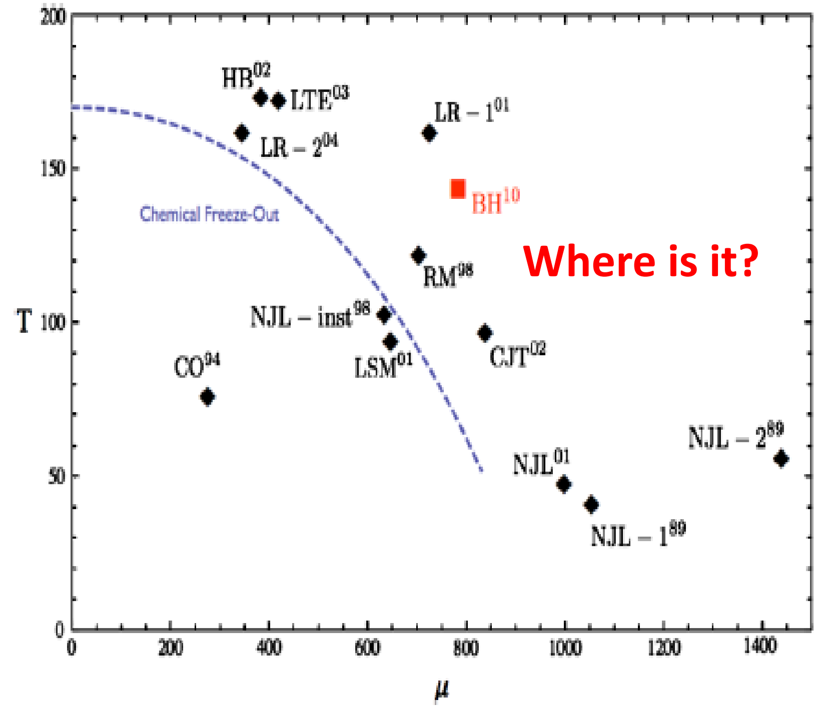




Boeckel and Schaffner-Bielich, PRD 85, 103506 (2012).

PHYSICAL REVIEW D 83, 086005 (2011)

DEWOLFE, GUBSER, AND ROSEN



Where is it?

Summary:

Suppression of sound wave near CP will have several consequences-

- affect the system life time

- suppress the Mach cone

Gradient of hydrodynamic quantities are more sensitive to CP

-Vorticity and consequently the spin polarization is found to be suppresses near the CP

1. Perturbations in Quark Gluon Plasma

Collaborators: **Golam Sarwar, Md Hasanujjaman, Mahfuzur Rahaman and Abhijit Bhattacharyya.**

Sources: ***Eur.Phys.J. C*** **82 (2022) 189,** ***Phys.Lett. B*** **820 (2021) 136583,** ***Eur.Phys.J.A*** **57 (2021) 283.**

Let Q (=energy density, net baryon density, flow velocity, ….)
be the a hydrodynamic quantity with its average value Q_0
and $\delta Q = Q - Q_0$ is linear perturbation. Fourier transformation of δQ:

$$\delta Q(\boldsymbol{k},\omega) = \int_{-\infty}^{\infty} d^3r \int_0^{\infty} dt e^{-i(\boldsymbol{k}.\boldsymbol{r}-\omega t)} \delta Q(\boldsymbol{r},t)$$

The set of linear algebraic equations obtained from hydrodynamic equations:

$$\delta Q(\boldsymbol{k},\omega) = \mathbb{M}\delta Q(k,0)$$

$$\left\langle \delta Q_i(\boldsymbol{k},\omega)\delta Q_j(\boldsymbol{k},0)\right\rangle = 0, \quad i \neq j$$

where

$$\mathbb{M} = \begin{bmatrix} i\omega & ikn_0 & 0 & 0 & 0 & 0 \\ \frac{ik}{h_0}\left(\frac{\partial p}{\partial n}\right)_T & i\omega & \frac{ik}{h_0}\left(\frac{\partial p}{\partial T}\right)_n & \frac{i\omega}{h_0} & \frac{ik}{h_0} & \frac{ik}{h_0} \\ 0 & ik\zeta & 0 & -ik\tilde{\alpha}_0\zeta & 1+i\omega\beta_0\zeta & 0 \\ 0 & -i\frac{4}{3}k\eta & 0 & i\frac{4}{3}\tilde{\alpha}_1 k\eta & 0 & 1+2i\omega\beta_2\eta \\ 0 & i\omega\kappa T_0 & ik\kappa & 1+i\omega\tilde{\beta}_1\kappa T_0 & ik\alpha_0\kappa T_0 & ik\tilde{\alpha}_1\kappa T_0 \\ -i\omega n_0\left(\frac{\partial s}{\partial n}\right)_T & 0 & i\omega n_0\left(\frac{\partial s}{\partial T}\right)_n & \frac{ik}{T_0} & 0 & 0 \end{bmatrix}$$

$$\delta Q(\boldsymbol{k},\omega) = \begin{bmatrix} \delta n(\boldsymbol{k},\omega) \\ \delta v_{\|}(\boldsymbol{k},\omega) \\ \delta\Pi(\boldsymbol{k},\omega) \\ \delta\pi_{\|}(\boldsymbol{k},\omega) \\ \delta q_{\|}(\boldsymbol{k},\omega) \\ \delta T(\boldsymbol{k},\omega) \end{bmatrix}; \quad \delta Q(\boldsymbol{k},0) = \begin{bmatrix} \delta n(\boldsymbol{k},0) \\ \delta v_{\|}(\boldsymbol{k},0) + \frac{1}{h_0}\delta q_{\|}(\boldsymbol{k},0) \\ i\omega\beta_0\zeta\delta\Pi(\boldsymbol{k},0) \\ -2\beta_2\eta\delta\pi_{\|}(\boldsymbol{k},0) \\ -\kappa T_0\delta v_{\|}(\boldsymbol{k},0) + \kappa T_0\tilde{\beta}_1\delta q_{\|}(\boldsymbol{k},0) \\ -n_0\left(\frac{\partial s}{\partial n}\right)_T\delta n(\boldsymbol{k},0) + n_0\left(\frac{\partial s}{\partial T}\right)_n\delta T(\boldsymbol{k},0) \end{bmatrix}$$

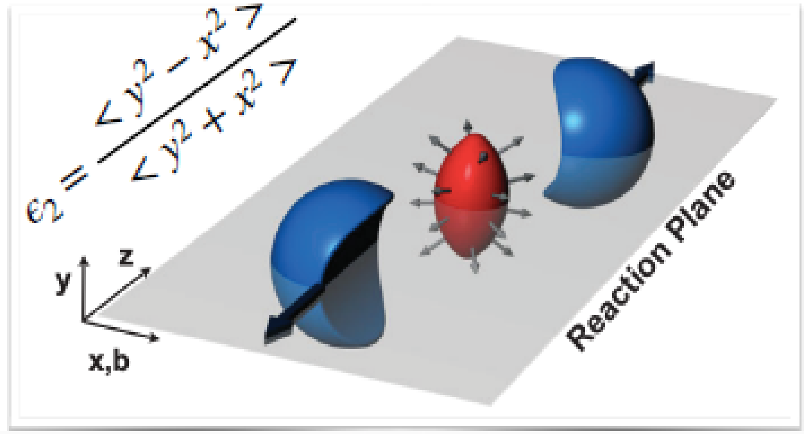
The expression for (baryon) density fluctuation:

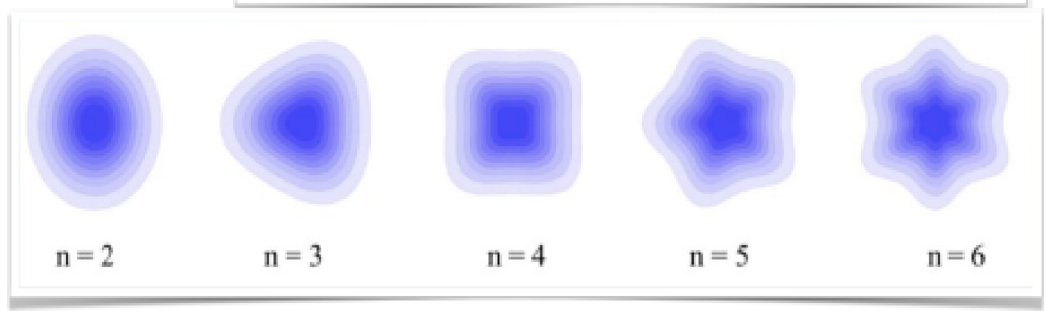
$$\delta n(\boldsymbol{k},\omega) = \left[\mathbb{M}^{-1}_{11} - n_0\left(\frac{\partial s}{\partial n}\right)_T \mathbb{M}^{-1}_{16}\right]\delta n(\boldsymbol{k},0) + \left[\mathbb{M}^{-1}_{12} - \kappa T_0\mathbb{M}^{-1}_{15}\right]\delta v_{\|}(\boldsymbol{k},0)$$
$$+\mathbb{M}^{-1}_{13}\left[i\omega\beta_0\zeta\right]\delta\Pi(\boldsymbol{k},0) - \mathbb{M}^{-1}_{14}\left[2\beta_2\eta\right]\delta\pi_{\|}(\boldsymbol{k},0)$$
$$\left[\frac{1}{h_0}\mathbb{M}^{-1}_{12} + \kappa T_0\tilde{\beta}_1\mathbb{M}^{-1}_{15}\right]\delta q_{\|}(\boldsymbol{k},0)\Big] + \mathbb{M}^{-1}_{16}\left[n_0\left(\frac{\partial s}{\partial T}\right)_n\right]\delta T(\boldsymbol{k},0)$$

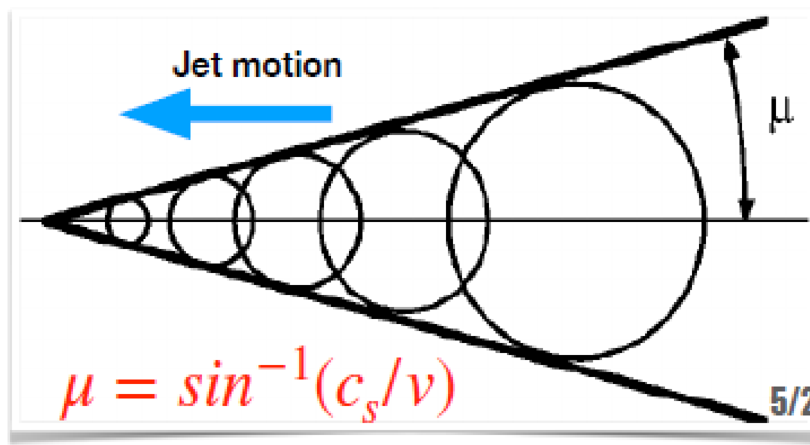
Introduction & Motivation

- The propagation of the perturbations: unveil the thermodynamic state of a fluid.
- The hydrodynamic response to the perturbation: imprinted on fluid and thus translated into the particle spectra.
- Promising observables
  - Flow harmonics, is attributed to the hydrodynamic response of the QGP to the initial geometry.
  - Formation of Mach cone in the medium due to the shock wave propagation.
- Consequences of the CEP on the hydrodynamic evolution if an isentropic trajectory passes through the critical region.

spatial anisotropy-> Flow







The correlation in density fluctuation: $S'_{nn}(k,\omega) = \left\langle \delta n(k,\omega)\delta n(k,0)\right\rangle$

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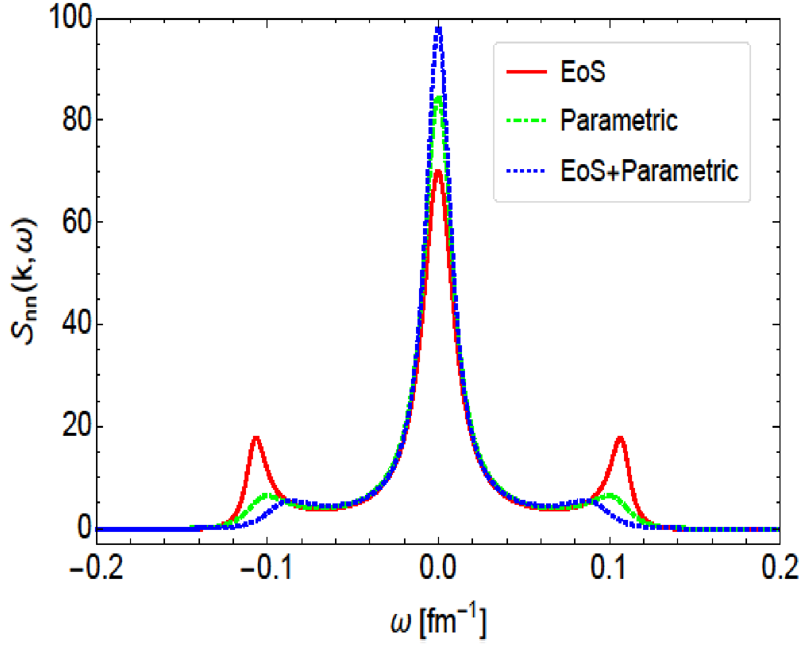
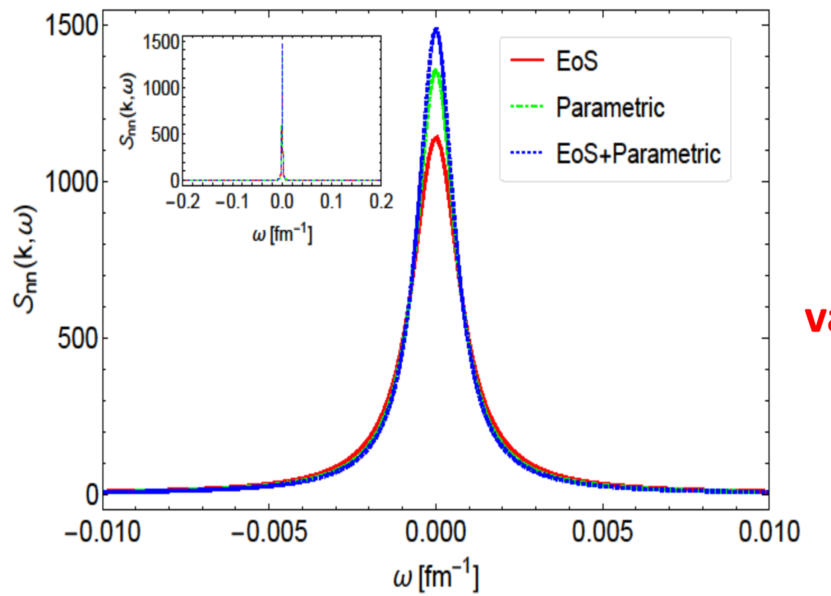




vanish

Dynamical spectral structure of density fluctuation away from the QCD critical point.

Near the QCD critical point

Nonlinear wave equations:

$$X = \frac{\sigma^{1/2}}{L}(x - c_s t) \quad \text{and} \quad Y = \frac{\sigma^{3/2}}{L}(c_s t)$$

To get the equations: Coordinate transformation and expansion

$$\hat{\epsilon} = \frac{\epsilon}{\epsilon_0} = 1 + \sigma \epsilon_1 + \sigma^2 \epsilon_2 + \sigma^3 \epsilon_3 + \ldots \quad (8$$

Here, we keep terms up to σ^3. We derive the required equation

Final Equations:

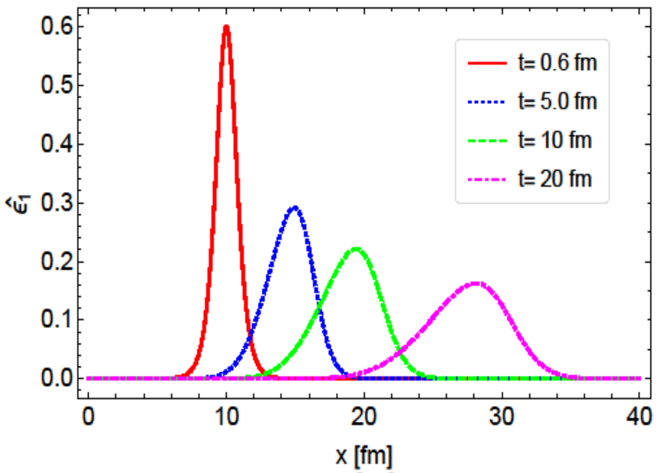
$$\frac{\partial \hat{\epsilon}_1}{\partial t} + [1 + (1 - c_s^2)\frac{\epsilon_0}{\epsilon_0 + p_0}\hat{\epsilon}_1]c_s \frac{\partial \hat{\epsilon}_1}{\partial x} - [\frac{1}{2(\epsilon_0 + p_0)}(\zeta + \frac{4}{3}\eta)]\frac{\partial^2 \hat{\epsilon}_1}{\partial x^2} = 0,$$

and

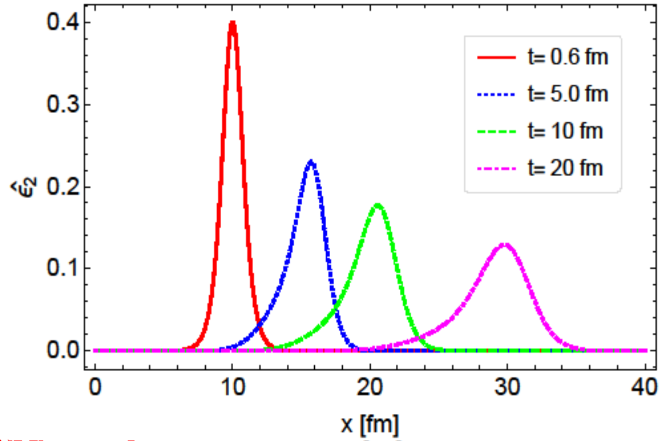
$$\frac{\partial \hat{\epsilon}_2}{\partial t} + \mathcal{S}_1 \frac{\partial \hat{\epsilon}_2}{\partial x} + \mathcal{S}_2 \frac{\partial \hat{\epsilon}_1}{\partial x} + \mathcal{S}_3 \frac{\partial^2 \hat{\epsilon}_1}{\partial x^2} + \mathcal{S}_4 \frac{\partial^3 \hat{\epsilon}_1}{\partial x^3} + \mathcal{S}_5 \frac{\partial^2 \hat{\epsilon}_2}{\partial x^2} = 0$$

$$\begin{aligned}
\mathcal{S}_1 &= \frac{1}{\epsilon_0 + p_0}\left[c_s\{\epsilon_0\left(1 - \hat{\epsilon}_1\left(c_s^2 - 1\right)\right) + p_0\}\right]; \\
\mathcal{S}_2 &= \frac{1}{\epsilon_0 + p_0}\left[\epsilon_0 c_s\{c_s^2 - 1\}\{\epsilon_0\left(\left(2c_s^2 + 1\right)\hat{\epsilon}_1{}^2 - \hat{\epsilon}_2\right) - p_0\hat{\epsilon}_2\}\right]; \\
\mathcal{S}_3 &= \frac{1}{12(\epsilon_0 + p_0)^2}\left[\epsilon_0\hat{\epsilon}_1\{3c_s^2(7\zeta + 8\eta) + 3\zeta + 4\eta\}\right]; \\
\mathcal{S}_4 &= -\frac{1}{72c_s c_V(\epsilon_0 + p_0)^2}\Big[4c_V c_s^2\{3\kappa T\epsilon_0(3\alpha_0\zeta + 4\alpha_1\eta) \\
&\quad + 3\kappa T(3\zeta + 4\eta) + 3\kappa T p_0(3\alpha_0\zeta + 4\alpha_1\eta) \\
&\quad + (\epsilon_0 + p_0)(9\beta_0\zeta^2 + 16\beta_2\eta^2)\} - c_V(3\zeta + 4\eta)^2 \\
&\quad + 12\kappa\{\epsilon_0 + p_0\}\{\epsilon_0(3\alpha_0\zeta + 4\alpha_1\eta) + 3\zeta \\
&\quad + 4\eta + p_0(3\alpha_0\zeta + 4\alpha_1\eta)\}\Big]; \\
\mathcal{S}_5 &= -\frac{3\zeta + 4\eta}{6(p_0 + \epsilon_0)} \qquad (11)
\end{aligned}$$

Propagation of non-linear perturbation through quark gluon plasma within the framework of 2nd order causal viscous hydrodynamics.

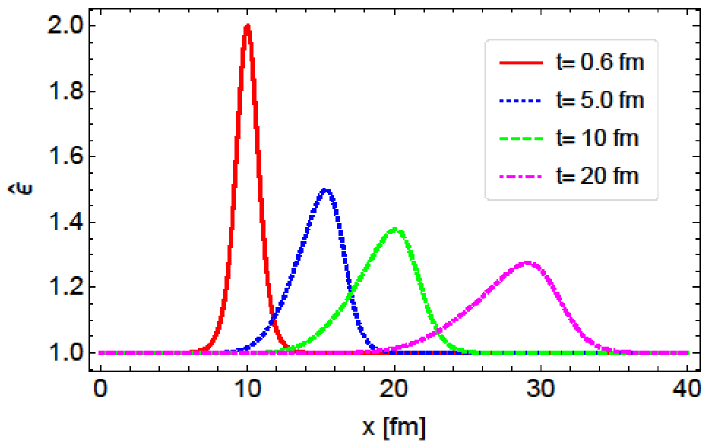


1st order



2nd order

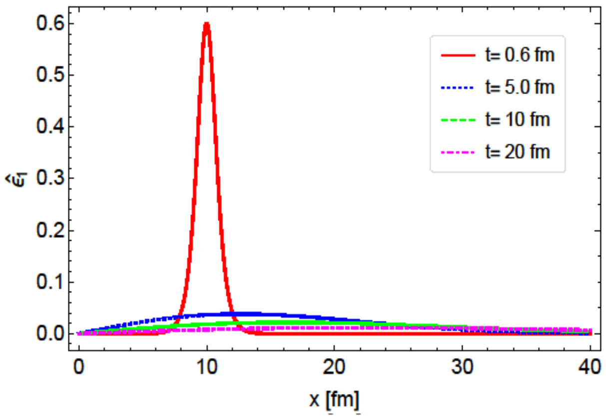
[1+1st order +2nd order]



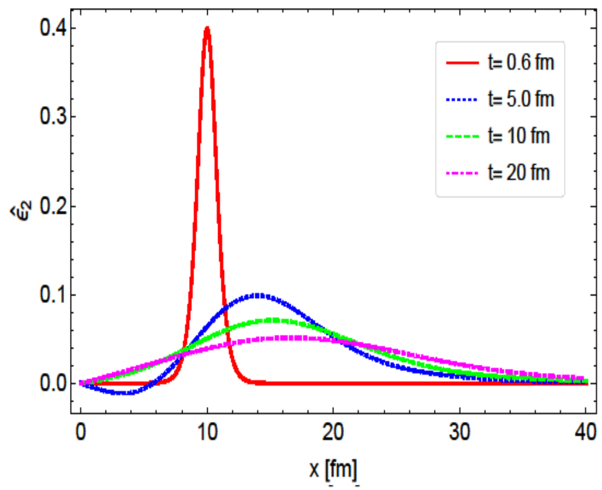
The system is away from the QCD critical point.

The perturbations survive.

Propagation of non-linear perturbation through quark gluon plasma within the framework of 2^{nd} order causal viscous hydrodynamics.

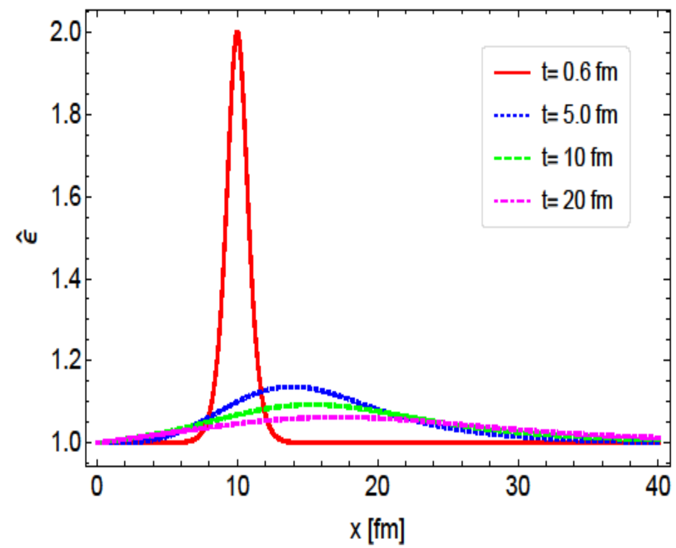


1^{st} order



2^{nd} order

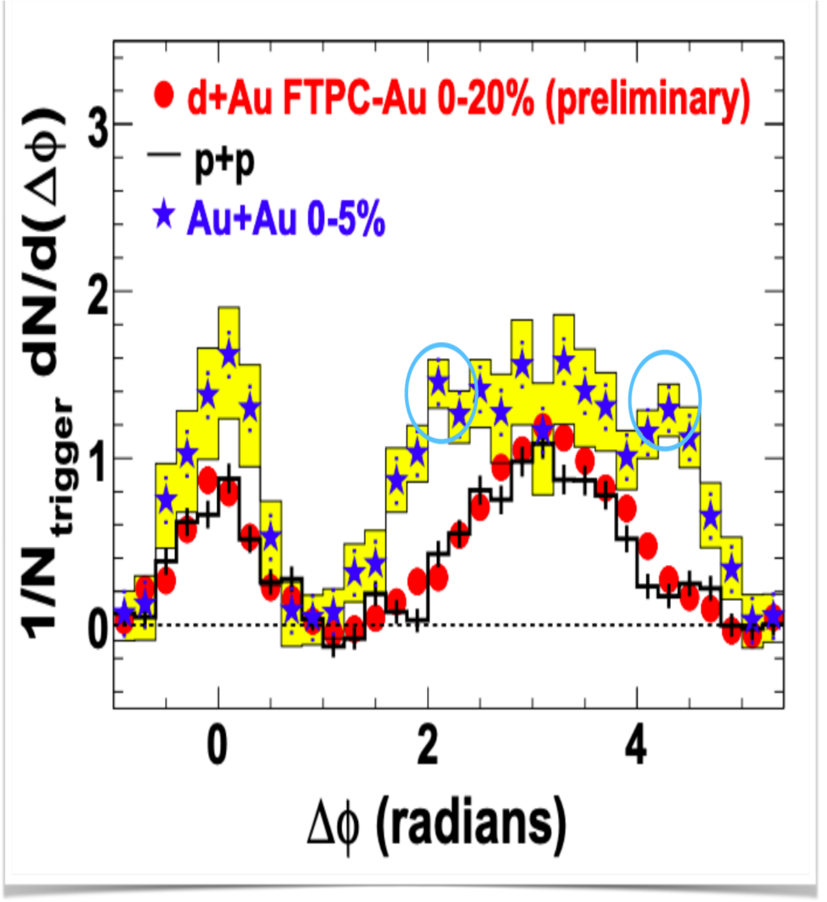
[$1+1^{st}$ order $+2^{nd}$ order]



The system is close to the QCD critical point.

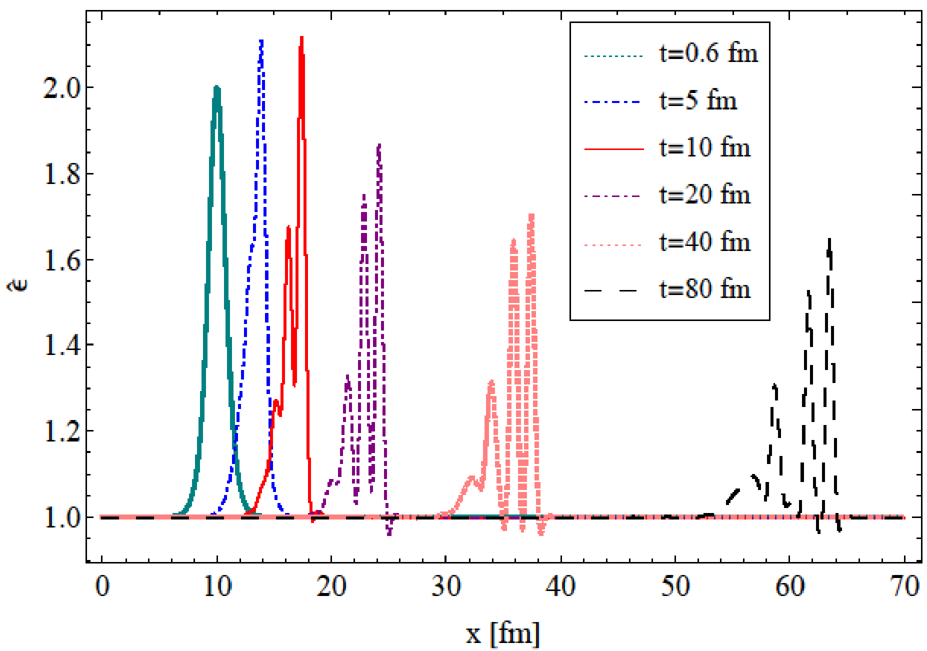
The perturbations get significantly suppressed in the presence of QCD CP.

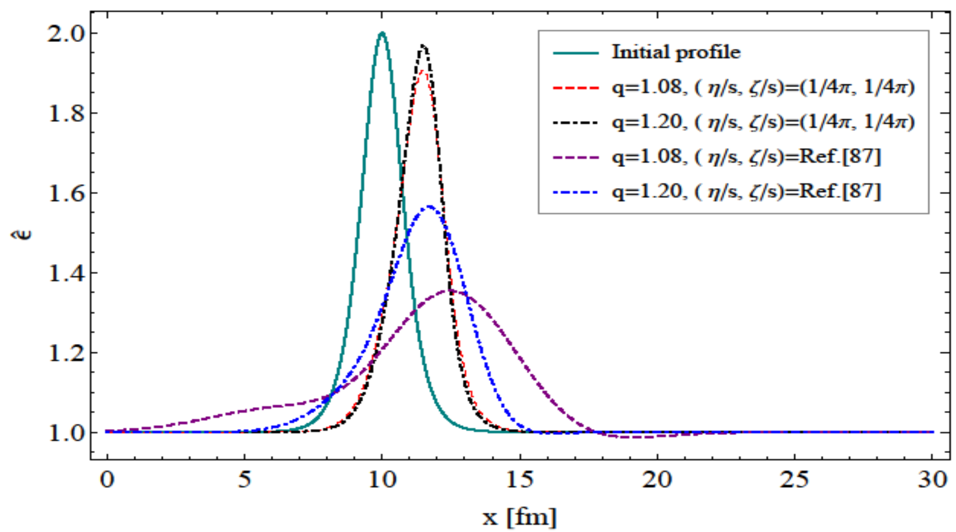
- The presence of the CEP will be resulting in the vanishing of Mach cone effects (or away side double-peak structure) and the broadening of the two and three-particle correlation.
- The suppression or collapse of elliptic flow near the CEP [1]. It will lead to large event-by-event fluctuation of flow harmonics between two events with and without the CEP.
- Proposed Signatures: A) the vanishing Mach cone effects (or away side double-peak structure) on the away side jet
- B) the enhancement of fluctuation of flow harmonics in event-by-event analysis accompanied by suppressed flow harmonics could be considered as signals of the CEP. Base line: anisotropy of particles that left the system before reaching the phase boundary.

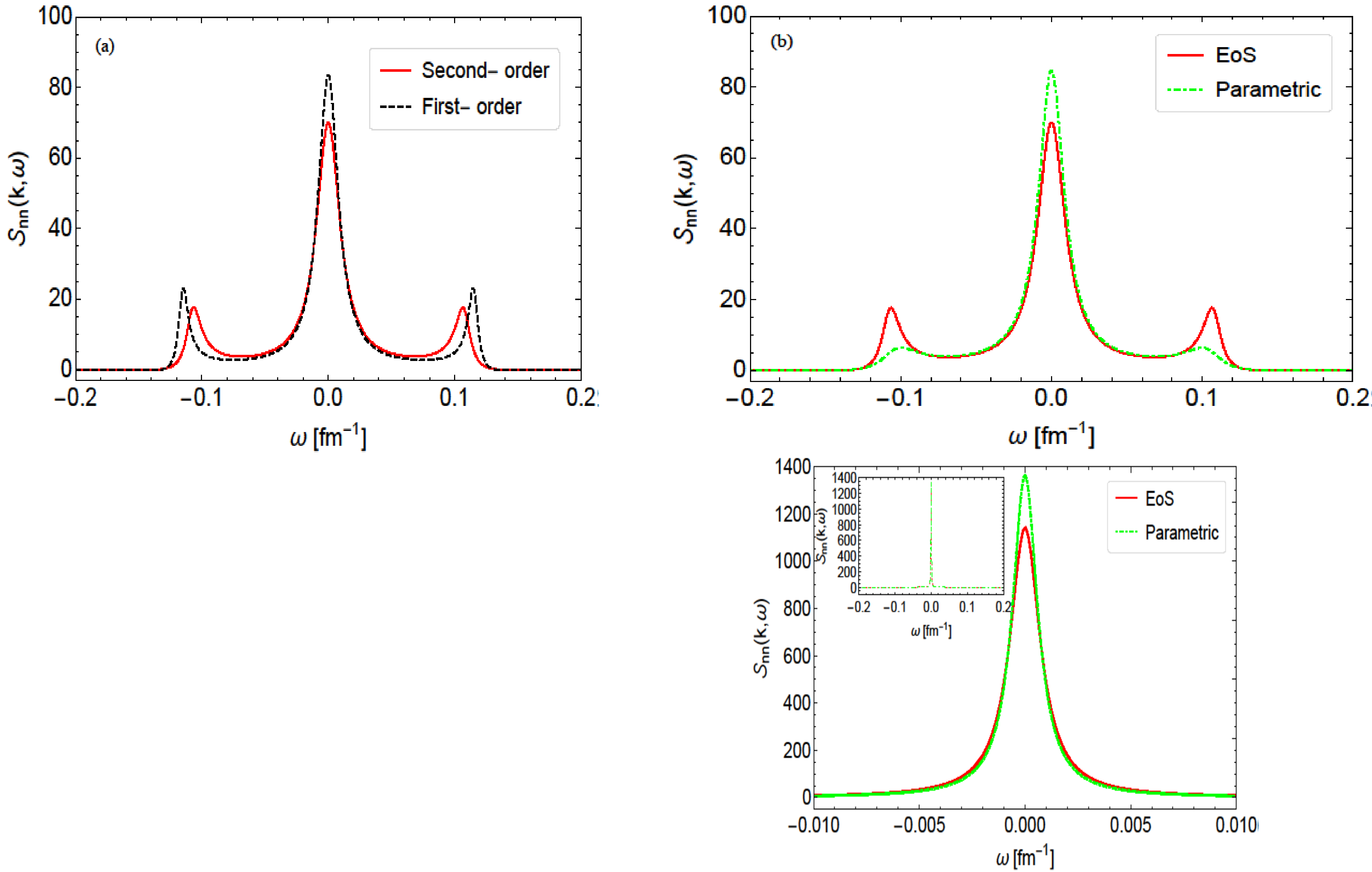


STAR Collaboration, NPA(2006)

- 1. M. Hasanujjaman, M. Rahman, A. Bhattacharyya, J. Alam, Phys. Rev. C, 102 (2020), Article 034910, arXiv:2008.03931v2


t=0.6 fm
t=5 fm
t=10 fm
t=20 fm
t=40 fm
t=80 fm
ε̂
x [fm]


Initial profile
q=1.08, (η/s, ζ/s)=(1/4π, 1/4π)
q=1.20, (η/s, ζ/s)=(1/4π, 1/4π)
q=1.08, (η/s, ζ/s)=Ref.[87]
q=1.20, (η/s, ζ/s)=Ref.[87]
ε̂
x [fm]


(a)
Second- order
First- order
$S_{nn}(k,\omega)$
ω [fm^{-1}]
(b)
EoS
Parametric
$S_{nn}(k,\omega)$
ω [fm^{-1}]
EoS
Parametric
$S_{nn}(k,\omega)$
ω [fm^{-1}]

$$\mathbb{M}\,\delta\mathcal{Q} = \mathcal{A}$$

$$\delta\mathcal{Q} = \begin{pmatrix} \delta n \\ \delta T \\ \delta\phi \\ \delta u_{||} \\ \delta q_{||} \\ \delta\pi \\ \delta\pi_{||||} \\ \delta\pi_{\perp\perp} \\ \delta u_{\perp} \\ \delta q_{\perp} \\ \delta\pi_{||\perp} \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} -\delta n(\boldsymbol{k}, t=0) \\ -\epsilon_n \delta n(\boldsymbol{k}, t=0) - \epsilon_\phi \delta\phi(\boldsymbol{k}, t=0) - e_T \delta T(\boldsymbol{k}, t=0) \\ -\delta\phi(\boldsymbol{k}, t=0) - ikT_0 \delta u_{||}(\boldsymbol{k}, t=0)\kappa_{q\pi} \\ (\epsilon_0 + P_0)\delta u_{||}(\boldsymbol{k}, t=0) - \delta q_{||}(\boldsymbol{k}, t=0) \\ -\chi\beta_1 T_0 \delta q_{||}(\boldsymbol{k}, t=0) - T_0\chi\delta u_{||}(\boldsymbol{k}, t=0) \\ -\frac{1}{3}\beta_0\zeta\delta\pi(\boldsymbol{k}, t=0) \\ -2\beta_2\eta\delta\pi_{||||}(\boldsymbol{k}, t=0) \\ -2\beta_2\eta\delta\pi_{\perp\perp}(\boldsymbol{k}, t=0) \\ (\epsilon_0 + P_0)\delta u_{\perp}(\boldsymbol{k}, t=0) - \delta q_{\perp}(\boldsymbol{k}, t=0) \\ -\chi\beta_1 T_0 \delta q_{\perp}(\boldsymbol{k}, t=0) - T_0\chi\delta u_{\perp}(\boldsymbol{k}, t=0) \\ -2\beta_2\eta\delta\pi_{||}\perp(\boldsymbol{k}, t=0) \end{pmatrix}$$

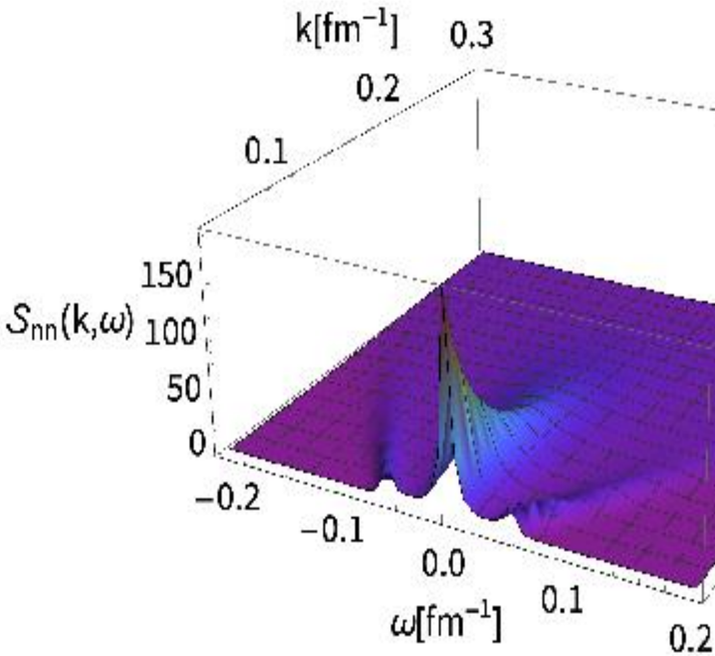
$$\mathbb{M} = \begin{pmatrix} \mathcal{L}_m & \mathbf{0} \\ \mathbf{0} & \mathcal{T}_m \end{pmatrix}$$

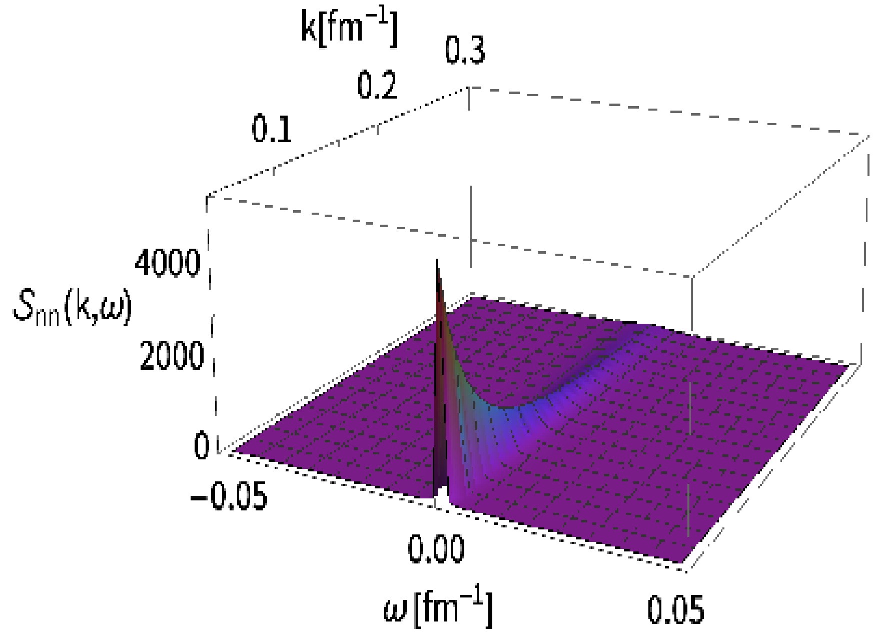
$$\mathcal{L}_m =$$

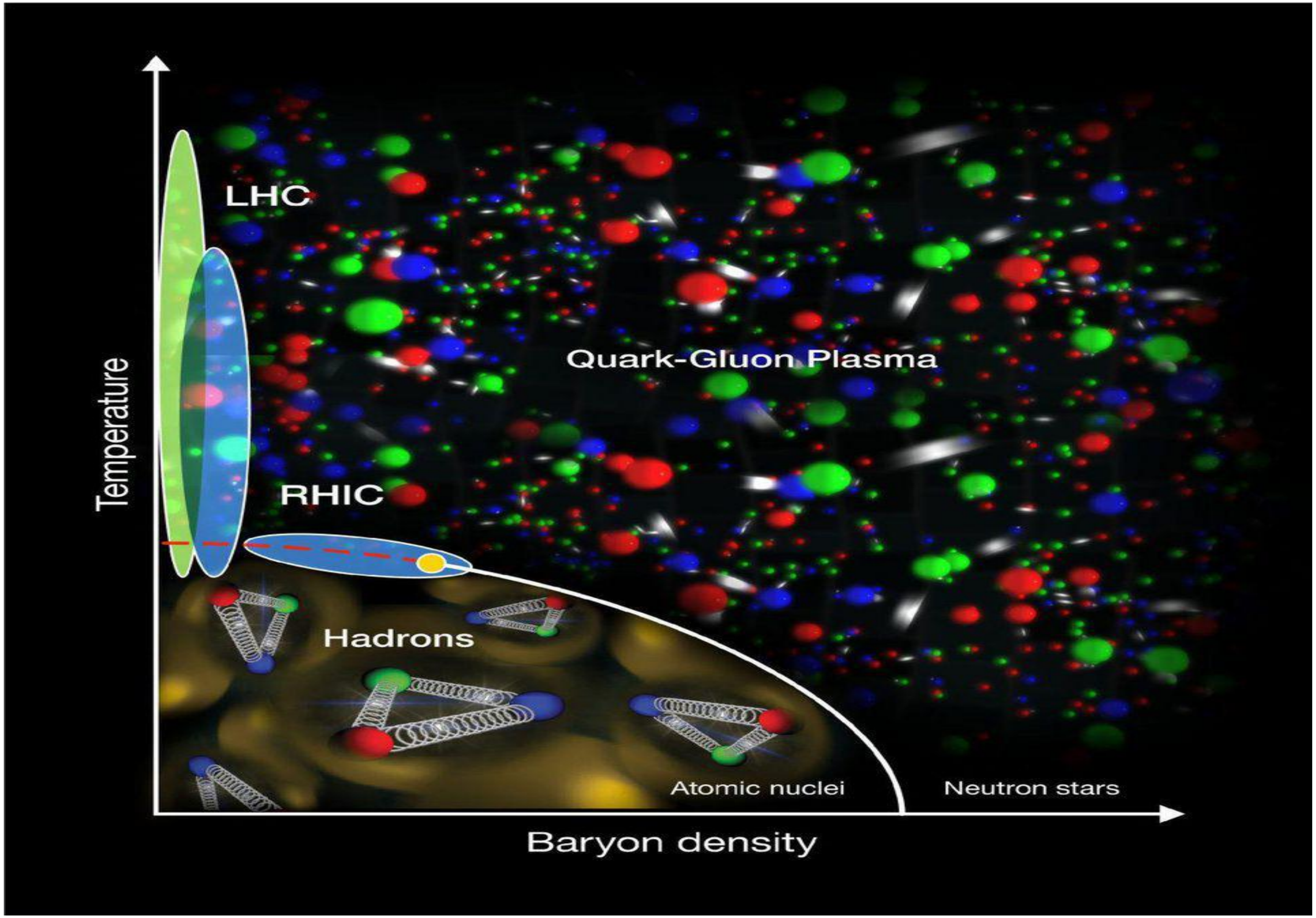
$$\begin{pmatrix}
i\omega & 0 & 0 & ikn_0 & 0 & 0 & 0 & 0 \\
i\omega\epsilon_n & i\omega\epsilon_T & i\omega\epsilon_\phi & ikw_0 & -ik & 0 & 0 & 0 \\
C_{\mathrm{n}\pi}B(k) & C_{\mathrm{T}\pi}B(k) + k^2\kappa_{\mathrm{q}\pi} & C_{\phi\pi}B(k) + i\omega & ikD_\phi(\omega) & 0 & 0 & 0 & 0 \\
-ikP_n & -ikP_T & -ikP_\phi & -i\omega w_0 & i\omega & -ik & -ik & 0 \\
-ikT_0^2C_{\mathrm{n}\pi}\kappa_{\mathrm{q}\pi} & ik\left(\chi - T_0^2C_{\mathrm{T}\pi}\kappa_{\mathrm{q}\pi}\right) & -ikT_0^2C_{\phi\pi}\kappa_{\mathrm{q}\pi} & iT_0\chi\omega & 1 + i\beta_1T_0\chi\omega & -\chi_T\tilde{\alpha}_0 & -\chi_T\tilde{\alpha}_1 & 0 \\
0 & 0 & 0 & \frac{i\zeta k}{3} & -\frac{1}{3}i\zeta k\tilde{\alpha}_0 & 1 + \frac{1}{3}i\beta_0\zeta\omega & 0 & 0 \\
0 & 0 & 0 & \frac{4i\eta k}{3} & -\frac{4}{3}i\eta k\tilde{\alpha}_1 & 0 & K(\omega) & 0 \\
0 & 0 & 0 & -\frac{4}{3}i\eta k & \frac{4}{3}i\eta k\tilde{\alpha}_1 & 0 & 0 & K(\omega)
\end{pmatrix}$$

$$\mathcal{T}_m = \begin{pmatrix}
-i\omega\left(\epsilon_0 + P_0\right) & i\omega & -ik \\
iT_0\chi\omega & 1 + i\beta_1T_0\chi\omega & -ikT_0\chi\tilde{\alpha}_1 \\
i\eta k & -i\eta k\tilde{\alpha}_1 & 1 + 2i\beta_2\eta\omega
\end{pmatrix}.$$

$$
\begin{aligned}
\delta \mathrm{n}\left(\boldsymbol{k},\omega\right) = & (-\epsilon_n \mathbb{M}^{-1}_{12} - \mathbb{M}^{-1}_{11})\delta n(\boldsymbol{k}, t=0) + \epsilon_T \left(-\mathbb{M}^{-1}_{12}\right)\delta T(\boldsymbol{k}, t=0) + (-\epsilon_\phi \mathbb{M}^{-1}_{12} - \mathbb{M}^{-1}_{13})\delta\phi(\boldsymbol{k}, t=0) \\
& +(\epsilon_0 \mathbb{M}^{-1}_{14} - ikT_0 \mathbb{M}^{-1}_{13}\kappa_{q\pi} + \mathbb{M}^{-1}_{14}P_0 - T_0\chi \mathbb{M}^{-1}_{15})\delta u_{||}(\boldsymbol{k}, t=0) \\
& + \left(-\beta_1 T_0 \chi \mathbb{M}^{-1}_{15} - \mathbb{M}^{-1}_{14}\right)\delta q_{||}(\boldsymbol{k}, t=0) + \frac{1}{3}\zeta \mathbb{M}^{-1}_{16}\beta_0 \delta\pi(\boldsymbol{k}, t=0) - 2\eta \mathbb{M}^{-1}_{17}\beta_2 \delta\pi_{||||}(\boldsymbol{k}, t=0) \\
& -2\eta \mathbb{M}^{-1}_{18}\beta_2 \pi_{\perp\perp}(\boldsymbol{k}, t=0) + (\epsilon_0 \mathbb{M}^{-1}_{19} + \mathbb{M}^{-1}_{19}P_0 - T_0\chi \mathbb{M}^{-1}_{110})\delta u_\perp(\boldsymbol{k}, t=0) \\
& + \left(-\beta_1 T_0 \chi \mathbb{M}^{-1}_{110} - \mathbb{M}^{-1}_{19}\right)\delta q_\perp(\boldsymbol{k}, t=0) - 2\eta \mathbb{M}^{-1}_{111}\beta_2 \delta\pi_{||\perp}(\boldsymbol{k}, t=0).
\end{aligned}
\tag{B22}
$$


k[fm^-1]
0.1
0.2
0.3
S_nn(k,ω)
150
100
50
0
−0.2
−0.1
0.0
0.1
0.2
ω[fm^-1]


k[fm^-1]
0.1
0.2
0.3
S_nn(k,ω)
4000
2000
0
−0.05
0.00
0.05
ω[fm^-1]



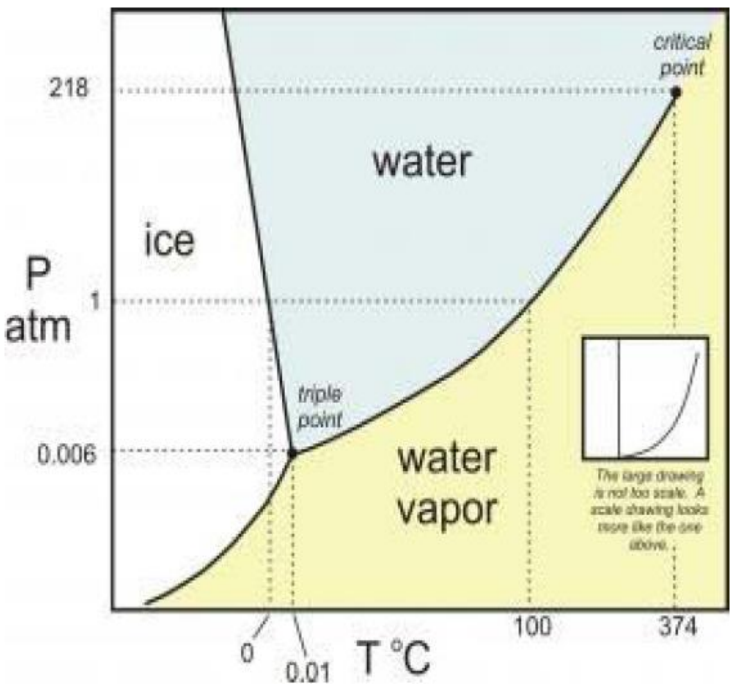
Source: https://www.forbes.com/sites/startswithabang/2018/08/15/what-was-it-like-when-we-lost-the-last-of-our-antimatter/?sh=ebf99b366608

$$S^{\mu}(x,p) = -\frac{1}{8m}(1-n_F)\epsilon^{\mu\nu\rho\sigma}p_{\nu}\varpi_{\rho\sigma}(x)$$

$$u^x = u^y = 0,$$

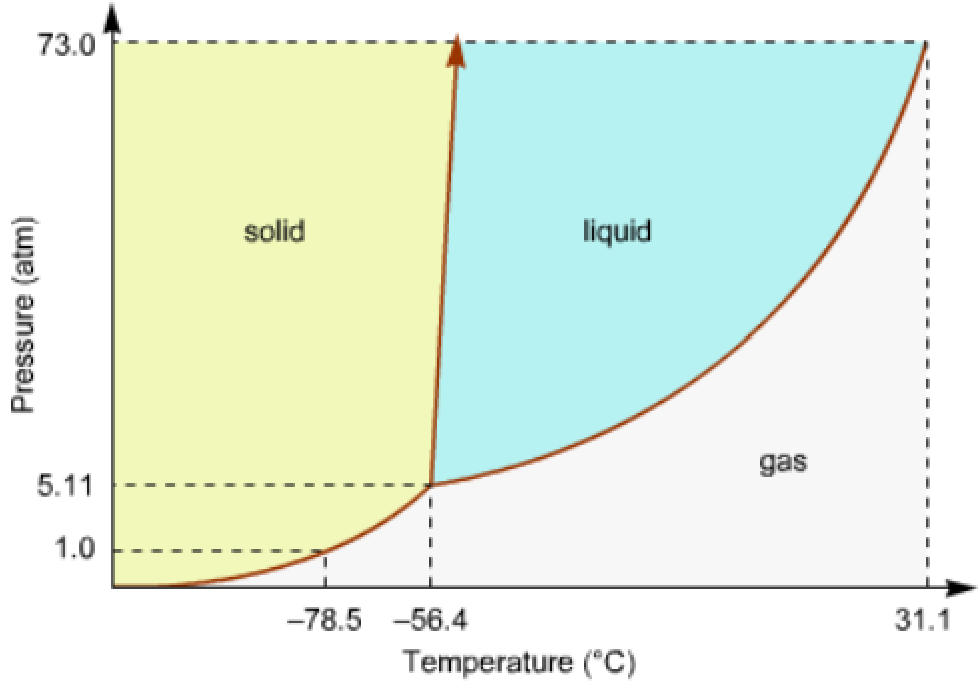
$$u^{\tau} = \cosh\left(\frac{y_L}{2}\right)\ ,\ u^{\eta} = \frac{1}{\tau_0}\sinh\left(\frac{y_L}{2}\right)$$

Phase diagram of H_2O

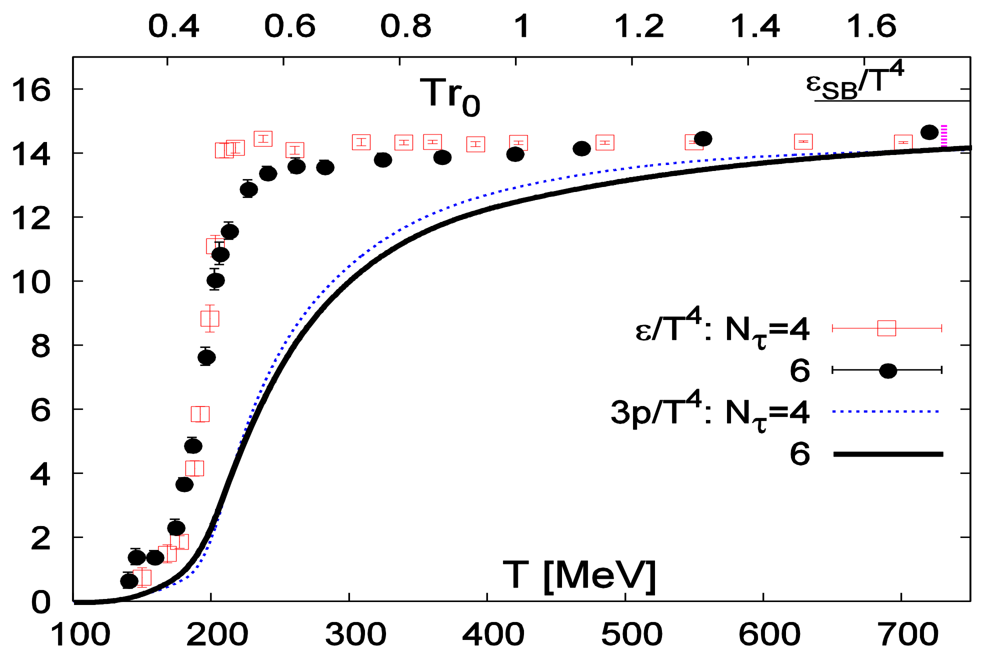


https://serc.carleton.edu/details/images/10201.html

Phase diagram of CO_2

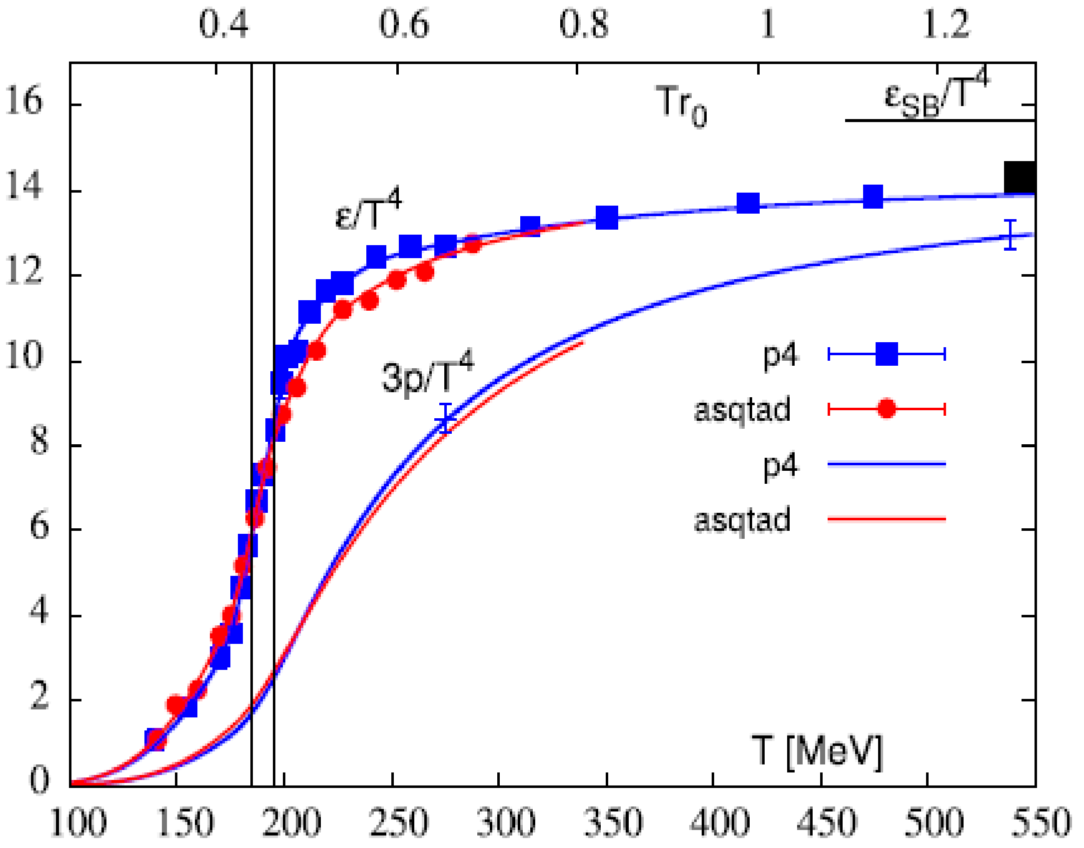


https://physics.stackexchange.com/questions/550010/is-there-any-relevance-between-phase-diagram-and-energy



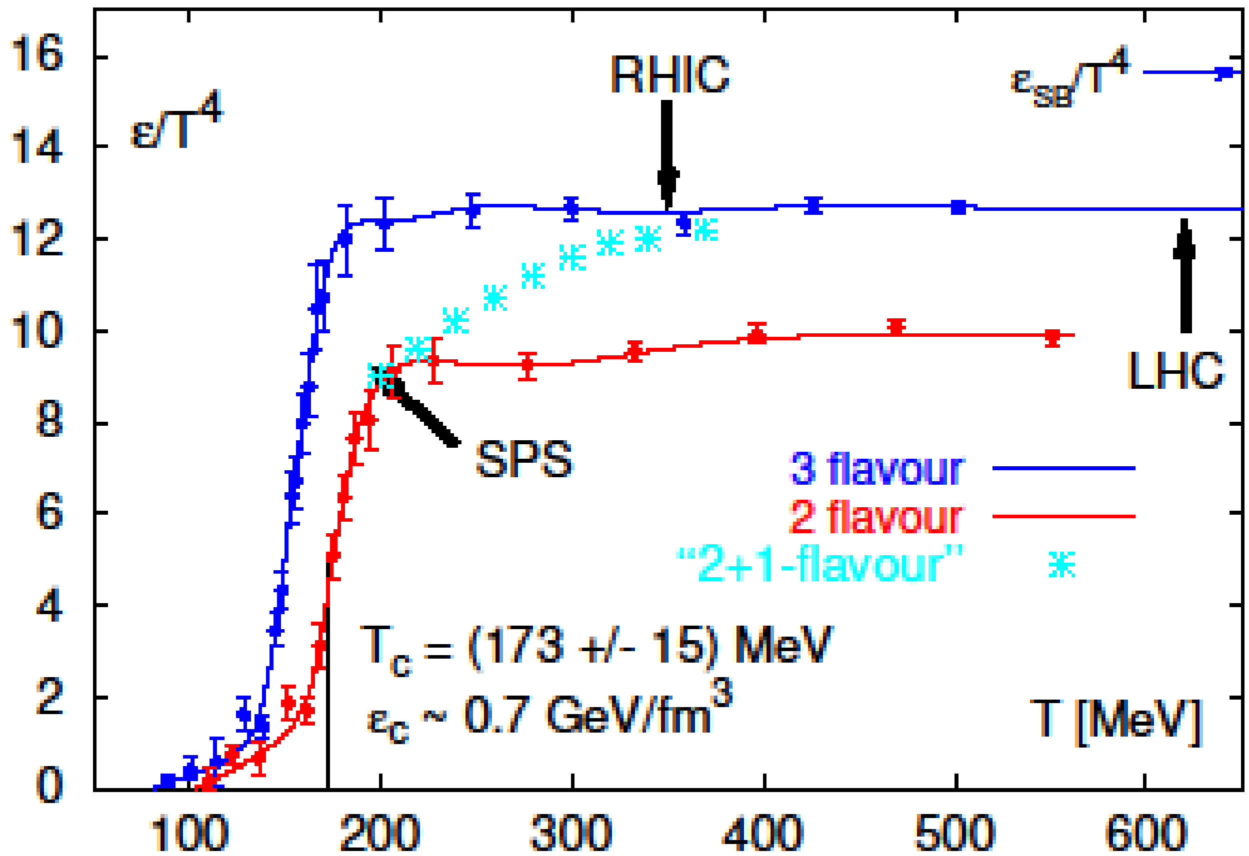
Fodor & Katz, arXiv:0908.341 [hep-ph]

Temperature variation of energy density and pressure from Lattice QCD



O. Philipsen, Prog. Part. & Nucl. Phys. 70 (2013)55

Temperature variation of energy density from Lattice QCD



Karsch & Laermann, 2004