EXPLORING GLUON TMDS AT THE ELECTRON-ION COLLIDER

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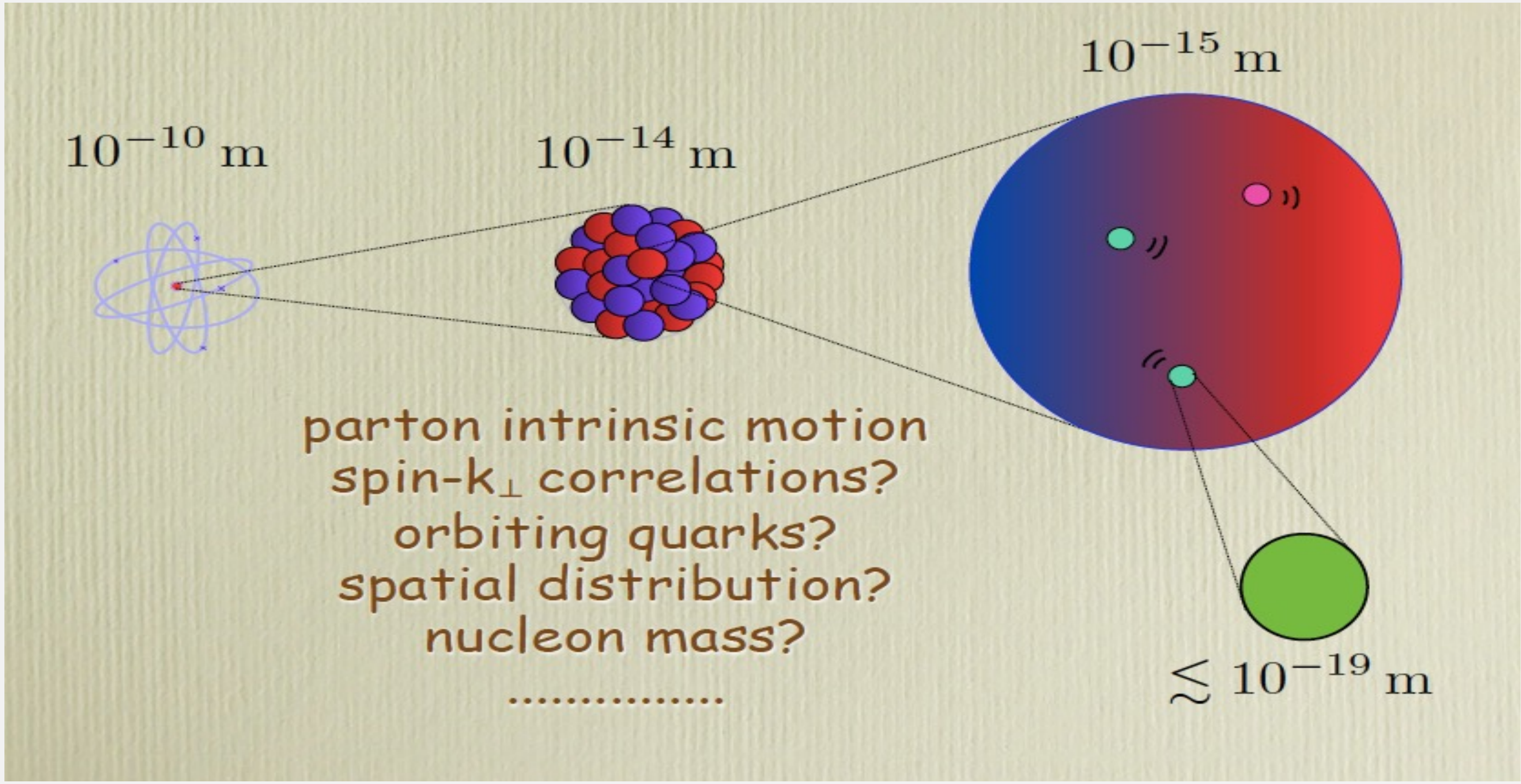
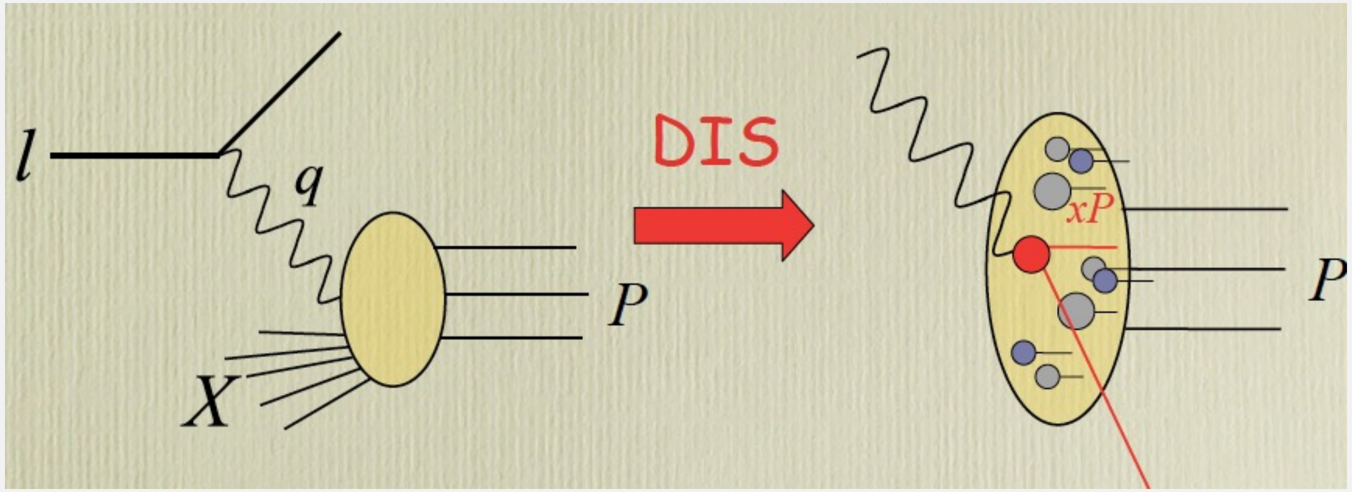
India-JINR workshop on elementary particle and nuclear physics, October 16-19, 2023

STRUCTURE OF THE NUCLEONS IN TERMS OF QUARKS AND GLUONS : STILL NOT UNDERSTOOD COMPLETELY



NUCLEON STRUCTURE : PROBED THROUGH ELECTRON-PROTON DEEP INELASTIC SCATTERING



Scattering through a virtual photon

Interacts with the quarks in the nucleon

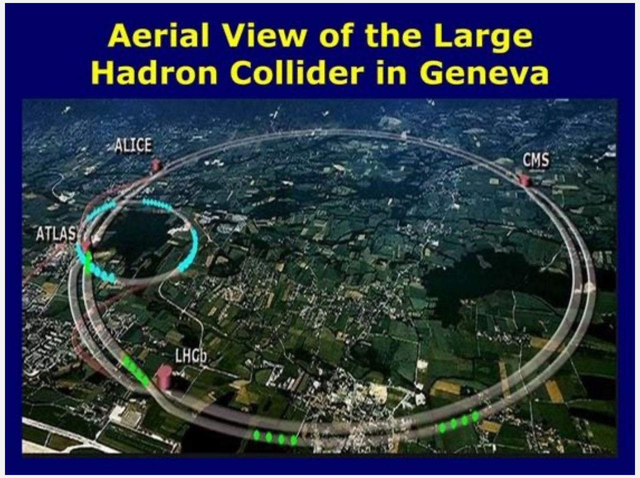
(collinear) parton distributions (pdfs) q(x) ; probability to find a parton in the nucleon with momentum fraction x ; depend also on the momentum scale

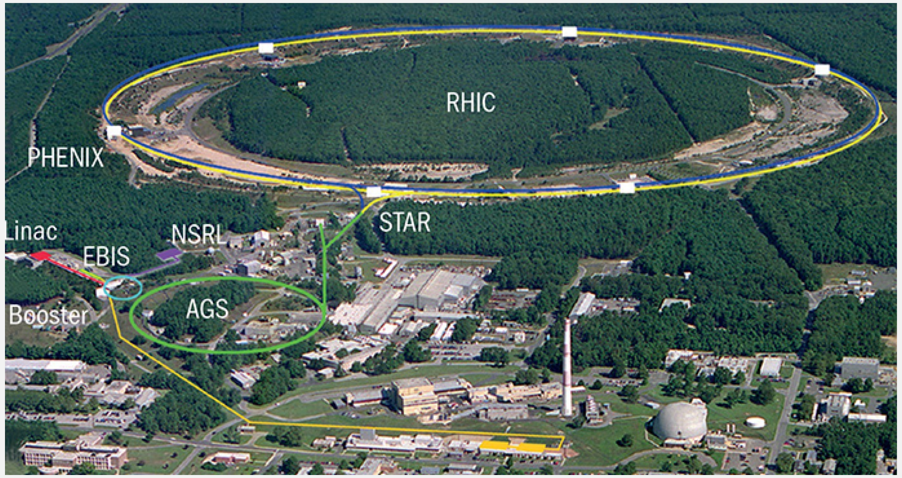
$$\ell\, p \to \ell\, X \qquad Q^2 = -q^2 \qquad x = \frac{Q^2}{2P\cdot q}$$

Cross section

$$\frac{d\sigma^{\ell p\to\ell X}}{dx\, dQ^2} = \sum_q q(x)\, \frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}$$

EXPERIMENTS AROUND THE WORLD





ELECTRON-ION COLLIDER (EIC)

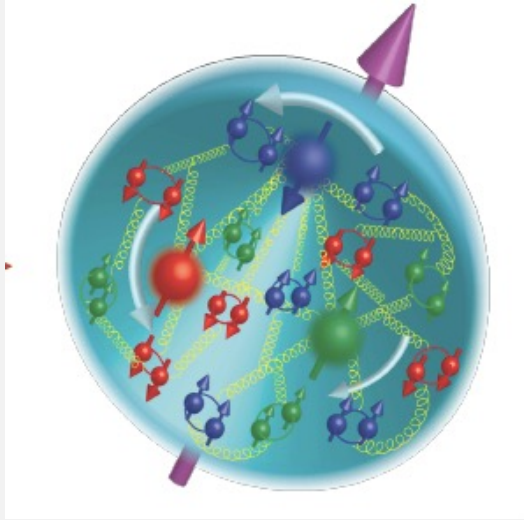
The EIC to be built at Brookhaven National Lab, USA will collide highly energetic electron beam with proton/heavy ion to take ‘snapshots’ at high accuracy --tomography of the nucleon

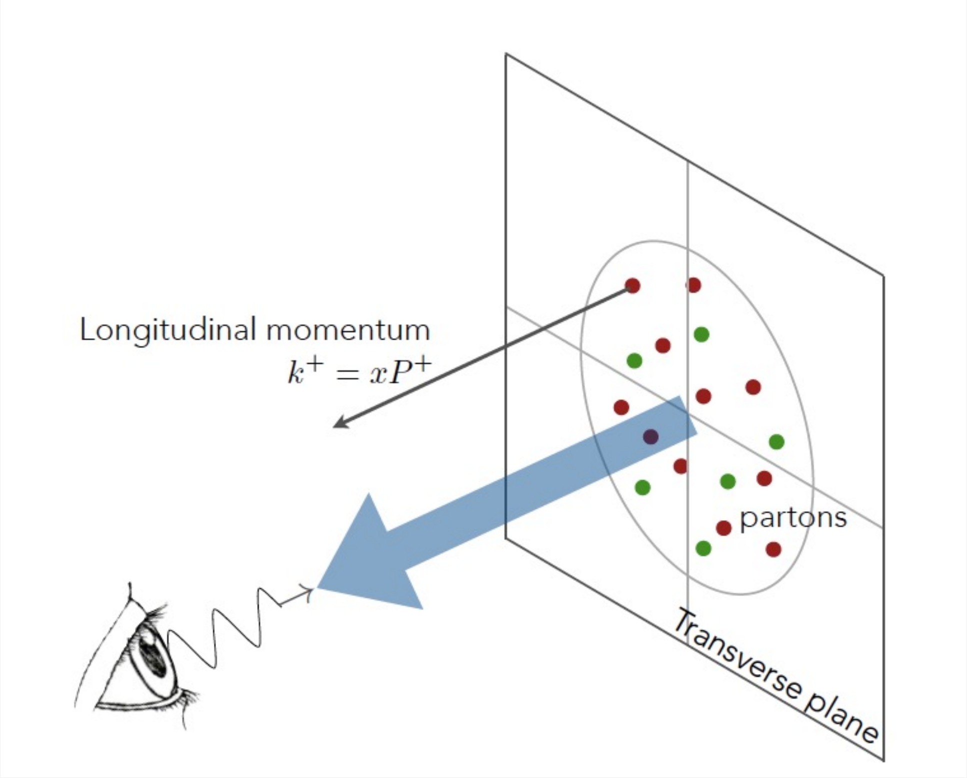
Will try to understand the glue (gluons) that binds us all

Will explore how the spin (1/2) of the proton is made from the spin and orbital angular momentum of the quarks and gluons

Will explore the correlations between spin/OAM and intrinsic transverse momentum

COLLINEAR PDFS : NUCLEON STRUCTURE IN 1-D





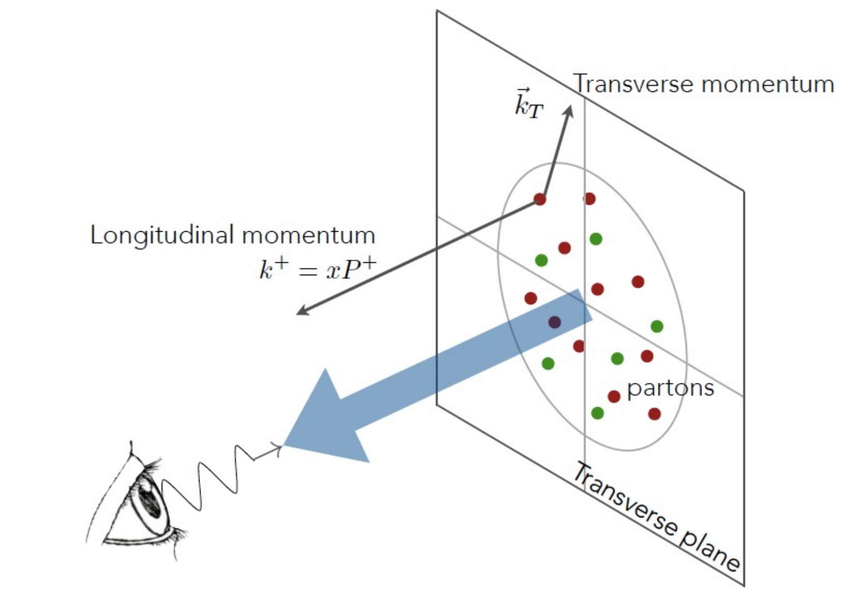
Motion of quarks in the transverse plane ignored

Non-perturbative :
Is extracted by fitting
Experimental data

Scale evolution of
pdfs can be
calculated

Independent of process : once extracted can be used to predict cross section of another process as the scale evolution is known

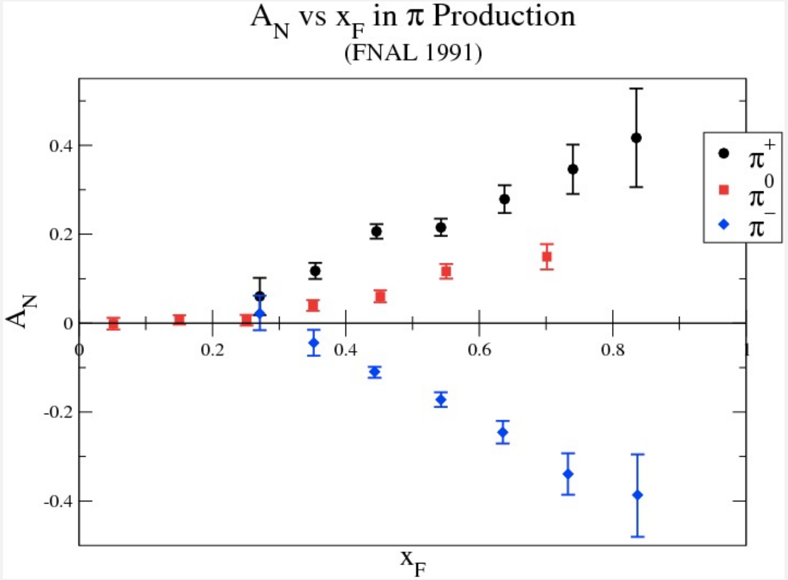
TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTIONS (TMDS)



Large (30-40%) Single transverse spin asymmetries were seen at FermiLab and RHIC experiments

Such large asymmetries cannot be explained in terms of collinear leading twist pdfs : need TMDs, or twist three pdfs

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

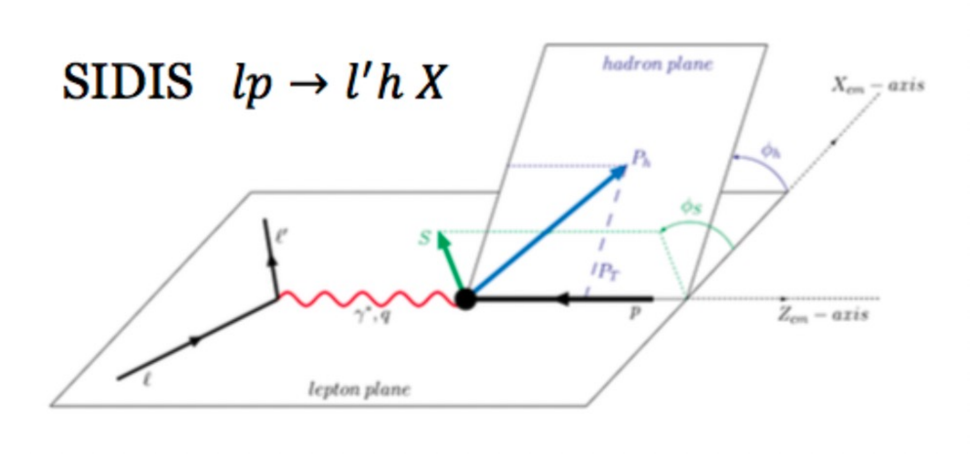


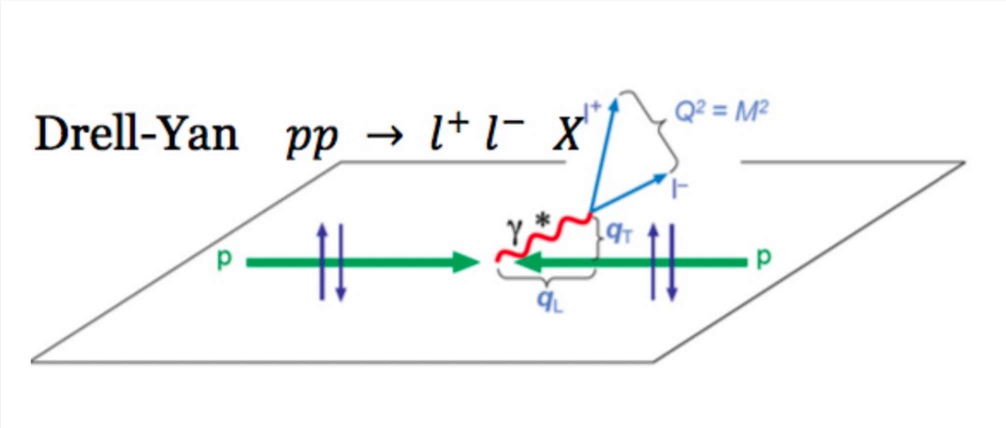
TMDs : functions of x and intrinsic transverse momentum : Gives a 3 D picture of the nucleon in momentum space

Correlations of spin, OAM and k_T : in terms of TMDs

TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

TMDs play a role in processes where two scales are present $Q^2 >> q_T^2$





For SIDIS and DY, TMD factorization is proven to all orders in α_s and leading twist

Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

For some processes, attempts have been made to prove TMD factorization at one loop and beyond leading twist

TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

$$d\sigma^{\ell p\to \ell h X} = \sum_q \underbrace{f_q(x, \boldsymbol{k}_\perp; Q^2)}_{\text{TMD-PDFs}} \otimes \underbrace{d\hat{\sigma}^{\ell q\to \ell q}(y, \boldsymbol{k}_\perp; Q^2)}_{\text{hard scattering}} \otimes \underbrace{D_q^h(z, \boldsymbol{p}_\perp; Q^2)}_{\text{TMD-FFs}}$$

TMDs play an important role in single spin and azimuthal asymmetries

Process dependent due to the gauge link or Wilson line in the operator

Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P, S \left| \overline{\psi}(0)\, \mathcal{U}^{\mathcal{C}}_{[0,\xi]}\, \psi(\xi) \right| P, S \right\rangle \qquad \mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu A^\mu(s)\right)$$

Φ : quark correlator, parametrized in terms of TMDs

Gauge link : resummation of initial and/or final state gluon exchanges : process dependent

QUARK TMDS

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T}, h_{1T}^{\perp}$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

There are eight quark TMDs at leading twist

Only three of them survive after transverse momentum integration

Two TMDs, Sivers function and Boer-Mulders function are odd under time reversal

TMDs contribute in different azimuthal angle asymmetries

Extraction of unpolarized TMD as well as the Sivers function Upto N^3LL

Pavia 2017, JHEP 06 (2017)
Scimemi, Vladimirov, JHEP 06 (2020)
MAP Collaboration, JHEP (2022)

Bury, Prokudin, Vladimirov, PRL 126 (2021)
Echevarria, Kang, Terry, JHEP 01 (2021)
Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

GLUON TMDS

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g,\ h_{1T}^{\perp g}$

$h_1^{\perp g}$: Linearly polarized gluon distribution in unpolarized hadron; T even

$f_{1T}^{\perp g}$ Gluon Sivers function in Transversely polarized proton

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

$h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2} h_{1T}^{\perp g}$ Vanish under p_T integration

In contrast to quark TMDs, very little is known about gluon TMDs

$$\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P,S|\,\mathrm{Tr}_c\big[\,F^{+\nu}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}^{\mathcal{C}'}_{[\xi,0]}\,\big]\,|P,S\rangle$$

Gluon TMDs need two gauge links for gauge invariance

Mulders, Rodrigues, PRD 63 (2001)
Buffing, Mukherjee, Mulders, PRD 88 (2013)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

SIMPLE EXAMPLE OF PROCESS DEPENDENCE OF TMDS : SIVERS EFFECT

Diff cross section for SIDIS with transversely polarized proton can be written as

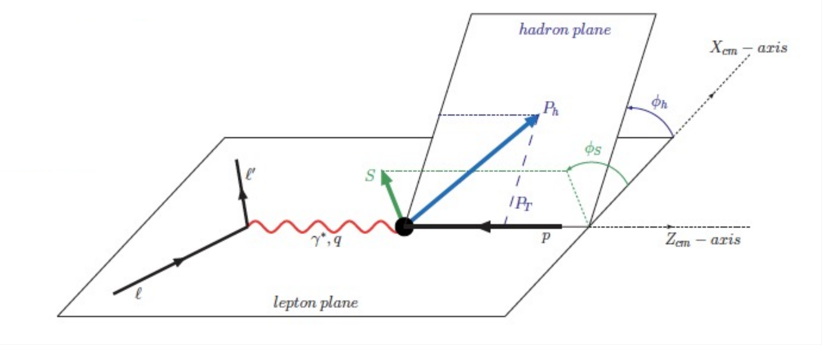
$$
\frac{d\sigma^{\ell+p(S_T)\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = \frac{2\,\alpha^2}{Q^4} \times
$$

$$
\Bigg\{ \frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + (1-y)\, \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}
$$

$$
+ \Bigg[\frac{1+(1-y)^2}{2} \sin(\phi_h-\phi_S)\, F_{UT}^{\sin(\phi_h-\phi_S)} + (1-y)\, \sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}
$$

$$
+ \ (1-y)\, \sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}
$$

$$
+ \ (2-y)\sqrt{1-y} \left(\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \right) \Bigg] \Bigg\}
$$

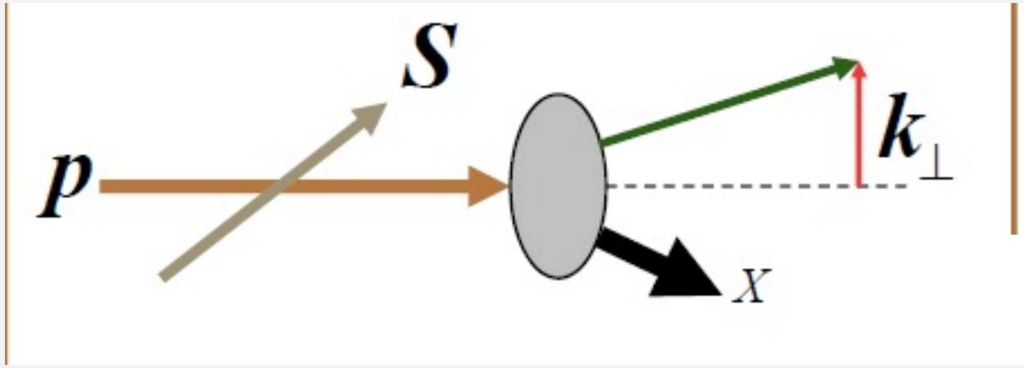


$$
F_{UT}^{\sin(\phi-\phi_S)} \sim \sum e_a^2\, f_{1T}^{\perp a} \otimes D_1^a
$$

Sivers Function

F functions contain different TMDs : each come with a different azimuthal modulation

SIVERS FUNCTION



$$f_{q/p,S}(x,k_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

Sivers function TMD (D. Sivers. (1990) is related to the probability to find an unpolarized quark inside a transversely polarized nucleon

It includes the correlation between the quark intrinsic transverse momentum and the transverse spin of the proton

In some models it is related to the orbital angular momentum of the quarks

Meissner, Metz, Goeke, PRD 76 (2007), 034002

T-odd function ; depends on gauge link

PROCESS DEPENDENCE OF TMDS

Gauge link is also present in collinear pdfs : but it is possible to choose a gauge (light-front gauge) where the gauge link becomes unity.

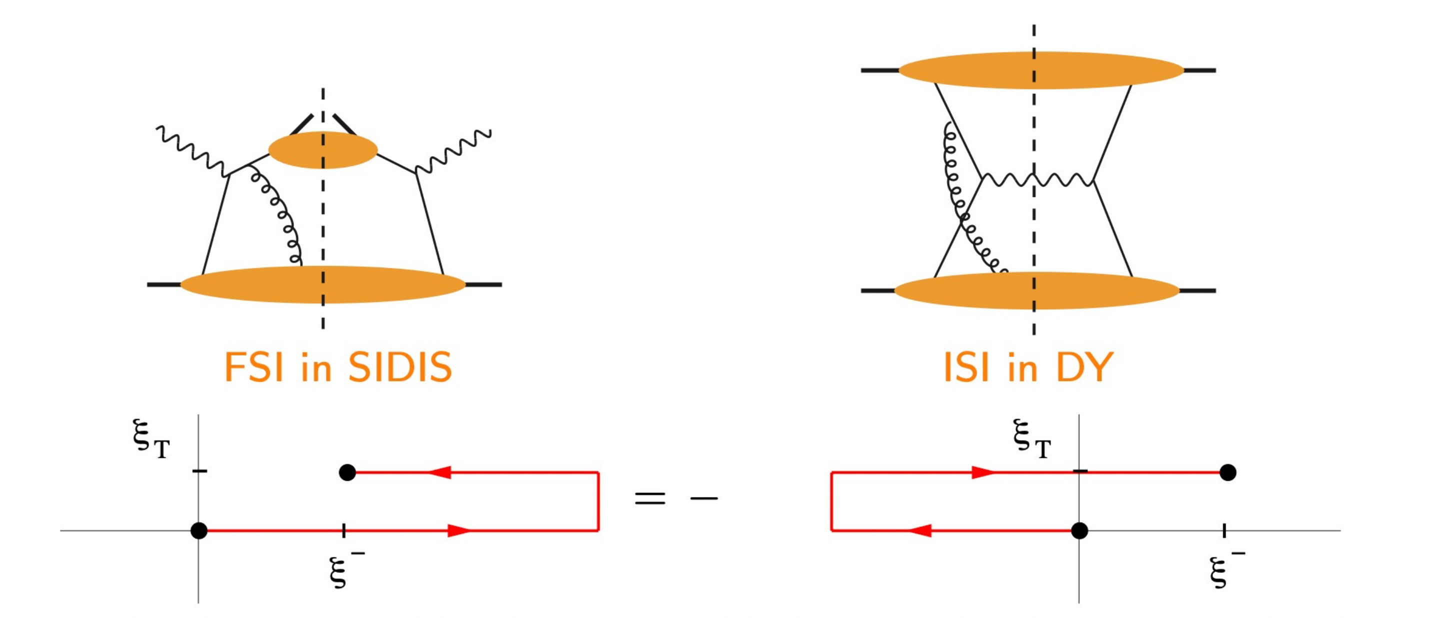
This is because the collinear pdf operator is bilocal only in the longitudinal direction but TMD operator is bilocal both in longitudinal and transverse direction

$\bar{\psi}(y^-)\Gamma\psi(0)$ $\bar{\psi}(y^-, y^\perp)\Gamma\psi(0)$ $y^- = y^0 - y^3$

For TMDs, even if one chooses the light cone gauge the effect of the gauge link remains and in fact plays a very important role for the T-odd TMDs like Sivers function.
Such TMDs would be zero if the gauge link is not taken into account

J. C. Collins, Phys. Lett. B 536 (2002) 43

SIVERS FUNCTION : PROCESS DEPENDENCE



Gauge link : depends on specific process . Example : SIDIS (final state interaction, future pointing gauge link) and Drell Yan (initial state interaction, past pointing gauge link)

Sivers function in Drell-Yan process is same in magnitude but opposite in sign compared to the Sivers function probed in semi-inclusive DIS

$$f_{1T}^{\perp\,[DY]}(x, \boldsymbol{k}_{\perp}^2) = -f_{1T}^{\perp\,[SIDIS]}(x, \boldsymbol{k}_{\perp}^2)$$

However, more complex processes have complex gauge link structure, and factorization is not always guaranteed

LINEARLY POLARIZED GLUON DISTRIBUTIONS

Linearly polarized gluon distributions were first introduced in

Mulders and Rodrigues, PRD 63, 094021 (2001)

Can be Weizsacker-Williams (WW) or dipole type depending on gauge link or color flow. Can be probed in different processes: extensive literature

Operator structure of these two unintegrated gluon distributions are different : WW distribution contains both past or both future pointing gauge links and dipole distributions contain one past and one future pointing gauge link

Linearly polarized gluon TMD : Measures an interference between an amplitude when the active gluon is polarized along x (or y) direction and a complex conjugate amplitude with the gluon polarized in y (or x) direction in an unpolarized hadron

Affects unpolarized cross section as well as generates a cos 2ϕ asymmetry

GLUON SIVERS FUNCTION (GSF)

Distribution of quarks and gluons in a transversely polarized proton is not left-right symmetric with respect to the plane formed by the momentum and spin directions – this generates an asymmetry called Sivers effect

D. Sivers, PRD 41, 83 (1990)

Highly sensitive to the color flow of the process and on initial/final state interactions (T-odd)

In some models, the Sivers function TMD is related to the orbital angular momentum of the quarks

Very little is known about GSF apart from a positivity bound

Depending on the gauge link in the operator structure there can be two different gluon Sivers function, f-type and d-type

Bomhof and Mulders, JHEP 02, 029 (2007),
Buffing, AM, Mulders, PRD 88, 054027 (2013)

Burkardt's sum rule, which states that the total transverse momentum of all quarks and gluon in a transversely polarized proton is zero, still leaves some room for GSF (30 %), moreover d type GSF is not constrained by it.

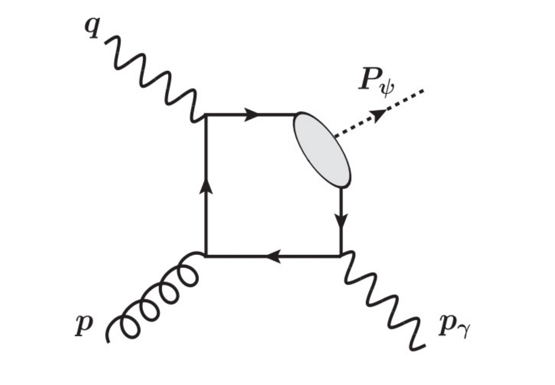
M. Burkardt, Phys. Rev. D 69, 091501 (2004)

J/ψ production and J/Ψ+ jet production at eP collision are effective ways to probe the GSF

GLUON TMDS IN J/ψ AND PHOTON PRODUCTION

We consider the following process where the proton can be unpolarized or transversely polarized

$$e(l) + p^{\uparrow}(P) \rightarrow e(l') + J/\psi(P_{\psi}) + \gamma(p_{\gamma}) + X,$$



$$P^{\mu} = n_{-}^{\mu} + \frac{M_p^2}{2} n_{+}^{\mu} \approx n_{-}^{\mu},$$

$$q^{\mu} = -x_B n_{-}^{\mu} + \frac{Q^2}{2x_B} n_{+}^{\mu} \approx -x_B P^{\mu} + (P \cdot q) n_{+}^{\mu},$$

$$l^{\mu} = \frac{1-y}{y} x_B n_{-}^{\mu} + \frac{1}{y}\frac{Q^2}{2x_B} n_{+}^{\mu} + \frac{\sqrt{1-y}}{y} Q \hat{l}_{\perp}^{\mu},$$

$$Q^2 = x_b y S \qquad S = (P+l)^2$$

$$y = \frac{P \cdot q}{P \cdot l} \qquad x_B = \frac{Q^2}{2P \cdot q}$$

$$\mathrm{d}\sigma = \frac{1}{2S} \frac{\mathrm{d}^3 \boldsymbol{l}'}{(2\pi)^3 2E_{l'}} \frac{\mathrm{d}^3 \boldsymbol{P}_{\psi}}{(2\pi)^3 2E_{\psi}} \frac{\mathrm{d}^3 \boldsymbol{p}_{\gamma}}{(2\pi)^3 2E_{\gamma}} \times \int \mathrm{d}x \mathrm{d}^2 \boldsymbol{p}_T (2\pi)^4 \delta^4(q + p - P_{\psi} - p_{\gamma}) \times \frac{1}{Q^4} L^{\mu\nu}(l,q) \Phi_g^{\rho\sigma}(x, \boldsymbol{p}_T) H_{\mu\rho} H_{\nu\sigma}^{*}.$$

Partonic subprocess : virtual photon-gluon fusion

J/ψ production in color singlet channel in virtual photon-photon fusion is suppressed due to a larger gluon density

Use TMD factorization

PRODUCTION OF J/Ψ IN NRQCD

In NRQCD the heavy quark pair is produced in the hard process either in color octet or in color singlet configuration

Then they hadronize to form a color singlet quarkonium state of given quantum numbers through soft gluon emission

Hard process is calculated perturbatively and soft process is given in terms of long distance matrix elements (LDMEs) that are determined from data

The LDMEs are categorized by performing an expansion in terms of the relative velocity of the heavy quark v in the limit $v \ll 1$

The theoretical predictions are arranged as double expansions in terms of v as well as α_s.

C. E. Carlson and R. Suaya, Phys. Rev. D 14, 3115 (1976).
E. L. Berger and D. L. Jones, Phys. Rev. D 23, 1521 (1981).
R. Baier and R. Ruckl, Phys. Lett. B 102B, 364 (1981).
R. Baier and R. Ruckl, Nucl. Phys. B201, 1 (1982).
E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995).
P. L. Cho and A. K. Leibovich, Phys. Rev. D 53, 150 (1996).

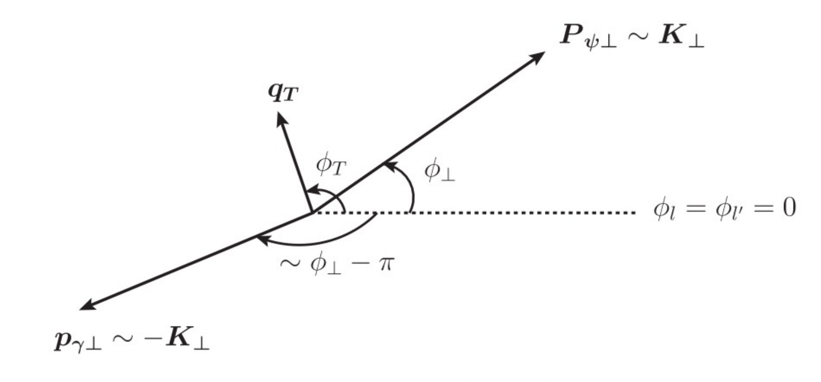
G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 51, 1125 (1995); 55, 5853(E) (1997).

GLUON TMDS IN J/ψ AND PHOTON PRODUCTION

J/ψ and photon almost back to back

$$\boldsymbol{q}_T \equiv \boldsymbol{P}_{\psi\perp} + \boldsymbol{p}_{\gamma\perp}, \qquad \boldsymbol{K}_\perp \equiv \frac{\boldsymbol{P}_{\psi\perp} - \boldsymbol{p}_{\gamma\perp}}{2}.$$

$$q_T \ll K_\perp$$



We use NRQCD to calculate J/ψ production. At leading order only one CO state ${}^3S_1^{(8)}$ contributes

Cross section depends on only one LDME at LO , and the asymmetry becomes independent of the LDME

$$\Phi_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{1}{2x}\left\{-g_T^{\mu\nu} f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp g}(x, \boldsymbol{p}_T^2)\right\}$$

$$\Phi_T^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{1}{2x}\left\{-g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, \boldsymbol{p}_T^2) + i\epsilon_T^{\mu\nu}\frac{p_T \cdot S_T}{M_p} g_{1T}^g(x, \boldsymbol{p}_T^2) + \frac{p_{T\rho}\epsilon_T^{\rho\{\mu} p_T^{\nu\}}}{2M_p^2}\frac{p_T \cdot S_T}{M_p} h_{1T}^{\perp g}(x, \boldsymbol{p}_T^2) - \frac{p_{T\rho}\epsilon_T^{\rho\{\mu} S_T^{\nu\}} + S_{T\rho}\epsilon_T^{\rho\{\mu} p_T^{\nu\}}}{4M_p} h_{1T}^g(x, \boldsymbol{p}_T^2)\right\}$$

In the back-to-back kinematics, we approximate

$$\boldsymbol{P}_{\psi\perp} \simeq -\boldsymbol{p}_{\gamma\perp} \simeq \boldsymbol{K}_\perp.$$

Mulders and Rodrigues, PRD 63, 094021 (2001)

CROSS SECTION AND ASYMMETRY

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}y\mathrm{d}x_B\mathrm{d}^2\boldsymbol{q}_T\mathrm{d}^2\boldsymbol{K}_\perp} \equiv \mathrm{d}\sigma(\phi_S,\phi_T,\phi_\perp) = \mathrm{d}\sigma^U(\phi_T,\phi_\perp) + \mathrm{d}\sigma^T(\phi_S,\phi_T,\phi_\perp).$$

Unpolarized proton

$$\mathrm{d}\sigma^U = \mathcal{N}\bigg[(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp) f_1^g(x,\boldsymbol{q}_T^2) + (\mathcal{B}_0\cos 2\phi_T + \mathcal{B}_1\cos(2\phi_T-\phi_\perp) + \mathcal{B}_2\cos 2(\phi_T-\phi_\perp) + \mathcal{B}_3\cos(2\phi_T-3\phi_\perp) + \mathcal{B}_4\cos(2\phi_T-4\phi_\perp))\frac{\boldsymbol{q}_T^2}{M_p^2} h_1^{\perp g}(x,\boldsymbol{q}_T^2)\bigg],$$

Transversely polarized proton

$$\mathrm{d}\sigma^T = \mathcal{N}|\boldsymbol{S}_T|\bigg[\sin(\phi_S-\phi_T)(\mathcal{A}_0 + \mathcal{A}_1\cos\phi_\perp + \mathcal{A}_2\cos 2\phi_\perp)\frac{|\boldsymbol{q}_T|}{M_p} f_{1T}^{\perp g}(x,\boldsymbol{q}_T^2) + \cos(\phi_S-\phi_T)(\mathcal{B}_0\sin 2\phi_T + \mathcal{B}_1\sin(2\phi_T-\phi_\perp) + \mathcal{B}_2\sin 2(\phi_T-\phi_\perp) + \mathcal{B}_3\sin(2\phi_T-3\phi_\perp) + \mathcal{B}_4\sin(2\phi_T-4\phi_\perp))\frac{|\boldsymbol{q}_T|^3}{M_p^3} h_{1T}^{\perp g}(x,\boldsymbol{q}_T^2) + (\mathcal{B}_0\sin(\phi_S+\phi_T) + \mathcal{B}_1\sin(\phi_S+\phi_T-\phi_\perp) + \mathcal{B}_2\sin(\phi_S+\phi_T-2\phi_\perp) + \mathcal{B}_3\sin(\phi_S+\phi_T-3\phi_\perp) + \mathcal{B}_4\sin(\phi_S+\phi_T-4\phi_\perp))\frac{|\boldsymbol{q}_T|}{M_p} h_{1T}^g(x,\boldsymbol{q}_T^2)\bigg],$$

Weighted asymmetry :

$$A^{W(\phi_S,\phi_T)} \equiv 2\frac{\int \mathrm{d}\phi_S\mathrm{d}\phi_T\mathrm{d}\phi_\perp W(\phi_S,\phi_T)\mathrm{d}\sigma(\phi_S,\phi_T,\phi_\perp)}{\int \mathrm{d}\phi_S\mathrm{d}\phi_T\mathrm{d}\phi_\perp \mathrm{d}\sigma(\phi_S,\phi_T,\phi_\perp)},$$

Using the weight factors, specific azimuthal modulations are isolated.

C. Pisano, D. Boer, S. J. Brodsky, M. G. A. Buffing, and P. J. Mulders, J. High Energy Phys. 10 (2013) 024.

ASYMMETRY

$$A^{\cos 2\phi_T} = \frac{\boldsymbol{q}_T^2}{M_p^2} \frac{\mathcal{B}_0}{\mathcal{A}_0} \frac{h_1^{\perp g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)},$$

$$A^{\cos 2(\phi_T - \phi_\perp)} = \frac{\boldsymbol{q}_T^2}{M_p^2} \frac{\mathcal{B}_2}{\mathcal{A}_0} \frac{h_1^{\perp g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}.$$

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)},$$

$$A^{\sin(\phi_S + \phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p} \frac{\mathcal{B}_0}{\mathcal{A}_0} \frac{h_1^g(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$

$$A^{\sin(\phi_S - 3\phi_T)} = -\frac{|\boldsymbol{q}_T|^3}{2M_p^3} \frac{\mathcal{B}_0}{\mathcal{A}_0} \frac{h_{1T}^{\perp g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}.$$

$$h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2} h_{1T}^{\perp g},$$

The coefficients A and B are calculated in NRQCD. Only one CO state contributes in the cross section namely

$$\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{(8)})|0\rangle$$

Using cross section data one can fit the LDME

The LDME cancels in the asymmetries : clean probe of gluon TMDs

UNPOLARIZED DIFF CROSS SECTION

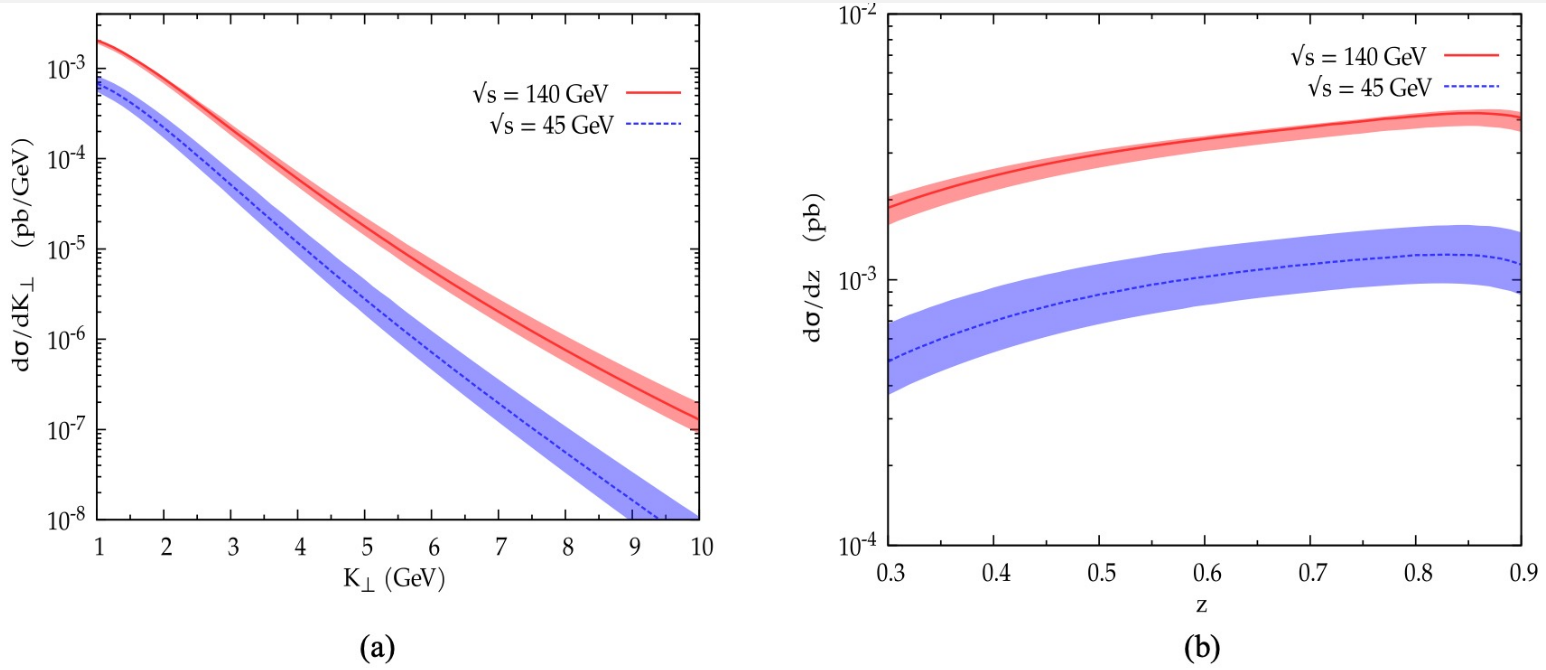


FIG. 3. Unpolarized differential cross section of $e + p \to e + J/\psi + \gamma + X$ process as a function of $K_\perp$ (a) and z (b) at $\sqrt{S} = 45$ and 140 GeV. The kinematical cuts are $1 < K_\perp < 10$ GeV, $0 < q_T < 1$ GeV, and $0.3 < z < 0.9$. For $\sqrt{s} = 140$ GeV we have taken $20 < W_{\gamma p} < 80$ GeV while for $\sqrt{s} = 45$ GeV, $10 < W_{\gamma p} < 40$ GeV. The bands are obtained by varying the factorization scale in the range $\frac{1}{2}\mu < \mu < 2\mu$.

D. Chakrabarti, R. Kishore, AM, S. Rajesh, Phys. Rev D *Phys.Rev.D* 107 (2023) 1, 014008

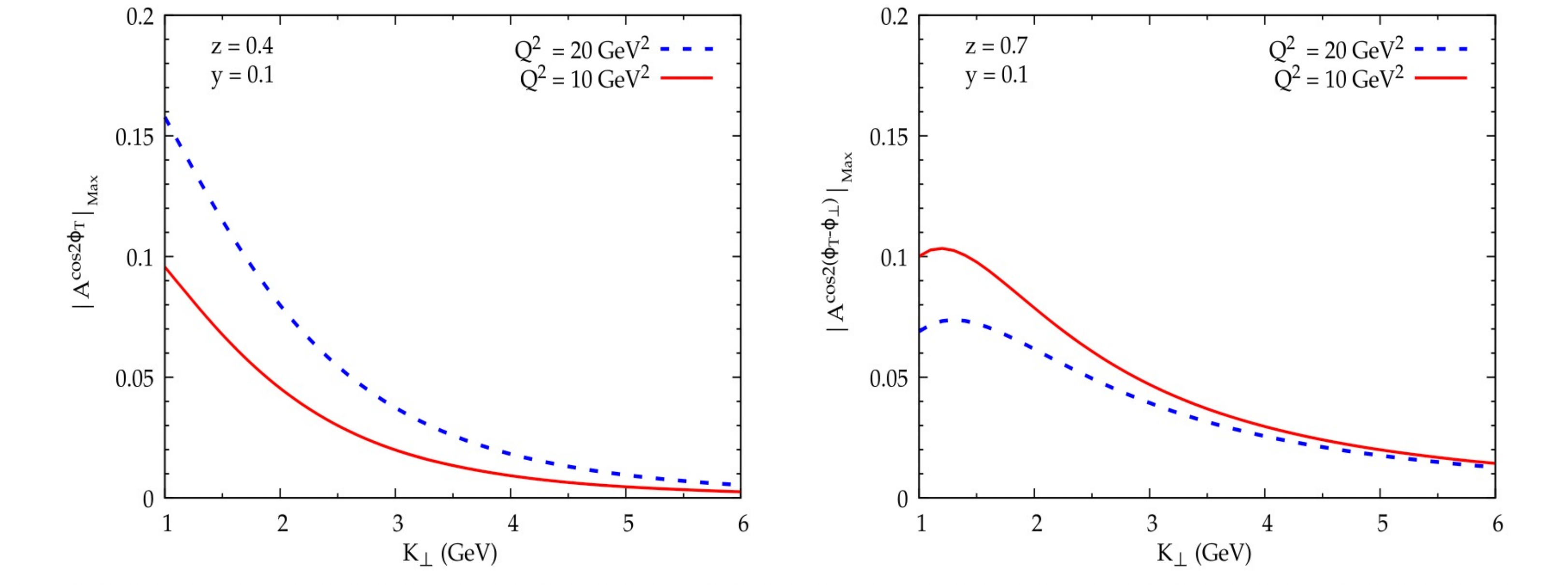
$$z = \frac{P.P_\psi}{P.q}$$

Higher cms energy probes smaller x region : high gluon density. Cross section larger

$$0.3 < z < 0.9.$$

Upper cutoff removes diffractive contribution and lower cutoff removes contribution from resolved photon

UPPER BOUND OF THE ASYMMETRIES



Upper bounds for two fixed values of Q^2

Model independent upper bound of the asymmetry is obtained by saturating the positivity bound of the TMDs

Upper bound is independent of CMS energy

$$|A^{\cos 2\phi_T}| \leq 2\frac{|\mathcal{B}_0|}{\mathcal{A}_0}, \qquad |A^{\cos 2(\phi_T-\phi_\perp)}| \leq 2\frac{|\mathcal{B}_2|}{\mathcal{A}_0}$$

Also independent of the LDME as it cancels in the ratio

Upper bound of the other two asymmetries are related to the upper bound of $A^{\cos 2\phi_T}$

D. Chakrabarti, R. Kishore, AM, S. Rajesh, Phys. Rev D *Phys.Rev.D* 107 (2023) 1, 014008

GAUSSIAN PARAMETRIZATION OF THE TMDS

Numerical estimate of the asymmetries are dependent on the TMD parametrization. We have used a Gaussian parametrization

$$f_1^g(x, \boldsymbol{q}_T^2) = f_1^g(x, \mu) \frac{e^{-\boldsymbol{q}_T^2/\langle q_T^2 \rangle}}{\pi \langle q_T^2 \rangle}$$

$f_1^g(x, \mu)$ Is the collinear unpolarized pdf at a scale $\langle q_T^2 \rangle = 1 \text{ GeV}^2$

$$\mu = \sqrt{M_\psi^2 + Q^2}$$

M. Anselmino, U. D'Alesio, and F. Murgia, Phys. Rev. D 67 (2003).

$$h_1^{\perp g}(x, \boldsymbol{q}_T^2) = \frac{M_p^2 f_1^g(x, \mu)}{\pi \langle q_T^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \frac{q_T^2}{r \langle q_T^2 \rangle}}$$

M_p is the proton mass, r=1/3

D. Boer et al, PRL 108 (2012), Boer and Pisano, PRD 86 (2012)

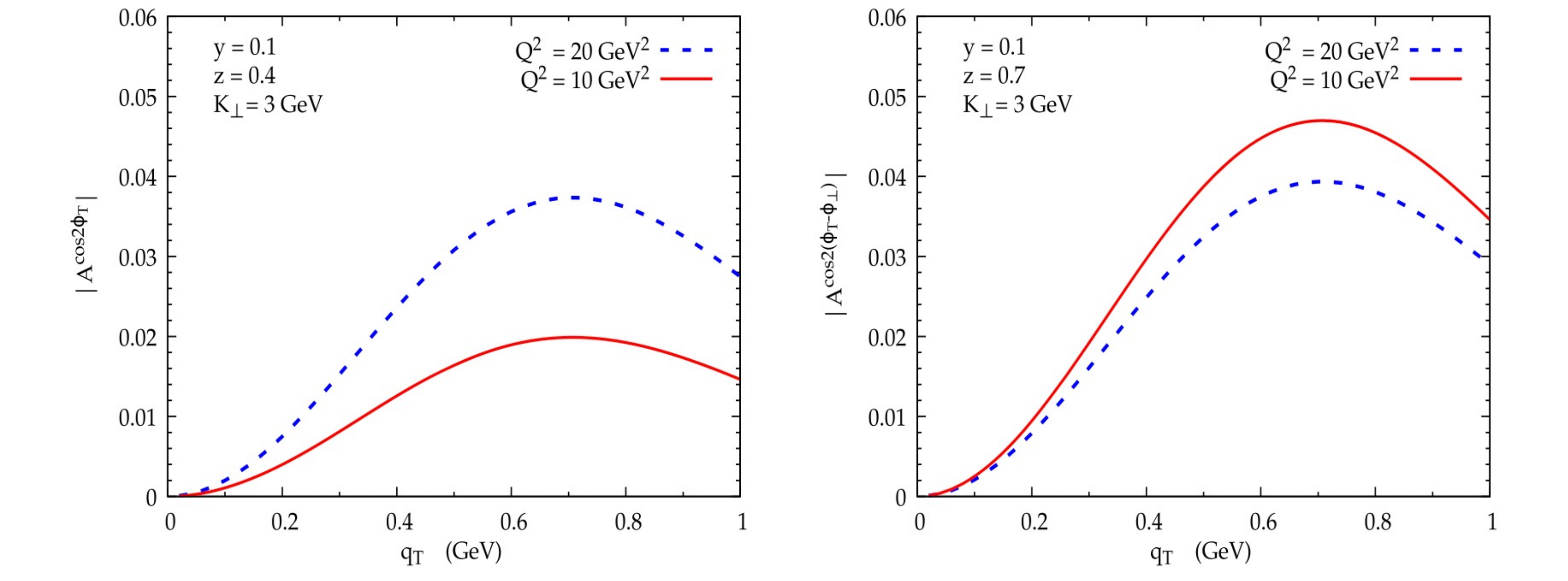
$$\Delta^N f_{g/p^\uparrow}(x, q_T) = \left(-\frac{2|\boldsymbol{q}_T|}{M_P}\right) f_{1T}^{\perp g}(x, q_T)$$
$$= 2\frac{\sqrt{2e}}{\pi} \mathcal{N}_g(x) f_{g/p}(x) \sqrt{\frac{1-\rho}{\rho}} q_T \frac{e^{-\boldsymbol{q}_T^2/\rho \langle q_T^2 \rangle}}{\langle q_T^2 \rangle^{3/2}}$$

$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

$$N_g = 0.25, \quad \alpha = 0.6, \quad \beta = 0.6, \quad \rho = 0.1.$$

U. D'Alesio, C. Flore, F. Murgia, C. Pisano, and P. Taels, Phys. Rev. D 99 (2019)

ASYMMETRY WITH GAUSSIAN PARAMETRIZATION OF TMDS



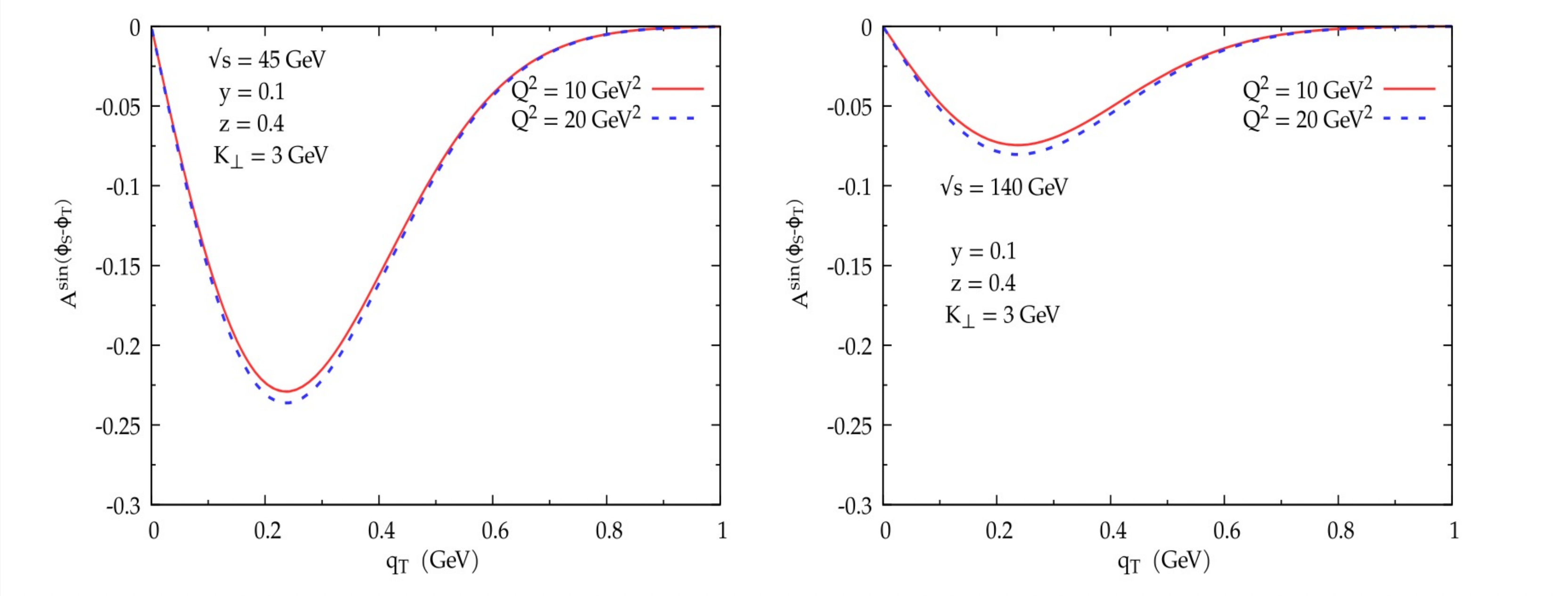
Asymmetry does not depend that much on CMS energy

Has peak value around q_T =0.7 GeV

kinematics chosen for back-to-back configuration

D. Chakrabarti, R. Kishore, AM, S. Rajesh, Phys. Rev D *Phys.Rev.D* 107 (2023) 1, 014008

SIVERS ASYMMETRY



Sivers asymmetry is negative, depends on the CMS energy

Does not depend much on the photon virtuality

Sivers asymmetry quite sizable at EIC kinematics and using Gaussian parametrization of the TMDs

D. Chakrabarti, R. Kishore, AM, S. Rajesh, Phys. Rev D *Phys.Rev.D* 107 (2023) 1, 014008

BACK-TO BACK PRODUCTION OF J/ψ AND JET

$$e^-(l) + p(\mathrm{P}) \to e^-(l') + J/\psi(\mathrm{P}_\psi) + \mathrm{jet}(\mathrm{P}_j) + X,$$

$$Q^2 = -q^2, \qquad s = (P+l)^2, \qquad W^2 = (P+q)^2,$$

$$x_B = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z = \frac{P\cdot \mathrm{P}_\psi}{P\cdot q}.$$

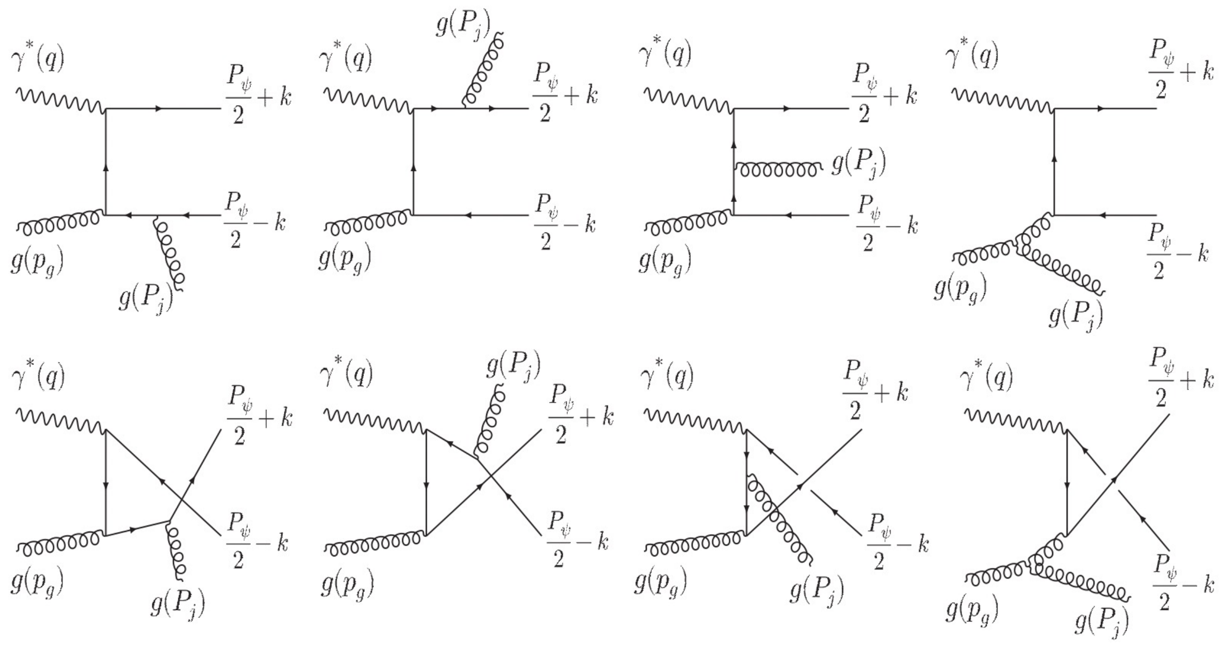


FIG. 1. Feynman diagrams for the partonic process $\gamma^*(q) + g(p_g) \to J/\psi(\mathrm{P}_\psi) + g(\mathrm{P}_j)$.

Use TMD factorization in the kinematics where the outgoing J/ψ and (gluon) jet are almost back-to back

Use NRQCD to calculate the J/ψ production

Also compare with the color singlet (CS) model result

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

BACK-TO-BACK PRODUCTION OF J/Ψ AND JET

$$d\sigma = \frac{1}{2s}\frac{d^3 l'}{(2\pi)^3 2E_{l'}}\frac{d^3 \mathrm{P}_\psi}{2E_\psi (2\pi)^3}\frac{d^3 \mathrm{P}_j}{2E_j (2\pi)^3} \times \int dx d^2\mathbf{p}_T (2\pi)^4 \delta^4(q+p_g-\mathrm{P}_j-\mathrm{P}_\psi) \times \frac{1}{Q^4} L^{\mu\mu'}(l,q)\Phi_g^{\nu\nu'}(x,\mathbf{p}_T^2)\mathcal{M}_{\mu\nu}^{g\gamma^*\to J/\psi g}\mathcal{M}_{\mu'\nu'}^{*g\gamma^*\to J/\psi g}.$$

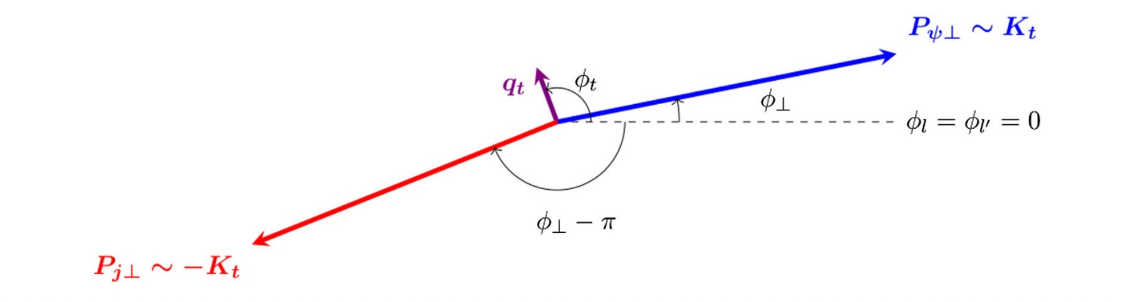
$$\mathcal{M}(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]g) = \sum_{L_z S_z}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\Psi_{LL_z}(\mathbf{k})\langle LL_z; SS_z|JJ_z\rangle \times \mathrm{Tr}[O(q,p_g,\mathrm{P}_\psi,k)\mathcal{P}_{SS_z}(\mathrm{P}_\psi,k)],$$

$$O(q,p_g,\mathrm{P}_\psi,k) = \sum_{i=1}^{8} C_i O_i(q,p_g,\mathrm{P}_\psi,k),$$

Contribution comes from the color singlet state $({}^3S_1^{(1)})$ And color octet states $({}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_{J(0,1,2)}^{(8)})$

In NRQCD, k, the relative momentum of the charm quark is small.
We have Taylor expanded the amplitude about k=0. The first term gives the S wave contribution and second term the p wave contribution

Formation of the bound state J/ψ from the heavy quark pair is encoded in the non-perturbative long distance matrix elements (LDMEs). These are obtained by fitting data



Upper bound of the asymmetries : U. D'Alesio, F. Murgia, C. Pisano, and P. Taels *Phys.Rev.D* 100 (2019) 9, 094016

ASYMMETRY

$$\mathbf{q}_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}, \qquad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}.$$

$$|\mathbf{q}_t| \ll |\mathbf{K}_t|.$$

$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = \frac{\int \mathrm{q}_t \mathrm{dq}_t \frac{\mathbf{q}_t^2}{M_p^2} \mathbb{B}_0 h_1^{\perp g}(x, \mathbf{q}_t^2)}{\int \mathrm{q}_t \mathrm{dq}_t \mathbb{A}_0 f_1^g(x, \mathbf{q}_t^2)}.$$

$$\frac{d\sigma}{dz dy dx_B d^2\mathbf{q}_t d^2\mathbf{K}_t} = \frac{1}{(2\pi)^4} \frac{1}{16 s z(1-z) Q^4} \Big\{ (\mathbb{A}_0 + \mathbb{A}_1 \cos\phi_\perp + \mathbb{A}_2 \cos 2\phi_\perp) f_1^g(x, \mathbf{q}_t^2) + \frac{\mathbf{q}_t^2}{M_P^2} h_1^{\perp g}(x, \mathbf{q}_t^2)(\mathbb{B}_0 \cos 2\phi_t + \mathbb{B}_1 \cos(2\phi_t - \phi_\perp) + \mathbb{B}_2 \cos 2(\phi_t - \phi_\perp) + \mathbb{B}_3 \cos(2\phi_t - 3\phi_\perp) + \mathbb{B}_4 \cos(2\phi_t - 4\phi_\perp)) \Big\}.$$

Gaussian parametrization of TMDs :

$$f_1^g(x, \mathbf{q}_t^2) = f_1^g(x, \mu) \frac{1}{\pi \langle \mathbf{q}_t^2 \rangle} e^{-\mathbf{q}_t^2/\langle \mathbf{q}_t^2 \rangle},$$

$$h_1^{\perp g}(x, \mathbf{q}_t^2) = \frac{M_p^2 f_1^g(x, \mu)}{\pi \langle \mathbf{q}_t^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \frac{\mathbf{q}_t^2}{r \langle \mathbf{q}_t^2 \rangle}},$$

Boer and Pisano, PRD, 2012

$$\langle \mathbf{q}_t^2 \rangle = 0.25\ \mathrm{GeV}^2. \qquad r=1/3$$

Spectator model :

$$F^g(x, \mathbf{q}_t^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}^g(x, \mathbf{q}_t^2; M_X).$$

M_X : mass of spectator : continuous

Spectral function

$$\rho_X(M_X) = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right],$$

$$L_X^2(\Lambda_X^2) = x M_X^2 + (1-x)\Lambda_X^2 - x(1-x)M^2.$$

$$\hat{f}_1^g(x, \mathbf{q}_t^2; M_X) = -\frac{1}{2} g^{ij} [\Phi^{ij}(x, \mathbf{q}_t, S) + \Phi^{ij}(x, \mathbf{q}_t, -S)]$$
$$= [(2Mxg_1 - x(M + M_X)g_2)^2 [(M_X - M(1-x))^2 + \mathbf{q}_t^2] + 2\mathbf{q}_t^2(\mathbf{q}_t^2 + xM_X^2) g_2^2 + 2\mathbf{q}_t^2 M^2 (1-x)(4g_1^2 - xg_2^2)] [(2\pi)^3 4xM^2 (L_X^2(0) + \mathbf{q}_t^2)^2]^{-1},$$

$$\hat{h}_1^{\perp g}(x, \mathbf{q}_t^2; M_X) = \frac{M^2}{\varepsilon_t^{ij} \delta^{jm} (p_t^j p_t^m + g^{jm} \mathbf{q}_t^2)} \varepsilon_t^{ln} \delta^{nr} [\Phi^{nr}(x, \mathbf{q}_t, S) + \Phi^{nr}(x, \mathbf{q}_t, -S)]$$
$$= [4M^2(1-x) g_1^2 + (L_X^2(0) + \mathbf{q}_t^2) g_2^2] \times [(2\pi)^3 x (L_X^2(0) + \mathbf{q}_t^2)^2]^{-1}.$$

A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taels, Eur. Phys. J. C 80, 733 (2020).

TMD EVOLUTION

Also incorportated TMD evolution in the asymmetry

TMD evolution is done in impact parameter space

Aybat and Rogers, PRD 83, 114042 (2011)

S_A and S_{np} are perturbative and non-perturbative Sudakov factors

$$\hat{f}(x, \mathbf{b}_t^2, Q_f^2) = \frac{1}{2\pi} \sum_{p=q,\bar{q},g} (C_{g/p} \otimes f_1^p)(x, Q_i^2) \times e^{-\frac{1}{2} S_A(\mathbf{b}_t^2, Q_f^2, Q_i^2)} e^{-S_{np}(\mathbf{b}_t^2, Q_f^2)},$$

$$S_A(\mathbf{b}_t^2, Q_f^2, Q_i^2) = \frac{C_A}{\pi} \int_{Q_i^2}^{Q_f^2} \frac{d\eta^2}{\eta^2} \alpha_s(\eta) \left(\log \frac{Q_f^2}{\eta^2} - \frac{11 - 2n_f/C_A}{6} \right) = \frac{C_A}{\pi} \alpha_s \left(\frac{1}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11 - 2n_f/C_A}{6} \log \frac{Q_f^2}{Q_i^2} \right).$$

$$S_{np} = \frac{A}{2} \log\left(\frac{Q_f}{Q_{np}}\right) \mathrm{b}_c^2, \qquad Q_{np} = 1 \text{ GeV}.$$

Boer, D'Alesio, Murgia, Pisano, and Taels, JHEP (2020) 40.

Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, EPJC(2020)

$$Q_f = \sqrt{M_\psi^2 + \mathrm{K}_t^2}. \qquad A = 2.3 \text{ GeV}^2$$

$$Q_i = 2e^{-\gamma_E}/\mathrm{b}_t$$

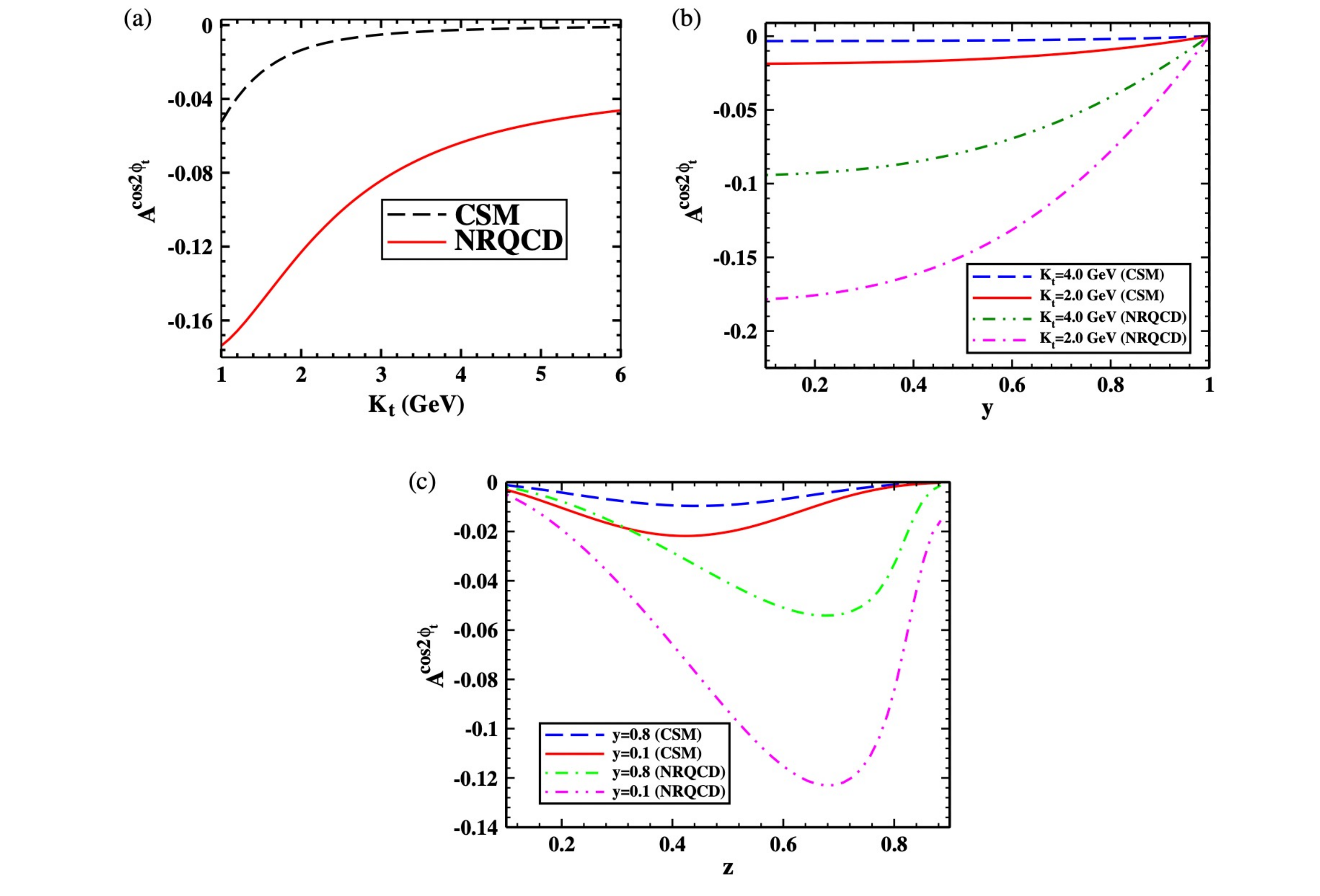
Used b_{t*} prescription to prevent Q_i larger than Q_f for low b_t

Final expressions are :

$$f_1^g(x, \mathbf{q}_t^2) = \frac{1}{2\pi} \int_0^\infty \mathrm{b}_t d\mathrm{b}_t J_0(\mathrm{b}_t \mathrm{q}_t) \Big\{ f_1^g(x, Q_f^2) - \frac{\alpha_s}{2\pi} \Big[\Big(\frac{C_A}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6} \log \frac{Q_f^2}{Q_i^2} \Big) f_1^g(x, Q_f^2) + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2) \log \frac{Q_f^2}{Q_i^2} - 2 f_1^g(x, Q_f^2) \Big] \Big\} \times e^{-S_{np}(\mathbf{b}_t^2)}.$$

$$\frac{\mathbf{q}_t^2}{M_p^2} h_1^{\perp g(2)}(x, \mathbf{q}_t^2) = \frac{\alpha_s}{\pi^2} \int_0^\infty d\mathrm{b}_t \mathrm{b}_t J_2(\mathrm{q}_t \mathrm{b}_t) \Big[C_A \int_x^1 \frac{d\hat{x}}{\hat{x}} \Big(\frac{\hat{x}}{x} - 1 \Big) f_1^g(\hat{x}, Q_f^2) + C_F \sum_{p=q,\bar{q}} \int_x^1 \frac{d\hat{x}}{\hat{x}} \Big(\frac{\hat{x}}{x} - 1 \Big) f_1^p(\hat{x}, Q_f^2) \Big] \times e^{-S_{np}(\mathbf{b}_t^2)}.$$

ASYMMETRY WITH GAUSSIAN PARAMETRIZATION



$\sqrt{s} = 140\ GeV$

$z = 0.7$ in (a) and (b)

$0.1 < y < 1$ in (a)

$K_t = 0.3\ GeV$ in (c)

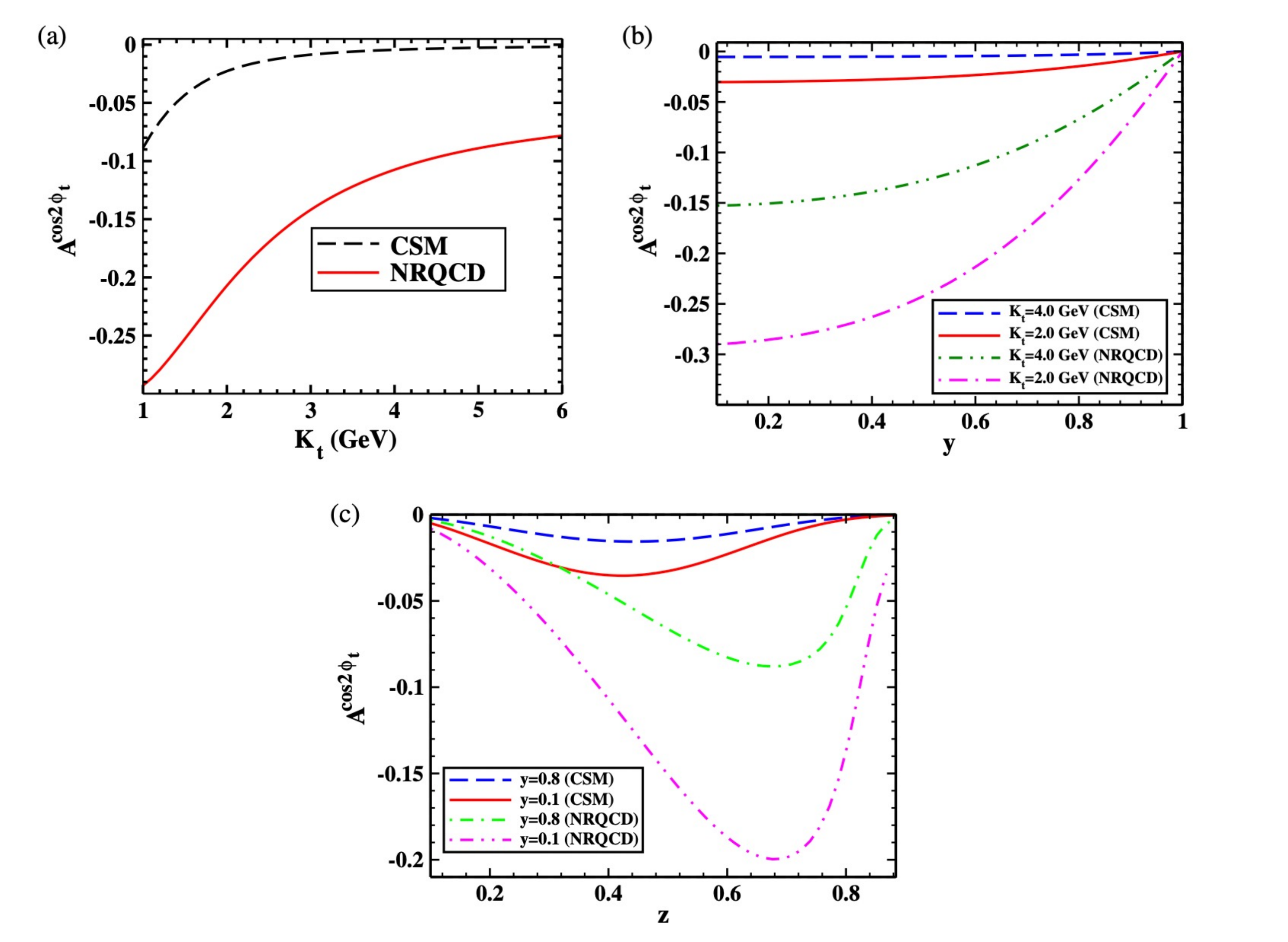
Magnitude of asymmetry does not change much if we take different value of cms energy

Showed both CSM and NRQCD results

LDMEs from Phys. Rev. Lett. 108, 242004 (2012).

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

ASYMMETRY IN SPECTATOR MODEL



$\sqrt{s} = 140\ GeV$

Kinematics same as previous plot

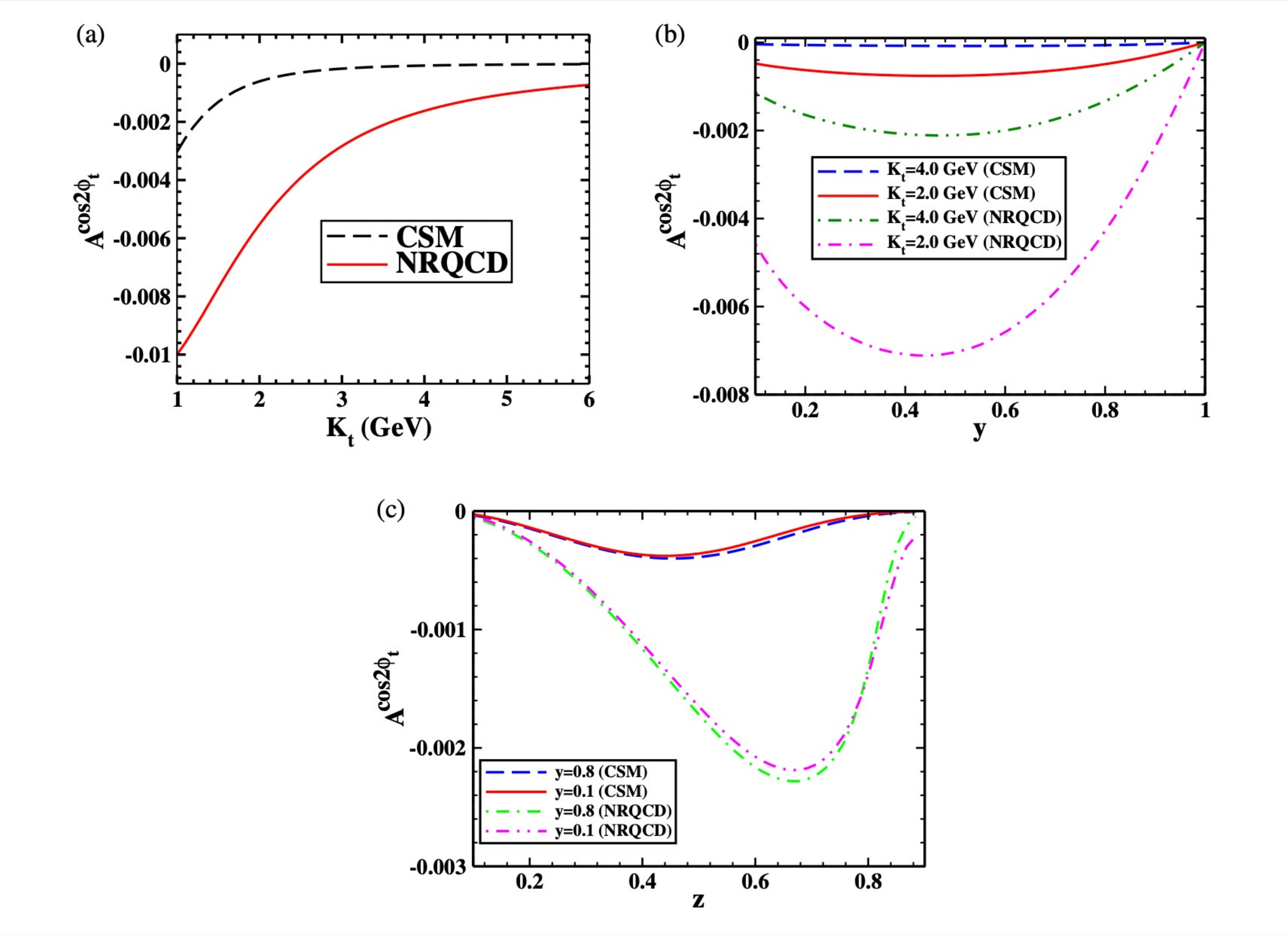
LDMEs from Phys. Rev. Lett. 108, 242004 (2012).

Magnitude of asymmetry in spectator model larger than in Gaussian parametrization of gluon TMDs

Sizable negative asymmetry in NRQCD, in contrast to CSM

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

ASYMMETRY AFTER INCORPORATING TMD EVOLUTION



Kinematics same as other plots

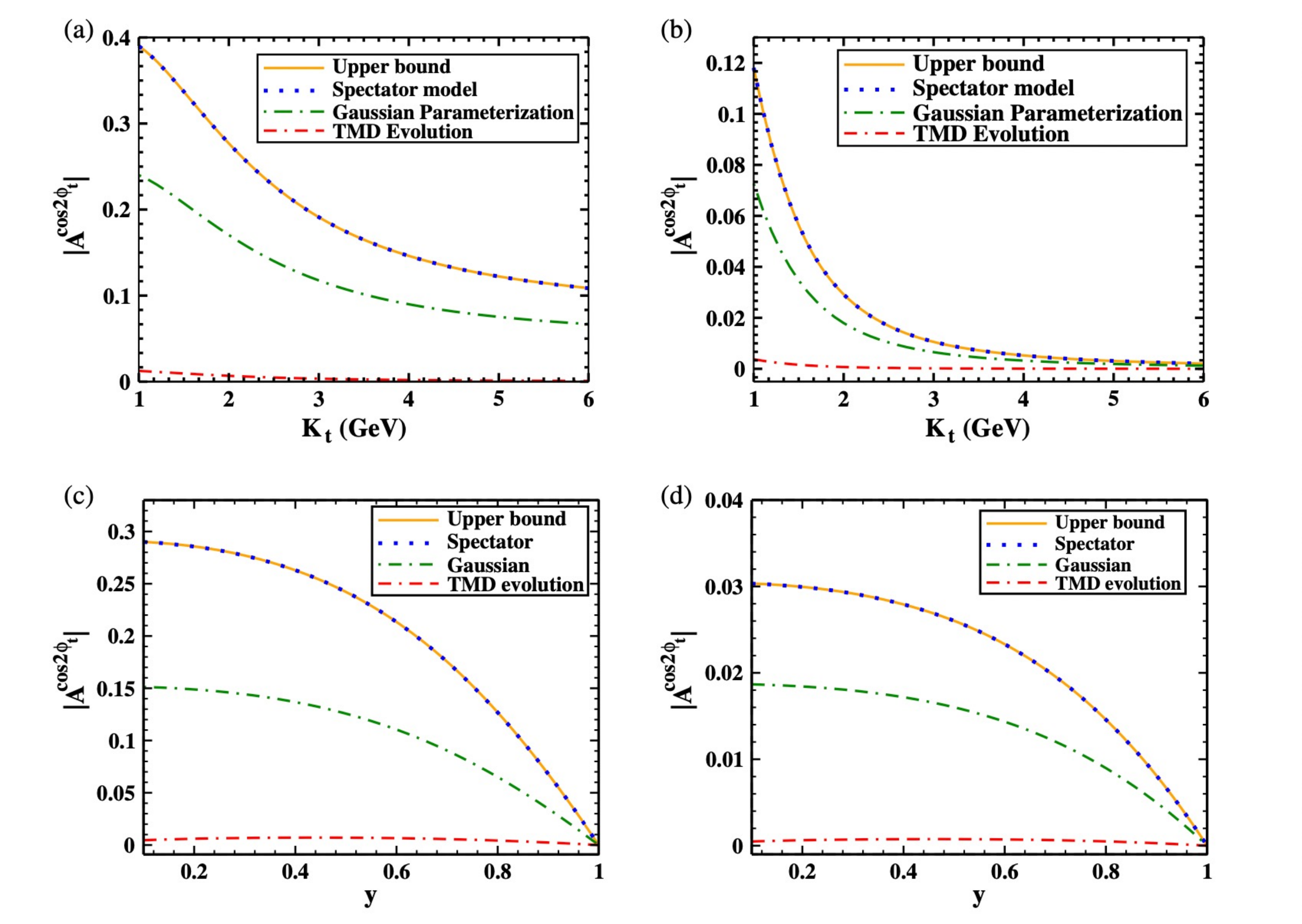
TMD evolution gives smaller asymmetry

This is because unpolarized TMD in the denominator has a LO term whereas the linearly polarized gluon TMD in the numerator gets contribution at NLO

Magnitude of asymmetry does not change much with CMS energy

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

UPPER BOUND OF THE ASYMMETRY COMPARED WITH DIFFERENT RESULTS



NRQCD

CS

$y = 0.3$ In upper panels

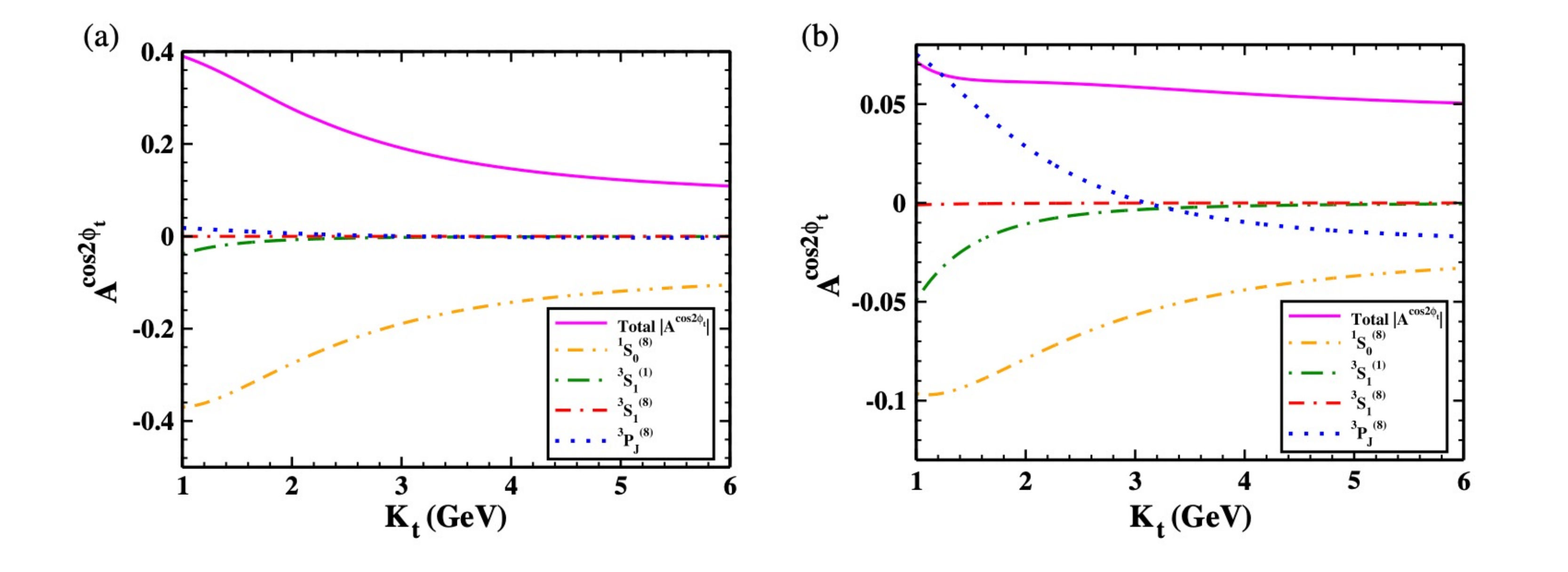
$K_t = 0.2\ GeV$ In lower panels

Result in spectator model in the kinematics considered overlaps with the upper bound saturating the positivity bound

Result is Gaussian parametrization lower than in spectator model

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

CONTRIBUTION TO ASYMMETRY FROM DIFFERENT STATES



$\sqrt{s} = 140\ GeV$

$$Q = \sqrt{M_\psi^2 + K_t^2},$$

y=0.3

(a) LDMEs from K.T. Chao et al, Phys. Rev. Lett. 108, 242004 (2012).

(b) LDMEs from Sharma and Vitev, Phys. Rev. C 87, 044905 (2013)

In (a) dominating contribution come from a single state whereas in (b) contributions come from several states

Raj Kishore, AM, Amol Pawar, M. Siddiqah *Phys.Rev.D* 106 (2022) 3, 034009

SUMMARY AND CONCLUSION

Upcoming EIC can probe the nucleons in three dimensions in terms of TMDs. In particular gluon TMDs can be probed in J/ψ production processes.

Discussed theoretical estimates of azimuthal asymmetries in back-to-back production of J/ψ -photon and J/ψ-jet in eP collision in the kinematics of EIC

Used NRQCD and CS mechanisms to calculate the J/ψ production rate

In J/ψ-photon production only one LDME contributes : asymmetry is independent of LDME choice. Robust probe of gluon TMDs

Cos 2ϕ asymmetry small but sizable Sivers asymmetry (Gaussian parametrization of gluon TMDs)

In J/ψ-jet production, significant contribution to cos 2ϕ asymmetry in NRQCD, smaller in CSM. Also considered effect of TMD evolution

Asymmetry depends on LDME sets chosen, does not depend much on CMS energy