FATE OF EARLY TIME ATTRACTOR SOLUTION UNDER FORCE

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MYSTERIOUS EFFECTIVENESS OF FLUID DYNAMICS

Applicability of fluid dynamics

One possibility: Knudsen number << 1

New definition: gradients around some reference configuration (e.g., local equilibrium) are small when compared to system temperature ~ stress tensor nearly isotropic.

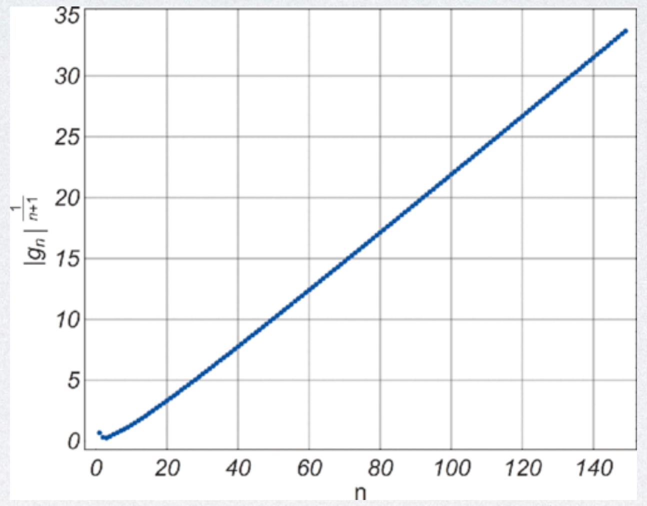
fluid dynamics can match exact results even if the gradient corrections are of order unity

Ultra-relativistic collisions of protons exhibits the same flow features as much larger systems produced in heavy-ion collisions

TOWARDS ATTRACTOR SOLUTION

rapid convergence of the hydrodynamic gradient expansion ?

not rapidly convergent, but is actually a divergent series!

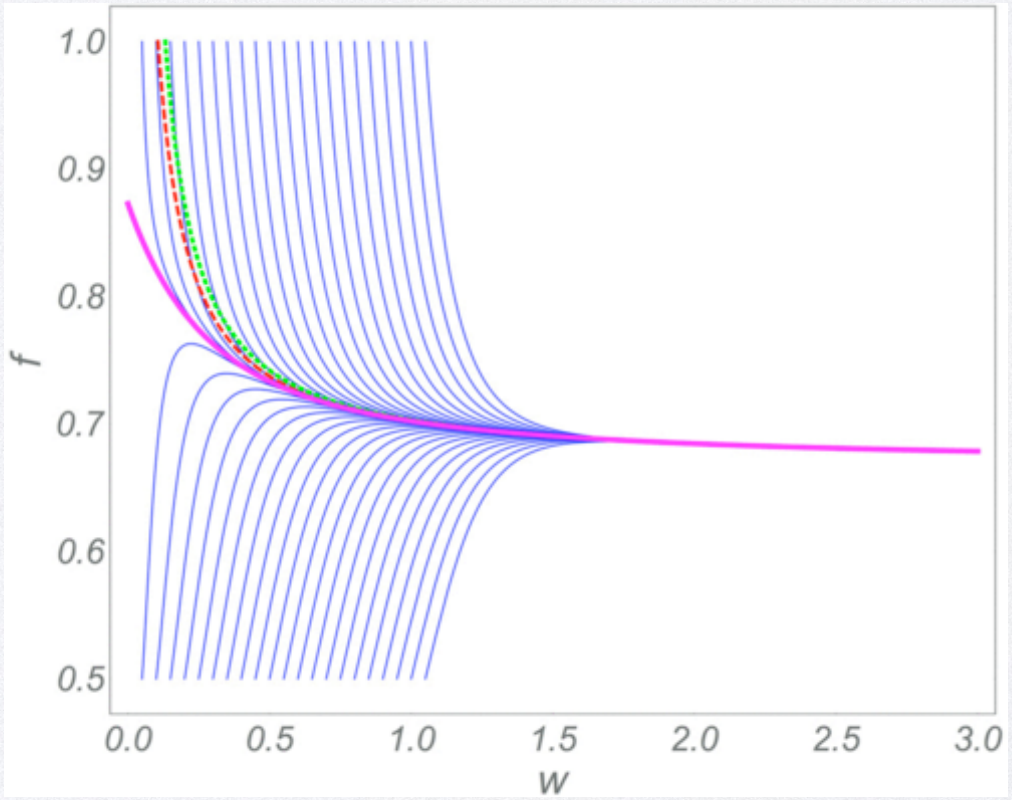


W Florkowski et al. Phys. Rev. D 94, 114025

Heller and Spalinski demonstrated that the Borel-resummed gradient series unique hydrodynamic attractor solution.

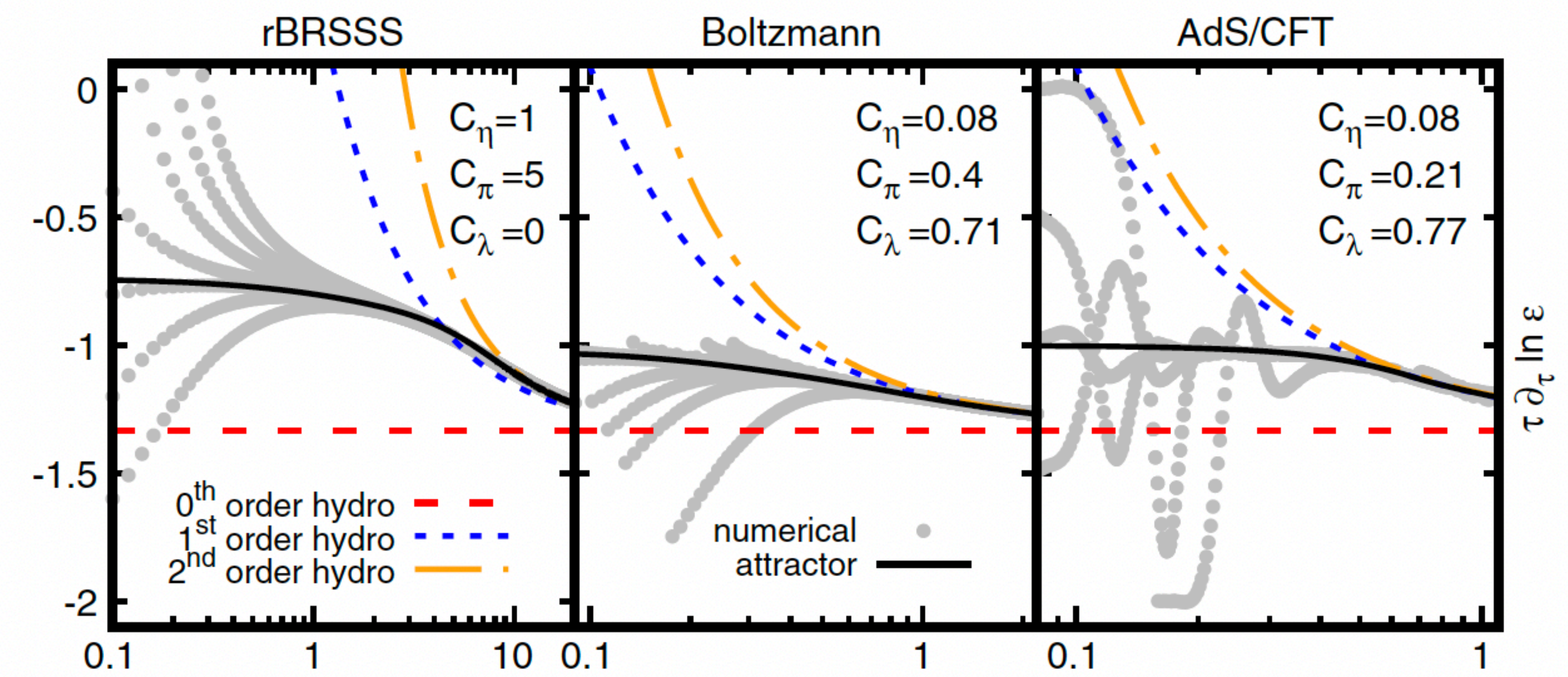
Hydrodynamics Beyond the Gradient Expansion: Resurgence and Resummation

Michal P. Heller and Michał Spaliński
Phys. Rev. Lett. **115**, 072501 – Published 14 August 2015



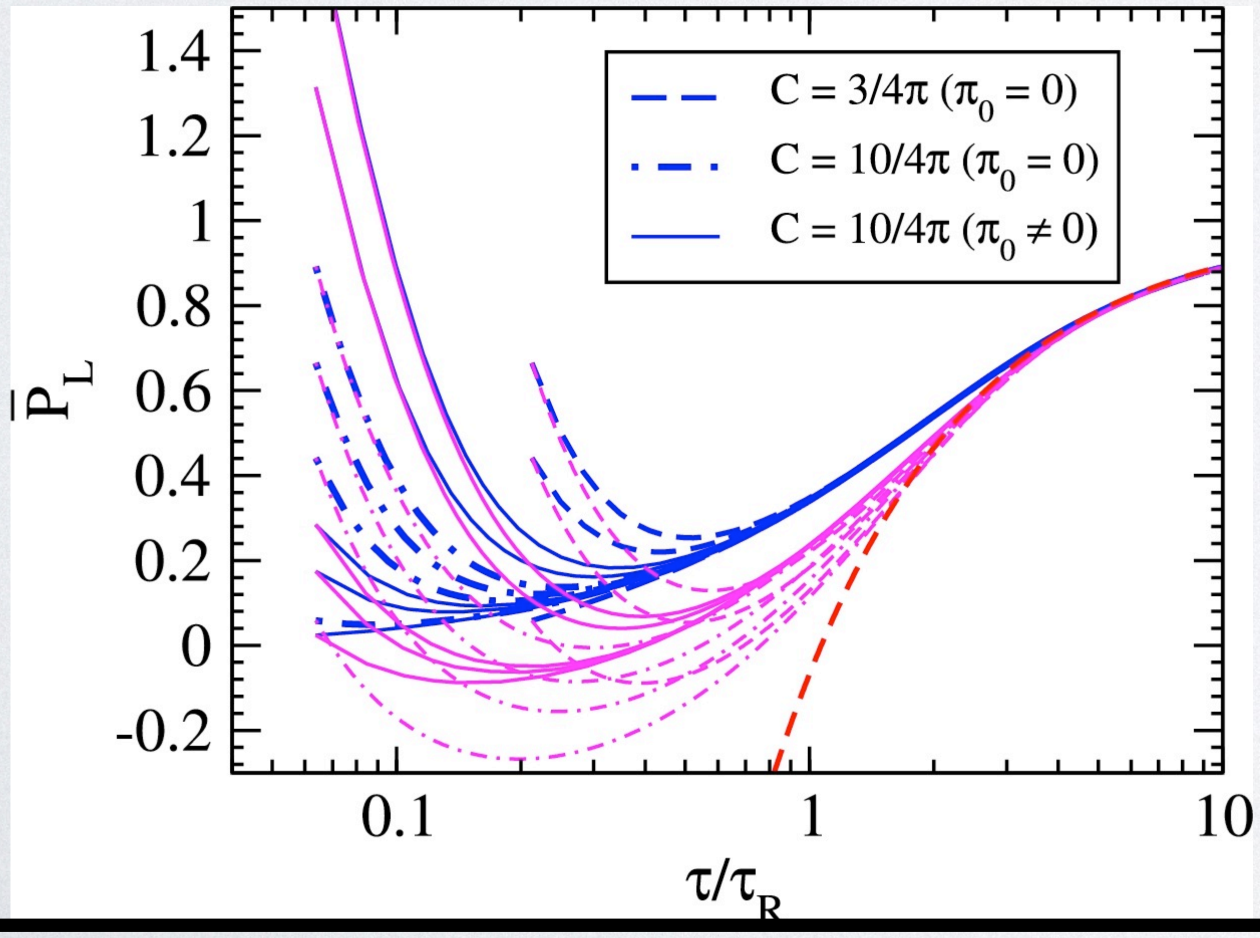
IS THAT A GENERAL FEATURE?

Conformal case with Bjorken symmetry P Romatschke PRL 120, 012301 (2018)



$$3\sigma^{\mu\nu}\sigma_{\mu\nu}/8T\nabla_\lambda u^\lambda = \frac{1}{\tau T}$$

Non-conformal case with Bjorken symmetry



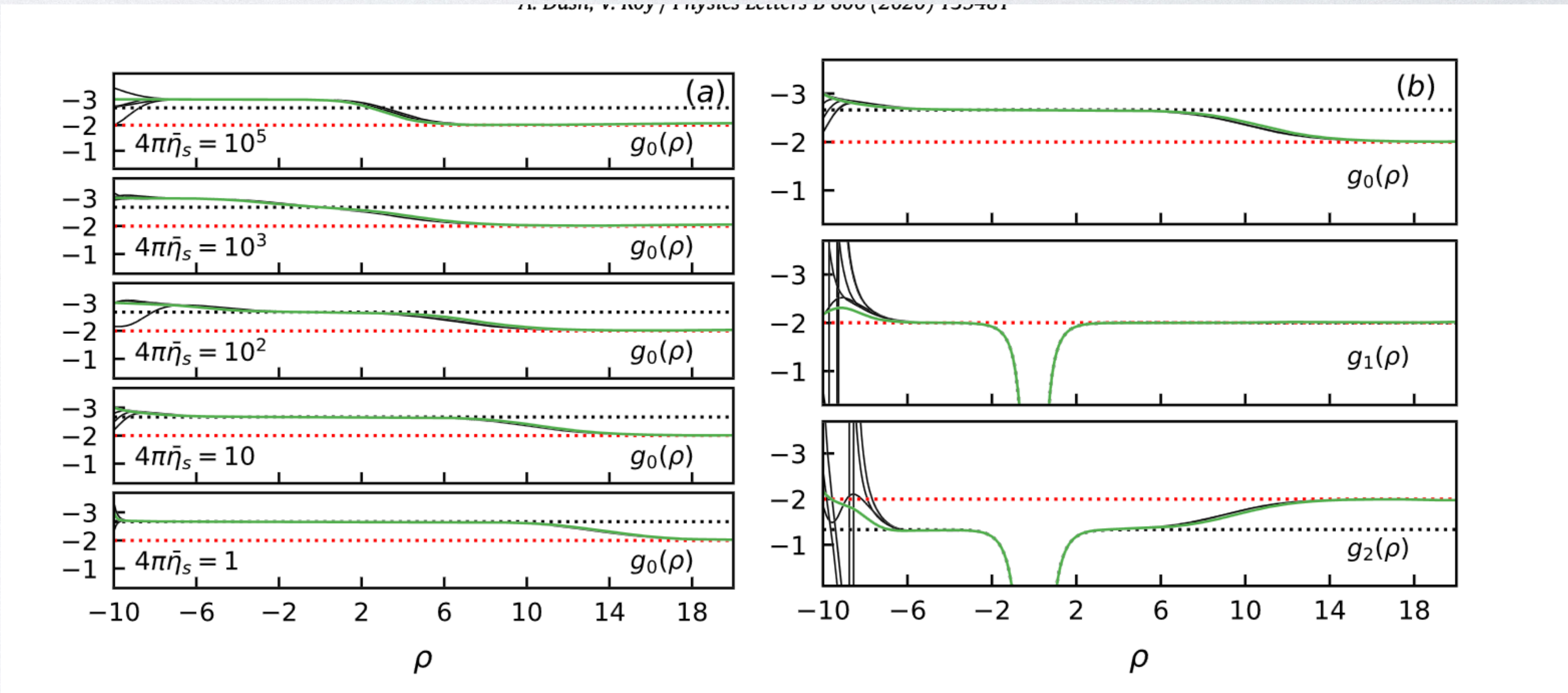
S Jaiswal et al. Physics Letters B Volume 824, 10 January 2022, 136820

Gubser-flow and attractor from angular moments

$$d\hat{s}^2 = -d\rho^2 + \cosh^2\rho\left(d\theta^2 + \sin^2\theta d\phi^2\right) + d\eta^2$$

$$\mathcal{L}_n = \int d\hat{P}\,(\hat{p}^\rho)^2 P_{2n}\left(\hat{p}_\eta/\hat{p}^\rho\right) f\left(\rho; \hat{p}_\Omega, \hat{p}_\eta\right)$$

$$g_n(\rho) = \frac{\partial \ln \mathcal{L}_n}{\partial \ln(\cosh\rho)}.$$



A. Dash, V. Roy, Physics Letters B 806 (2020) 135481

FATE OF EARLY TIME ATTRACTOR IN PRESENCE OF FORCE

$$p^\mu \frac{\partial f}{\partial x^\mu} + mK^\mu \frac{\partial f}{\partial p^\mu} - \Gamma^\sigma_{\mu\nu} p^\mu p^\nu \frac{\partial f}{\partial p^\sigma} = C(f)$$

$$C(f) = -\, u \cdot p \frac{(f - f_{eq})}{\tau_R}$$

Milne coordinate with Bjorken symmetry

$$\frac{\partial f(\tau, p_T, w)}{\partial \tau} = -\frac{\left(f - f_{eq}\right)}{\tau_R}$$

$$w \equiv p_\eta = \tau p^z = t p^z - z p^t$$

Exact solution

$$f(\tau, w, p_T) = D(\tau, \tau_0) f_0(w, p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_R(\tau')} D(\tau, \tau') f_{eq}(\tau', w, p_T)$$

EXACT SOLUTION FOR FLUID VARIABLES

Evolution of energy density, longitudinal and transverse pressure

$$\mathcal{E}(s) = D(s,s_0)\frac{\Lambda_0^4}{4\pi N_0}H_\epsilon^F\left(s',s,\xi_0,\frac{\alpha}{\Lambda_0},\frac{m}{\Lambda_0}\right) + \int_{s_0}^{s}\frac{ds'}{\tau_R(s')}D(s',s)T^4(s')H_\epsilon^F\left(s',s,\frac{\alpha}{T},\frac{m}{T}\right)$$

$$P_L(s) = D(s,s_0)\frac{\Lambda_0^4}{4\pi N_0}H_L^F\left(s',s,\xi_0,\frac{\alpha}{\Lambda_0},\frac{m}{\Lambda_0}\right) + \int_{s_0}^{s}\frac{ds'}{\tau_R(s')}D(s',s)T^4(s')H_L^F\left(s',s,\frac{\alpha}{T},\frac{m}{T}\right)$$

$$P_T(s) = D(s,s_0)\frac{\Lambda_0^4}{4\pi N_0}H_T^F\left(s',s,\xi_0,\frac{\alpha}{\Lambda_0},\frac{m}{\Lambda_0}\right) + \int_{s_0}^{s}\frac{ds'}{\tau_R(s')}D(s',s)T^4(s')H_T^F\left(s',s,\frac{\alpha}{T},\frac{m}{T}\right)$$

Iterative solution with conformal τ_R

INITIALISATION

Romatschke-Strickland form of initial anisotropic distribution

$$f_0 = \frac{2}{(2\pi)^3 N_0} \exp - \frac{\sqrt{p_T^2 + (1+\xi_0)p_z^2 + m^2}}{\Lambda_0}$$

$\xi_0 \to 0$ ⟶ $\Lambda_0 \to$ temperature

Initial conditions are set for different scaled shear and bulk viscosity

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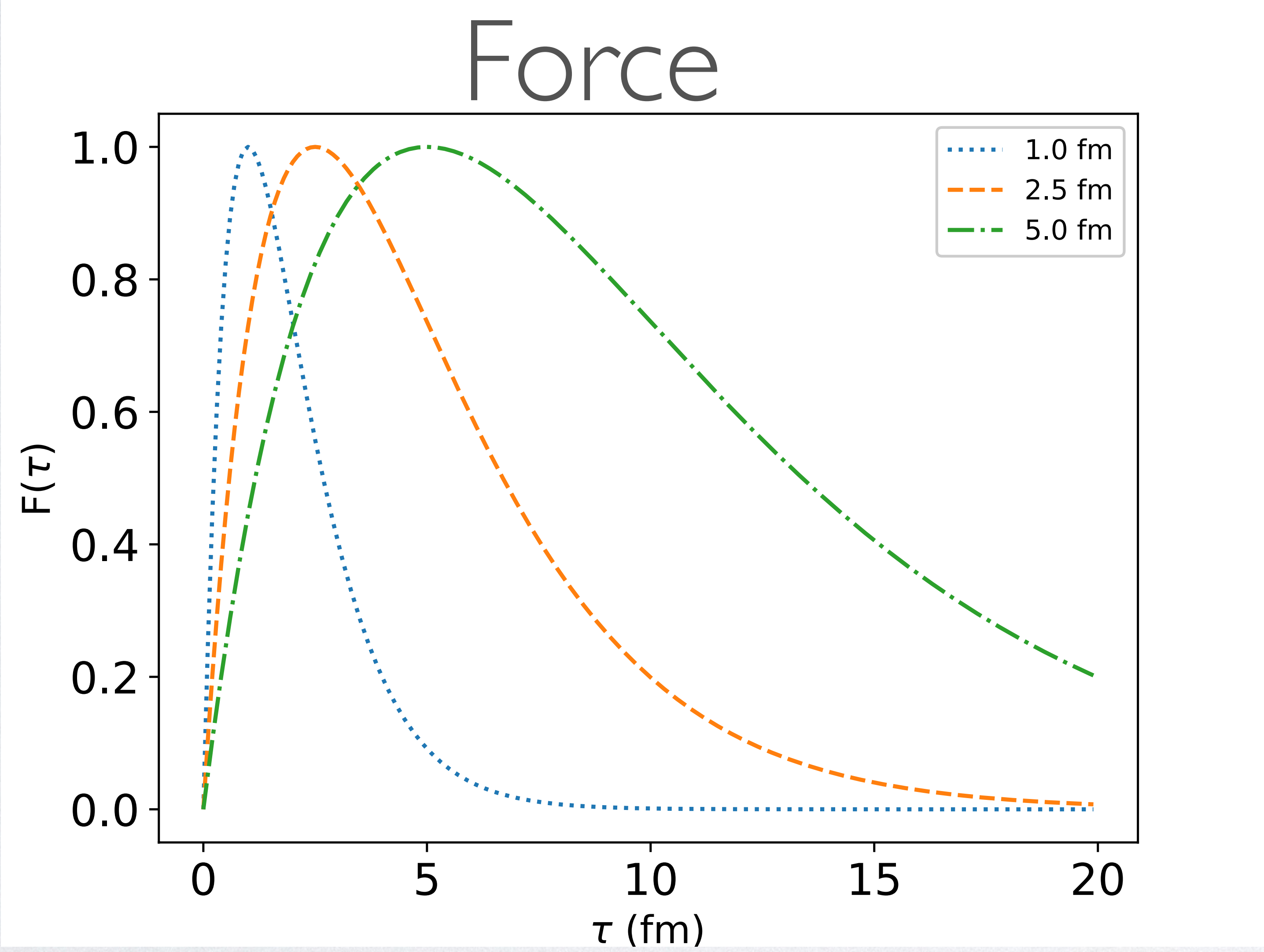
No.	0	1	2	3	4	5	6
$(\Pi/P)_0$	0	-0.25	-0.37	0	0	0.25	-0.85
$(\pi/P)_0$	-1	-1	-1	0.99	-1.8	0	0

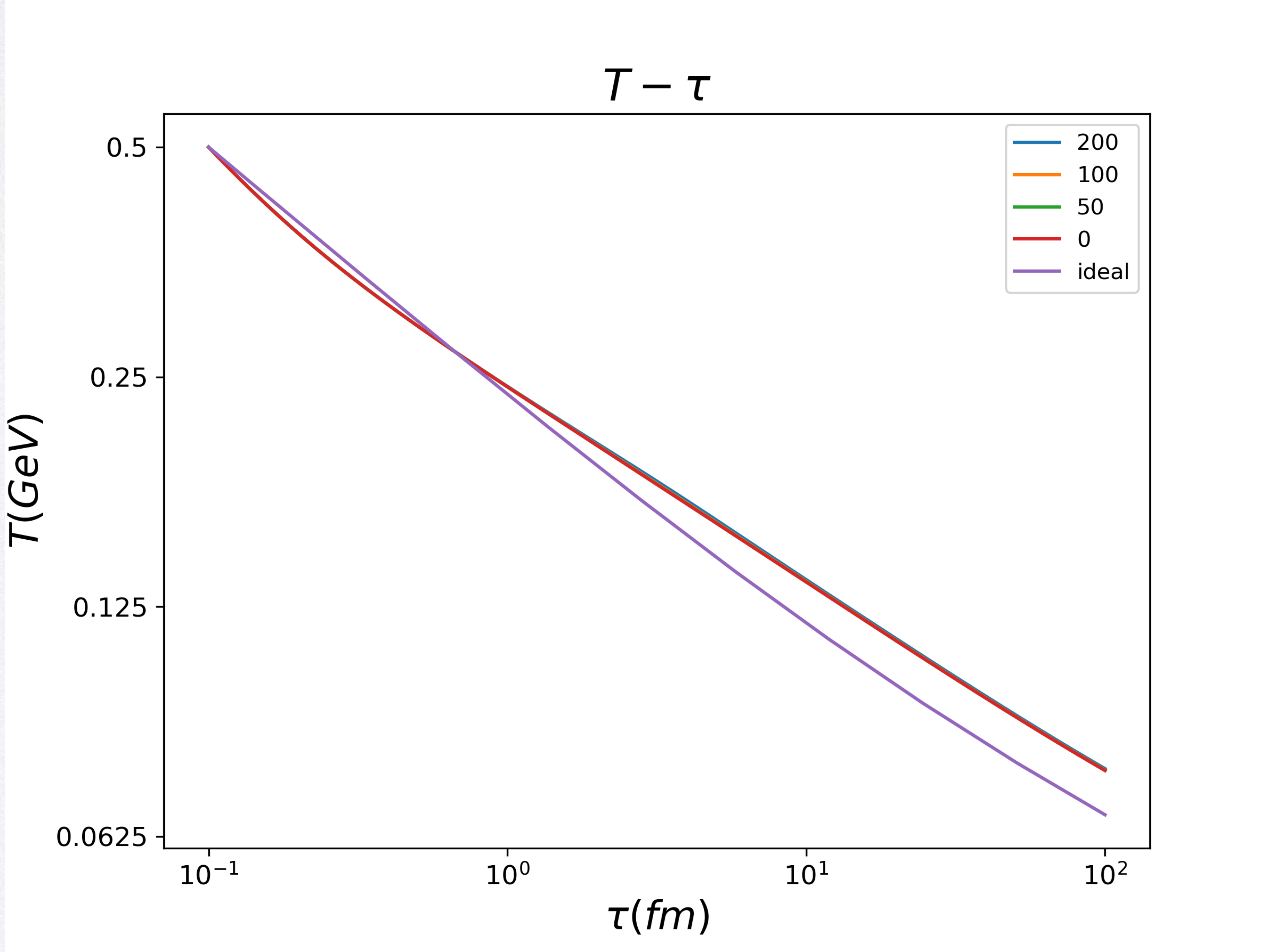
No.	0	1	2	3	4	5	6
Λ_0	321.74	24.98	3.45	681.526	99.74	90.21	0.310
N_0	0.655	4×10^{-5}	2.5×10^{-8}	0.078	0.0632	1.06×10^{-3}	1.48×10^{-13}
ξ_0	-0.832	-0.908	-0.949	1208.05	-0.987	0	0

EFFECTS ON TEMPERATURE EVOLUTION



$$\mathscr{F} = \frac{\alpha}{\tau_F} F(\tau)$$

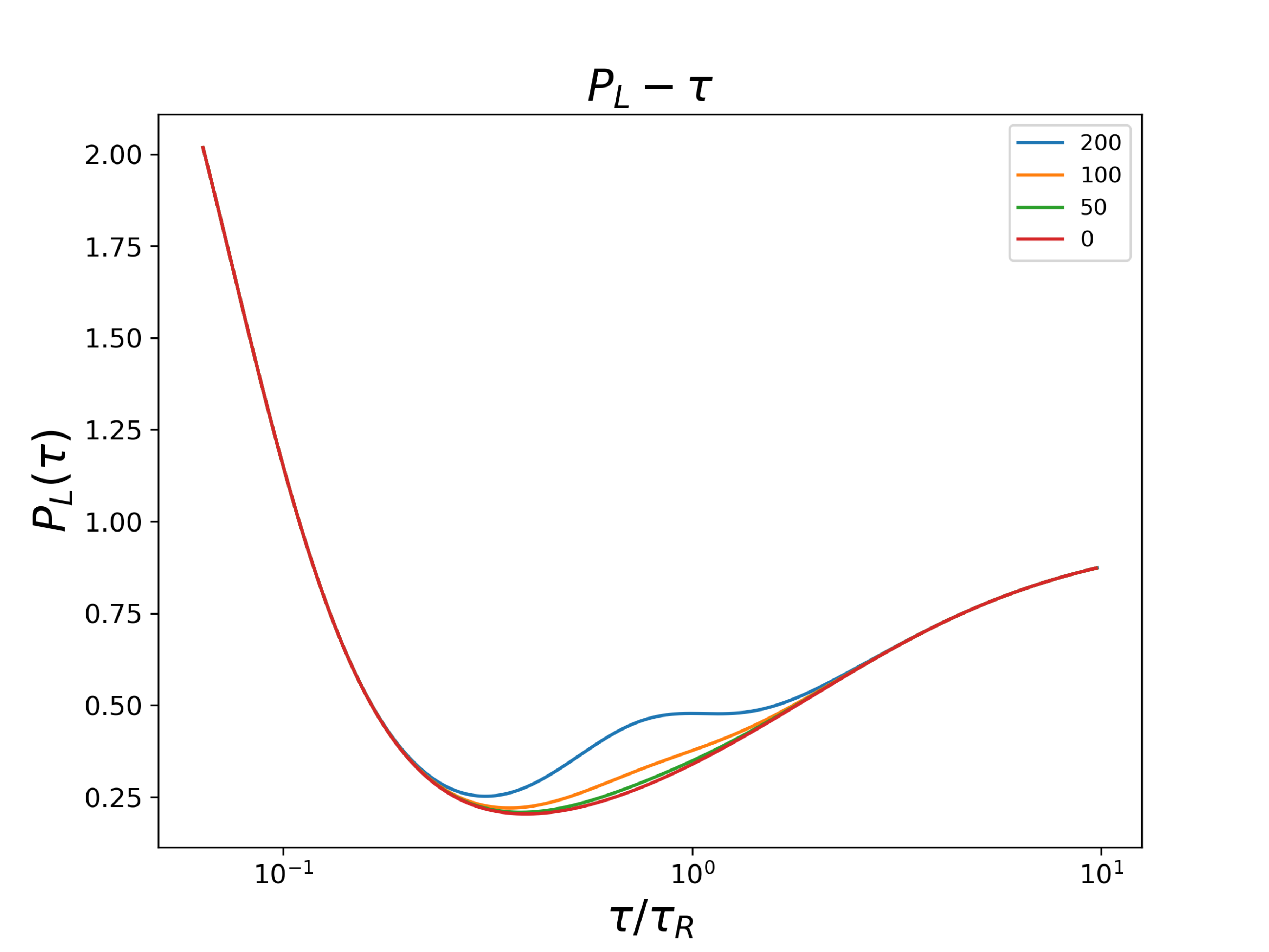
$$F(\tau) = e \frac{\tau}{\tau_F} \exp\left\{-\frac{\tau}{\tau_F}\right\}$$

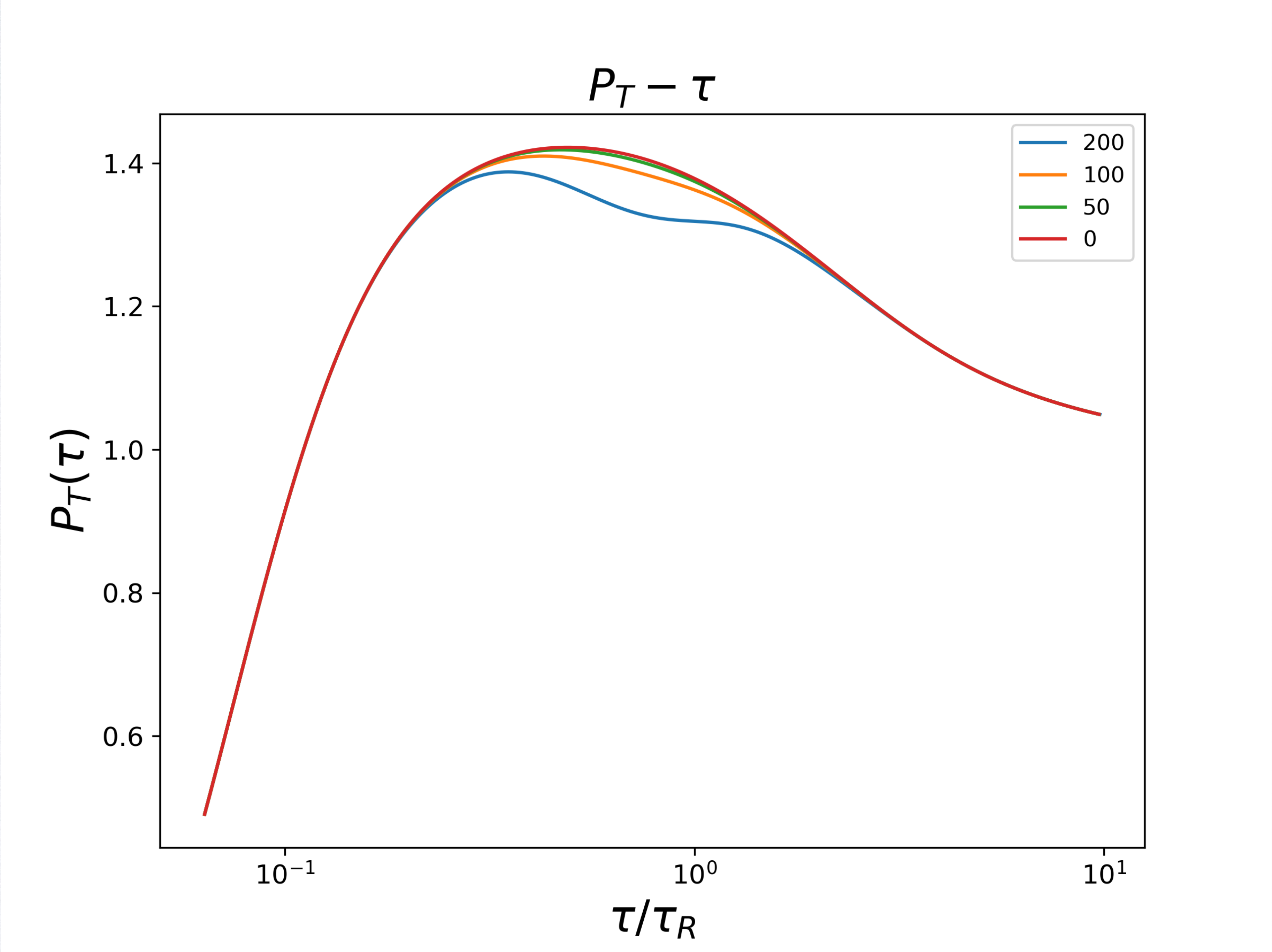


Iterative solution

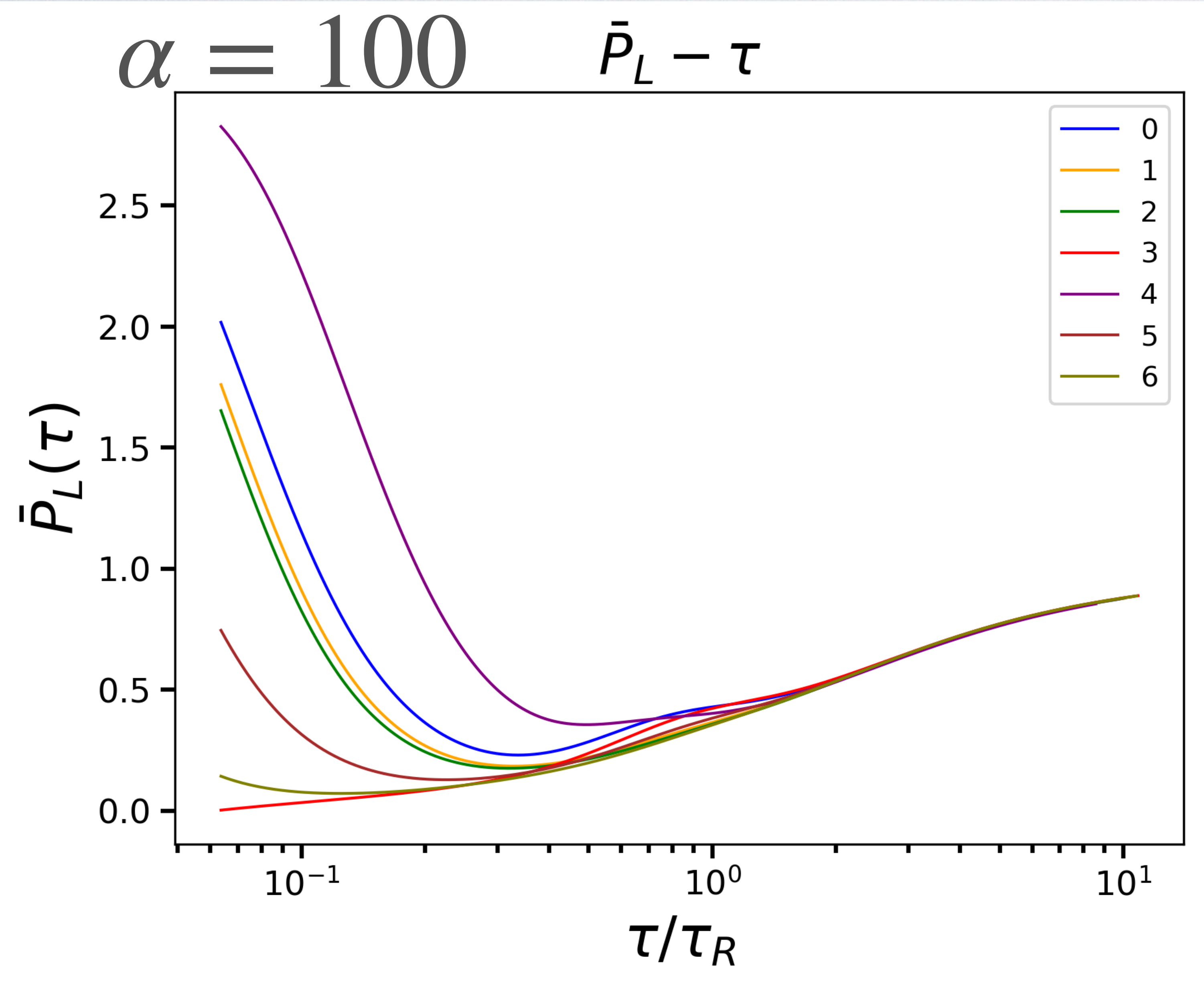
$$\mathscr{E}(T) = \frac{3T^4}{\pi^2}\left(\frac{z^2}{2}K_2(z) + \frac{z^3}{6}K_1(z)\right)$$

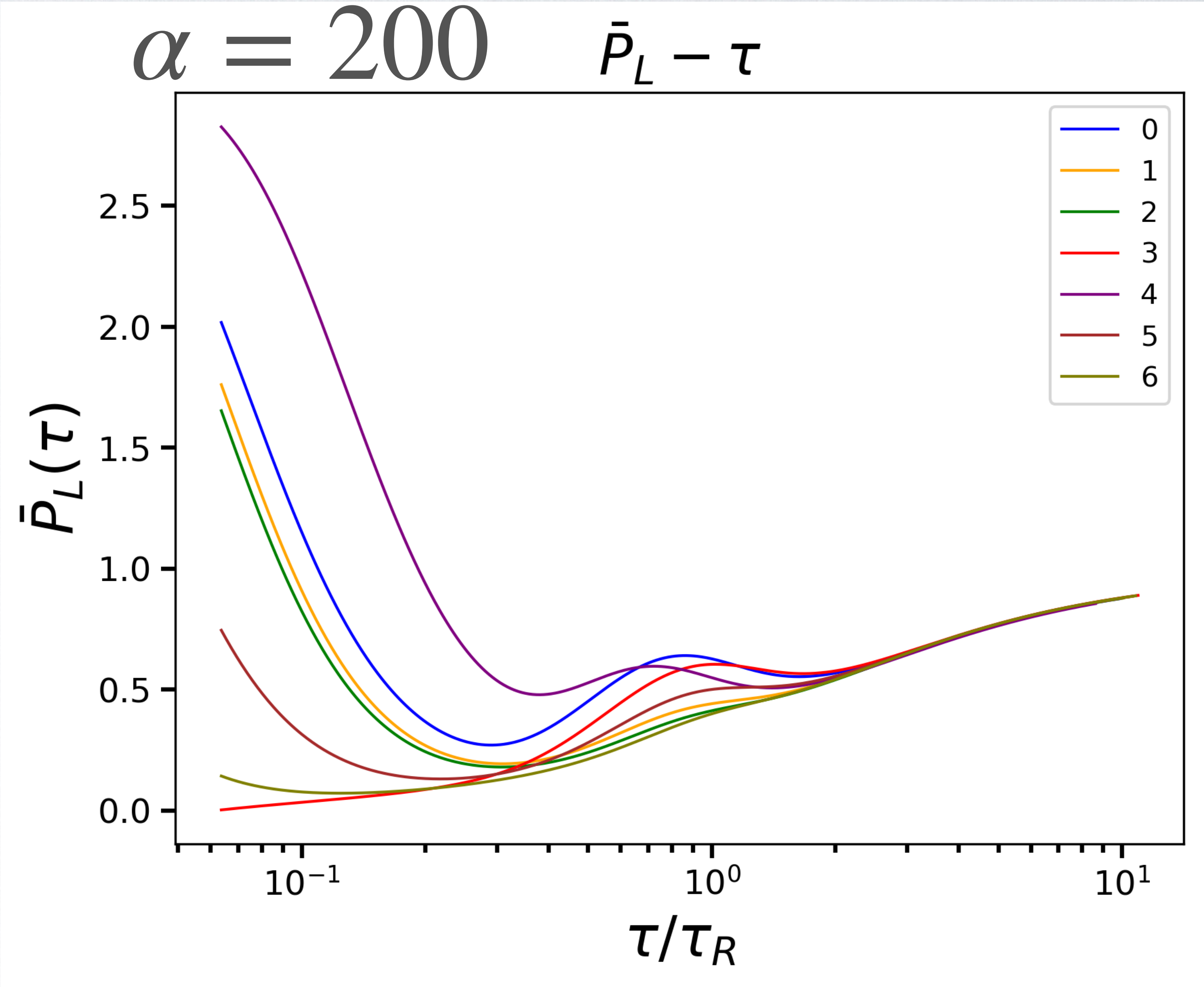
LONGITUDINAL AND TRANSVERSE PRESSURE

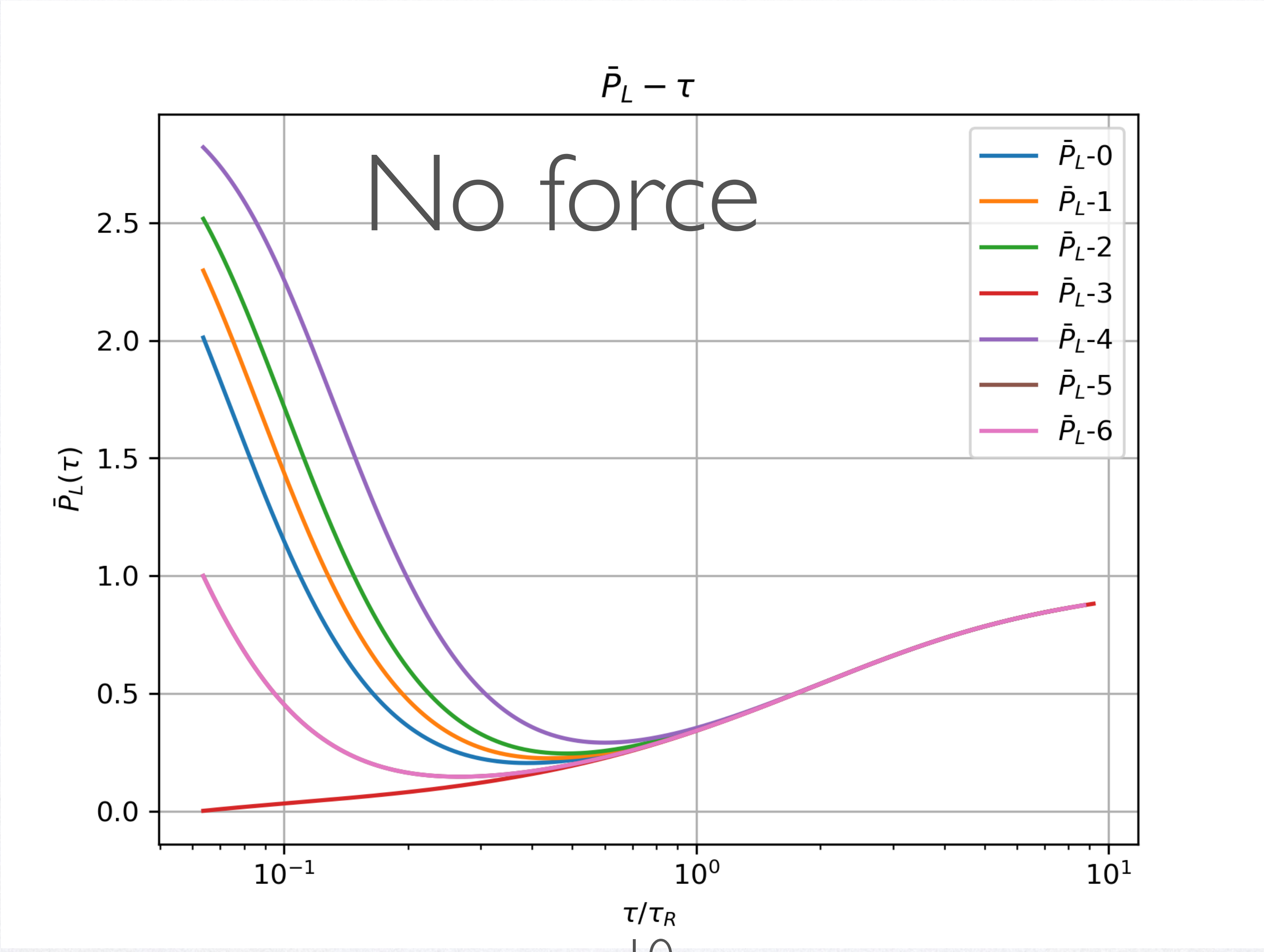




SCALED LONGITUDINAL PRESSURE WITH FORCE







SUMMARY

- Early time attractor seems to be influenced by force.

- Force, however violates the exact Bjorken symmetry.

- Breaking of Bjorken symmetry could be improved.

- Numerical simulations with relaxed symmetry is preferable.

Спасибо

EXTRA SLIDE

H functions $$\tilde{H}^F_\epsilon\left[s', s, m, \alpha\right] = \int du u^3 e^{-\sqrt{u^2+z^2}} H^F_\epsilon\left(u, z, s, s', \alpha\right)$$

$$H^F(u,z,s,s',\alpha) = -\bar{h}\sqrt{(1-\bar{h}^2)}N^2\left\{\frac{1}{2}\left[\sin^{-1}\frac{G-1}{N} - \sin^{-1}\frac{(G+1)}{N}\right]\right.$$
$$\left. +\frac{1}{2}\left[-\frac{1+G}{N}\sqrt{1-\left(\frac{1+G}{N}\right)^2} + \frac{G-1}{N}\sqrt{1-\left(\frac{G-1}{N}\right)^2}\right]\right\}$$

$$H_L(u,z,s,s',\alpha) = -\frac{\bar{h}^3}{\sqrt{1-\bar{h}^2}}\left\{\left(\frac{N^2}{2} + (g+G)^2\right)\left[\sin^{-1}\frac{G-1}{N} - \sin^{-1}\frac{1+G}{N}\right]\right.$$
$$+\frac{N^2}{2}\left[\frac{1+G}{N}\sqrt{1-\left(\frac{1+G}{N}\right)^2} - \frac{G-1}{N}\sqrt{1-\left(\frac{G-1}{N}\right)^2}\right]$$
$$\left. -2N(g+G)\left[\sqrt{1-\left(\frac{1+G}{N}\right)^2} - \sqrt{1-\left(\frac{1-G}{N}\right)^2}\right]\right\}$$

$$H_T(u,z,s,s',\alpha) = -\frac{\bar{h}}{\sqrt{(1-\bar{h}^2)}}\left\{\left(1-G^2-\frac{N^2}{2}\right)\left[\sin^{-1}\frac{G-1}{N} - \sin^{-1}\frac{1+G}{N}\right]\right.$$
$$-\frac{N^2}{2}\left[\frac{1+G}{N}\sqrt{1-\left(\frac{1+G}{N}\right)^2} - \frac{G-1}{N}\sqrt{1-\left(\frac{G-1}{N}\right)^2}\right]$$
$$\left. +2NG\left[\sqrt{1-\left(\frac{1+G}{N}\right)^2} - \sqrt{1-\left(\frac{1-G}{N}\right)^2}\right]\right\}$$