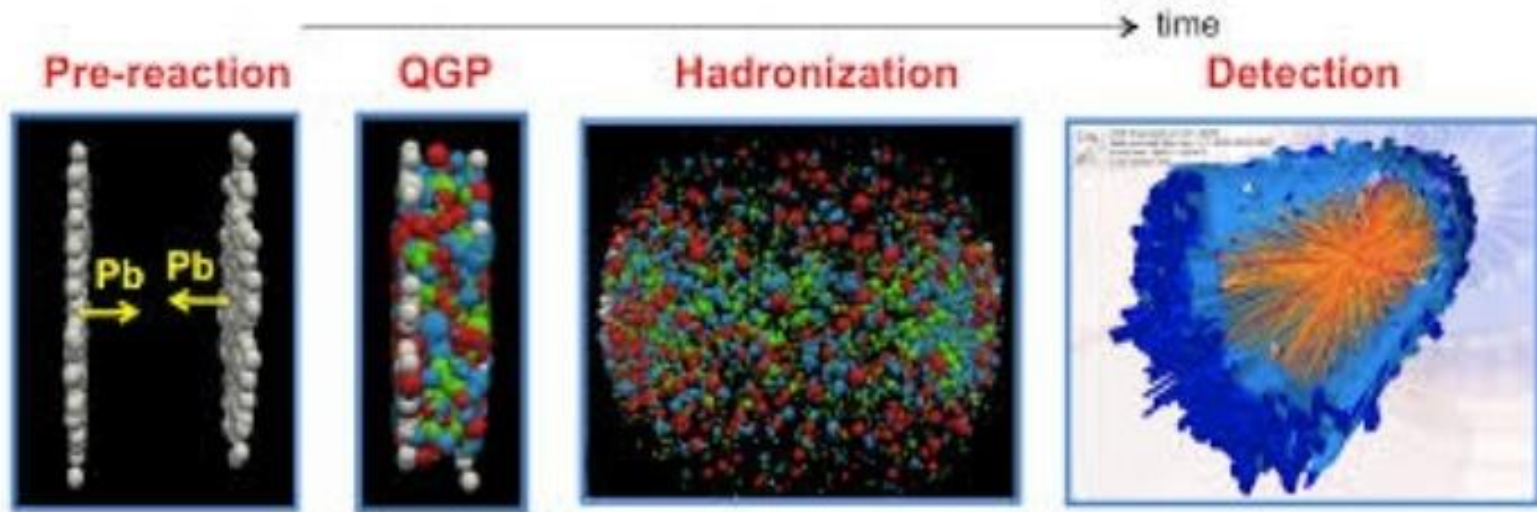


Impact of Memory on Heavy Quark Diffusion in Hot QCD Matter



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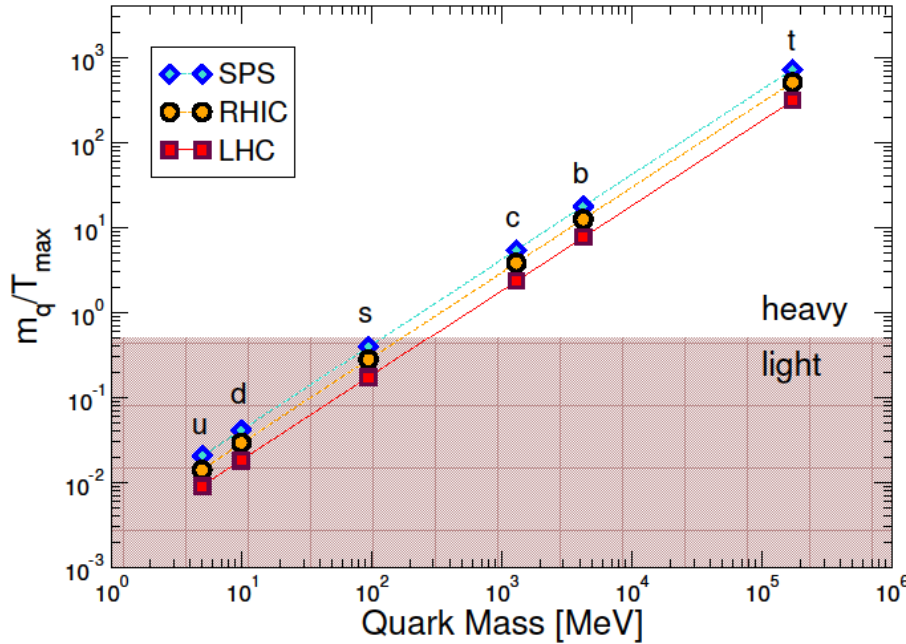
India-JINR workshop on elementary particle and nuclear physics, and condensed matter research



OUTLINE

- ❑ Introduction**
- ❑ Heavy quark diffusion in QGP**
- ❑ Exponential and Power-law Memory**
- ❑ Thermalization & Diffusion with memory**
- ❑ Summary and outlook**

Heavy Quark & QGP



SPS to LHC

$$\sqrt{s} = 17.3 \text{ GeV to } 2.76 \text{ TeV} \sim 100 \text{ times}$$

$$T_i = 200 \text{ MeV to } 600 \text{ MeV} \sim 3 \text{ times}$$

$$M_{c,b} \gg \Lambda_{QCD}$$

Produced by pQCD process (before equilibrium)
(Early production)

$$\tau_{c,b} \gg \tau_{QGP}$$

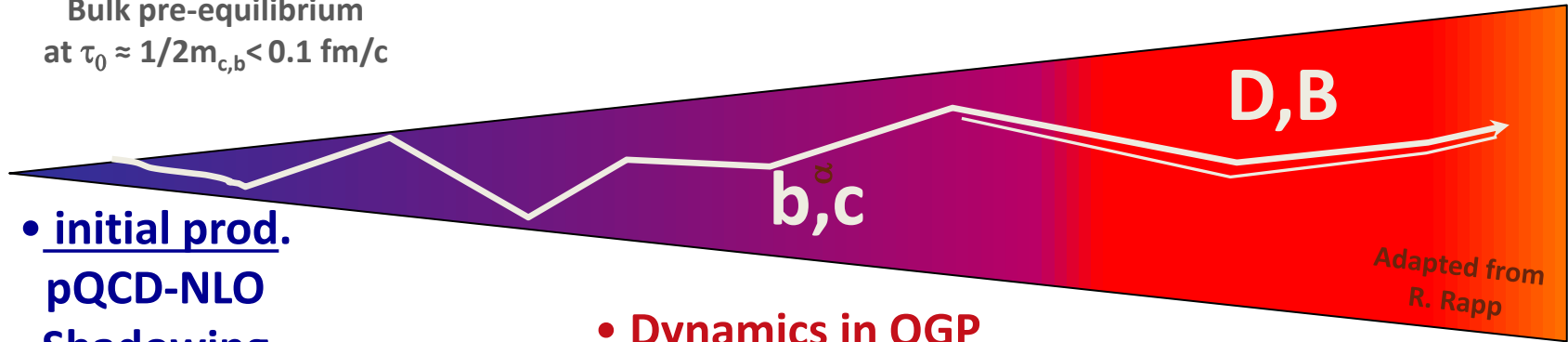
They go through all the QGP life time

$$M_{c,b} \gg T_0$$

No thermal production

Studying the HF dynamics in HIC

Bulk pre-equilibrium
at $\tau_0 \approx 1/2m_{c,b} < 0.1 \text{ fm}/c$



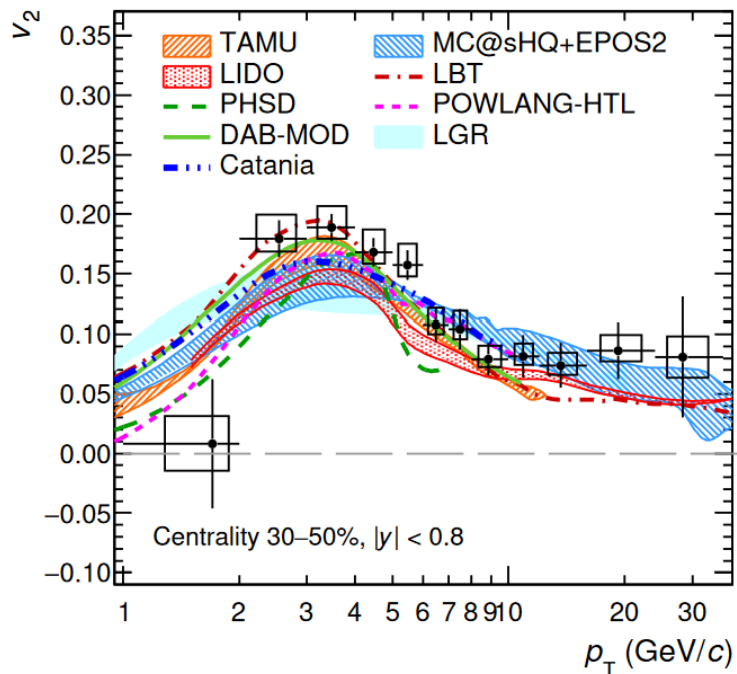
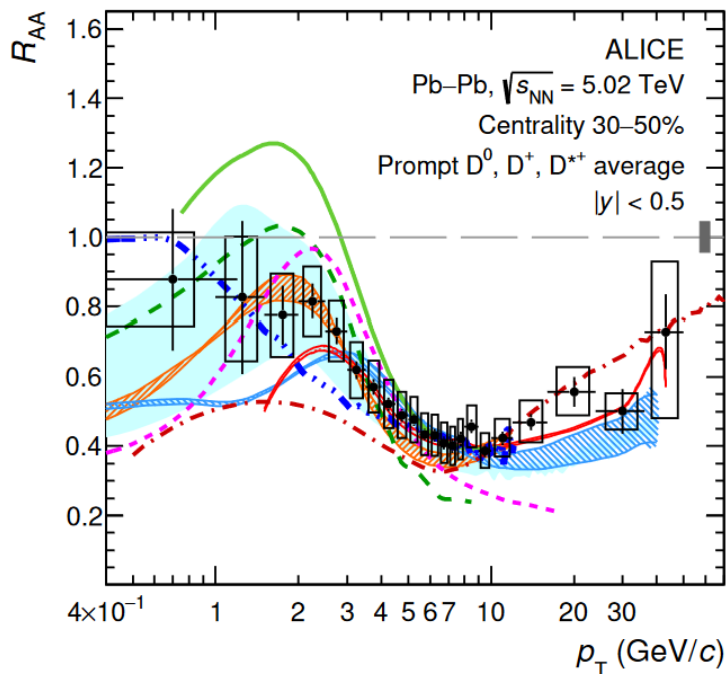
- initial prod.
pQCD-NLO
Shadowing
Pre-equilibrium
Effect/Glasma
Electromagnetic
field

- Dynamics in QGP
Heavy quark QGP interaction
Transp. coeff. of QCD matter
-> thermalization ?!
Mass & color in Jet quenching
Heavy quark momentum evol.
(Langevin/Boltzmann/E. loss model)

- hadronization:
coalescence and/or
fragmentation.
Hadronic rescattering

R_{AA} and v_2

Comparison with models

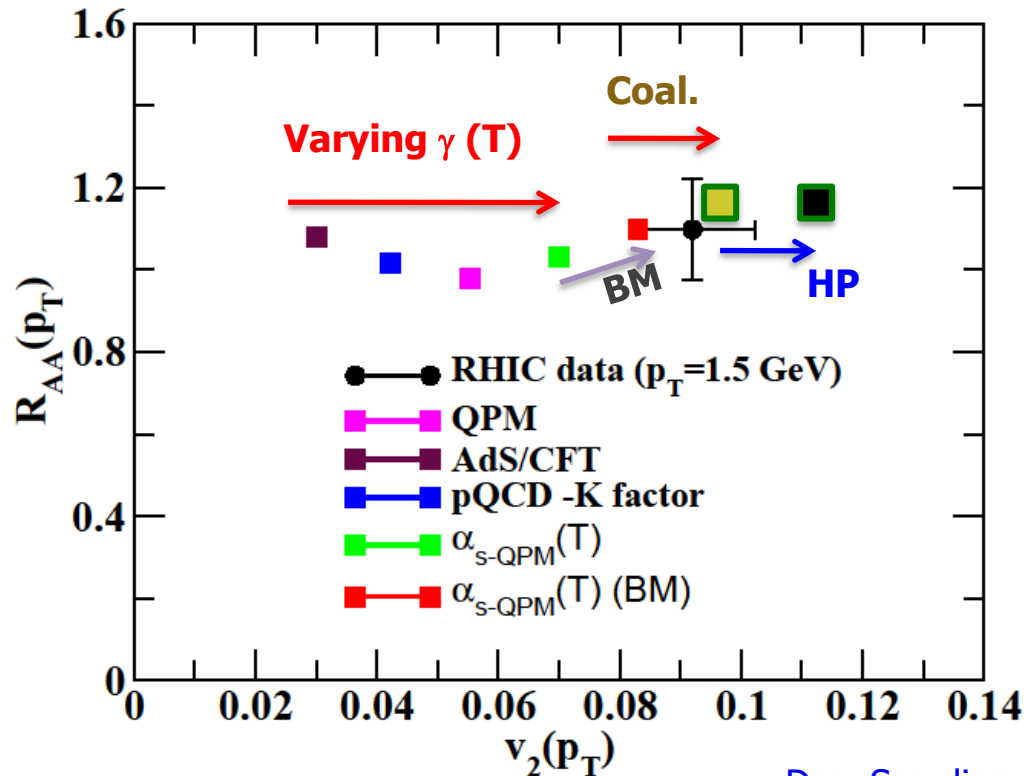


ALICE, JHEP 01 (2022) 174

Most of the models able to describe both R_{AA} and v_2 in certain p_T domain

Simultaneous description of R_{AA} and v_2 is still a challenge in the whole measured p_T and centrality ranges

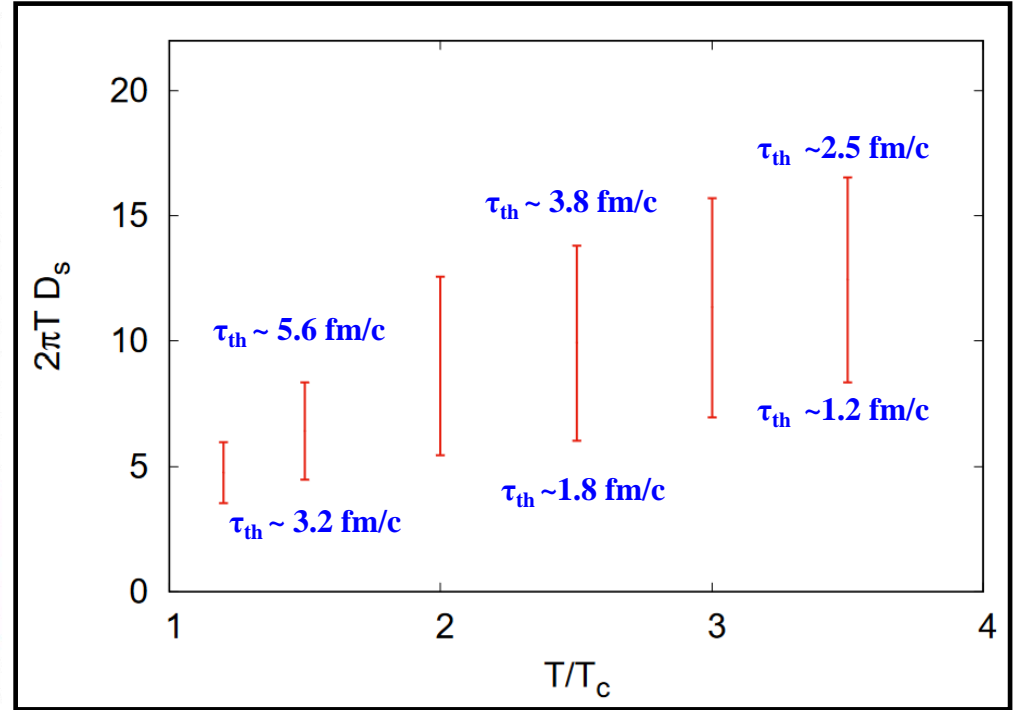
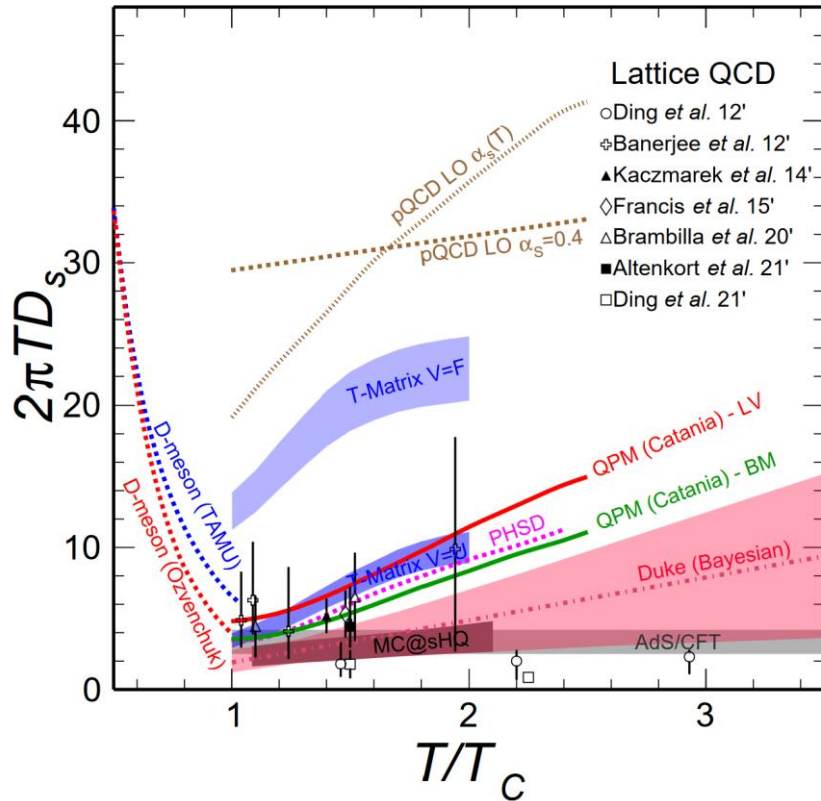
Summary on the build-up of v_2 at fixed R_{AA}



Das, Scardina, Plumari, Greco
 Phys. Lett. B 747 (2016)260-264

R_{AA} and V_2 are correlated but still one can have
 R_{AA} about the same while V_2 can change up to a factor 2-3
 $\gamma(T)$ + Boltzmann dynamics+ hadronization+ hadronic phase

Heavy quark diffusion



Banerjee, Datta, Gavai, Majumdar
arxiv: 2206.15471 [hep-ph]

$$\tau_{th} = \frac{M}{2\pi T^2} (2\pi T D_s) \cong 1.8 \frac{2\pi T D_s}{(T/T_c)^2} \text{ fm/c}$$

He, Fries, Rapp, PRL,110, 112301 (2013)

$2\pi T D_s \propto T^2$, **corresponds to a constant thermalization time.**

Scardina, Das, Minissale, Plumari, Greco
PRC,96, 044905 (2017)

Heavy quark dynamics with memory effect

- Langevin Equation

$$\frac{d\mathbf{p}}{dt} = -\gamma\mathbf{p} + \eta(t)$$

- $\langle \eta(t) \rangle = 0$
- $\langle \eta(t)\eta(t') \rangle = 2D\delta(t - t')$

- Generalized Langevin Equation

$$\frac{dp}{dt} = - \int_0^t \gamma(t, t')p(t')dt' + \eta(t)$$

- The correlation of fluctuations

$$\langle \eta(t)\eta(t') \rangle = 2Df(|t - t'|)$$

$$\langle \eta(t) \rangle = 0$$

$$f(|t - t'|) = \frac{1}{2\tau} e^{-|t-t'|/\tau}$$

- Ancillary process

$$\frac{dh}{dt} = -\alpha h + \alpha \rho$$

$$\eta(t) = \sqrt{\frac{2D}{\tau}} h(t)$$

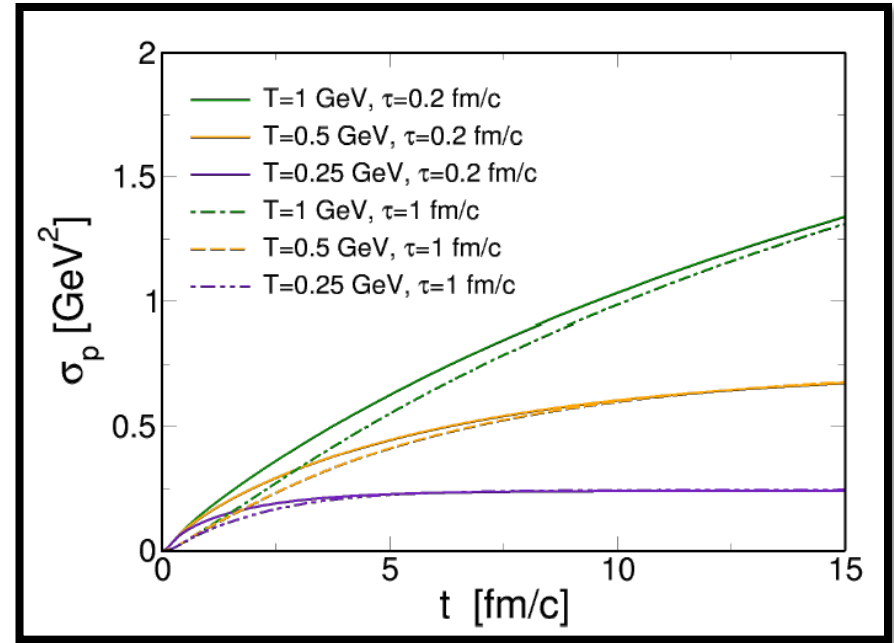
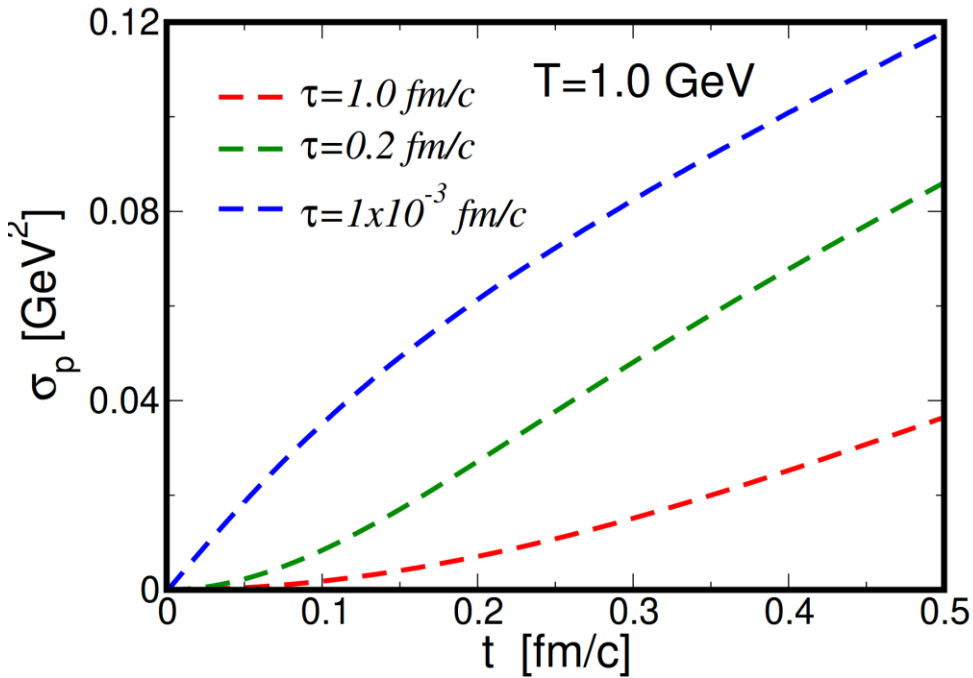
$$\alpha = \frac{1}{\tau}$$

$$\langle h(t)h(t') \rangle \approx \frac{e^{-\alpha|t-t'|}}{2}$$

The drag from FDT:

$$\gamma(t, t') = \frac{2D}{ET} \frac{e^{-|t-t'|/\tau}}{2\tau}$$

Impact of memory on heavy quark thermalization



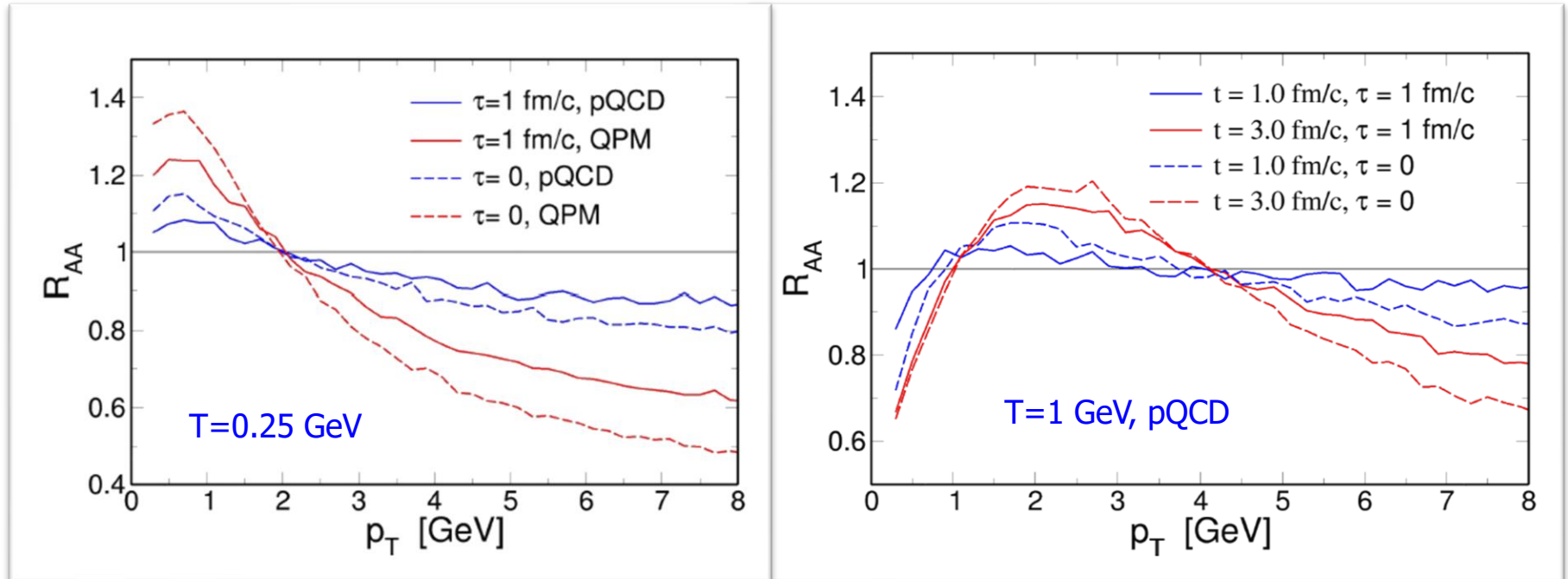
$$\sigma_p = \langle (p_T - \langle p_T \rangle)^2 \rangle$$

Memory delay the thermalization time

Liu, Das, Greco, Ruggieri, PRD 103, 034029 (2021)

Ruggieri, Pooja, Jai Prakash, Das, PRD, 106 (2022) 3, 034032

Impact of memory on heavy quark suppression



Formation of R_{AA} are slowed down by memory

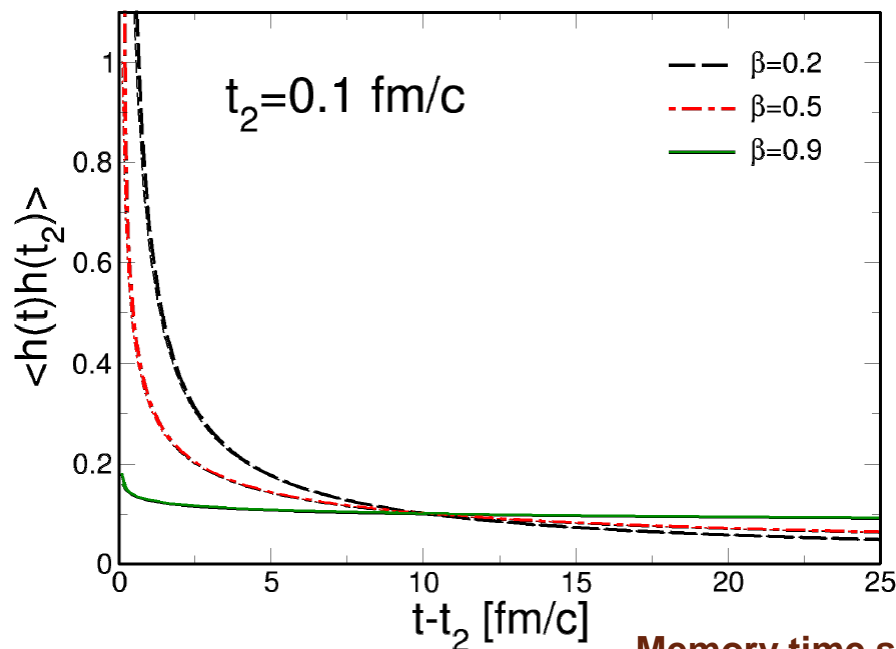
Longtail memory: time correlations decaying with a power law

$$h(t) = \sqrt{\kappa} \frac{\sqrt{\beta}}{\tau^\beta} \int_0^t (t-u)^{\beta-1} \xi(u) du, \quad 0 < \beta < 1$$

β fixes the power law at which correlations decay.

ξ is a standard Gaussian noise

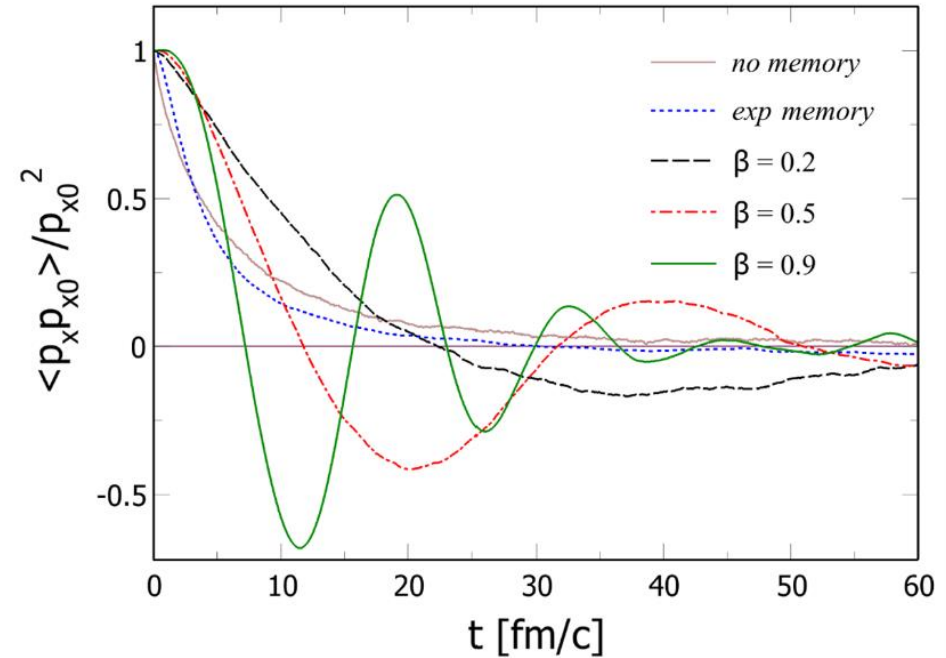
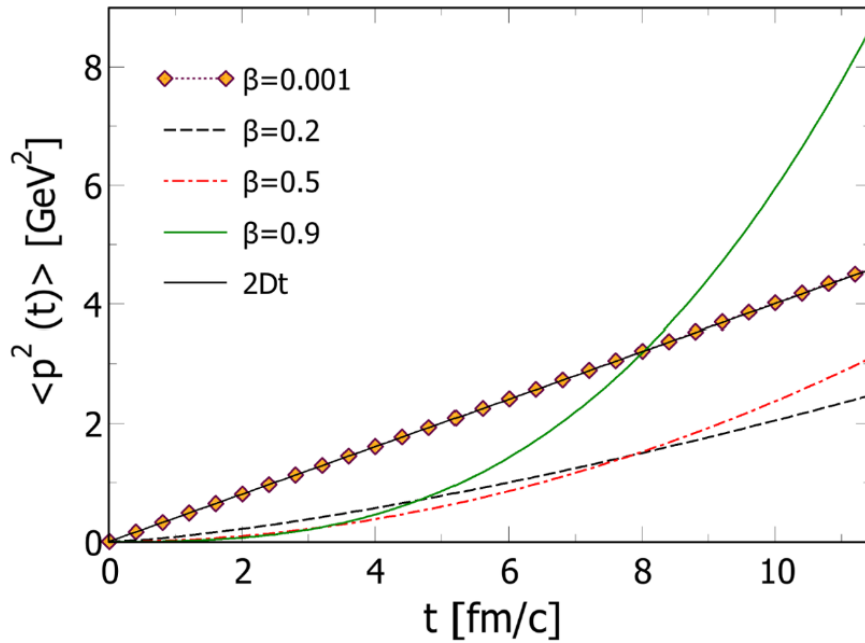
$$\langle h(t_1)h(t_2) \rangle = \kappa \tau^{-2\beta+1} \beta \int_0^{t_{\min}} (t_1-u)^{\beta-1} (t_2-u)^{\beta-1} du,$$



Pooja, Das, Greco, Ruggieri
PRD 108, 054026 (2023)

Memory time set the time scale at which correlations decay.

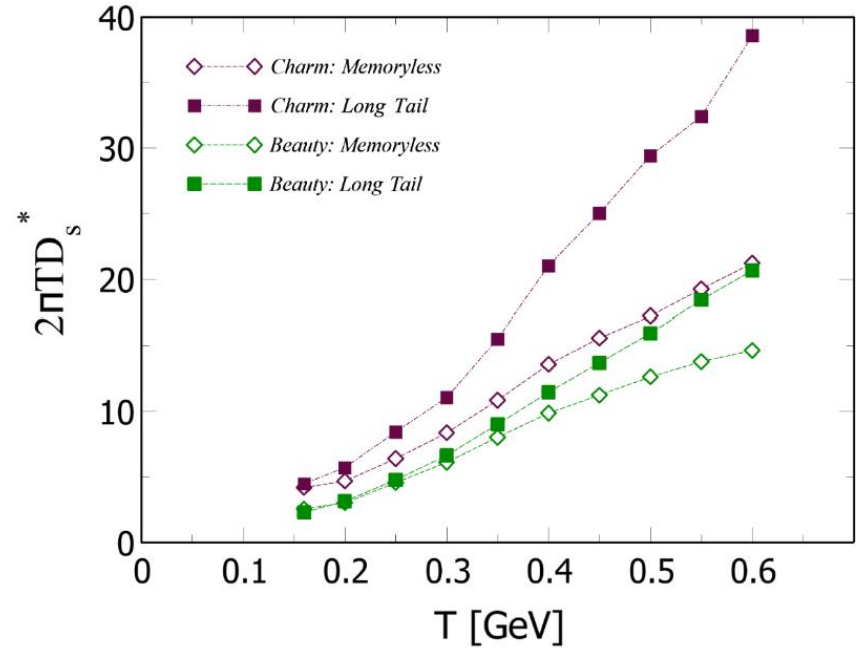
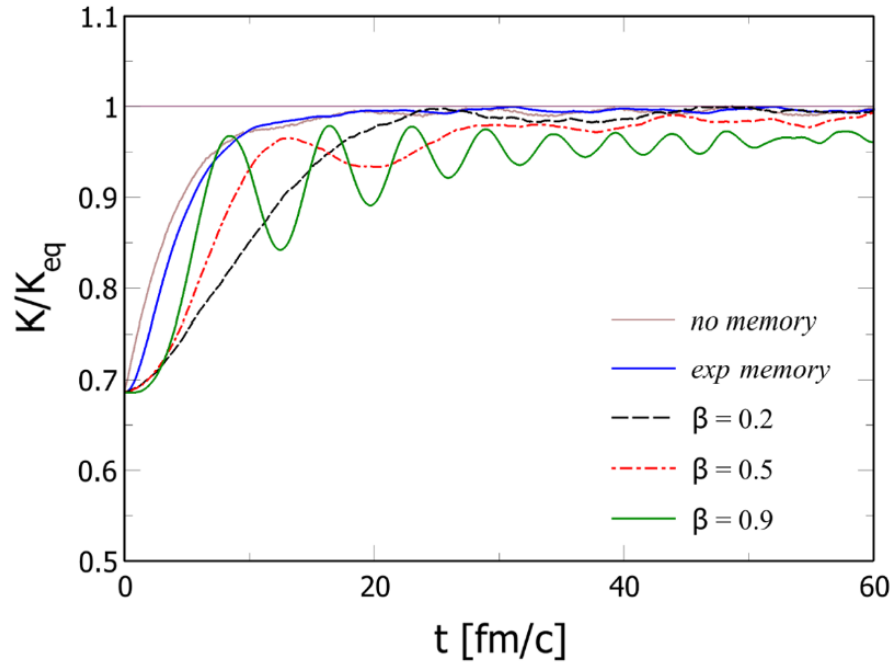
Longtail memory: Momentum broadening and randomization



For larger β the $\langle p^2(t) \rangle$ increases with a power of time larger than one.

The drag kernel is proportional to the correlator of h , and in the early evolution, this gets suppressed if $\beta \rightarrow 1$. Consequently, the effect of the damping given by the drag force is lowered by increasing β . The resulting motion is qualitatively similar to that of a damped oscillator.

Longtail memory: Thermalization and D_s



$$K_{eq} \equiv \frac{\int \frac{d^3p}{(2\pi)^3} (\sqrt{p^2 + m^2} - m) e^{-\beta\sqrt{p^2+m^2}}}{\int \frac{d^3p}{(2\pi)^3} e^{-\beta\sqrt{p^2+m^2}}}$$

Need higher momentum diffusion coefficient D or, equivalently, a smaller D_s is in order to reproduce the experimental data.

Pooja, Das, Greco, Ruggieri
PRD 108, 054026 (2023)

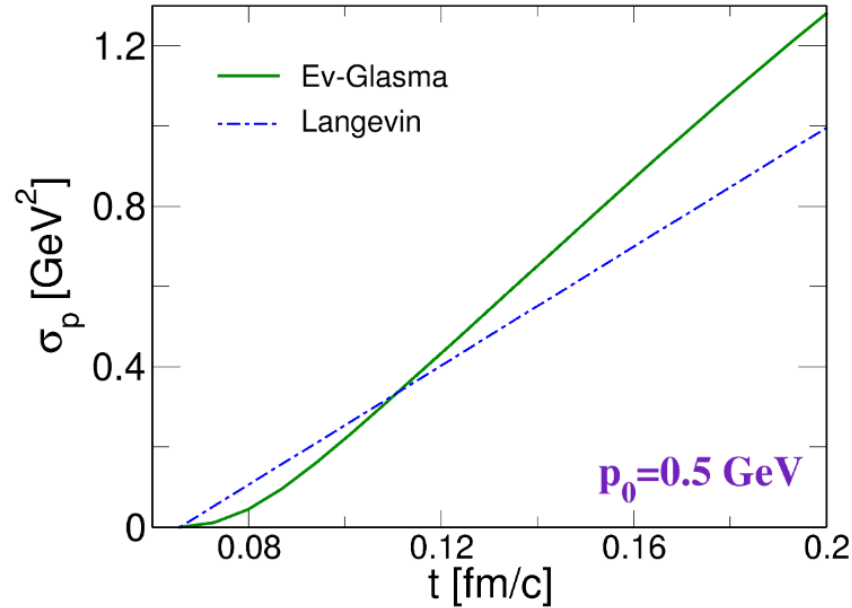
Conclusions and Perspectives:

- ❖ We have studied heavy quark diffusion with exponential and power-law memory.
- ❖ Memory slows down the momentum evolution of heavy quarks.
- ❖ Formation of R_{AA} and v_2 are slowed down by memory
- ❖ Thermalization time of the heavy quarks become larger.
- ❖ Requires smaller D_s To reproduce the data.

Thank You



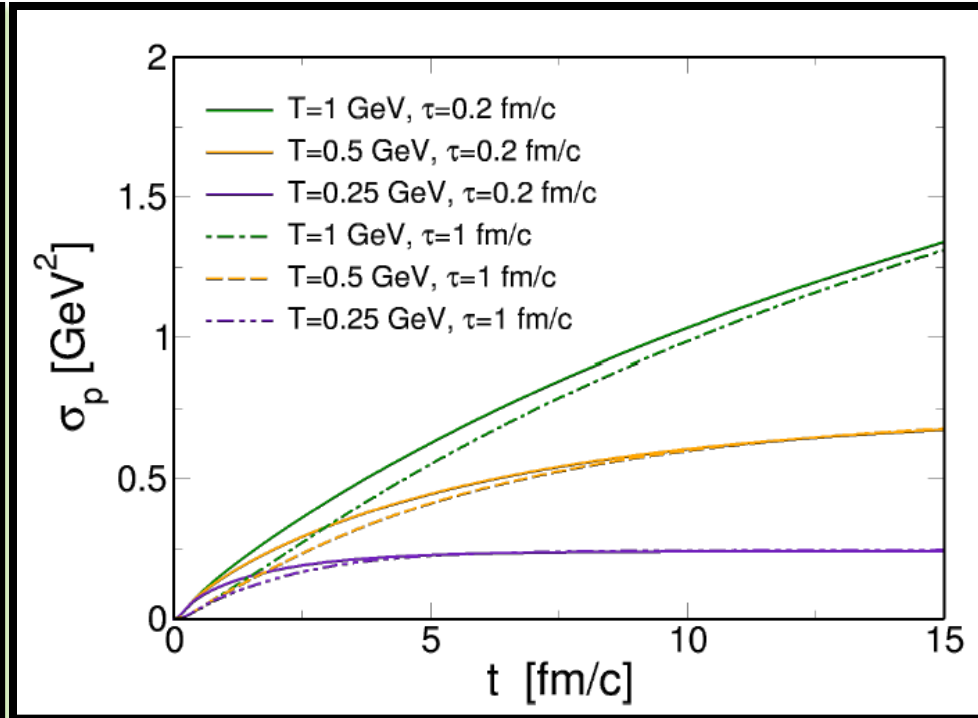
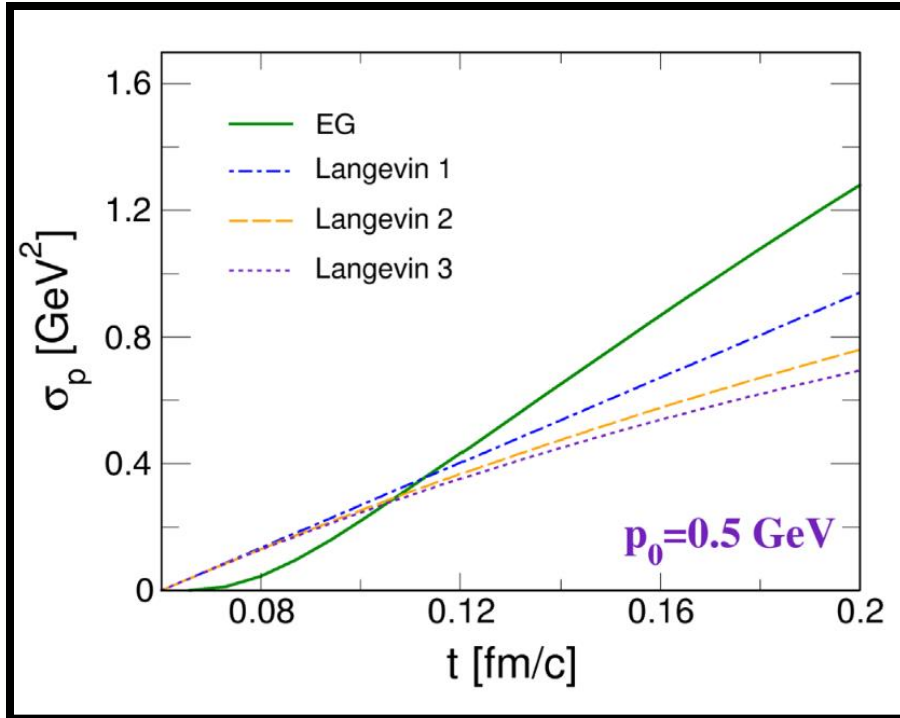
Impact of memory on heavy quark thermalization



$$\sigma_p = \frac{1}{2} \langle (p_x(t) - p_{0x})^2 + (p_y(t) - p_{0y})^2 \rangle$$

Liu, Das, Greco, Ruggieri, PRD 103, 034029 (2021)

Impact of memory on heavy quark thermalization



$$\sigma_p = \frac{1}{2} \langle (p_x(t) - p_{0x})^2 + (p_y(t) - p_{0y})^2 \rangle$$

$$\sigma_p = \langle (p_T - \langle p_T \rangle)^2 \rangle$$

Memory delay the thermalization time

Liu, Das, Greco, Ruggieri, PRD 103, 034029 (2021)

Ruggieri, Pooja, Jai Prakash, Das, arxiv: 2203.06712 [hep-ph]