Heavy quarkonium potential in presence of magnetic field

NAJMUL HAQUE

School of Physical Science (SPS),
National Institute of Science Education and Research (NISER),
India

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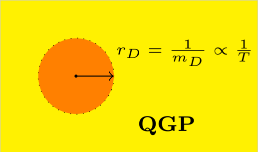
Quarkonia in QGP



- In 1986, Matsui and Satz proposed that the quarkonia is suppressed if QGP is formed because the binding potential becomes short-range due to color Debye screening.
- Debye screening makes the potential short-range, and when the screening radius r_D becomes less than the quarkonia radius, it dissociates.
- As the temperature increases, the Debye radius decreases and which leads to sequential suppression of quarkonia, one of the most striking signatures of the QGP.

T. Matsui and H. Satz, Phys. Lett. B ***178****, 416-422 (1986)*

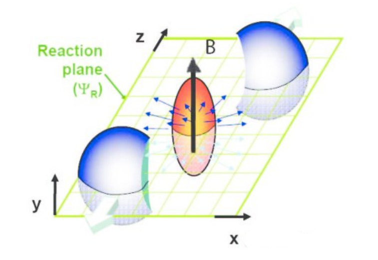
- The interaction between quark and anti-quark can be expressed by the Cornell potential $V(r) = -\frac{\alpha}{r} + \sigma r$ in vacuum.

- The vacuum potential in momentum-space becomes $V_p = \sqrt{\frac{2}{\pi}}\frac{\alpha}{p^2} - 2\sqrt{\frac{2}{\pi}}\frac{\sigma}{p^4}$.

- So, quarknium potential in thermal medium becomes

$$\begin{aligned} V(\boldsymbol{r}) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}} \left(e^{i\boldsymbol{p}\cdot\boldsymbol{r}} - 1\right) \frac{V_p(\boldsymbol{p})}{\epsilon(\mathbf{p})} \\ &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}} \left(e^{i\boldsymbol{p}\cdot\boldsymbol{r}} - 1\right) \frac{V_p(\boldsymbol{p})}{p^2 + m_D^2} \\ &= -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D - \frac{2\sigma}{m_D^2 r}\left(1 - e^{-m_D r}\right) + \frac{2\sigma}{m_D}. \end{aligned}$$

Magnetic field in HIC



- The non-central heavy ion collision produces a very strong magnetic field normal to the reaction plane.
- At LHC energies, the strength of the magnetic field is estimated to be as high as $eB = 15m_{\pi}^2 = 1.5 \times 10^{19}$Gauss, the largest magnetic field ever produced in the laboratory.

V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009)

Permittivity in presence of magnetic field

- In presence of a magnetic field

$$V(\boldsymbol{r}) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}} \left(e^{i\boldsymbol{p}\cdot\boldsymbol{r}} - 1\right) \frac{V_p(\boldsymbol{p})}{\epsilon(\mathbf{p}, B)}$$

- The dielectric permittivity of QGP medium in a magnetic field can be calculated using the Schwinger proper time formalism.

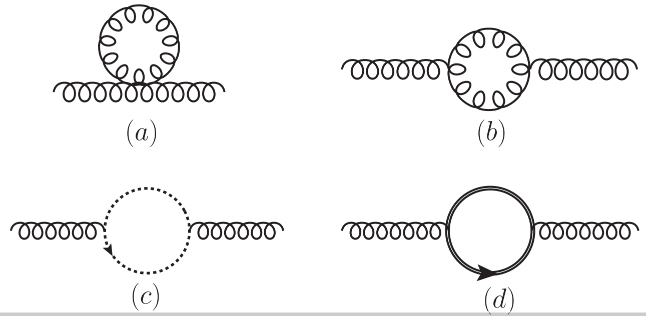
$$\epsilon^{-1}(\mathbf{p}, B) = \frac{p^2}{p^2 + \Pi^L} - i\pi T \frac{p\,\Pi^L}{(p^2 + \Pi^L)^2},$$

- Π^L is the longitudinal polarisation tensor, which is B dependent.

$$\Pi^L = \Pi^{00}(p_0 \to 0)$$

where Π^{00} is the temporal component of the one-loop gluon self energy $\Pi^{\mu\nu}$

Gluon self-energy in presence of magnetic field



$$\Pi^L_{a+b+c}(\omega, \mathbf{p}) = \frac{g^2T^2N_c}{3}\left[1 - \frac{\omega}{2p}\ln\left(\frac{\omega+p}{\omega-p}\right) + i\pi\frac{\omega}{2p}\Theta(p^2-\omega^2)\right].$$

Quark propagator

- Quark propagator:

$$\begin{aligned}\widetilde{S}_l(\mathbf{k}) &= -i\int_0^\infty ds\, e^{-s[\hat{\omega}_l^2+k_3^2+k_\perp^2\tanh(|q_fB|s)/|q_fB|s+m^2]} \\ &\times \{(-\hat{\omega}_l\gamma_4 - k_3\gamma_3 + m)[1 - i\gamma_1\gamma_2\tanh(|q_fB|s)] \\ &- k_\perp\gamma_\perp[1-\tanh^2(|q_fB|s)]\},\end{aligned}$$

- Gluon self-energy(quark-loop):

$$\begin{aligned}\Pi_n^{\mu\nu}(\mathbf{p},B) &= -g^2T\sum_f\int\frac{d^3\mathbf{k}}{(2\pi)^3}\sum_{l=-\infty}^{\infty}\mathrm{tr}\{\gamma^\mu\widetilde{S}_l(\mathbf{k}) \\ &\times \gamma^\nu\widetilde{S}_{l-n}(\mathbf{k}-\mathbf{p})\} + Q^{\mu\nu}(p),\end{aligned}$$

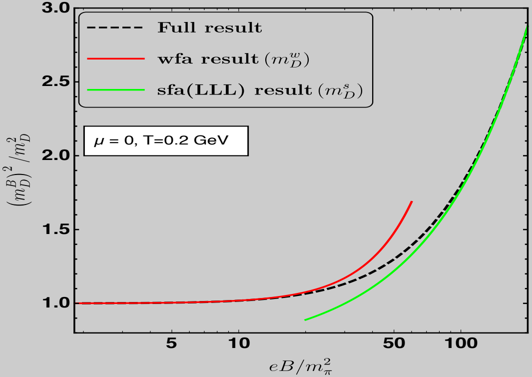
$$\begin{aligned}\Pi_q^L(\mathbf{p},B) &= \sum_f \frac{g^2 q_f B}{32\pi^2}\int_0^\infty du \int_{-1}^1 dv \exp\left[-\frac{1}{4}p^2\cos^2\theta\, u(1-v^2)\right.\\ &\quad \left. -\frac{p^2\sin^2\theta}{2\,q_f B\sinh(q_f Bu)}(\cosh(q_f Bu)-\cosh(q_f Buv))\right]\\ &\times \left[2\coth(q_f Bu)\ \frac{\partial}{\partial u}\vartheta_4(0,e^{-\frac{1}{4T^2u}}) + \left(1-\vartheta_4(0,e^{-\frac{1}{4T^2u}})\right)\right.\\ &\quad \times\left\{p^2\sin^2\theta\cosh(q_f Buv)\mathrm{csch}(q_f Bu)\right.\\ &\quad \left.\left. +\coth(q_f Bu)\left(p^2\cos^2\theta(1-v^2)-v\,p^2\sin^2\theta\ \frac{\sinh(q_f Buv)}{\sin(q_f Bu)}\right)\right\}\right].\end{aligned}$$

Debye mass

- The Debye mass is QCD is obtained as

$$m_D^2 \;=\; \Pi_L(p_0 = 0, p \to 0) = \frac{g^2T^2}{3}\left[C_A + \frac{N_f}{2}\left(1 + \frac{3\mu^2}{\pi^2T^2}\right)\right],$$

- In presence of the magnetic field:



- The real part of the potential:

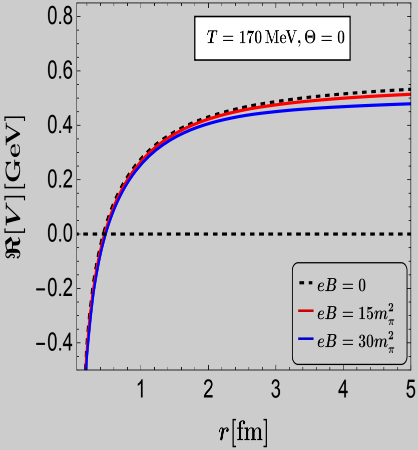
$$\Re V(r,T,B,\Theta) = -\frac{1}{\pi}\int \frac{\sin\theta\, d\theta\, dp}{p^2+\Pi^L}\Big[(\alpha\,\Pi^L - 2\,\sigma) \times(\alpha\, p^2 + 2\,\sigma) e^{ipr\cos\theta\cos\Theta} J_0(pr\sin\theta\sin\Theta)\Big],$$

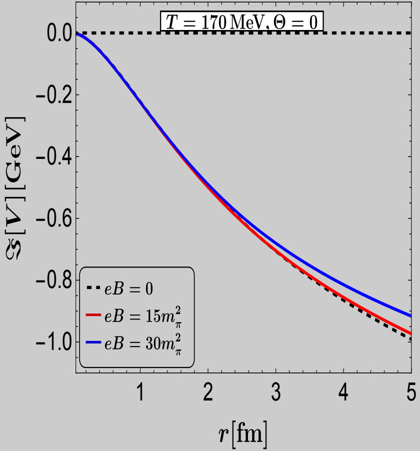
- The imaginary part of the potential:

$$\Im V(r,\Theta,T,B) = -T\int \frac{\sin\theta\, d\theta\, dp}{\left(p^2+\Pi^L\right)^2}\Pi^L\left[\alpha\, p + \frac{2\sigma}{p}\right] \times\left\{1 - e^{ipr\cos\theta\cos\Theta} J_0(pr\sin\theta\sin\Theta)\right\}.$$

- The angle between the magnetic field and dipole is Θ.

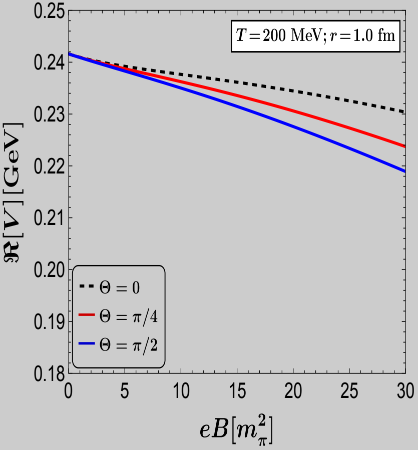
Results: Radial dependence

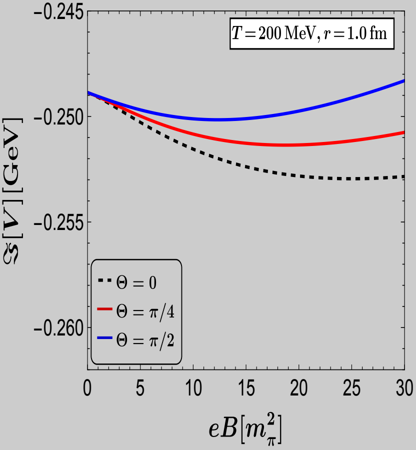




- The magnetic field dependence is very small and is insignificant even at $eB = 15m_\pi^2$.

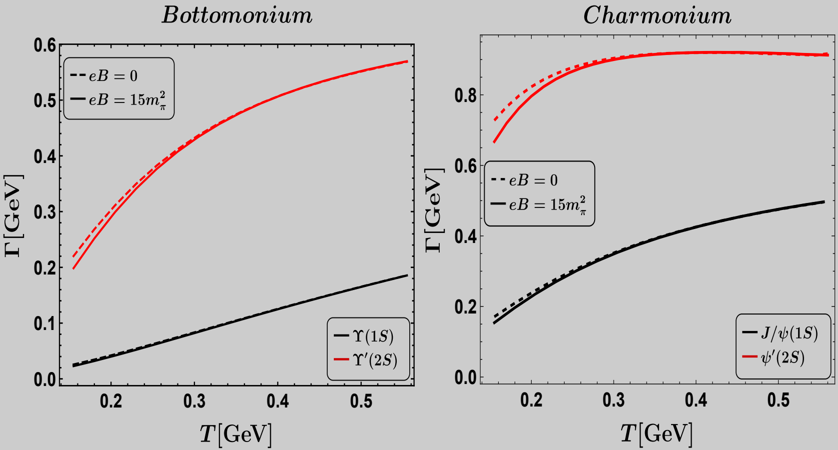
Results: Radial dependence





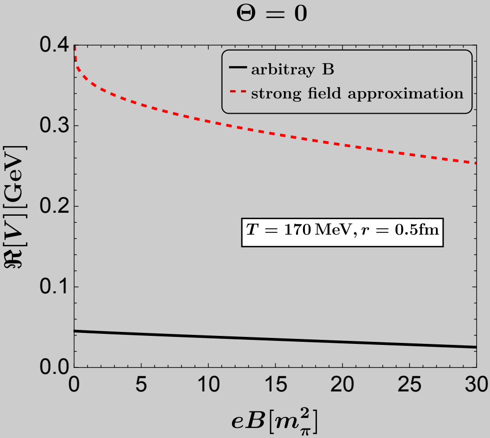
- The potential variation with the magnetic field is different in different directions. Here Θ is the angle between the quark-antiquark axis and the magnetic field.

Results: Thermal width



- The thermal widths are more sensitive to the magnetic field at lower temperatures..
- The magnetic field effects decrease with the increase of heavy quark mass.

Results: Strong field approximation



- The longitudinal component of the gluon self-energy in the strong magnetic field approximation ($T \ll \sqrt{eB}$) is computed and used to calculate the potential.
- The potential with approximation differs in large values from the exact potential for any realistic magnetic field magnitude.
- Few hundreds of m_π^2 of magnetic field is required for SFA to be valid.

Thank you for your attention.