Kinetic freeze-out in low energy relativistic nuclear collisions

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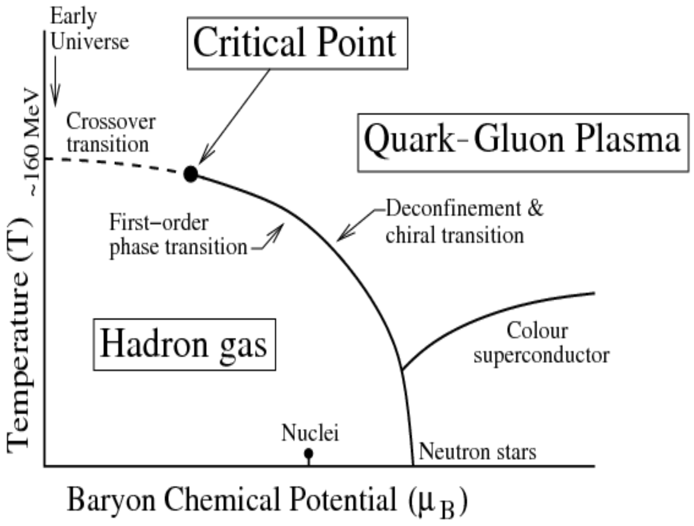
Relativistic Heavy-Ion Collisions : Big Picture



Collide heavy-ions at relativistic energies: hot and dense nuclear matter in the laboratory

Tune collision energy: QCD matter at wide range of temperature and densities

Experimental mapping the QCD phase diagram

Ultra-relativistic collisions @ RHIC/LHC:
Medium with high T negligible μ_B : Lattice QCD studies: cross over

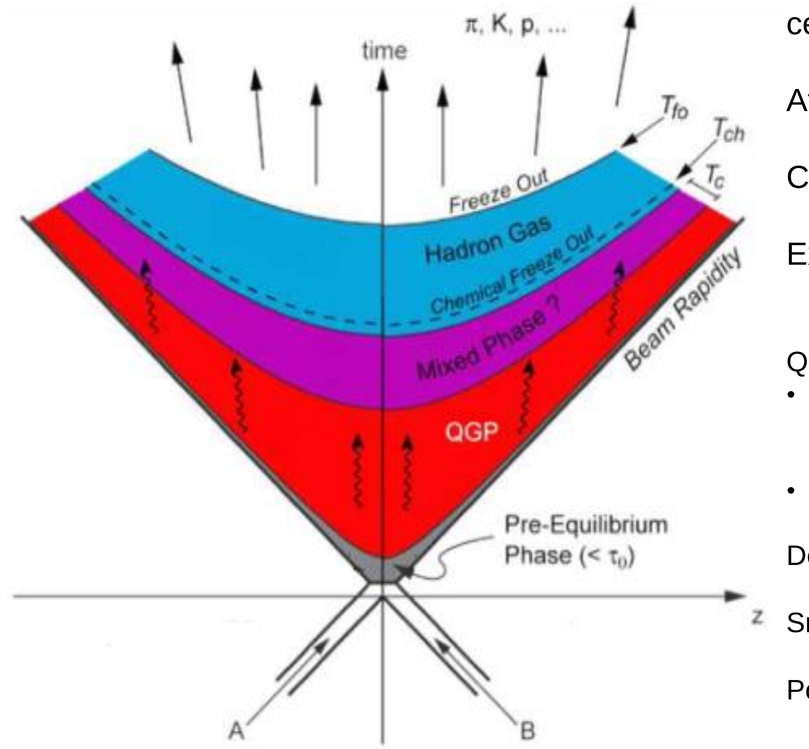
Lower beam energy:
QCD matter with moderate T finite μ_B: limited applicability of lQCD

Theoretical models predict 1st order phase transition, CEP
Renewed experimental interest: RHIC-BES, SPS, CBM@ FAIR, MPD@NICA …

Optimal utilization of future facilities demand systematic investigation of existing data from past experiments at AGS and SPS at lower beam energies

Determination of freeze-out conditions of the fireball at various beam energies:
one of the non-trivial issues (out of many) in relativistic heavy-ion collision experiments

Space-time evolution : Standard Picture



At chemical freeze-out particle chemistry stabilizes : inelastic scattering ceases

At kinetic freeze-out hadron momenta frozen: elastic collisions ceases

CFO precedes KFO due to larger mean-free path of inelastic collisions

Experimentally particle ratios for CFO and p_T spectra for KFO parameters

Quark flavor dependent multiple (sequential) freeze-out scenario:

- Strange hadrons fix their chemical compositions earlier than light hadrons (S. Chatterjee et. al., PLB (2013))
- Also observed for charmonia and bottomonia (D. Kumar et. al., PRC (2023))

Do such hierarchal structure prevail for kinetic decoupling?

Smaller medium induced momentum change of heavy hadrons than light hadrons

Possibility of hadron mass dependent hierarch in kinetic freeze-out

Blast Wave Model

Hydrodynamics inspired phenomenological model to study observables connected to collective expansion of nuclear fireball

Widely used to analyse momentum distribution of final state hadrons and provide thermodynamic information of the matter at collective freeze-out

Different variants of blast wave models in literature

Main assumption: produced particles (locally) thermalized till emitting the fireball and collective expansion of the system with common radial velocity field undergoing an instantaneous common freeze-out

Expansion profile dependent on beam energy:
Very low (few GeV) energy regime (HADES @ SIS18): spherically expanding source: isotropic radial expansion
(P. J. Siemens and J. O. Rasmussen, PRL (1979))

Ultra-relativistic energy (RHIC, LHC):
- Stronger longitudinal flow: cylindrical expansion: boost-invariant blast wave model (E. Schnedermann, J. Sollfrank and U. Heinz, PRC (1993))
- Reasonable description of transverse spectra with two parameters: T_{kin} and β_T

What happens at intermediate energies (AGS, SPS)?
- Rapidity distribution is Gaussian
- Longitudinal boost invariance does not hold good
- Any reliable description of particle spectra needs to relax the assumption of boost invariance
- Non boost-invariant blast wave model: Explicitly break the cylindrical symmetry via suitable modification of system boundary for low energies (H. Dobler, J. Sollfrank and U. Heinz, PLB (1999))
- Simultaneous description of transverse (p_T) and rapidity (y) spectra

Kinetic freeze-out in non-boost-invariant blast-wave model

For an expanding fireball, common instantaneous freeze-out at t_F, thermal single particle spectrum in transverse mass m_T and rapidity y:

$$\frac{dN}{m_T dm_T dy} = \frac{g}{2\pi} m_T \tau_F \int_{-\eta_{\max}}^{+\eta_{\max}} d\eta \cosh(y-\eta) \times \int_0^{R(\eta)} r_\perp \, dr_\perp \, \mathrm{I}_0\left(\frac{p_\mathrm{T} \sinh\rho(r_\perp)}{T}\right) \times \exp\left(\frac{\mu - m_\mathrm{T}\cosh(y-\eta)\cosh\rho(r_\perp)}{T}\right)$$

- $\eta = \tanh^{-1}(z/t)$, is space-time rapidity.
- T is Kinetic freeze-out temperature.
- ρ is transverse rapidity.
- $\beta_T = \tanh(\rho)$, is transverse velocity.

- Relaxation of boost-invariance by introducing a dependence of the transverse size of the fireball on the space-time rapidity (η):

$$R(\eta) = R_o \sqrt{\left(1 - \frac{\eta^2}{\eta^2{}_{max}}\right)}$$

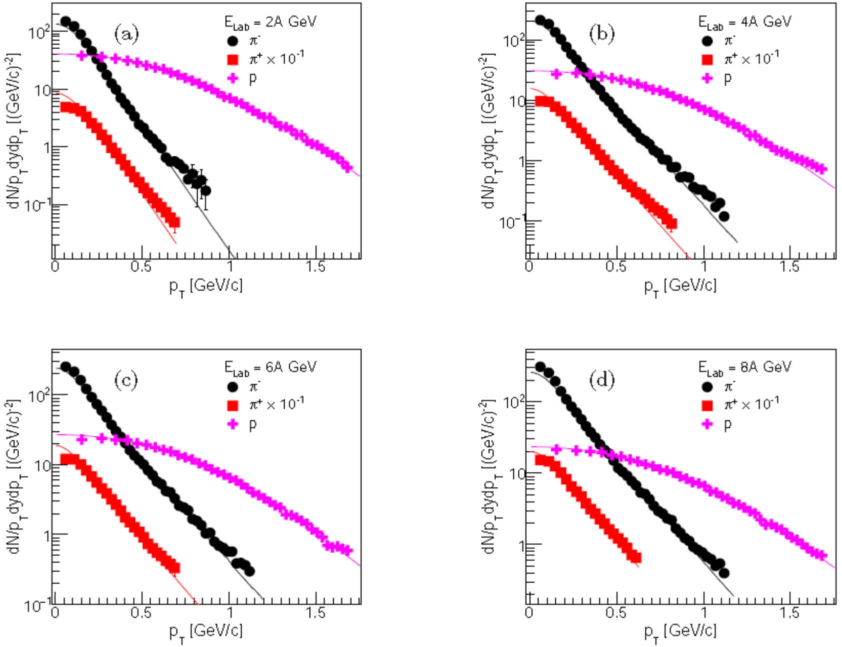
- Boost is restricted between $|\eta| \leq \eta_{\max}$, $\eta_{\max}$ is maximum space-time rapidity
- Parametrization of transverse flow: Hubble like expansion: $\beta_T(r_\perp) = \beta_s\left(\frac{r_\perp}{R(\eta)}\right)$ ⇨ $<\beta_T> = \frac{2}{3}\beta_s$
- Transverse flow vanishes at the centre, maximum at the surface
- Exponential expansion of the fireball in the transverse direction
- Use this variant to analyze transverse and longitudinal spectra in central collisions at AGS and SPS energies

Fitting strategy: iterative method

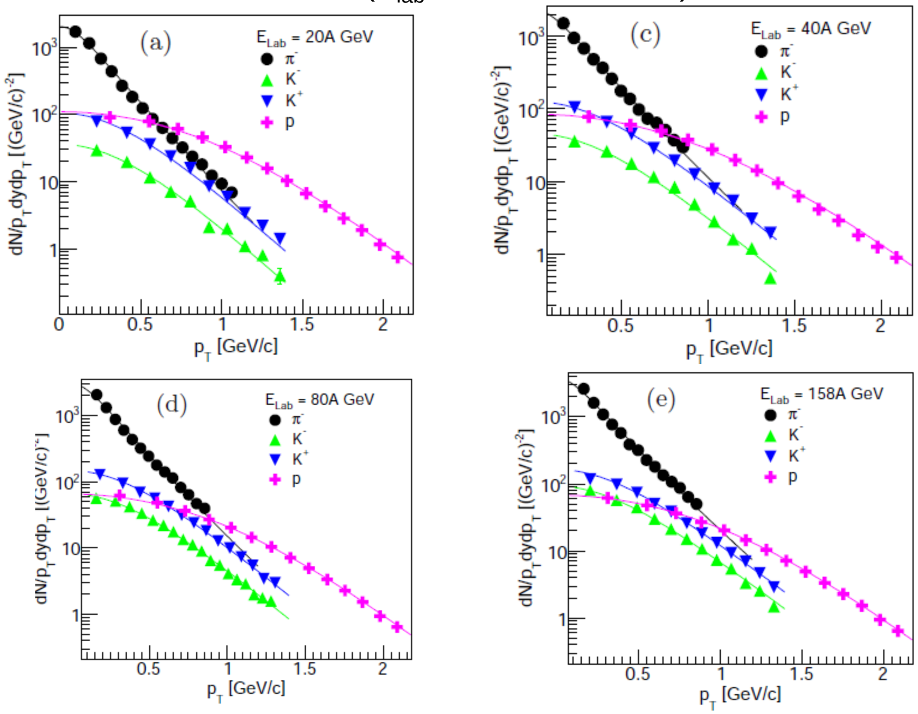
- Separate simultaneous fit of p_T and y spectra of identified light hadrons (π, K, p) and heavy strange hadrons ($\Lambda, \phi, \Xi, \Omega$)
- Strange hadrons barely available at AGS (E_b = 2A – 11A GeV) energies with poor statistics
- Strange analysis only at SPS energies (E_b=20A -160A GeV)
- Three (simultaneous) fit parameters (T_{kin}, β_T and η_{max}) in contrast to two (T_{kin}, β_T) for boost-invariant model
- p_T spectra sensitive to (T_{kin}, β_T), y spectra sensitive to η_{max}
- Follow a recursive method of fitting
- Fit y spectra to get η_{max}, with guess values of T and β_T
- Use this η_{max} value to fit p_T spectra to get (T_{kin}, β_T)
- (T_{kin}, β_T) further used to fit y spectra to refine η_{max}
- Iterations continue until convergence between fit parameter

Light hadron p_T spectra

AGS (E_{lab}: 2A – 8A GeV)

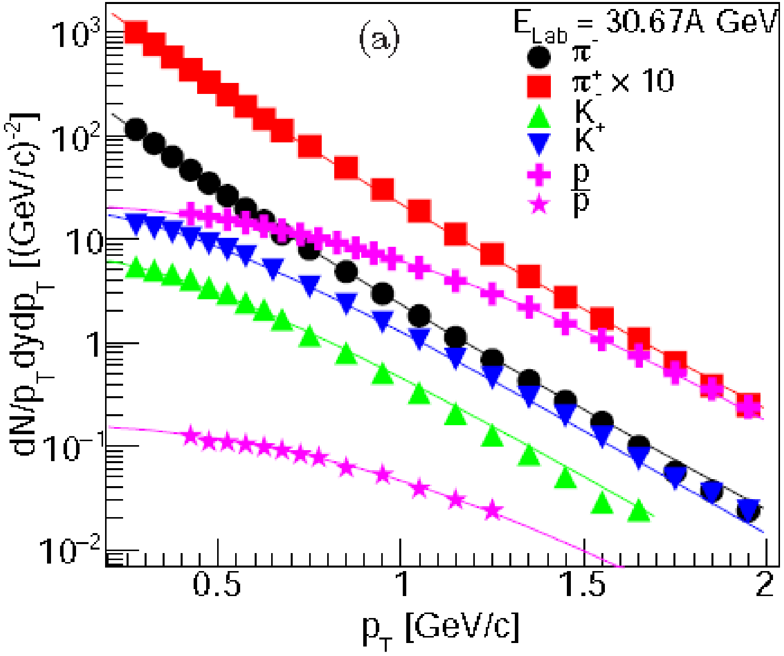


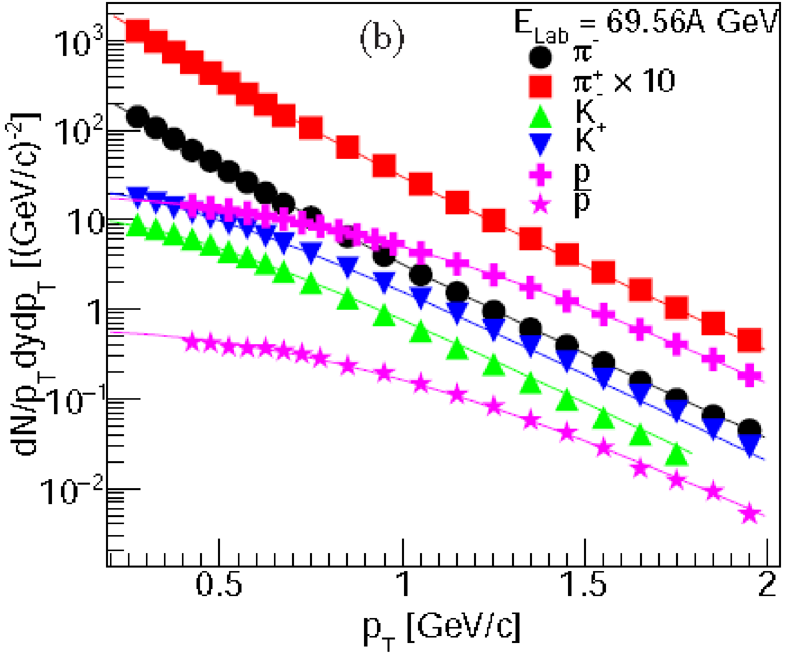
SPS (E_{lab}: 20A – 158A GeV)



- 0 – 5 % (0 – 7 %) central Au+Au (Pb+Pb) collisions
- Resonance decay (2body and 3 body) contribution to pion
- Decay contribution from hadrons beyond Δ (1232) neglected
- Fitted parameters: T_{kin}~ 55 – 85 MeV $<\beta_T>$ ~ 0.48c – 0.55c η_{max} ~ 0.99 – 2.6

RHIC-BES (STAR): light hadron p_T spectra

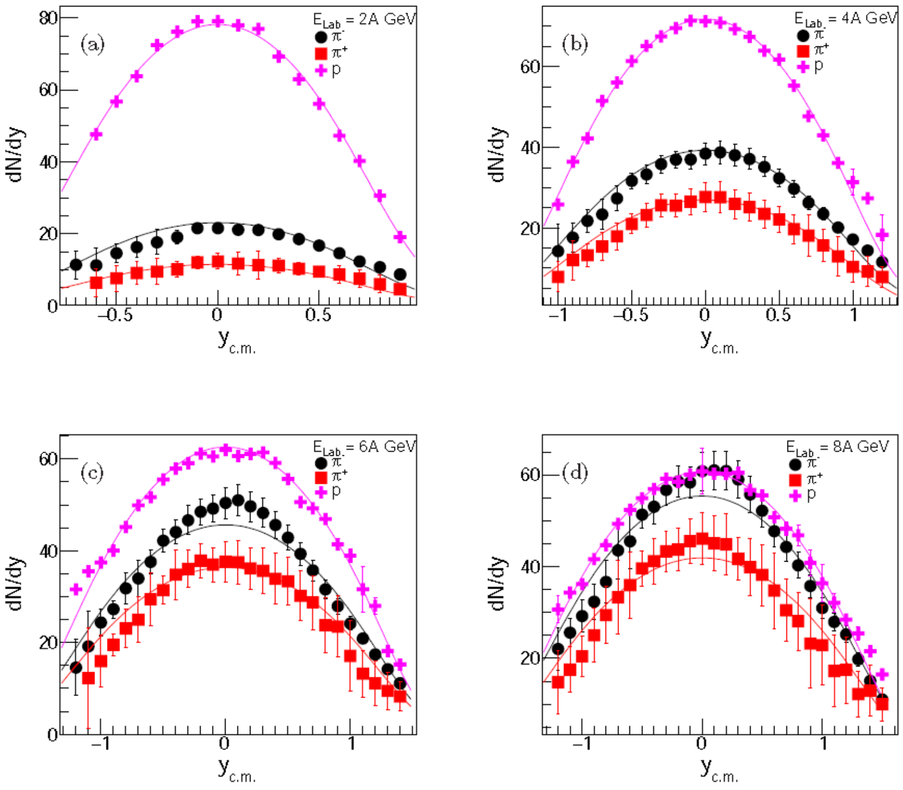




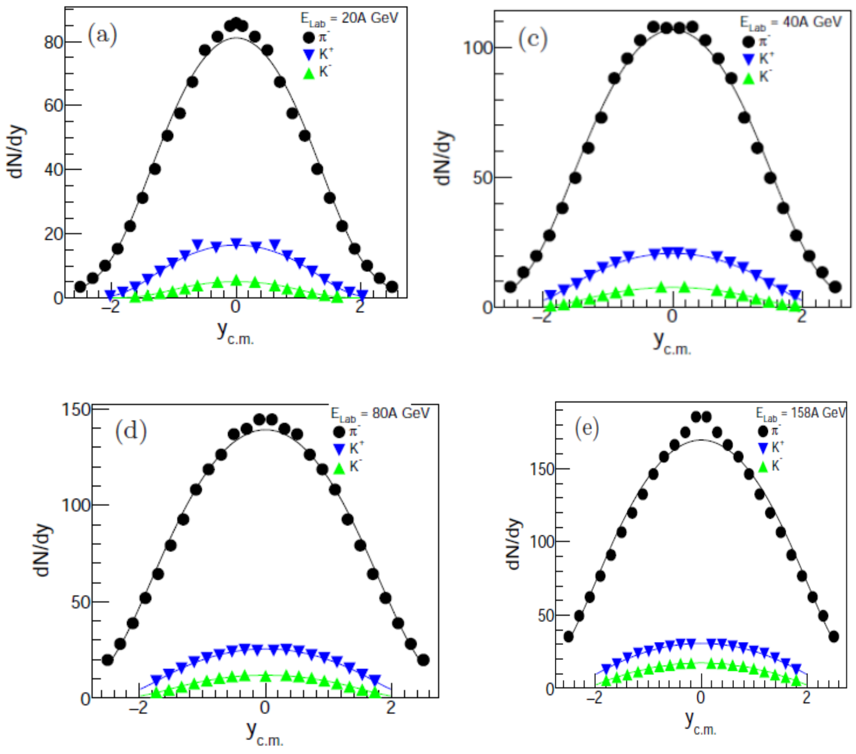
- Central (0 – 5 %) Au+Au collisions
- Mid-rapidity ($-0.1 < y_{c.m.} < 0.1$) spectra
- Lower p_T cut (0.2 GeV/c) in data: resonance decay not included

Light hadron rapidity spectra

AGS



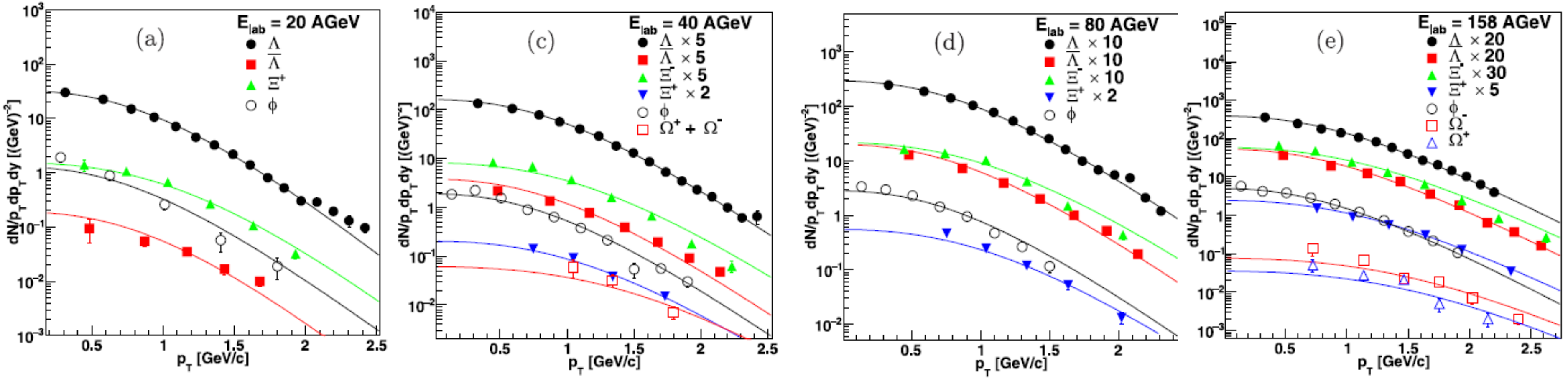
SPS



- Spectra rather insensitive to temperature
- No rapidity spectra from RHIC-BES program available

SPS: heavy strange hadron p_T spectra

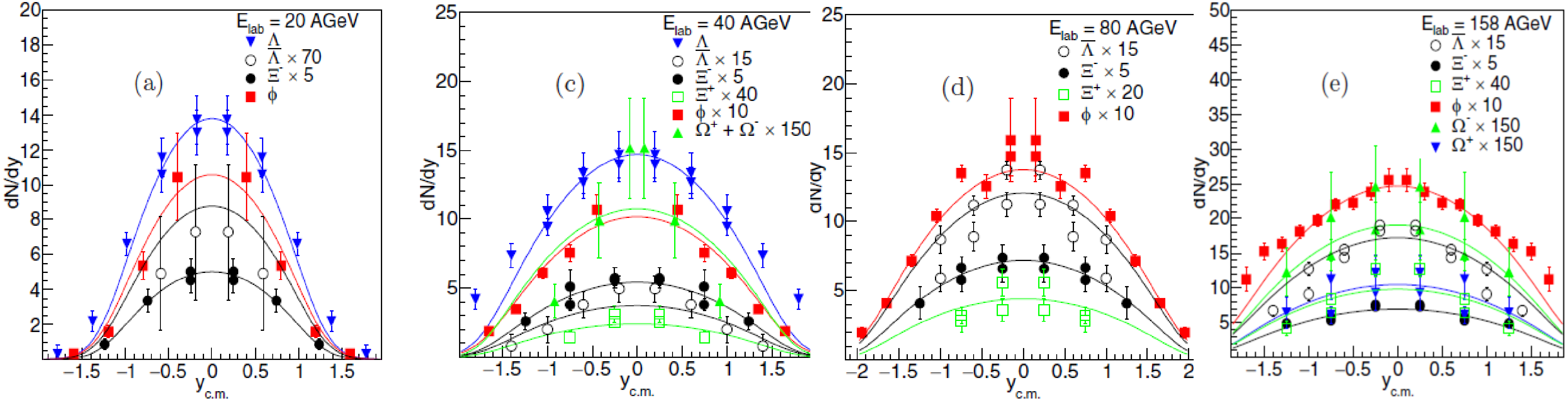
S. P. Rode, PPB, A. Jaiswal and A. Roy, Phys. Rev. C **102**, 054912 (2020)



- Simultaneously fitted p_T spectra at different SPS beam energy (E_b=20-158 A GeV)
- Most central (0- 7 %) Pb+Pb collisions
- Available statistical error bars
- Best fit parameters: T_{kin} : 93 – 110 MeV $\langle\beta_T\rangle$: 0.44 - 0.47c

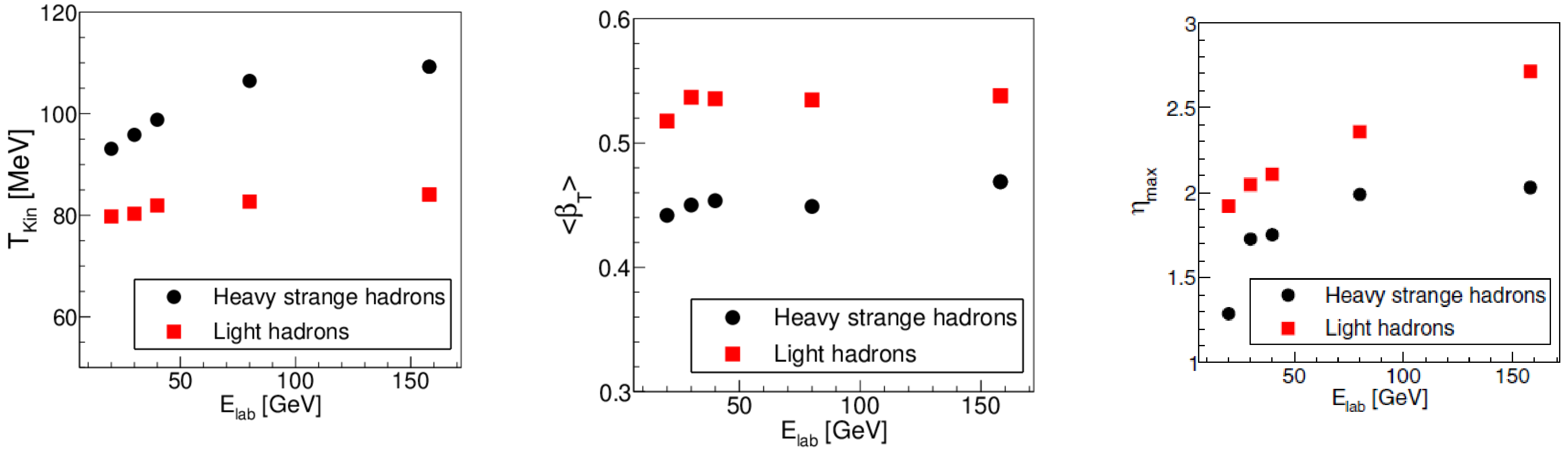
SPS: heavy strange hadron rapidity spectra

S. P. Rode, PPB, A. Jaiswal and A. Roy, Phys. Rev. C **102**, 054912 (2020)



- Simultaneously fitted y spectra at different SPS beam energy (E_b=20 - 158 A GeV)
- Most central Pb+Pb collisions, larger centrality bins for heavier species
- Available statistical error bars
- Best fit parameters: T_{kin} ~ 93 – 110 MeV $\langle\beta_T\rangle$ ~ 0.44 - .47c η_{max} ~ 1.29 – 2.03

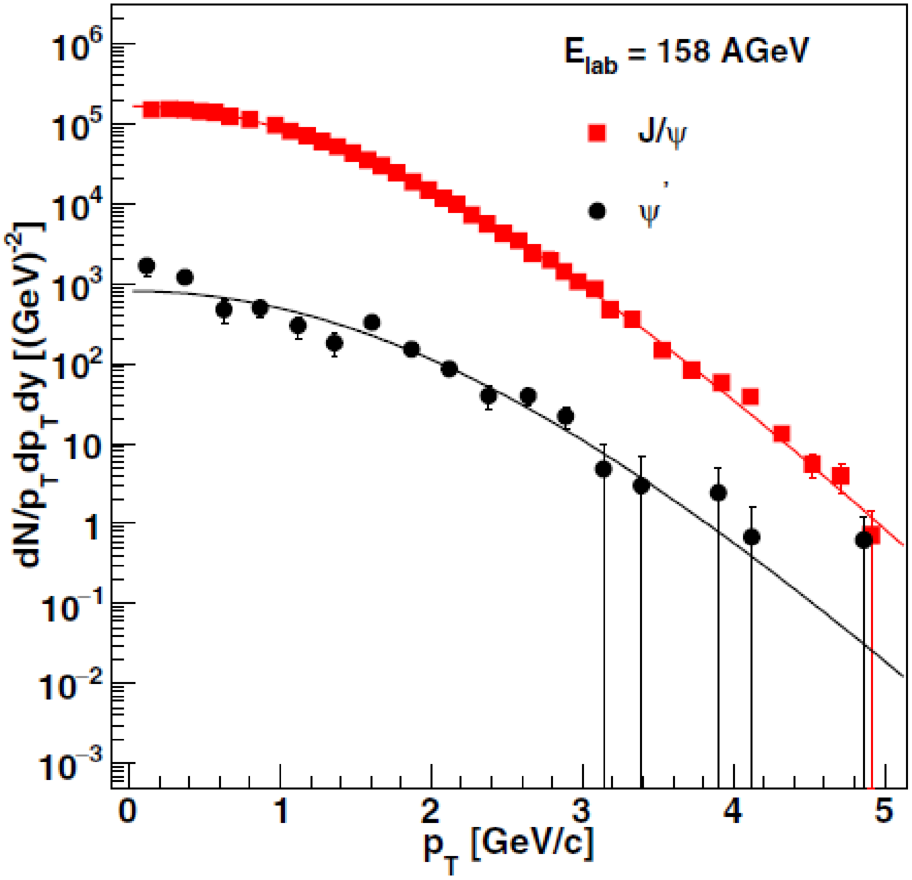
Mass hierarchy in freeze-out parameters



- $T^{LH}_{kin} < T^{SH}_{kin}$ → **suggest earlier kinetic decoupling of the strange hadrons**
- Similarly, $<\beta_T>^{LH} > <\beta_T>^{SH}$.: stronger radial flow at late phase
- Indication of **mass-dependent hierarchy** in the kinetic freeze-out parameters

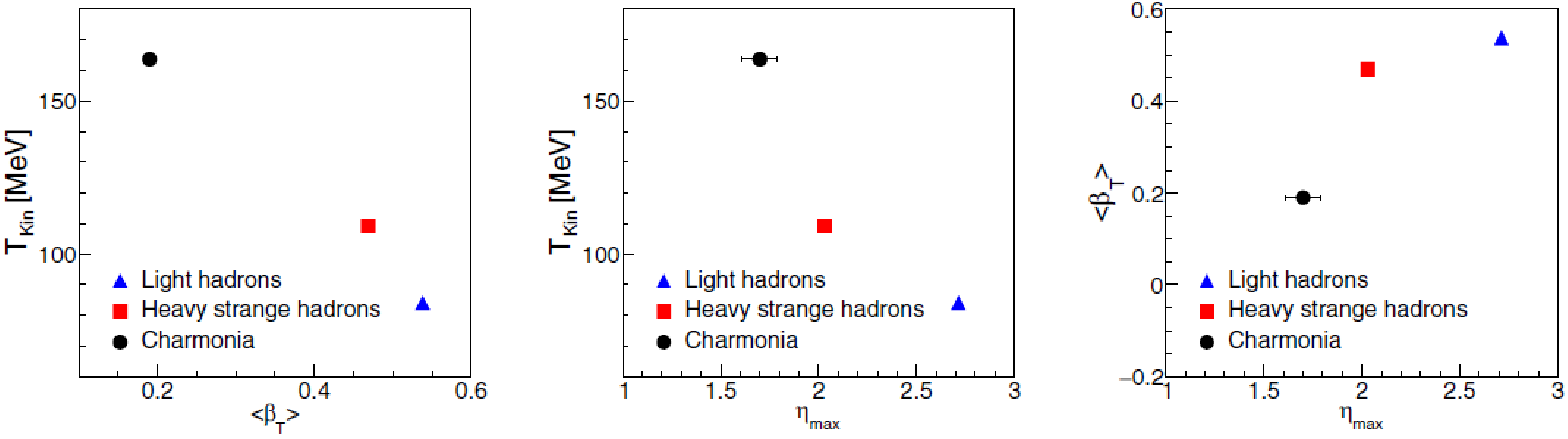
Phys. Rev. C 98, no. 2, 024907 (2018)
Phys. Rev. C 102 (2020) 5, 054912.

Charmed hadron p_T spectra



- Simultaneous fit to J/ψ and ψ' p_T spectra
- Thermal fit over entire available p_T domain
- No rapidity spectra available
- η_{max} fixed from p_T spectra only
- Data from SPS-NA50 Collaboration
- Central (0 -10%) Pb+Pb collisions
- No data below 158 A GeV
-
- No open charm data at SPS available
- Errors indicate uncorrelated statistical and systematic errors added in quadrature
- T_{kin} ~ 164 $\langle\beta_T\rangle$ ~ 0.22
- Stable fit parameters to the choice upper cut in p_T

(Partial) expansion history of the fireball



- Kinetic freeze-out points measured from hidden charm, heavy strange and bulk hadron spectra
- 158 A GeV central Pb+Pb collisions
- Defines path of the expanding system
- Monotonous nature indicates mass dependent hierarchy
- Heaviest charmed hadrons decoupling earliest from the fireball due to negligible scattering cross section in the hadronic phase
- Systematic investigation of freeze-out parameters: trace expansion history of the fireball in nuclear collisions
- No charm data below top SPS energy
- Foreseen measurements of charmonia in NA60+ @ SPS below 158 A GeV would further clarify the picture

Inclusion of Fluctuation

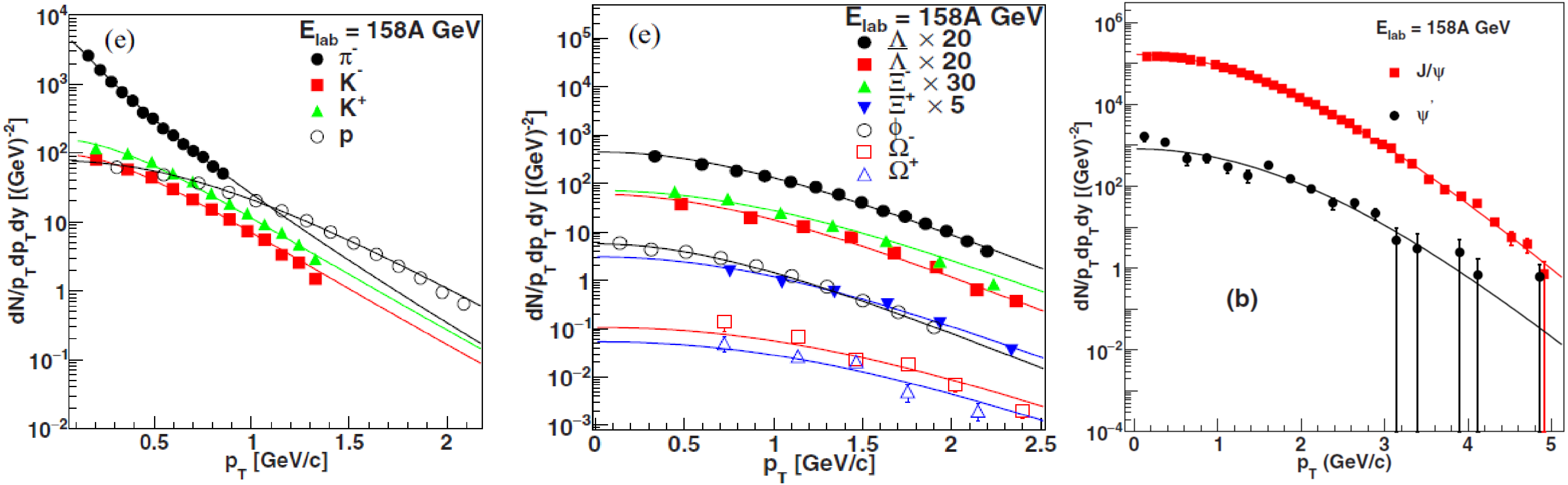
- Finite size of the fireball produced in relativistic heavy-ion collisions
- Possibility of large fluctuations in the initial stage, even in central collisions
- Fluctuations may survive till freeze-out

$$\frac{dN}{m_T dm_T dy} \propto \frac{g}{2\pi} m_T \tau_F \int_{\beta_s{}^{min}}^{\beta_s{}^{max}} d\beta_s F(\beta_s) \times \int_{-\eta_{max}}^{+\eta_{max}} d\eta \cosh(y-\eta) \times \int_0^{R(\eta)} r_\perp dr_\perp\, I_o\left[\frac{p_T \sinh\rho(r_\perp)}{T}\right] \times \exp\left[\frac{\mu - m_T\cosh(y-\eta)\cosh\rho(r_\perp)}{T}\right]$$

Parametrization of β_s : $F(\beta_s) = \begin{cases} 1 & : Uniform : \beta_s^{min} < \beta_s < \beta_s^{max} \\ \exp\left[-\frac{(\beta_s - \beta_s^o)^2}{\delta^2}\right] & : Gaussian : 0 < \beta_s < 1 \end{cases}$

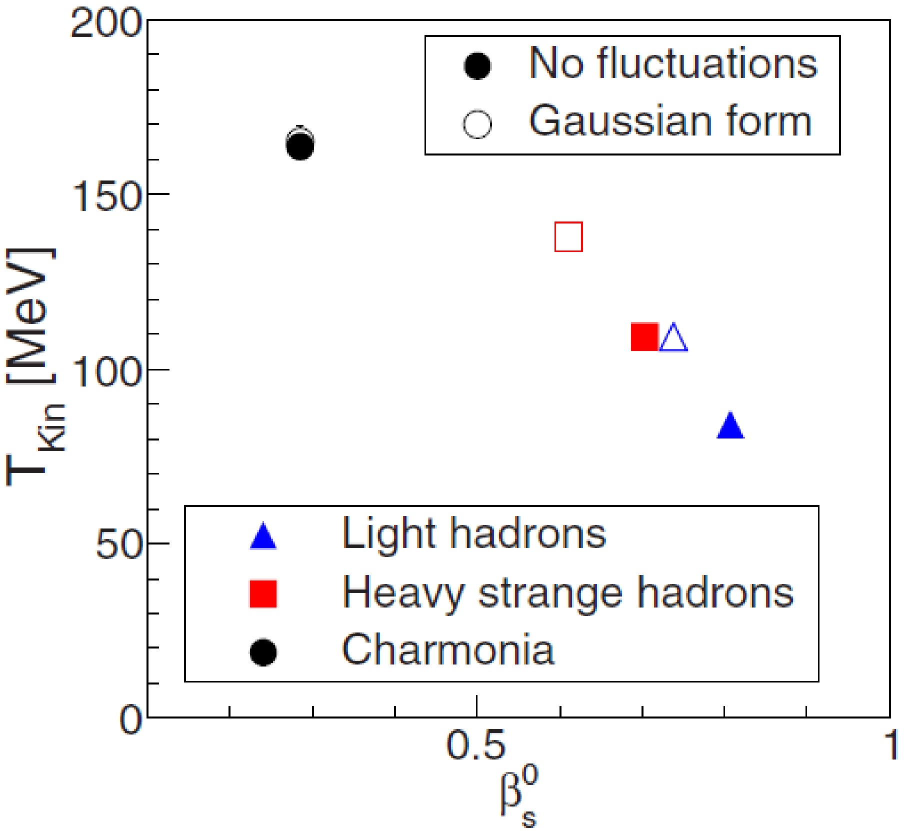
S. V. Akkelin, P. Braun-Munzinger and Y. M. Sinyukov, Phys. Rev. C **81** (2010), 034912

Gaussian flow fluctuation



- Hadrons categorized according to their masses
- Fluctuation incorporated in transverse flow: y spectra and η_{max} remain unaltered
- Resonance decay contribution included
- Best fit values of T_{kin} ($<\beta_T>$) higher(lower) than default (no-fluctuation) and uniform velocity distribution cases for light and heavy strange hadrons
- For charmonia, fit parameters almost insensitive to the inclusion of fluctuation

Updated (partial) expansion history

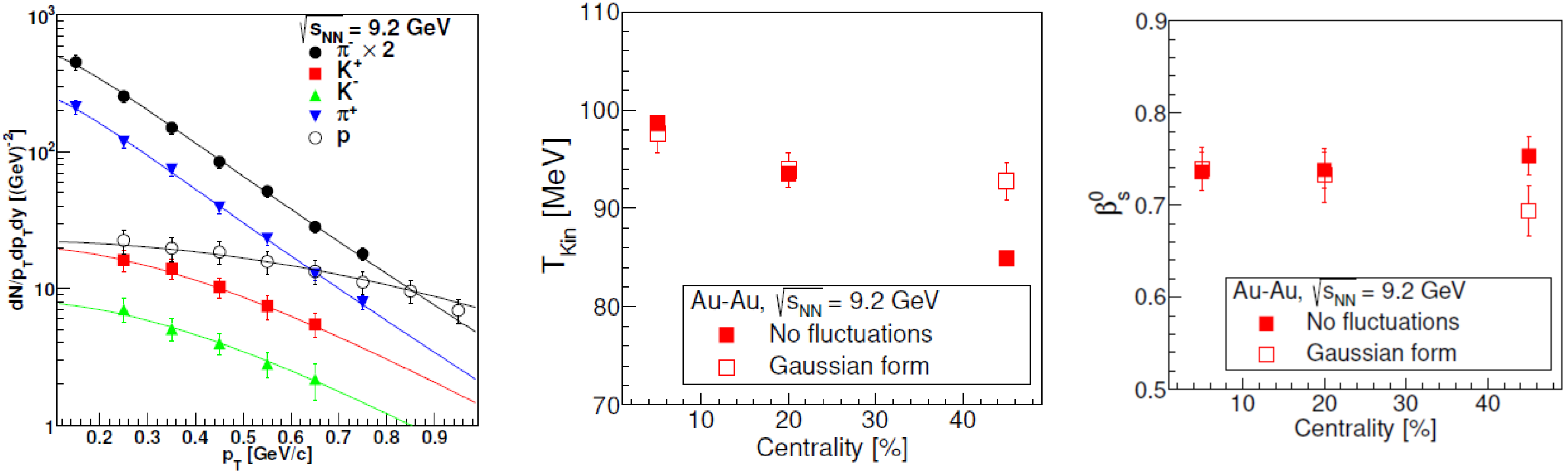


- (Partial) expansion history of the fireball in presence of flow fluctuation
- Mass hierarchy in kinetic freeze-out preserved in presence of flow fluctuations
- Minimal effect of fluctuation on charmed hadrons
- Small scattering cross section in hadronic phase
- Momentum distribution of charmonia frozen near phase boundary
- Radial flow and associated fluctuation underdeveloped and show insensitivity

S. P. Rode, PPB, A. Jaiswal, PRC (2023)

A possible case for NICA

Sudhir P. Rode, PPB & A. Jaiswal (under preparation)



- Light hadron p_T spectra analysed in $\sqrt{s_{NN}} = 9.2$ GeV in Au+Au collisions from STAR @ RHIC-BES
- Analysis performed for both no fluctuation and Gaussian fluctuation scenarios
- Three different (0-10 %, 10 – 30 %, 30 – 60 %) centrality intervals
- For less central collisions, smaller system size, more prominent effect of flow fluctuations

Would be useful to make prediction for upcoming Bi+Bi collisions at NICA at same energy

Summary and Outlook

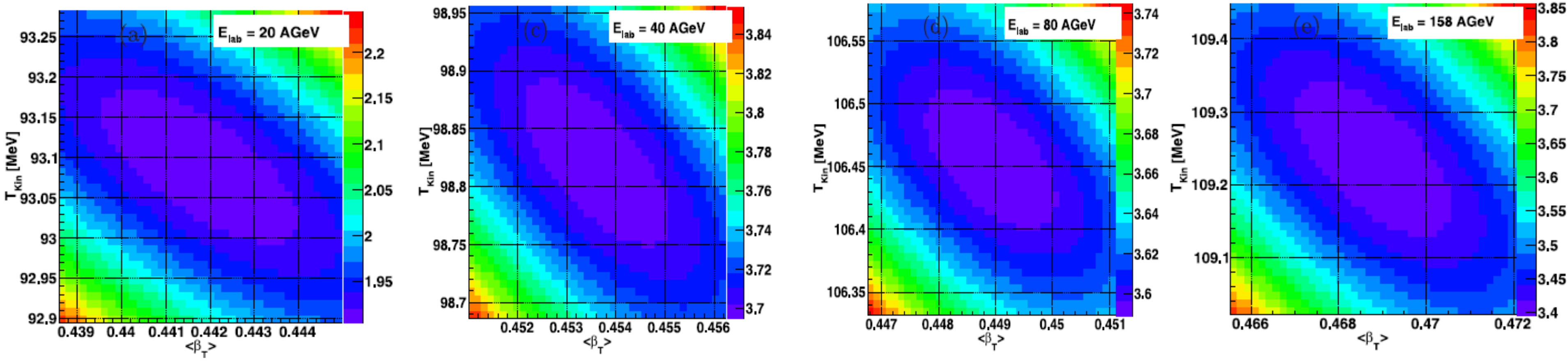
- Kinetic freeze-out conditions in low energy (2A-158A GeV) nuclear collisions using non-boost-invariant blast wave model
- Boost-invariance explicitly broken by introducing dependence of transverse size of the fireball on space-time rapidity
- Both T_{kin} and β_T lead to hardening of the transverse spectra
- Accounting for resonance decays lead to effective cooling of spectra: relatively lower T_{kin}
- Clear mass dependent hierarchy in the kinetic freeze-out parameters
- Inclusion of flow fluctuations lead to higher value of T_{kin} and lower β_T : mass hierarchy is preserved
- Minimal effect of fluctuations on charm hadron spectra
- Results useful for upcoming measurements at NA60+ at SPS, CBM at FAIR and MPD at NICA

In future we plan to:

- Extend the model to non-central collisions
- Investigate the flow fluctuations more closely at different centralities and anisotropic flow coefficients
- Universal model to include both spherical and cylindrical expansion profiles to study freeze-out from AGS/HADES to LHC

Thank You

Constant χ^2 contour: 2-D projection



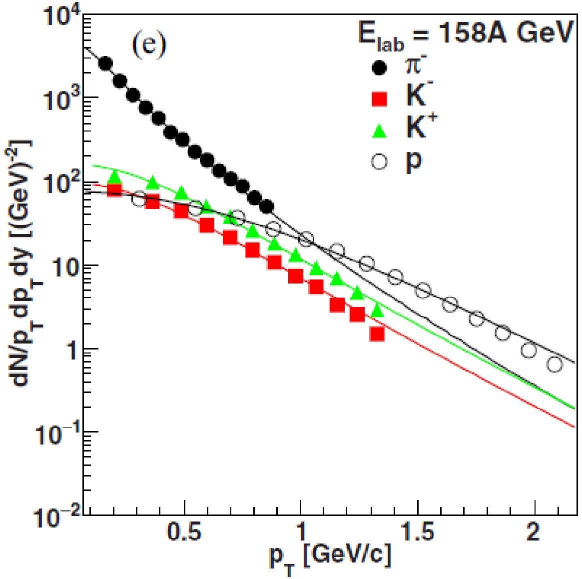
2D projections of χ^2 contour plots in $\langle\beta_T\rangle$-T_{kin} plane

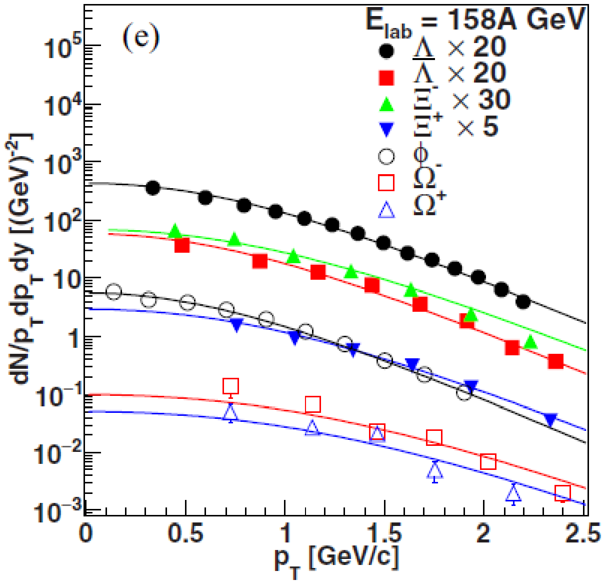
Different colours correspond to different values of χ^2/N_{DOF}

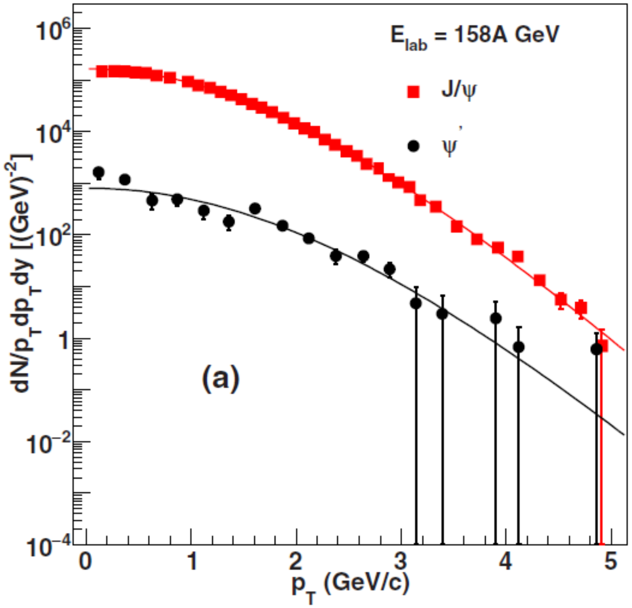
Anti correlation between T_{kin} and $\langle\beta_T\rangle$

S. P. Rode, PPB, A. Jaiswal and A. Roy, Phys. Rev. C **102**, 054912 (2020)

Uniform flow fluctuation







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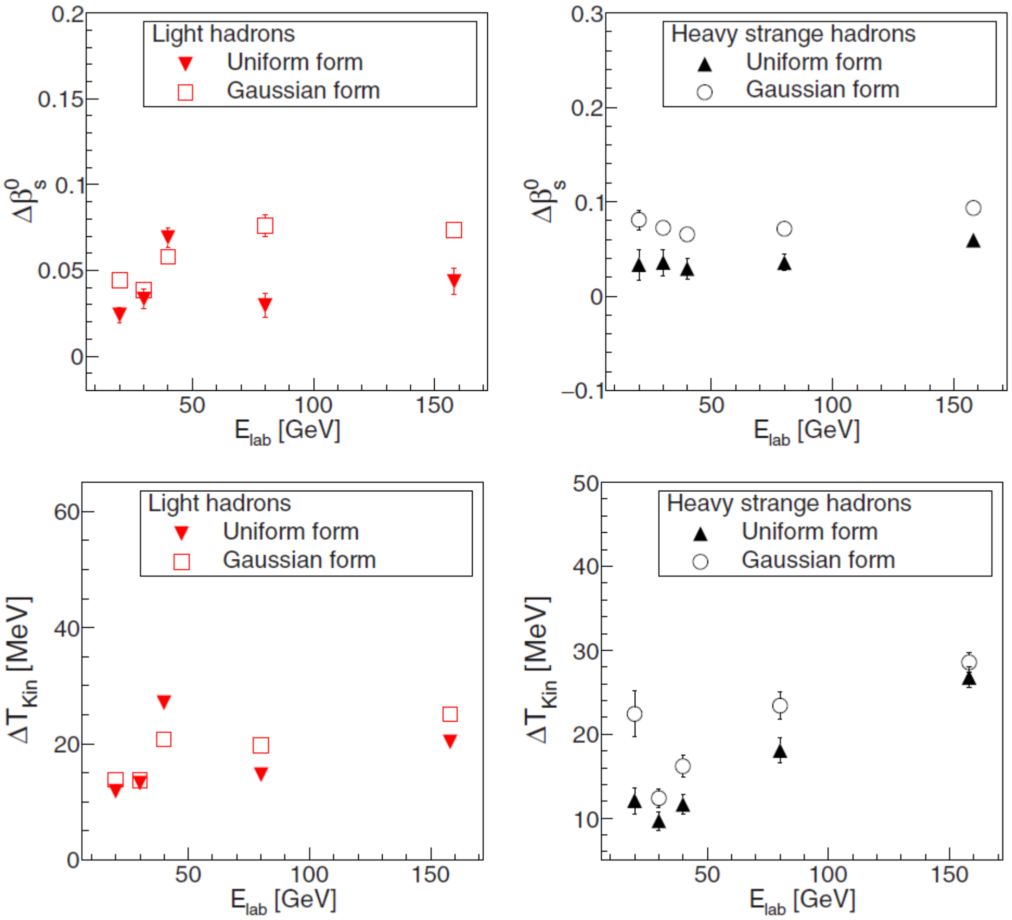
S. P. Rode, PPB, A. Jaiswal, PRC (2023)

Comparison of different scenarios



$\Delta T_{kin} = T_{kin}^{UF/GF} - T_{kin}^{NF}$

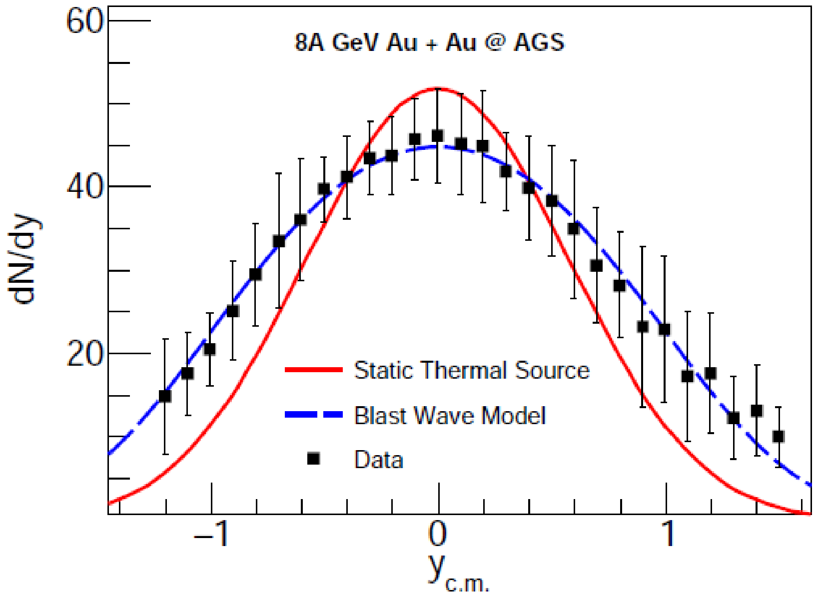
Non-monotonic beam energy dependence

Minima/Maxima around 30 – 40 A GeV

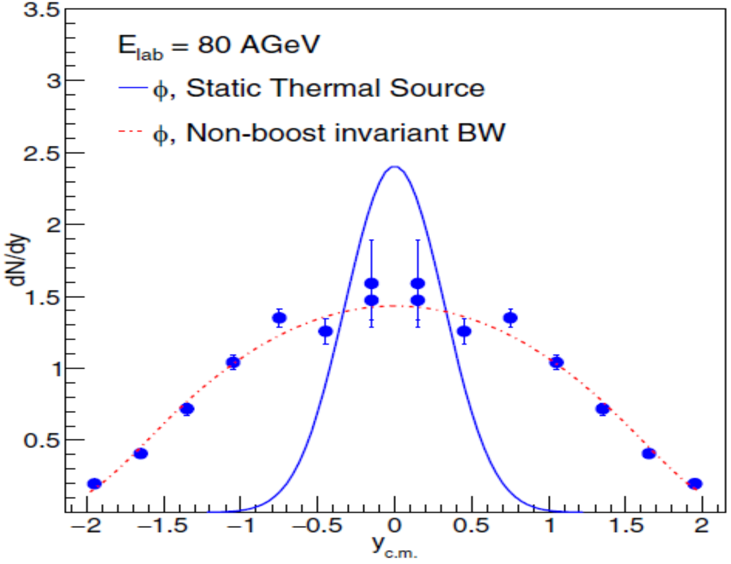
Does indication of any special feature?

Rapidity distribution: a closer look

π^- @ AGS



ϕ @ SPS



Static thermal source: $$\frac{dn_{th}}{dy} = \frac{V}{(2\pi)^2} T^3 \left(\frac{m^2}{T^2} + \frac{m}{T}\frac{2}{\cosh y} + \frac{2}{\cosh^2 y}\right) \times \exp\left(-\frac{m}{T}\cosh y\right)$$

- Emission from static thermal source cannot explain the data: faster fall off
- Longitudinal flow of the source required to reproduce the shape of measured distribution