Exotic hadrons from Lattice QCD

Nilmani Mathur

tata institute of fundamental research

Collaborators: Parikshit Junnarkar (GSI, Germany), M. Padmanath (IMSc.) and Archana Radhakrishnan (TIFR)

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Exotic Hadrons

- Hadrons whose quantum numbers require a valence quark content beyond **qqq** or $q\bar{q}$ are called as **"exotics"**, e.g. $cc\bar{u}\bar{d}$, **glueball**

- Hadrons whose spin, parity and charge conjugation are forbidden in the non-relativistic quark model are also often termed "exotics" (spin exotics)

- Cryptoexotics :
  - mass/width does not fit with meson or baryon spectra
  - overpopulation of the spectra
  - production or decay properties incompatible with standard mesons/baryons

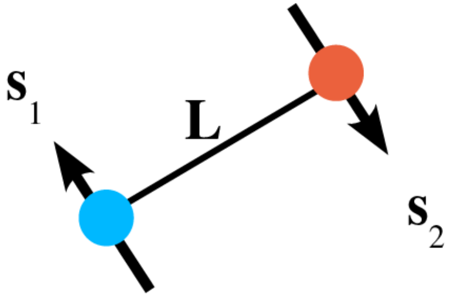
A constituent picture of Hadrons

- QCD : Fundamental degrees of freedoms are quarks (6 flavours) and gluons (8 degrees of freedom)
- Confinement conjecture: quarks and gluons must be combined into colour-neutral combinations of hadrons

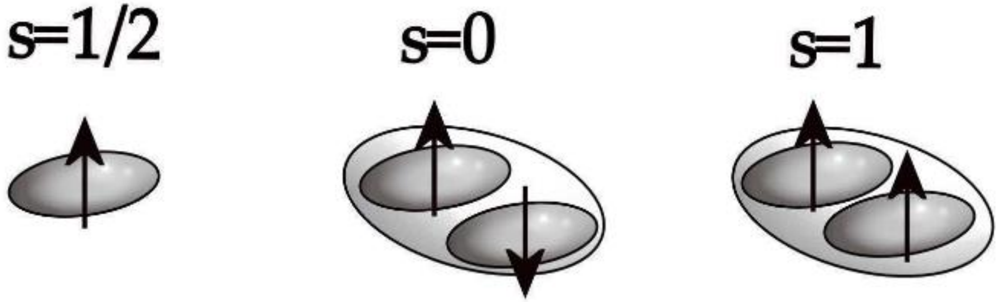
Constituents	**Combinations**	**Naming convention (quark model)**
$\mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{8}$	Meson
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$	Baryon
$\mathbf{8} \otimes \mathbf{8}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Glueball
$\bar{\mathbf{3}} \otimes \mathbf{8} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Hybrid
$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Tetraquark/molecule
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27} \oplus \mathbf{35} + \cdots$	Pentaquark
………..	………..	?

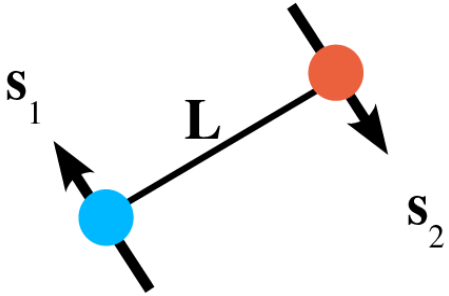
A constituent model of hadrons

- However, there can be strong mixings between different hadrons with the same quantum numbers



S = 0, 1





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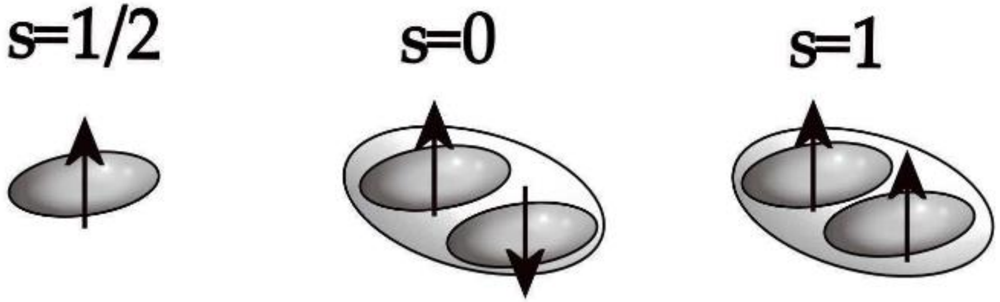
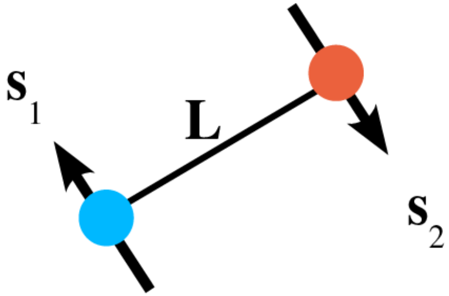


Combine with orbital angular momentum ***L***

$$\vec{J} = \vec{L} \oplus \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$



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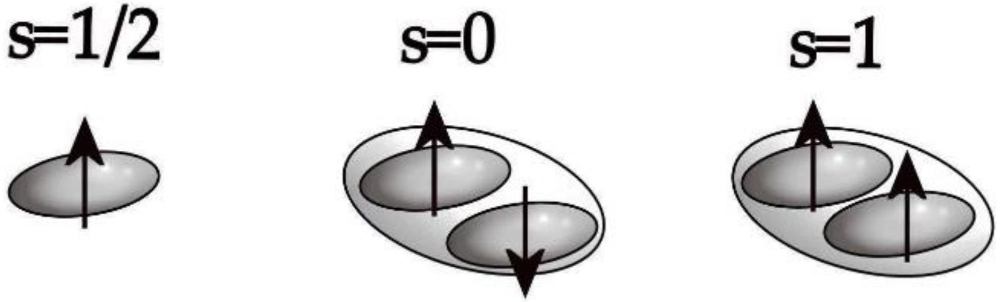


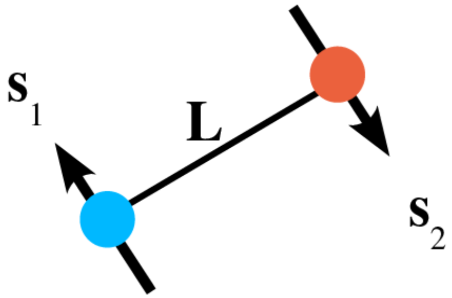
Combine with orbital angular momentum ***L***

$$\vec{J} = \vec{L} \oplus \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

	L = 0	L = 1	L = 2	L = 3	....
Singlet (S = 0)	0^{-+}	1^{+-}	2^{-+}	3^{+-}	....
Triplet (S = 1)	1^{--}	$\{1,2,3\}^{++}$	$\{1,2,3\}^{--}$	$\{2,3,4\}^{++}$	....
	S-wave	P-wave	D-wave	F-wave	

Allowed : $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{-+}, 2^{++}, ...$

Are these all?



$$\vec{J} = \vec{L} \oplus \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

Forbidden (within such a model) quantum numbers :

J^{PC}: $0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, 4^{+-}, \ldots.$

$odd^{-+}, even^{+-}$

Any meson with these quantum numbers will be called **EXOTIC MESON (spin exotic)**

Example of an Exotic

- States with quantum number : $\mathbf{1^{-+}}$
- It is not possible to write an interpolating field for this state with a form : $\bar{q}\Gamma q$
- Possible operators :

$$\vec{q}^{\,a}\gamma_4 E_j^{ab} q^b,$$

$$i\varepsilon_{jkl}\vec{q}^{\,a}\gamma_k B_l^{ab} q^b \Rightarrow \rho \otimes B$$

$$i\varepsilon_{jkl}\vec{q}^{\,a}\gamma_4\gamma_k B_l^{ab} q^b$$

$$\varepsilon_{jkl}\vec{q}^{\,a}\gamma_5\gamma_4\gamma_k E_l^{ab} q^b$$

$$\vec{q}\gamma_4 \overleftrightarrow{D} q$$

$$\vec{q}^{\,a}_{\alpha}\gamma_5 q^a_{\beta}\vec{q}^{\,b}_{\beta}\gamma_5\gamma_i q^b_{\lambda} \Rightarrow \pi \otimes a_1$$

$$\varepsilon_{ijk}\vec{q}\gamma_5\gamma_4\gamma_j \overleftrightarrow{D}_k q$$

$$\varepsilon_{ijk}\vec{q}\gamma_j \vec{B}_k q, \quad \vec{B}_i = \varepsilon_{ijk}\vec{D}_j\vec{D}_k$$

$$\varepsilon_{ijk}\vec{q}\gamma_4\gamma_j \vec{B}_k q$$

$$\begin{aligned}\mathbb{B}_i &= \epsilon_{ijk}\overrightarrow{D}_j\overrightarrow{D}_k \\ &= \epsilon_{ijk}\tfrac{1}{2}\Big([\overrightarrow{D}_j, \overrightarrow{D}_k] + \{\overrightarrow{D}_j, \overrightarrow{D}_k\}\Big) \\ &= \epsilon_{ijk}\tfrac{1}{2}[\overrightarrow{D}_j, \overrightarrow{D}_k] \\ &= -\tfrac{i}{2}\epsilon_{ijk}F^{jk}\end{aligned}$$

$$\mathbb{E}_i \overset{-}{=} \mathbb{Q}_{ijk}\overleftrightarrow{D}_j\overleftrightarrow{D}_k$$

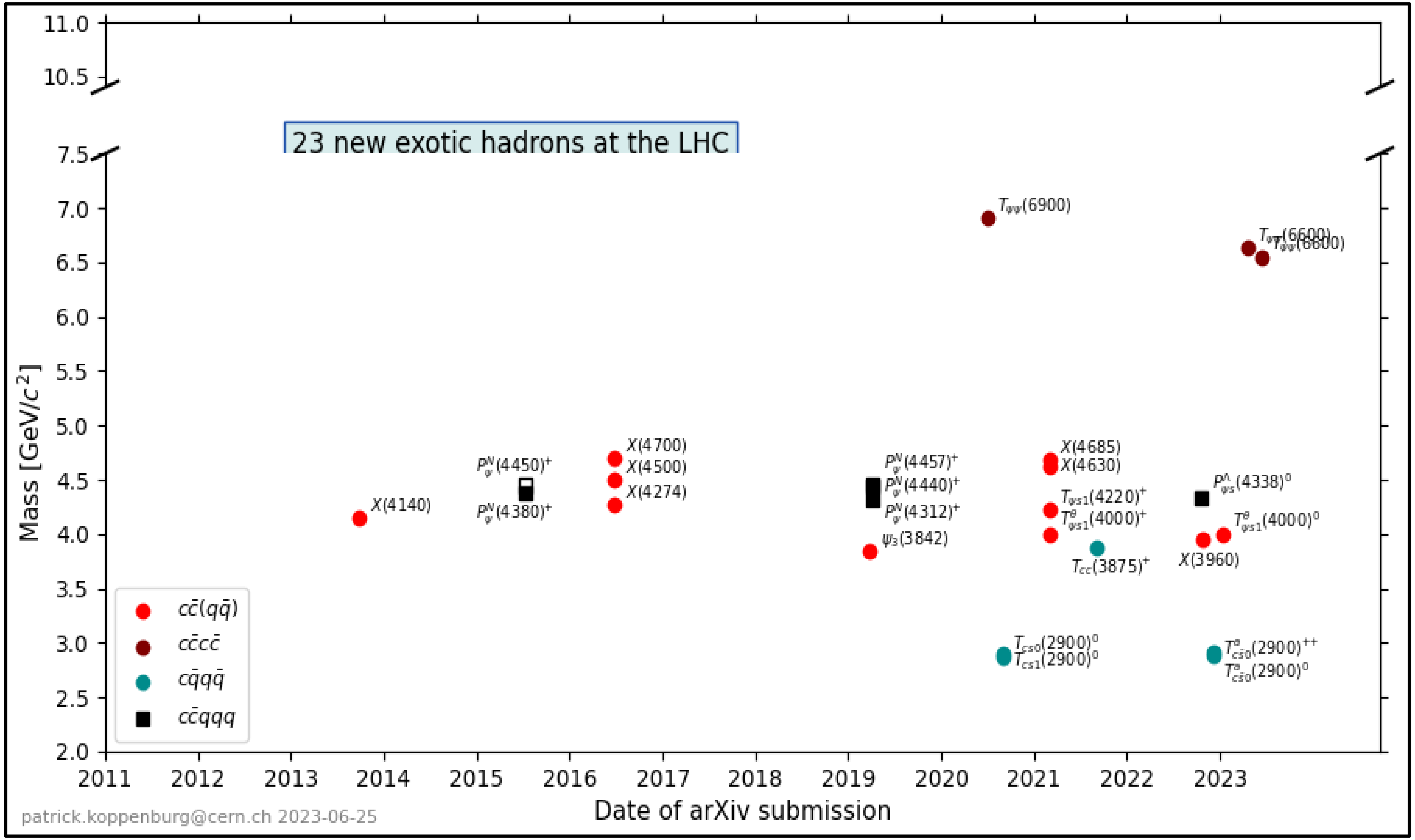
Hadspec (2008)

Quest

- Does nature permit what QCD allows?

- Are there subatomic particles beyond mesons and baryons valence structures?

Exotic hadrons at LHC

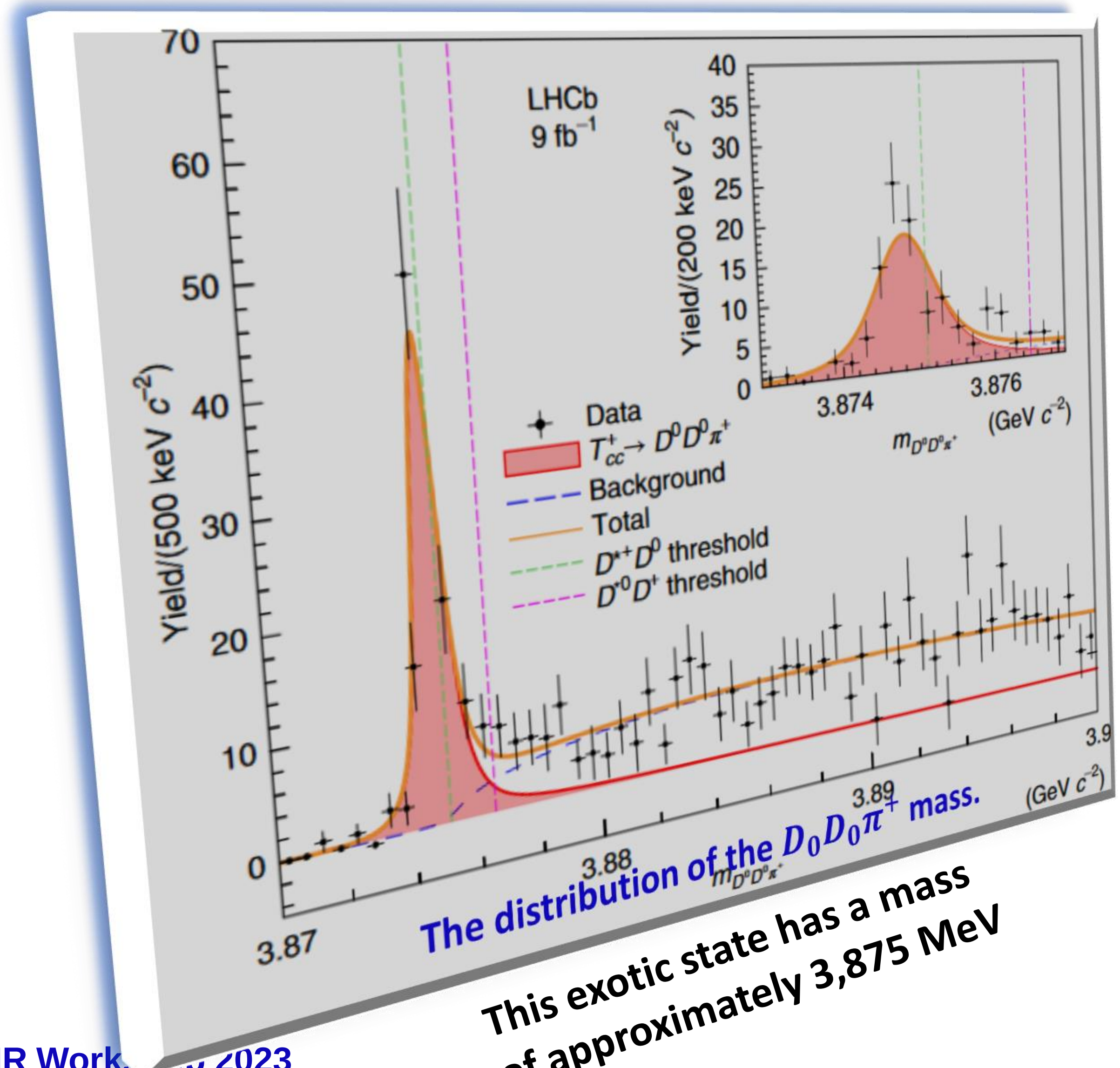


patrick.koppenburg@cern.ch 2023-06-25

@LHCb

$T_{cc}^{+}(cc\bar{u}\bar{d})$

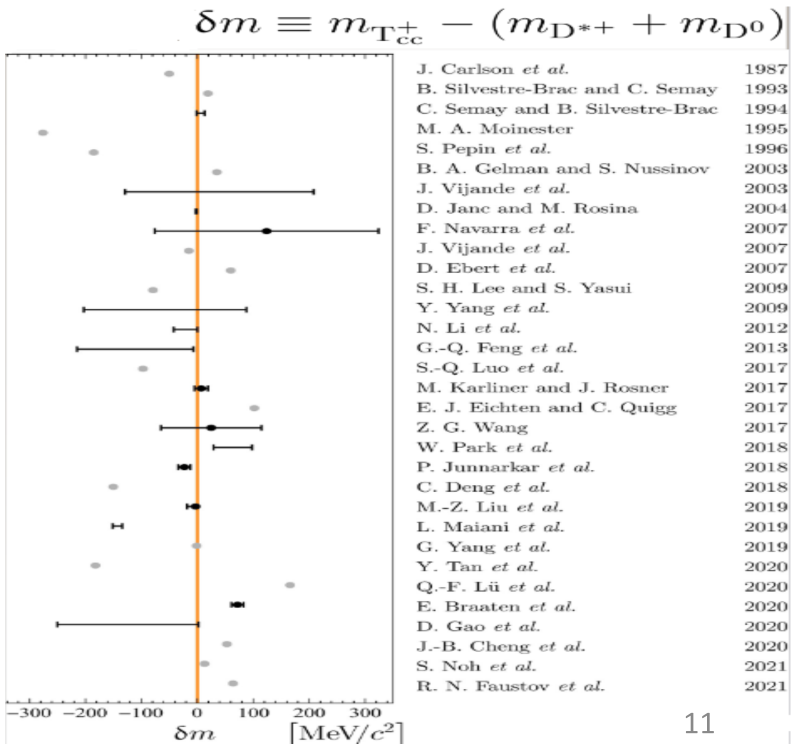
Nature Physics,18, 751(2022)



The distribution of the $D_0D_0\pi^+$ mass.

This exotic state has a mass of approximately 3,875 MeV

Parameter	Value
N	117 ± 16
δm_{BW}	−273 ± 61 keV c^{-2}
Γ_{BW}	410 ± 165 keV



Exotic hadrons and lattice QCD

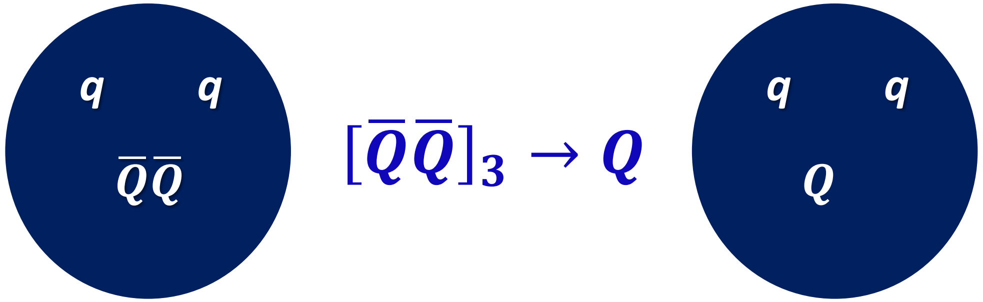
- Tetraquark and pentaquark hadrons have been observed experimentally with heavy quark contents. ....LHC, Belle, BES

- Are their possibilities to find more of those? And other multiquark states?

- What are the structures and properties of these exotic hadrons?

- What can lattice studies do?
    - Can predict more exotic states with possible valence structures and energies
    - Can decipher structures and properties of exotic hadrons

Heavy four-quark states



$$[\bar{Q}\bar{Q}]_3 \rightarrow Q$$

A possible structure:

How about ?

$$\underset{\{qq'\}}{\underset{\downarrow}{(qC\gamma_5 q')}}\underset{\{\bar{Q}\bar{Q}'\}}{\underset{\downarrow}{(\bar{Q}C\gamma_i\bar{Q}')}}$$

Possible states? : $\bar{b}\bar{b}ud, \bar{b}\bar{b}us, \bar{b}\bar{b}uc, \bar{b}\bar{b}sc,$
$\bar{b}\bar{c}ud, \bar{b}\bar{c}us\ \ etc.$

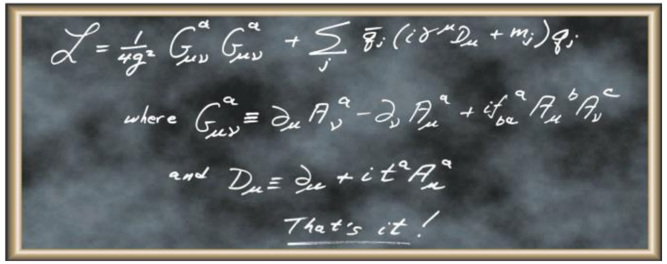
$J = 1, l_1 l_2 \bar{Q}\bar{Q}$ $\qquad$ $J = 0, ll\bar{Q}\bar{Q}$

LQCD: bound states of

$$T_{bb}(\bar{b}\bar{b}ud), T_{bbs}(\bar{b}\bar{b}us), T_{bc}(\bar{b}\bar{c}ud), T_{cc}(\bar{c}\bar{c}ud)$$

Expt: $T_{cc}(\bar{c}\bar{c}ud)$

QCD



$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

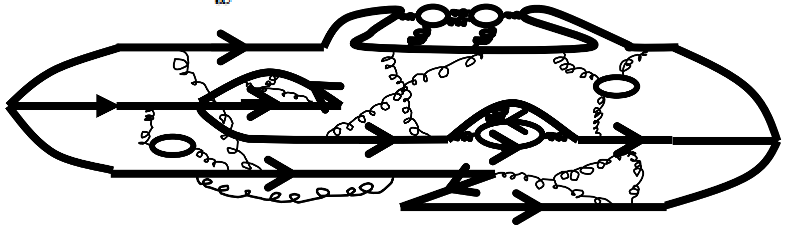
and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

That's it!

$$S_{QCD} = \int d^4x \, L_{QCD}(m_{q,} g_s)$$

$$\langle C \rangle = \frac{\int DGDqD\bar{q}Ce^{-S_{QCD}}}{\int DGDqD\bar{q}\, e^{-S_{QCD}}}$$

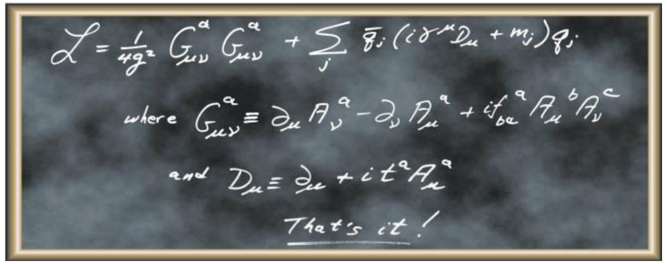
$$C_{\mathcal{O}}(t_i, t_f) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle 0|\mathcal{O}(\vec{x}_f, t_f)\bar{\mathcal{O}}(\vec{x}_i, t_i)|0\rangle$$



large time $\sim e^{-E_0 t}$

QCD → LQCD

Euclidean time →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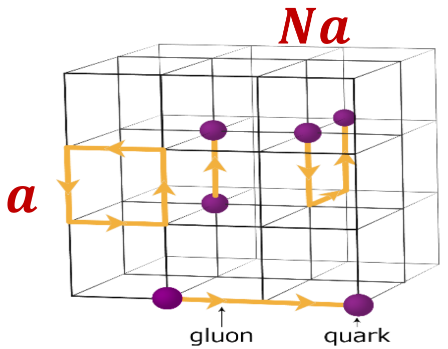

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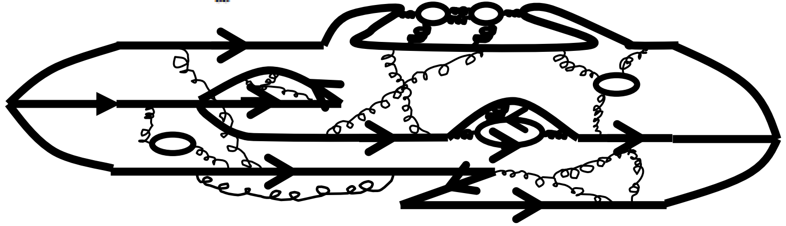
$$S^E_{QCD} = S^E_{QCD}[U, qi, D(U), m_{q_i}, a]$$

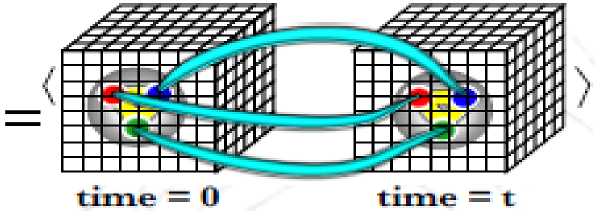
$$\langle C \rangle = \frac{\int DUDqD\bar{q}Ce^{-S^E_{QCD}}}{\int DUDqD\bar{q}\, e^{-S^E_{QCD}}} \approx \frac{1}{N}\sum_n C(D^{-1}(U_n))$$

$$\Delta C = \frac{1}{\sqrt{N}} + \text{systematics}$$

$$C_{\mathcal{O}}(t_i, t_f) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle 0 | \mathcal{O}(\vec{x}_f, t_f) \bar{\mathcal{O}}(\vec{x}_i, t_i) | 0 \rangle$$

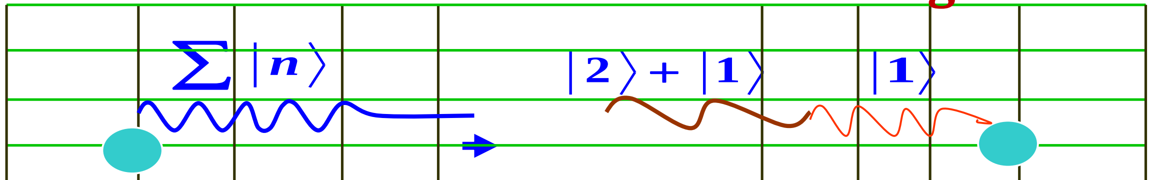


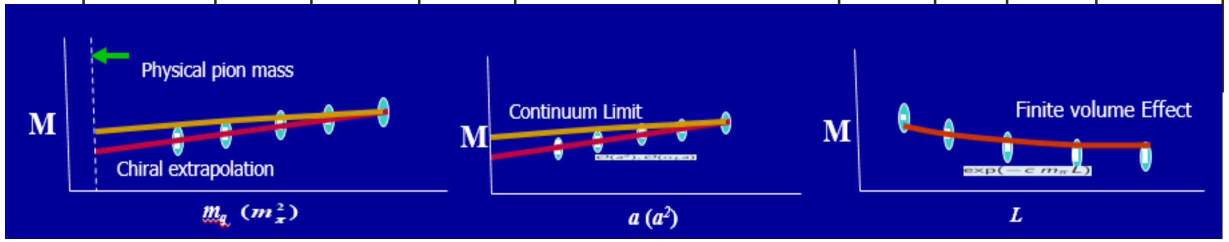
large time $\sim e^{-E_0 t}$



$$\left\langle C^{2pt}_{ab}(t, \vec{P}) \right\rangle = \quad = \sum_n Z_{b,n} Z^\dagger_{a,n} e^{-E_n t}$$

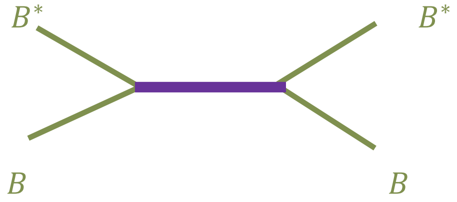
τ large $\sim e^{-E_0 \tau}$

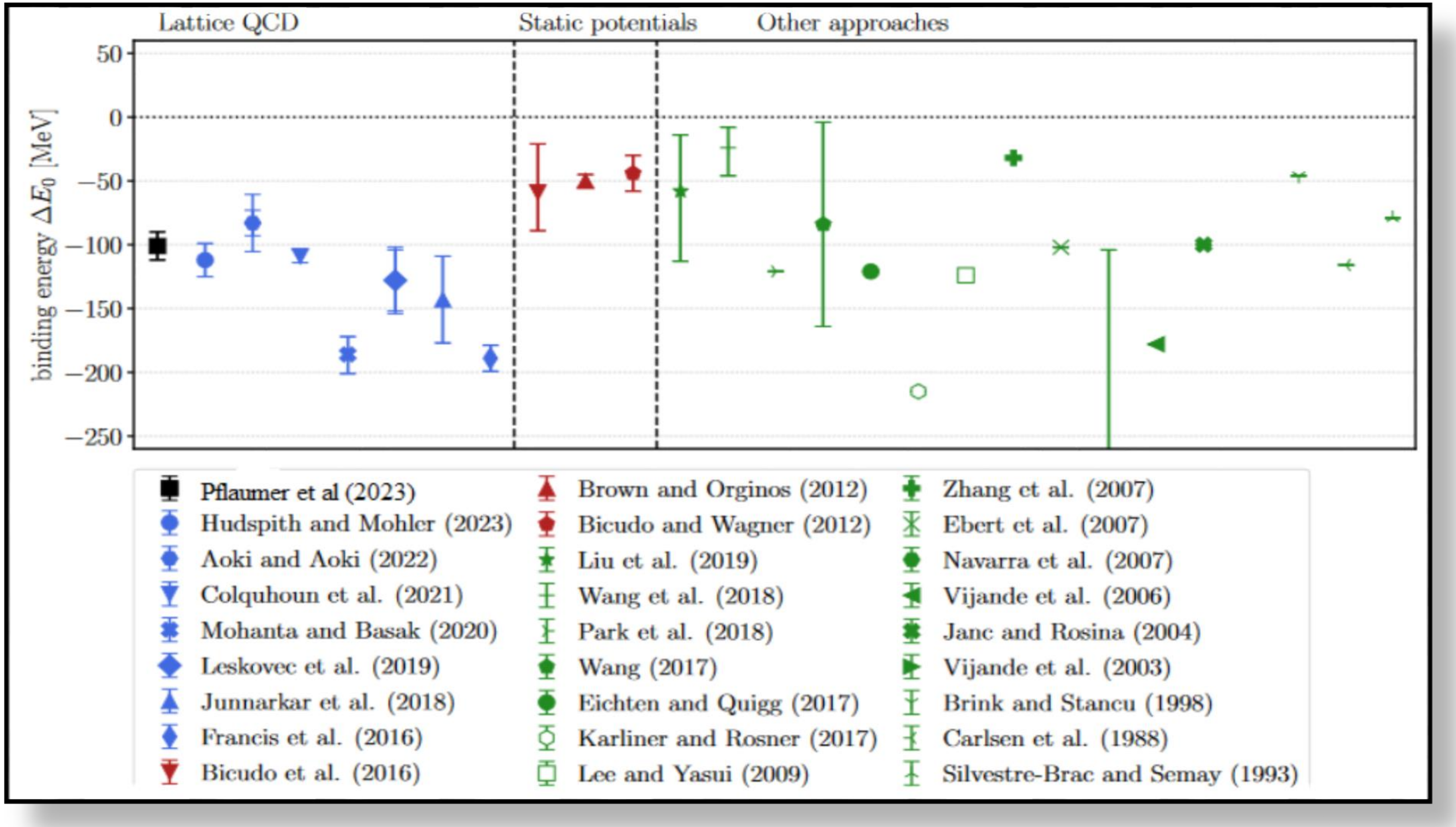




$T_{bb} \equiv \bar{b}\bar{b}ud$

Results so far

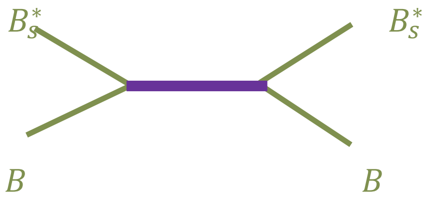


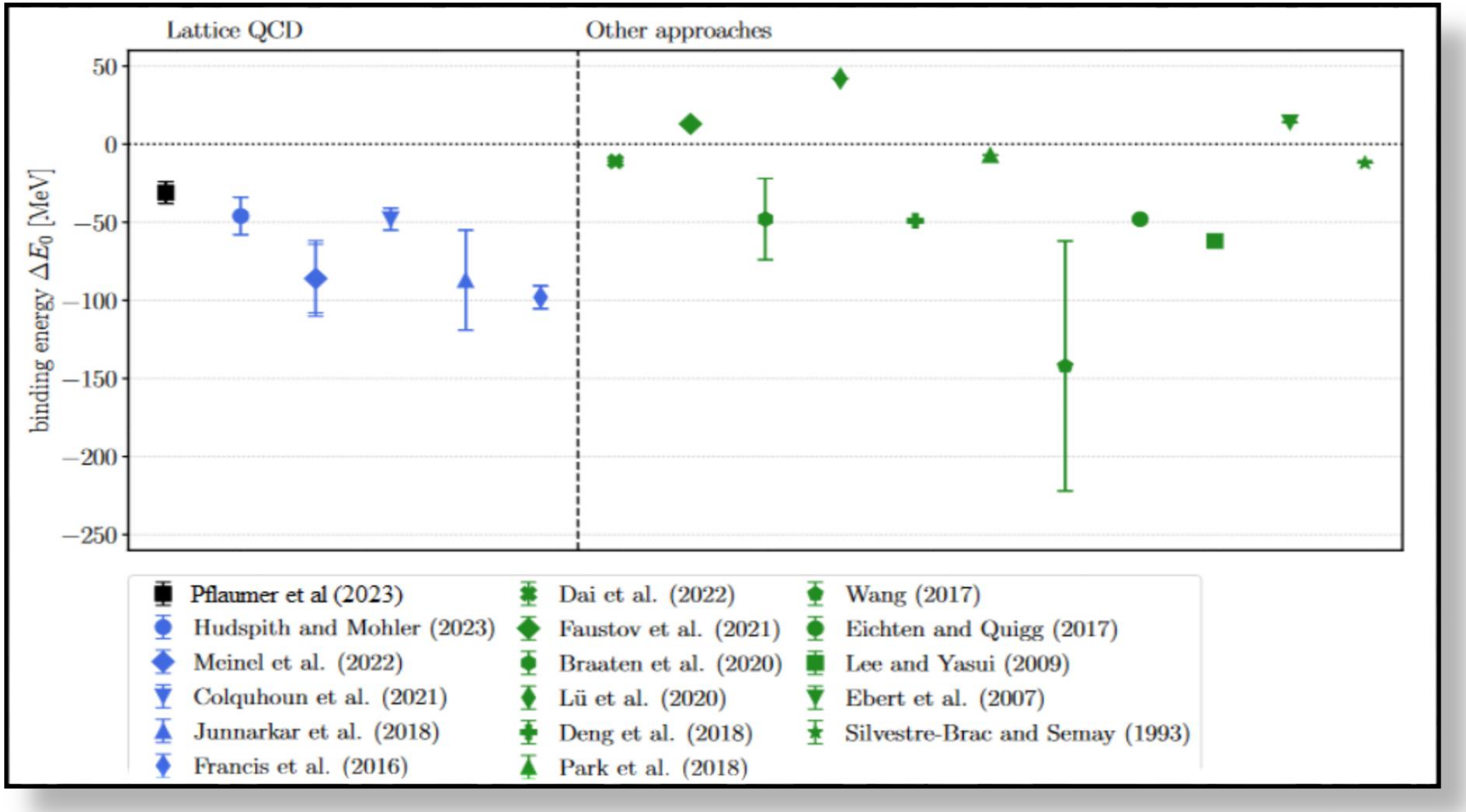


Summary plot: Pflaumer (2023)

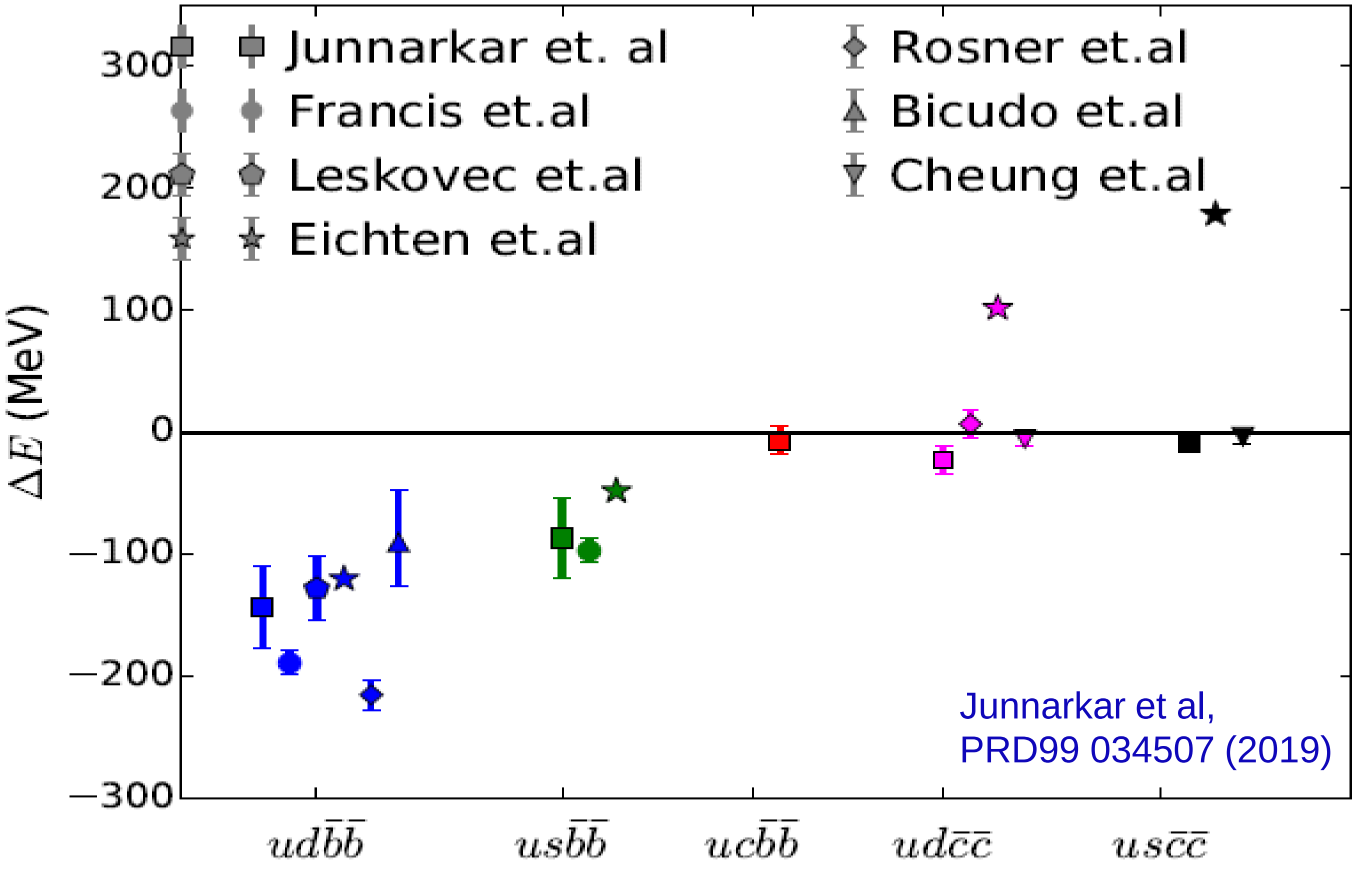
$T_{bb} \equiv \bar{b}\bar{b}us$

Results so far





Summary plot: Pflaumer (2023)

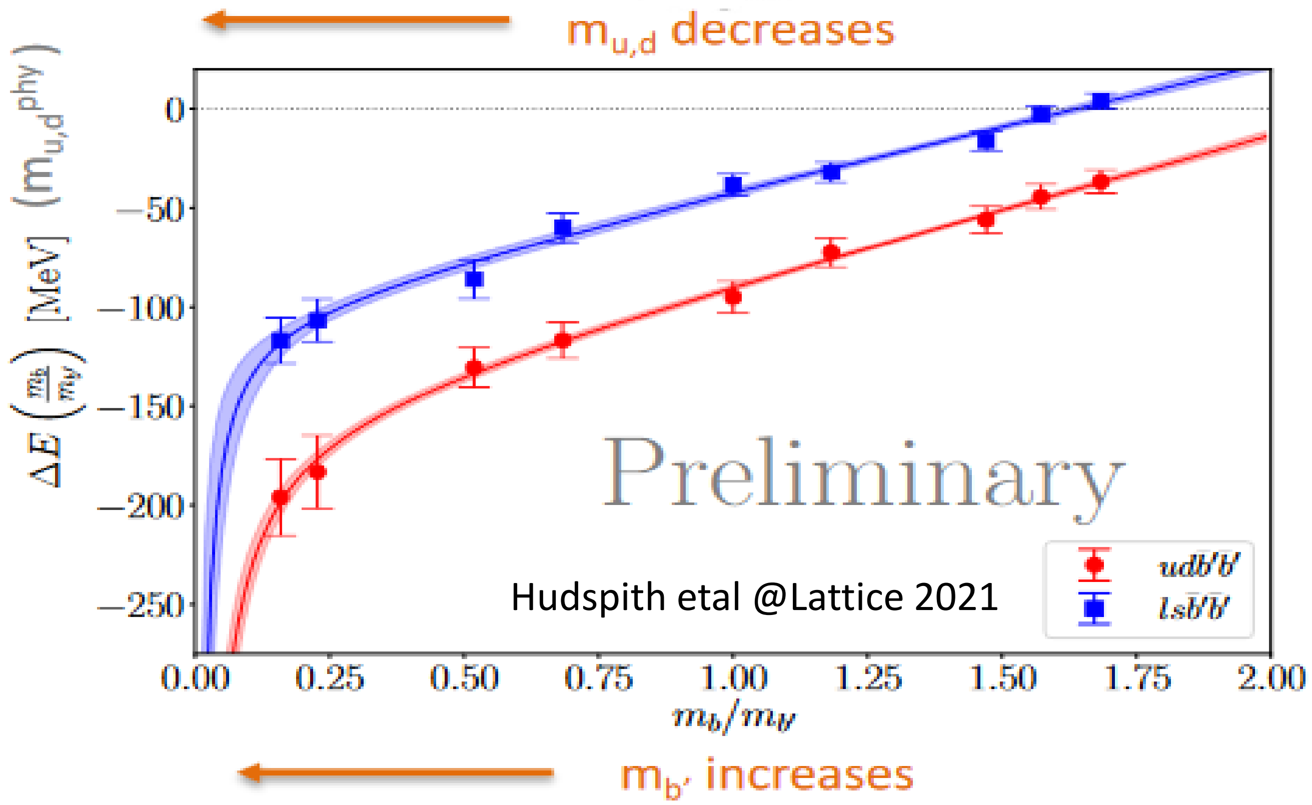


Junnarkar et al,
PRD99 034507 (2019)

State	ΔE^1 [MeV]	State	ΔE^1 [MeV]
$ud\bar{b}\bar{b}$	-143(34)	$us\bar{b}\bar{b}$	-87(32)
$uc\bar{b}\bar{b}$	-6(11)	$sc\bar{b}\bar{b}$	-8(3)
$ud\bar{c}\bar{c}$	-23(11)	$us\bar{c}\bar{c}$	-8(8)

State	ΔE^0 [MeV]	State	ΔE^0 [MeV]
$uu\bar{b}\bar{b}$	-5(18)	$uu\bar{c}\bar{c}$	26(11)
$ss\bar{b}\bar{b}$	3(9)	$ss\bar{c}\bar{c}$	14(4)
$cc\bar{b}\bar{b}$	16(1)		

$m_{u,d}$ decreases



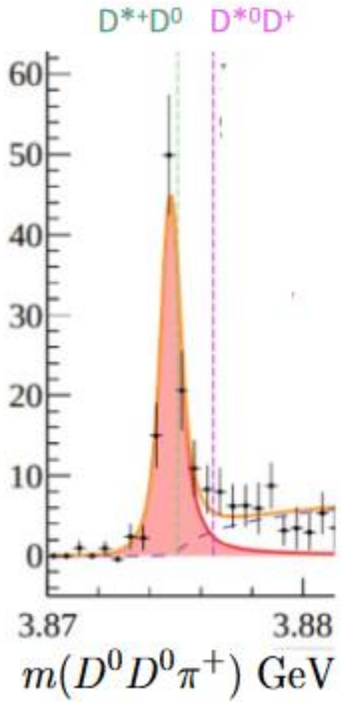
Hudspith etal @Lattice 2021

$m_{b'}$ increases

- ➢ **Heavier the heavy quark masses, deeper the binding**
- ➢ **Lighter the light quark masses, deeper the binding**

T_{cc}

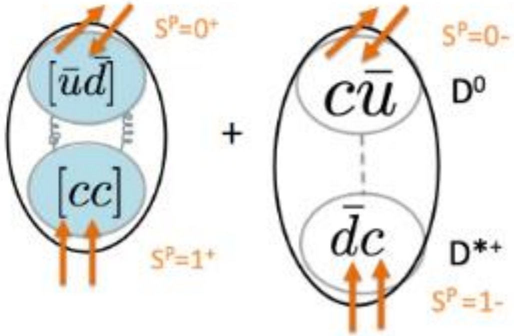
Doubly charm tetraquark T_{cc} $cc\bar{d}\bar{u}$



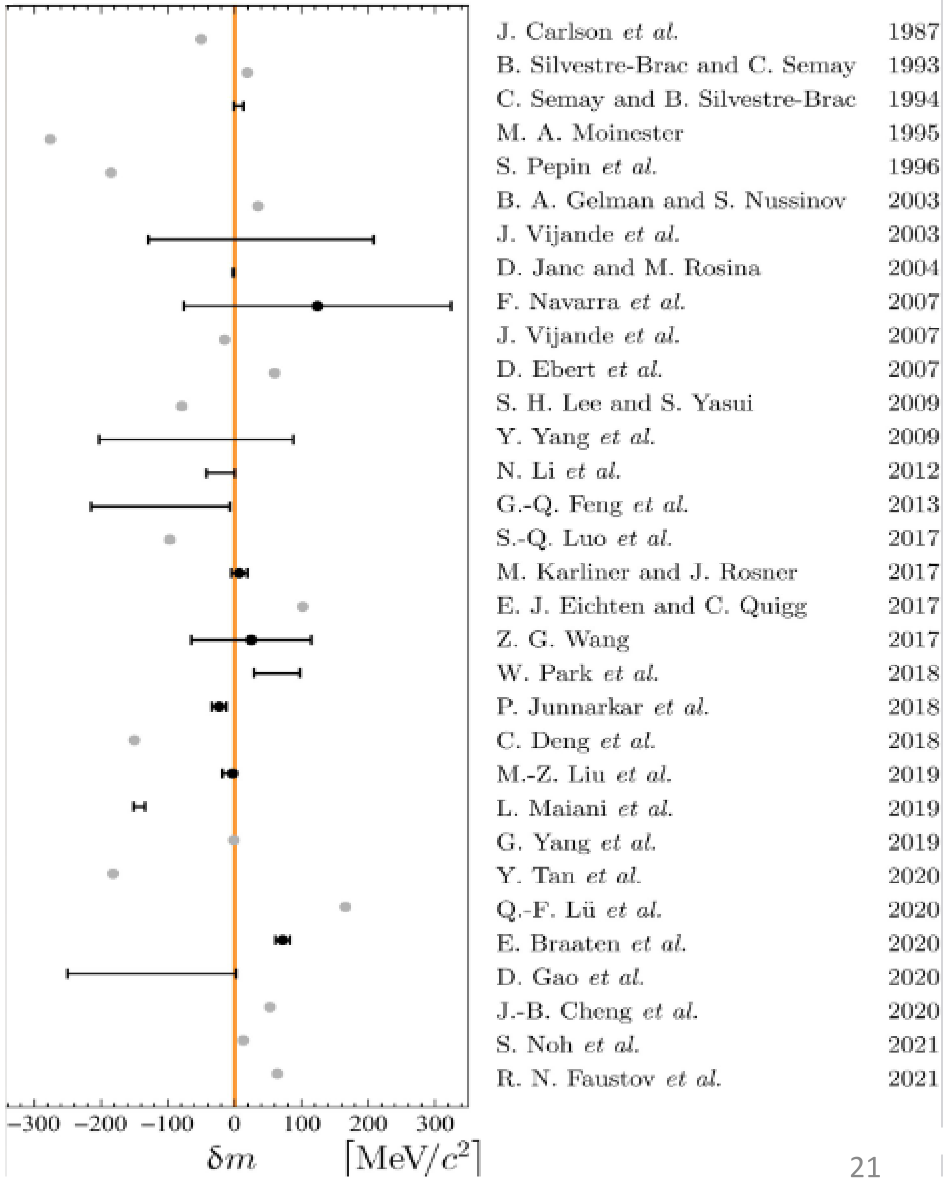
$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056



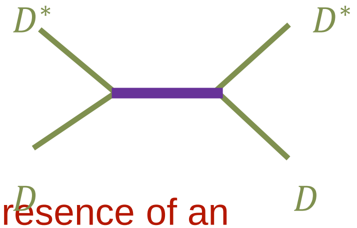
$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$



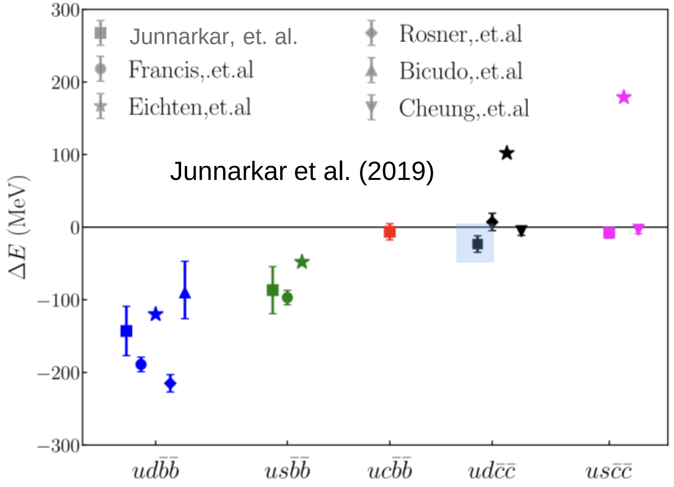
What does LQCD tell us?

States near threshold

$cc\bar{u}\bar{d}\quad 0(1^+)$



A lattice calculation had seen presence of an energy level below lowest threshold- **later discovered by LHCb!**



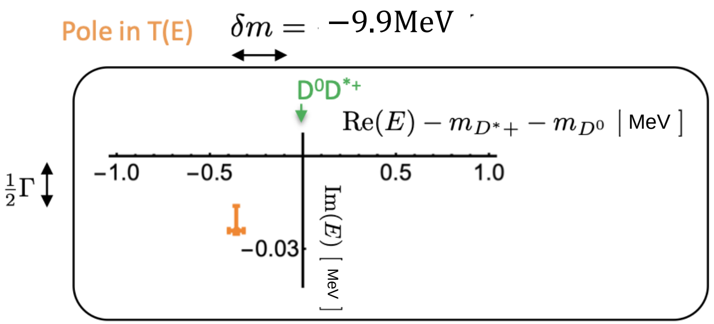
Exploring tetraquarks with bottom and charm may be accessible to experiments

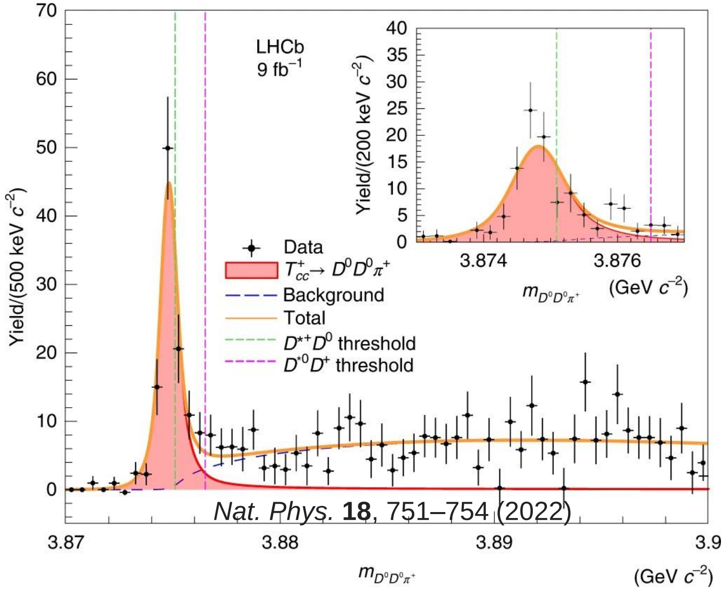
Motivates the study of $bc\bar{u}\bar{d}$

but significant finite volume effects possible - close to threshold

Need to find the poles in the scattering amplitude to extract (virtual) bound poles:

Padmanath et al. Phys.Rev.Lett. 129 (2022)



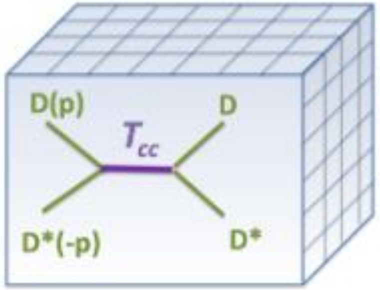


Nat. Phys. **18**, 751–754 (2022)

Other recent lattice studies confirm the presence the bound state:

Lyu et al. arXiv.2302.04505
Chen et al. j.physletb.2022.137391

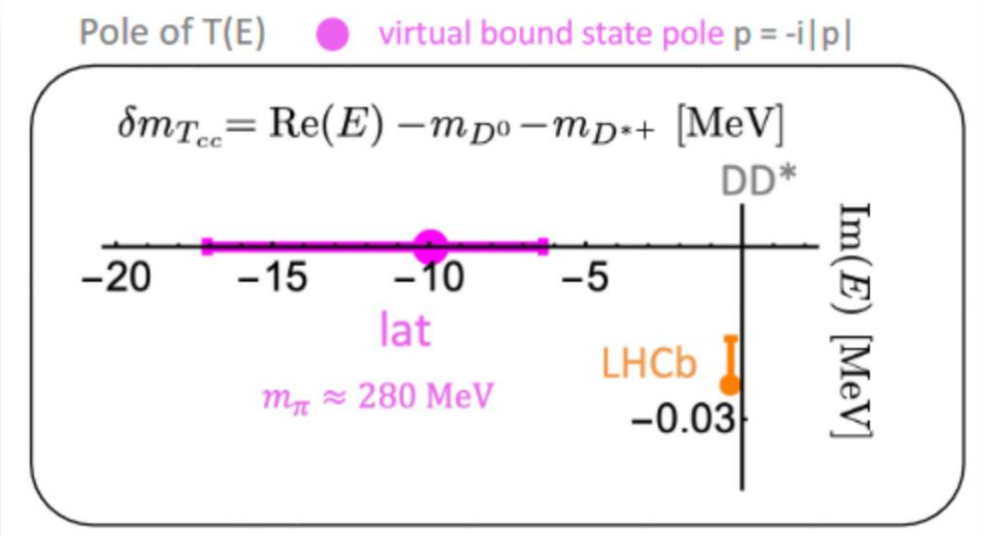
T_{cc} on lattice

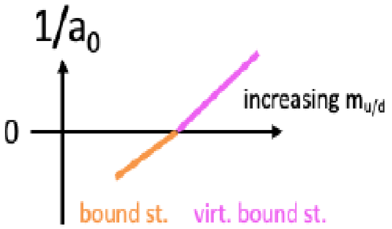


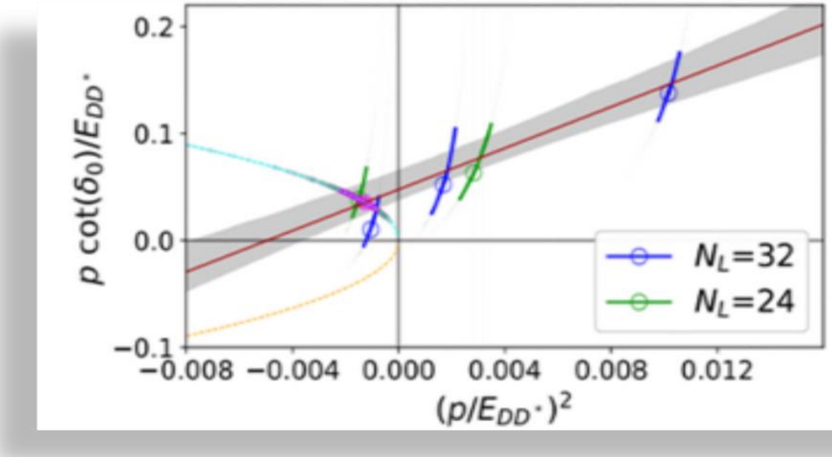
PRL 129, 032002 (2022)
Padmanath and Prelovsek

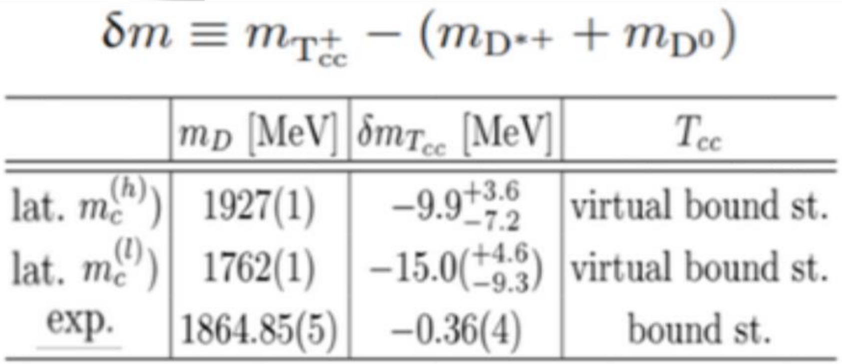
$$t_l^{(J)} = \frac{E_{\rm cm}}{2}\frac{1}{p\cot\delta_l^{(J)} - ip}, \qquad p^{2l+1}\cot\delta_l^{(J)} = \frac{1}{a_l^{(J)}} + \frac{r_l^{(J)}}{2}p^2,$$

$$p\cot\delta_{l=0}^{(J=1)} = \frac{1}{a_0^{(1)}} + \frac{1}{2}r_0^{(1)}p^2,$$





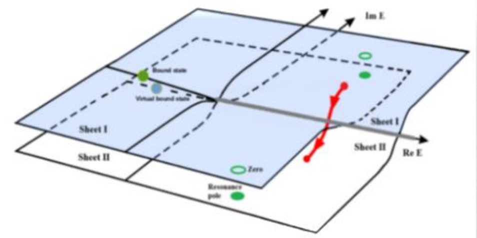




$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	−0.36(4)	bound st.

$\delta m = E^p_{cm} - E_{th}$



$$T \propto (p\cot\delta_0 - ip)^{-1}$$

- Bound state: $p = i|p| \rightarrow e^{ipr} = e^{-|p|r}$
- Virtual bound state $p = -i|p| \rightarrow e^{ipr} = e^{|p|r}$ like the spin-singlet dineutron

India-JINR Workshop 2023

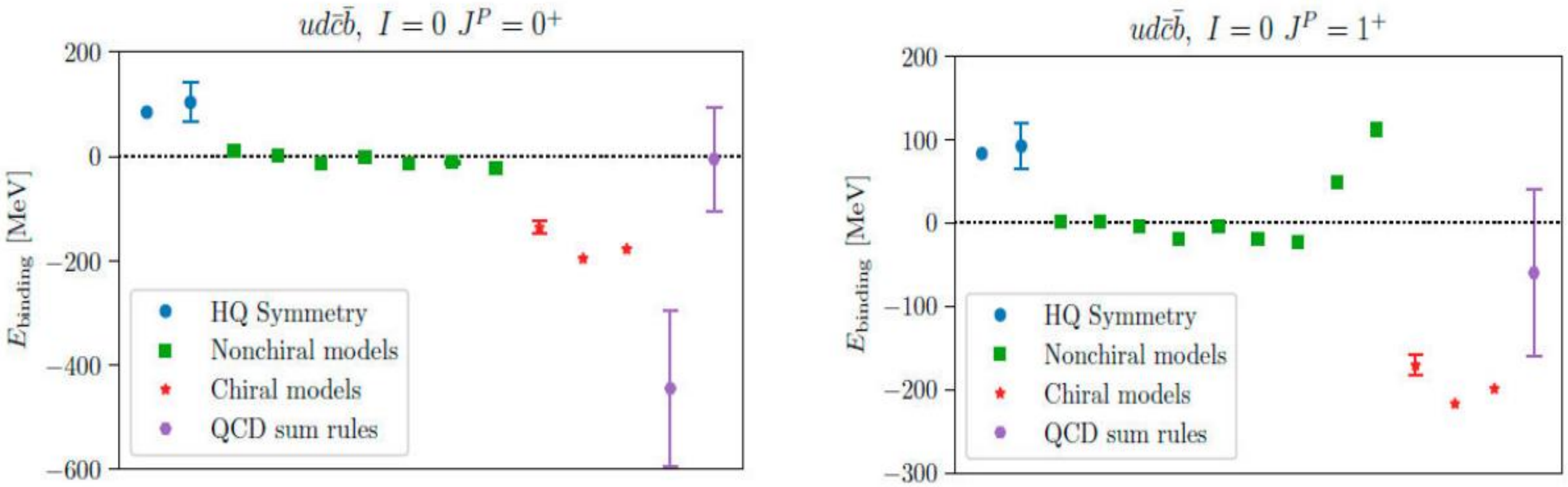
Slide thanks: Padmanath and Sasa

What about T_{bc}: $\bar{b}\bar{c}q_1q_2$?

Various models predicted mixed results for $ud\bar{b}\bar{c}$ (1^+):

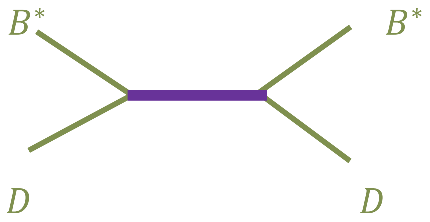
- HQ-symmetry inspired and non-chiral models: mostly unbound or very weekly bound
- QCD sum rule, chiral models: a bound state (both for 0 and 1-isospins) with binding over a wide range ~ 20-400 MeV !

Hudspith et al, Phys. Rev. D102, 114506 (2020)

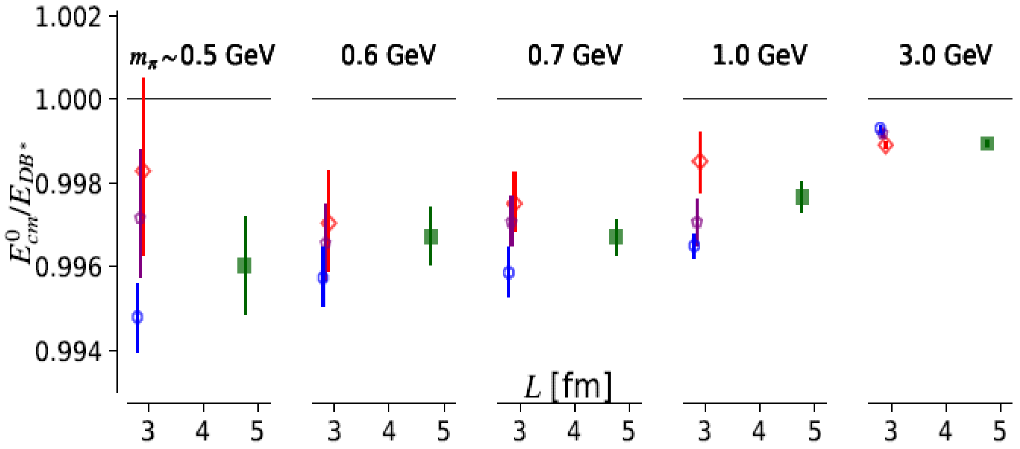


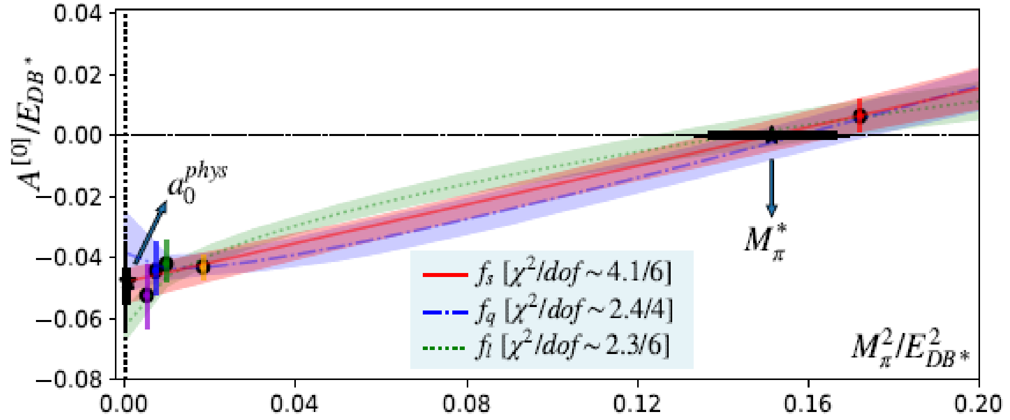
B. Colquhoun et al, Rev. Mex. Fis. Suppl. 3 (2022) 3, 0308044

$bc\bar{u}\bar{d}\ 0(1^+)$

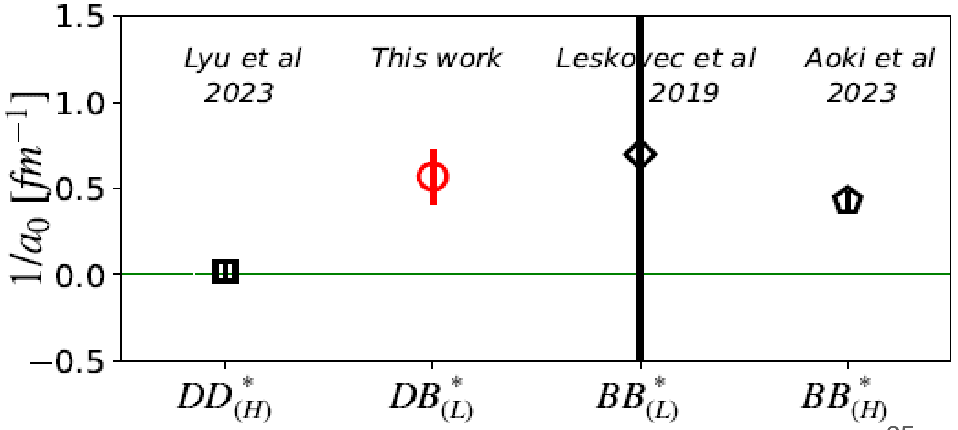


States near threshold/bound?





Strong indication of a bound state of about 40 MeV binding energy



Tetraquarks with charm and bottom may be accessible to experiments - theoretical predictions can help in such searches!

M Padmanath, A Radhakrishnan, N Mathur
arXiv:2307.14128

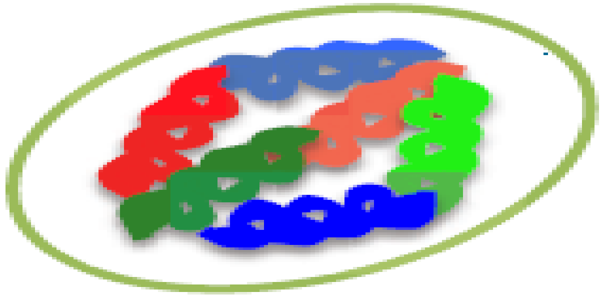
Glueball

- A glueball is a gluonic bound state.

- In the theory of QCD gluon self coupling admits the existence of such a state.

- No conclusive experimental evidence of glueball as yet though the f_o states are indicative. Difficult to detect due to mixing but the searches are ongoing

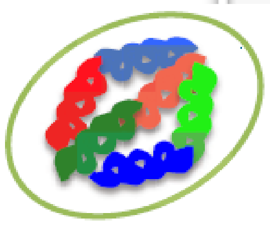
- However, lattice QCD calculations can tell us about glueball spectra



$8 \otimes 8$	$1 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27$

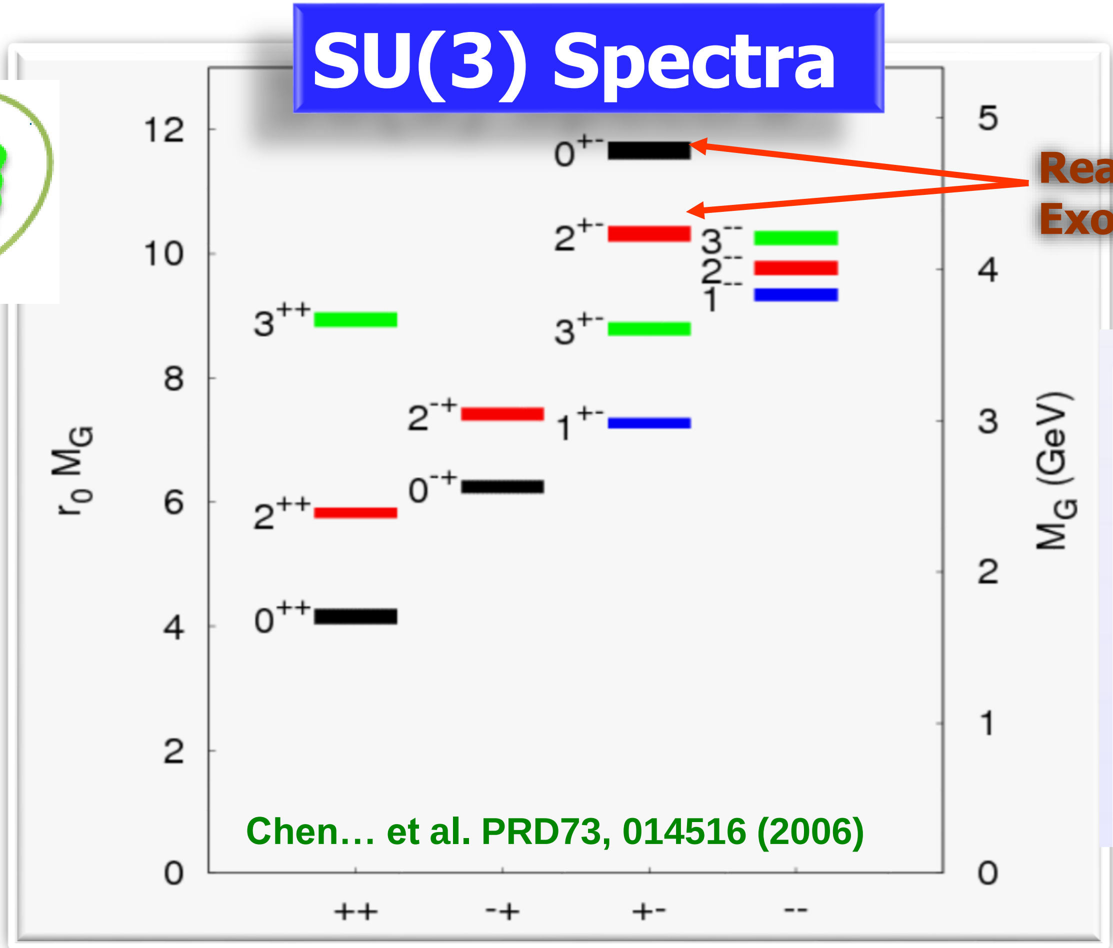
Glueball

- A glueball is a gluonic bound state.



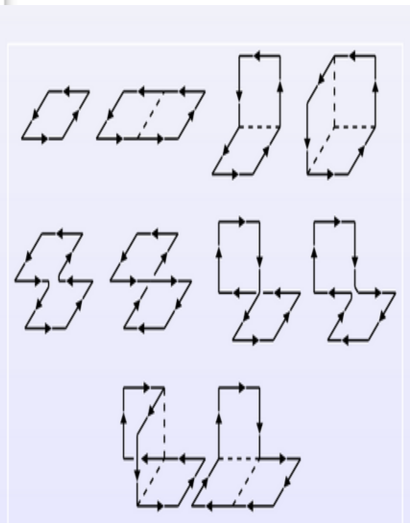
- In the theory of QCD gluon self coupling admits the existence of such a state.
- No conclusive experimental evidence of glueball as yet though the *f_o* states are indicative. Difficult to detect due to mixing but the searches are ongoing

SU(3) Spectra



Really Exotic!

Chen… et al. PRD73, 014516 (2006)



- Signal-to-noise ratios in lattice glueball correlation functions with dynamical quarks are still very poor.
- Multiple channels with glueball, two-quarks and four-quarks with the same quantum numbers need to be addressed together
- **Precise calculations with dynamical quarks including mixing effects are necessary**

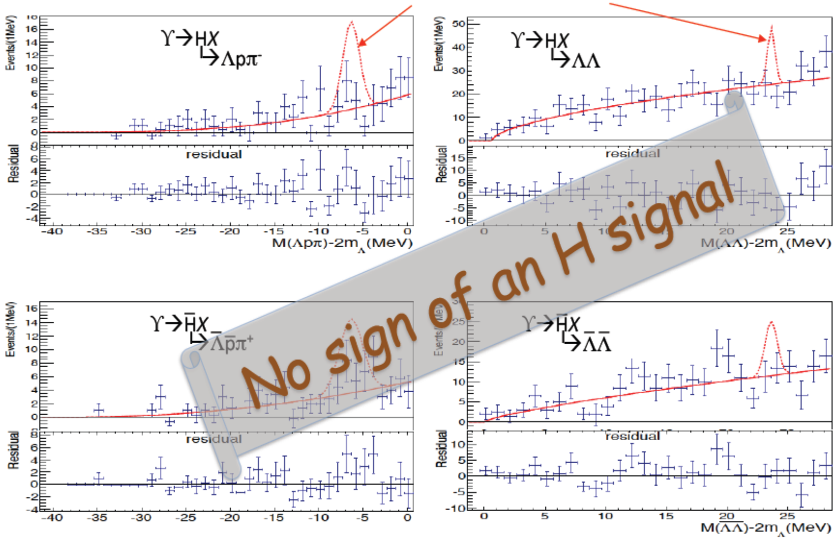
H Dibaryon

Bound state of two Λ $\Lambda\Lambda\ (udssud)$ Proposed by Jaffe (1976)

- Has to be below the two proton threshold. Then it will be bound
- If it exists it is extremely stable and could be a candidate for SM dark matter?
  (May not be as oxygen may not exist with that!)

No H dibaryon

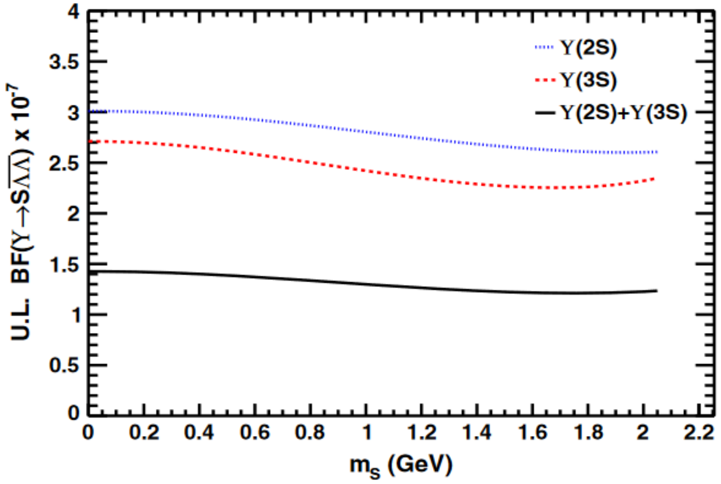
expected signals for Bf(ϒ→HX)=1/20Bf(ϒ→$\bar{d}$X)



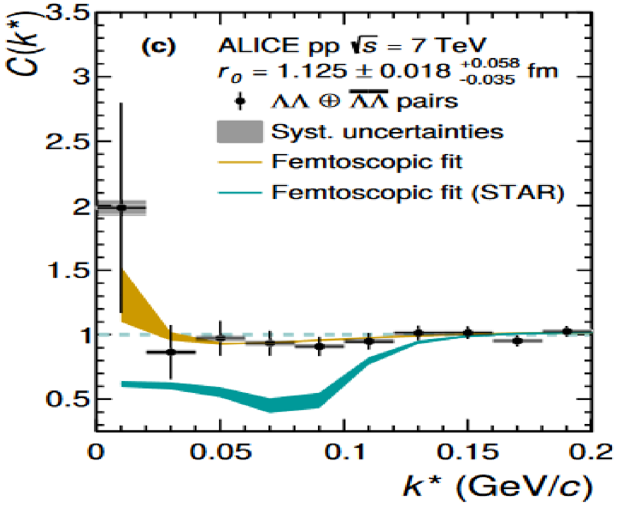
No sign of an H signal

M(Λpπ⁻)-2m$_\Lambda$ B.H. Kim *et al* (Belle) PRL 110, 222002 (2013) M(ΛΛ)-2m$_\Lambda$

Belle: PRL 110,222002(2013)

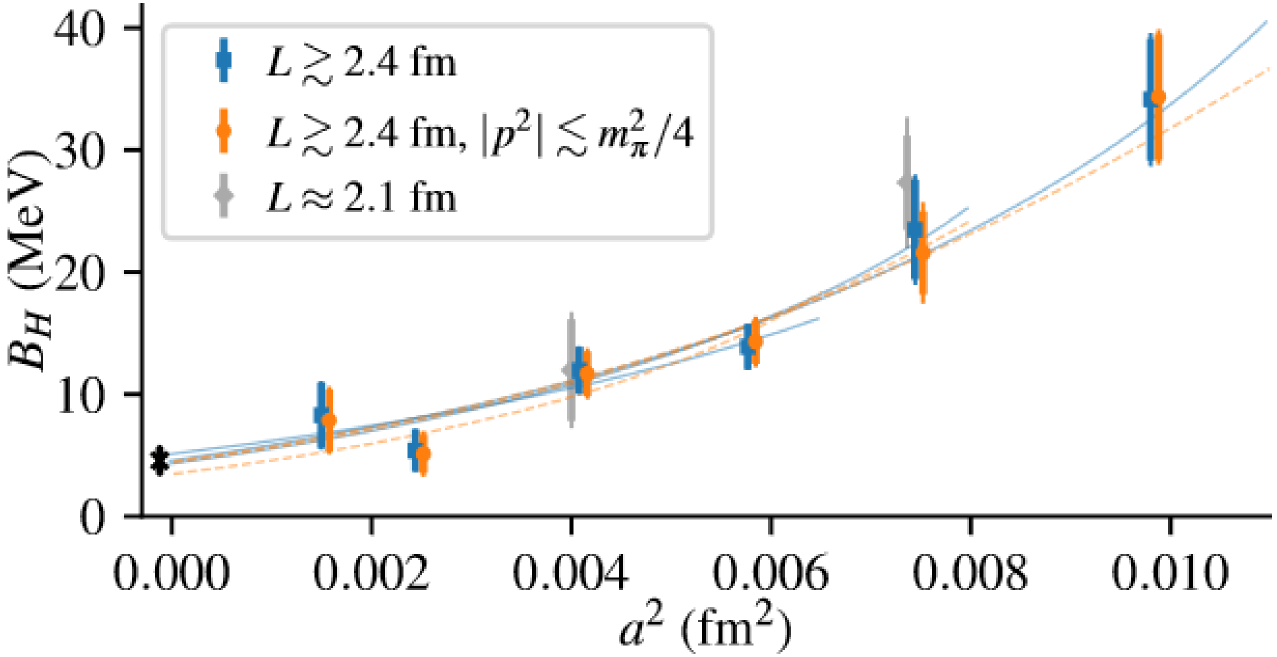


BABAR Collaboration:
Phys. Rev. Lett. 122, 072002 (2019)
No signal is observed
in Υ **decays** (90% confidence limit)



Alice: Phys. Rev. C 99, 024001 (2019)

H-dibaryon at $\mathrm{SU}(3)_\mathrm{F}$ symmetric point

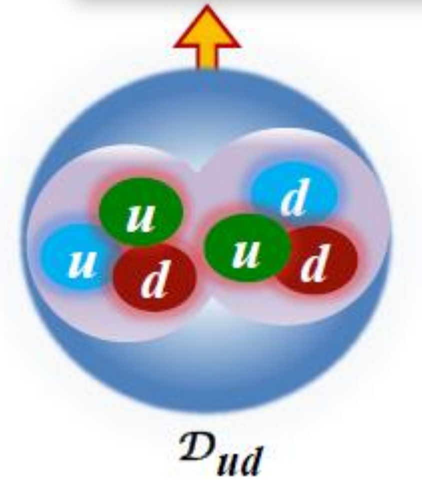


$$\check{B}_H^{\mathrm{SU(3)_f}} = 4.56 \pm 1.13_{\mathrm{stat}} \pm 0.63_{\mathrm{syst}}\ \mathrm{MeV}.$$

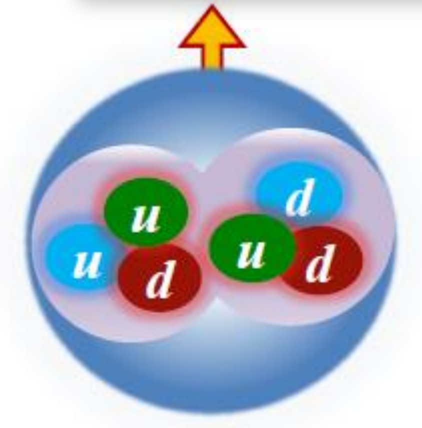
Green et al : *Phys. Rev. Lett.* 127 (2021) 24, 242003

Are there heavy dibaryons?

Deuteron-like heavy dibaryons



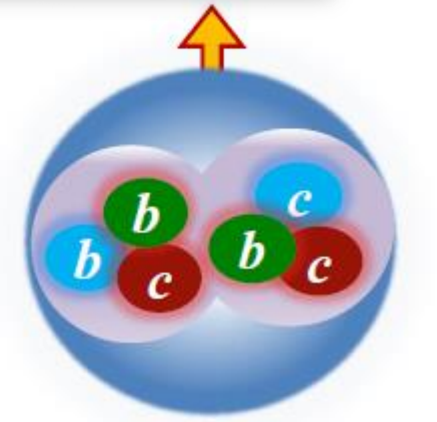
Deuteron-like heavy dibaryons



$\mathcal{D}_{ud}$

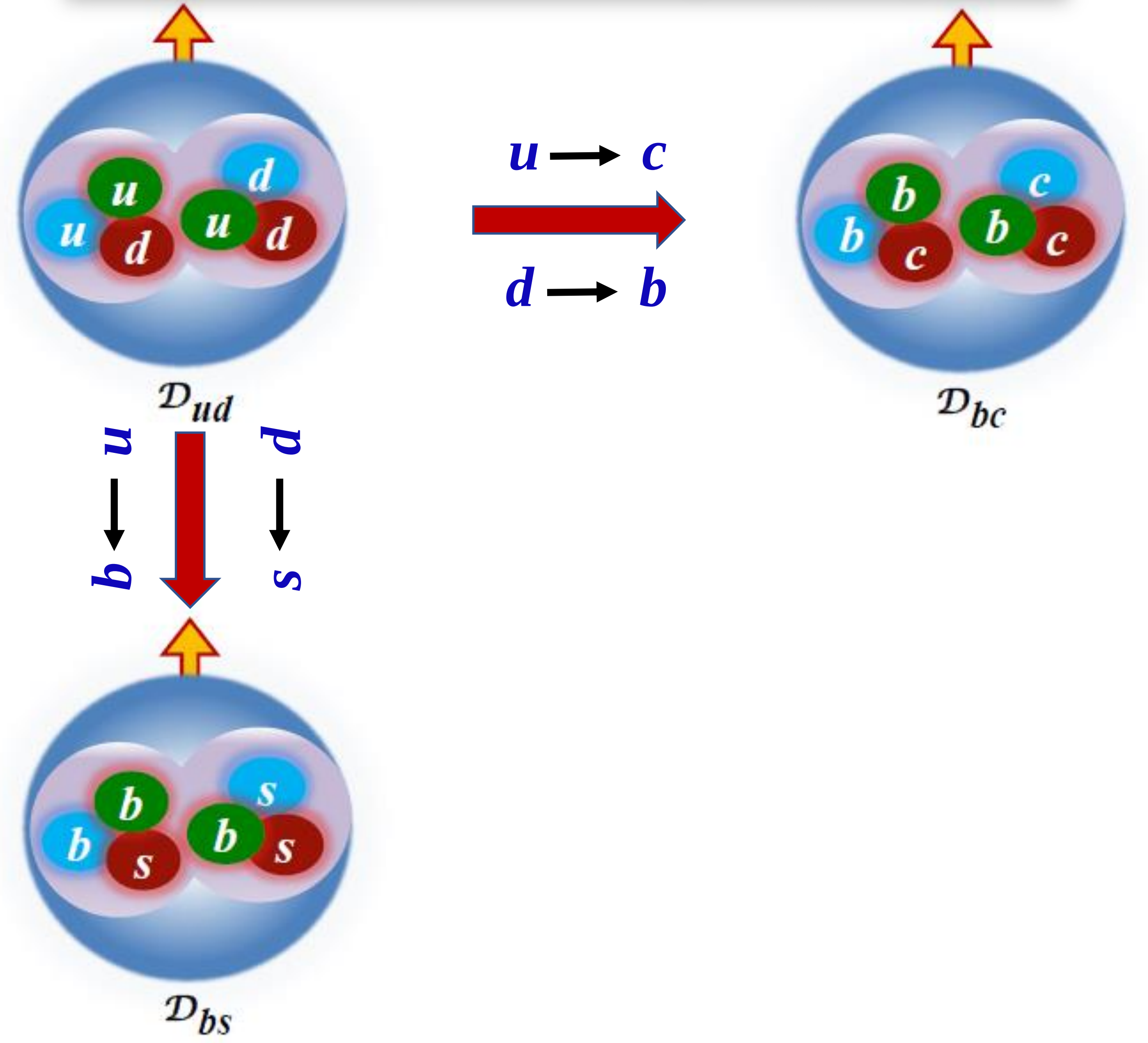
$u \longrightarrow c$

$d \longrightarrow b$

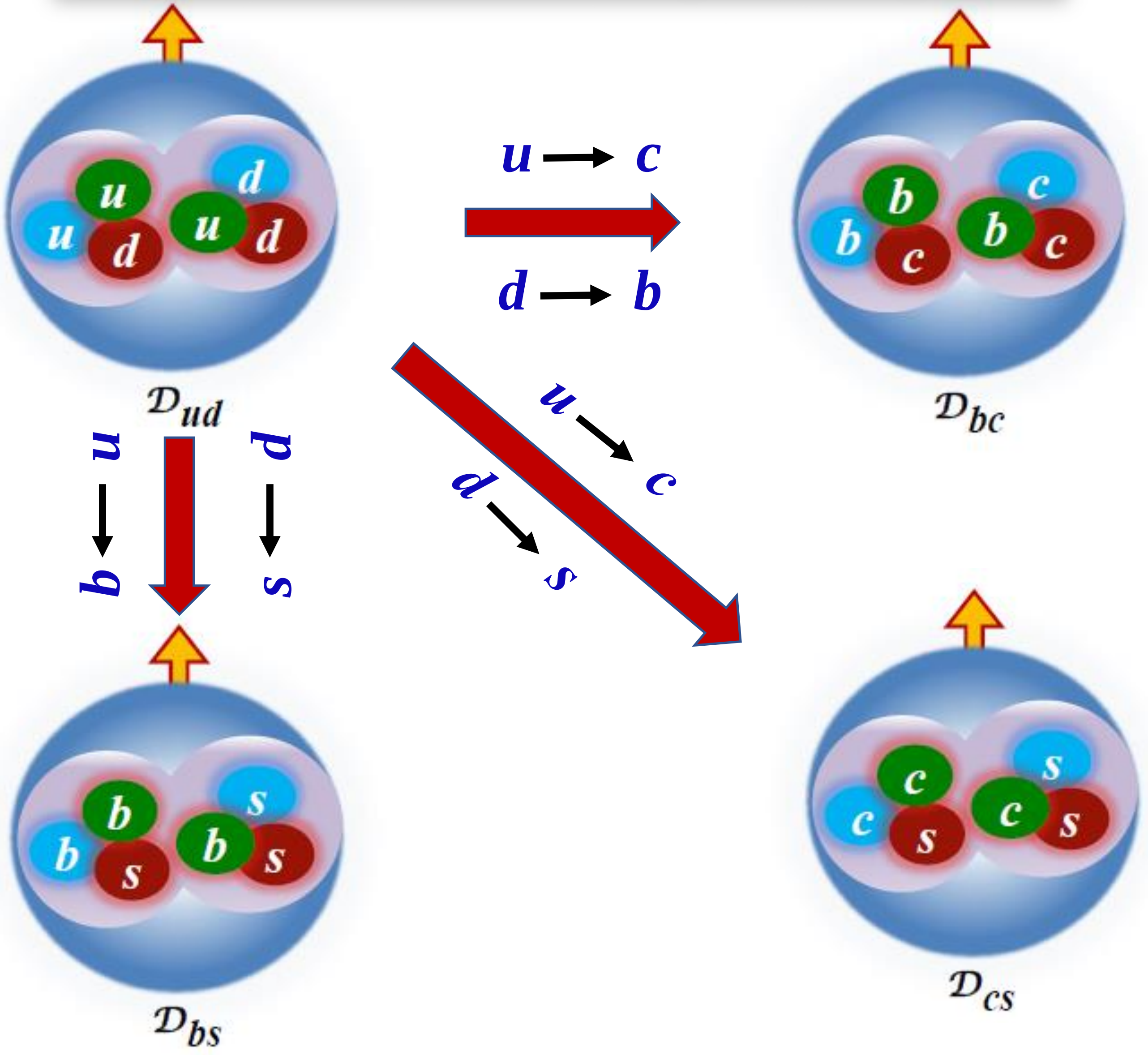


$\mathcal{D}_{bc}$

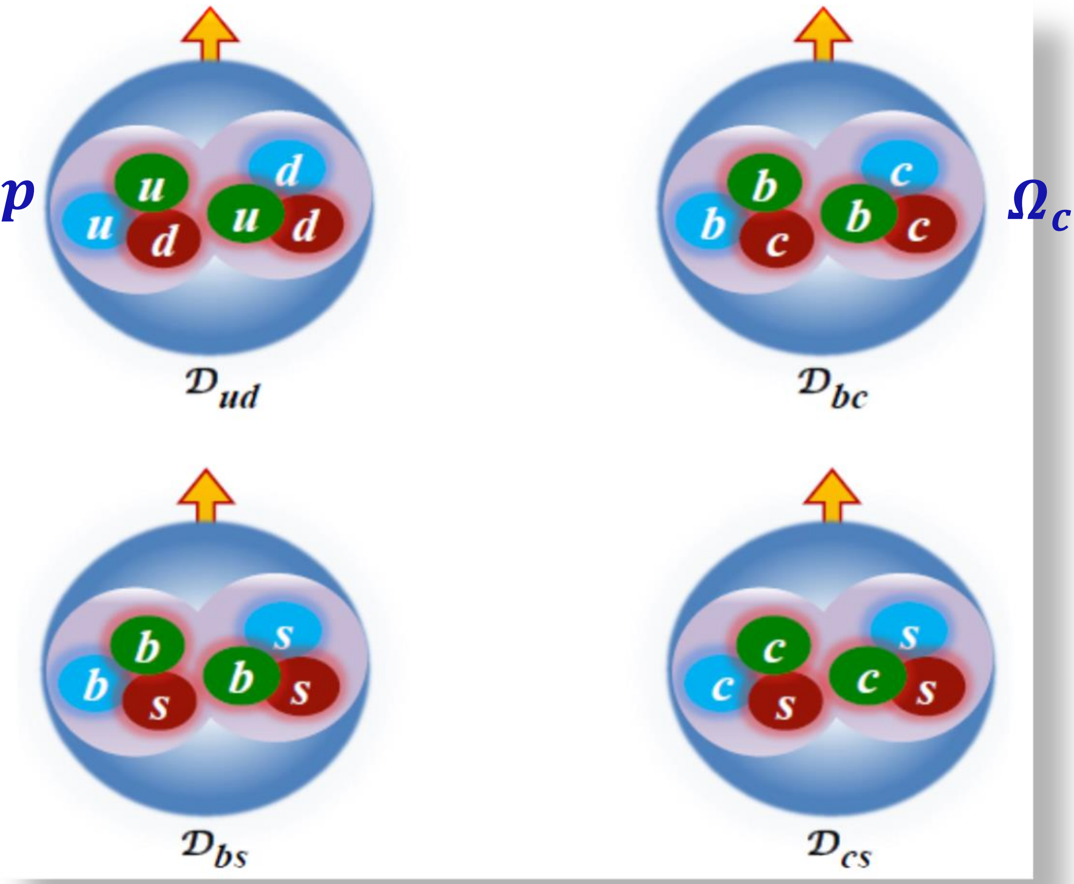
Deuteron-like heavy dibaryons



Deuteron-like heavy dibaryons



Deuteron-like heavy dibaryons



np

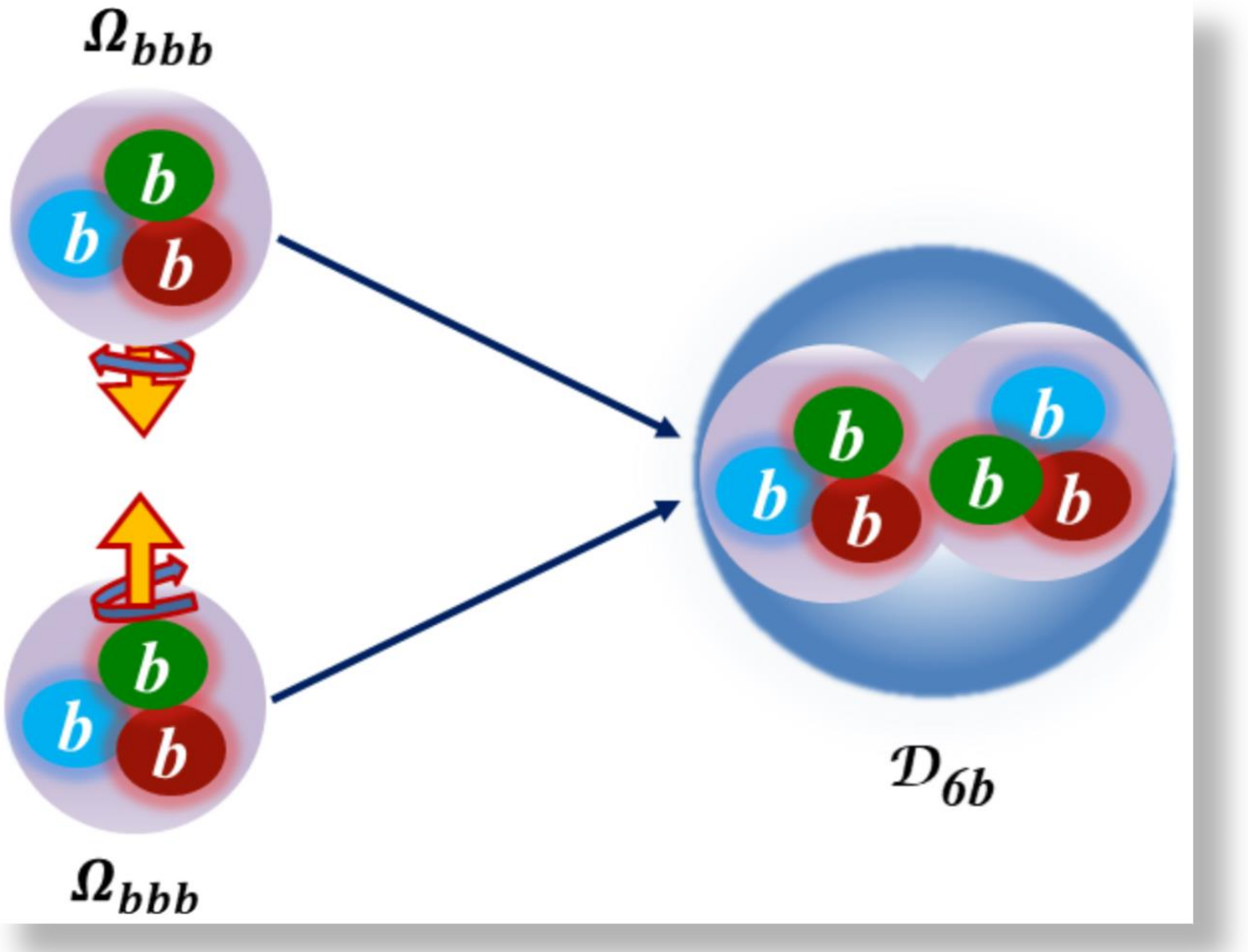
$\Omega_{ccb}\Omega_{cbb}$

$\Omega_{b}\Omega_{bb}$

$\Omega_{c}\Omega_{cc}$

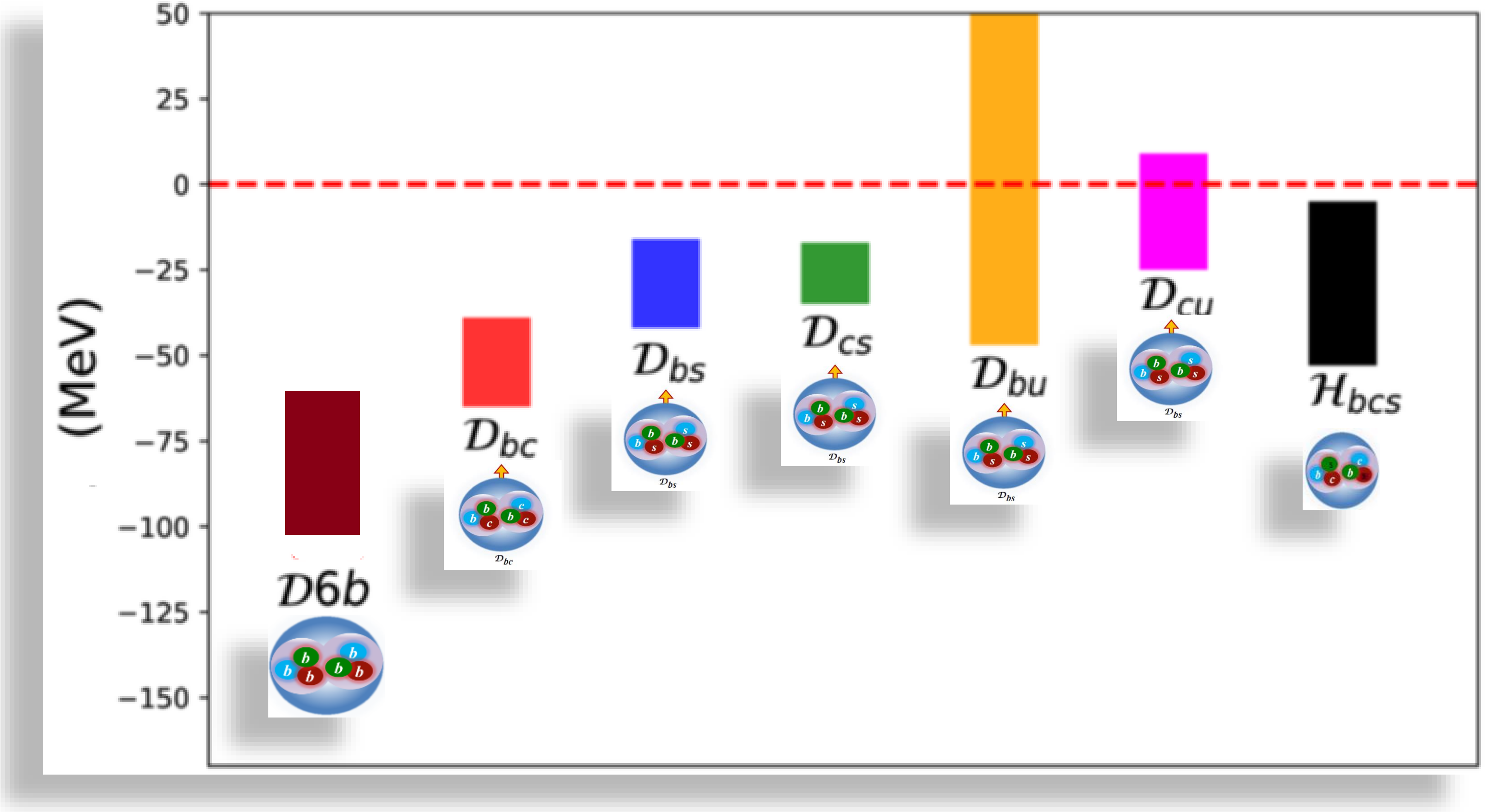
Junnarkar and NM : Phys. Rev. Lett. 123, 162003(2019)

Most beautiful dibaryons!



NM, Padmanath and Chakraborty: PRL 130, 111901 (2023)

Heavy Dibaryon Candidates?

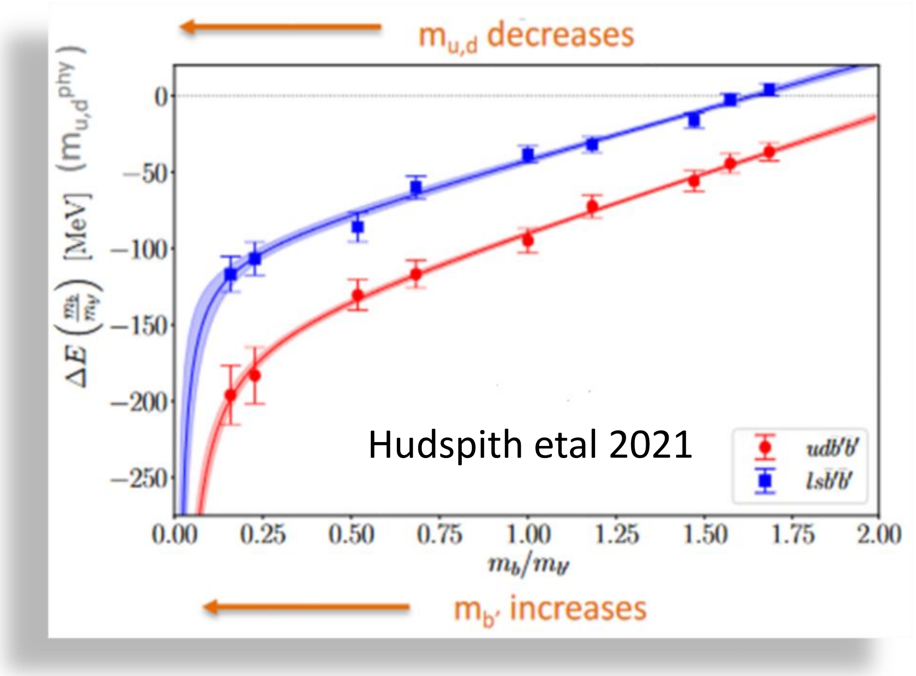


PRL 123,162003 (2019): Junnarkar, NM

PRL 130, 111901 (2023): NM, Padmanath and Chakraborty

PRD 106, 054511 (2019): Junnarkar, NM

Four-quark hadrons $(Q_1Q_2\bar{q}_1\bar{q}_2)$



$m_{u,d}$ decreases

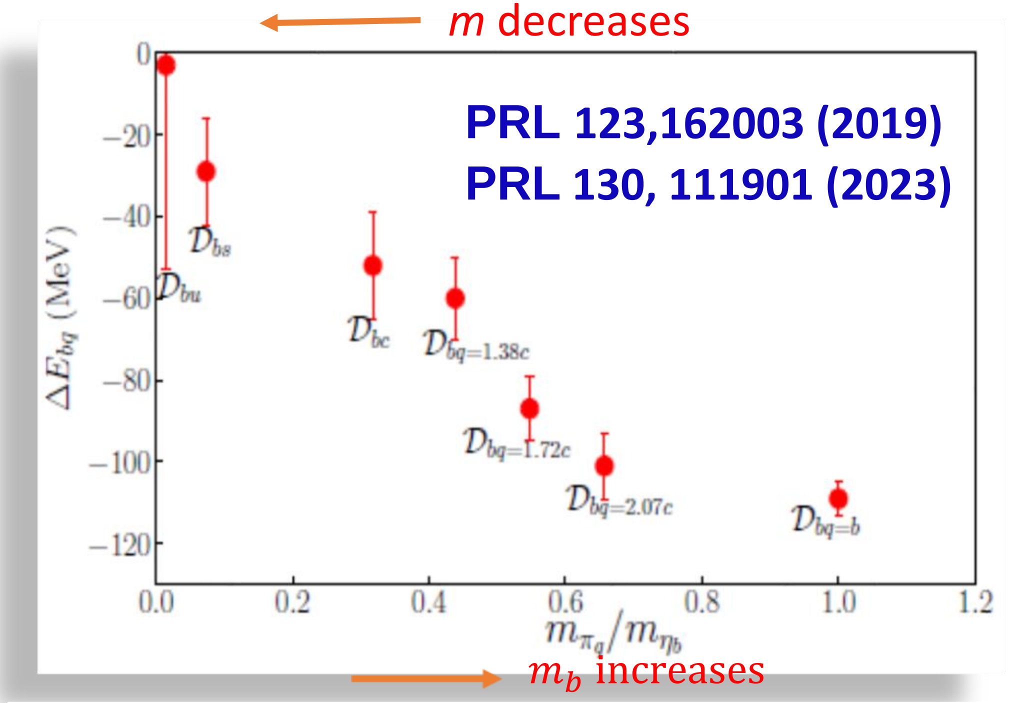
Hudspith etal 2021

$m_{b'}$ increases

- Heavier the heavy quark masses, stronger the binding
- Lighter the light quark masses, stronger the binding

Six-quark hadrons $(Q_1Q_2q_1Q_1Q_2q_2)$

m decreases



PRL 123,162003 (2019)
PRL 130, 111901 (2023)

m_b increases

- Heavier the heavy quark masses, stronger the binding
- Heavier the light quark masses, deeper the binding

Conclusions and Outlooks

- Exotic hadrons beyond the meson and baryon configurations, such as tetraquarks and pentaquarks, have been discovered recently. Most of these exotic hadrons have one or more heavy valence quark contents.

- Lattice QCD provides a rigorous approach to hadron spectroscopy. Lattice QCD calculations have predicted and postdicted some of these exotic hadrons.

- Lattice QCD calculations have predicted more such exotic hadrons which could be discovered in future.

- Lattice QCD calculations are essential to understand the structures and properties of these exotic hadrons.