Effects of finite volume on the QCD phase diagram

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Based on work done with Adiba Shaikh and Ranjita Mohapatra

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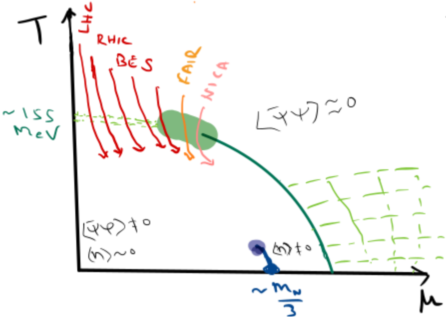
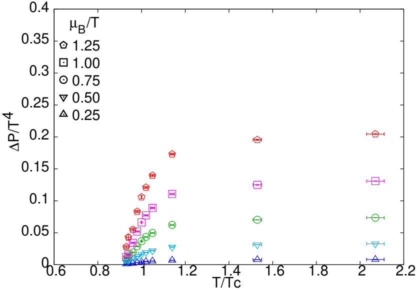
Introduction

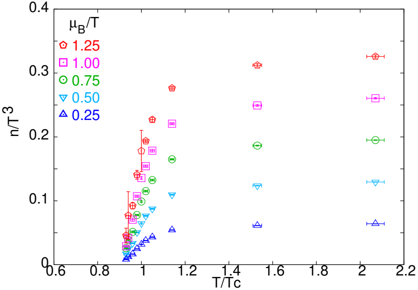


- At $\mu_q = \mu_B/3 \sim 0$ strong evidence of creation of a deconfined, chiral symmetry restored phase in AA collision in LHC and RHIC.
- Theory input: lattice QCD. Crossover, chiral symmetry restoring. $T_d \sim 155$ MeV, $\epsilon_d \sim 0.3$ GeV/fm^3.
- large μ_B: interesting phase diagram expected.
- BES at RHIC, CBM at FAIR
  NICA at JINR: collider and fixed target mode:
  Expected to explore the critical and 1st order regimes.

Theoretical input

Theoretical studies at finite μ_B more difficult: extrapolate from $\mu_B = 0$
Reliable results only at moderate μ_B





$N_f = 2$ results for $\Delta P(\mu, T) = P(\mu, T) - P(0, T)$ and $n(\mu, T) = \frac{\partial P(\mu, T)}{\partial \mu_B}$
S. Datta, R. Gavai, S. Gupta, PRD 95 (2017) 054512

Recent $N_f = 2 + 1$ results in HotQCD, PRD 105 (2022) 074511

At high μ_B lattice studies difficult: need to use QCD-motivated models.
Uncertainty in the location of the QCD critical point.

Experimental situation

- Lattice results, where available, give the results for an **infinite volume system** at **thermal equilibrium**.
- Experimental results are for a rapidly evolving system.
- System size $\sim$ a few fm not too large compared to the hadronic scale, or the characteristic scale $\sim 1/T$, near T_c.
- This is particularly relevant for NICA, where we will have $Xe - Xe$ collisions and the collision energy is small.
- Finite volume effect is most important near a critical point, where the correlation length becomes $\sim$ system size L.
- Non-Poissonian baryon number distribution in the critical zone.
- In actual system evolution, the system does not have enough time for the correlation length to grow sufficiently.

  Berdnikov & Rajagopal, PRD 61 (2000) 105017.
- Look at higher order cumulants to locate critical growth.

  M. Stephanov, PRL 102(2009) 032301; S. Gupta, PoS CPOD 2009 (2009) 025.
- Finite volume can cause a shift of the location of the critical point.

Finite volume effect using QCD-based models

- We will look at the volume dependence of the phase diagram using a Nambu Jona-Lasinio (NJL) type lagrangian.
- NJL can be considered as an effective model for studying the chiral symmetry breaking transition in QCD.
- The lagrangian consists of only quarks; interaction effects are taken into account by chiral symmetry preserving 4-fermion terms.

$$\mathcal{L} = \bar{\psi} \left(i \gamma_\mu \partial^\mu - m_q\right) \psi + G \left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\, i\, \gamma_5 \vec{\tau} \psi\right)^2\right]$$

- The lagrangian is non-renormalizable: results depend on the regularization, as well as G and m_q.
- G and m_q have to be tuned in a regulator-dependent way.
- We will use Schwinger's proper-time regularization, and two sets of parameters:

	Λ_{UV}[MeV]	m_q[MeV]	G [GeV^{-2}]
I	1080	5	3.2
II	645	15	17.2

- The parameters are chosen to get the pion mass, pion decay constant and the chiral condensate in the correct ballpark.

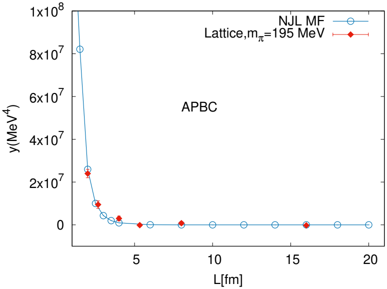
H. Kohyama, D. Kimura, T. Inagaki, NP B 896 (2015) 682.

NJL model study

- We study the model at mean field level.

$$Z \implies Z_{MF} = \int \mathcal{D}\bar{\Psi}\, \mathcal{D}\Psi \; e^{-\bar{\Psi}\,[i\not{\partial} - m_q - 2G\,\sigma_0]\,\Psi}, \qquad \sigma_0 = \langle \bar{\Psi}\,\Psi \rangle \equiv \frac{\partial \log Z}{\partial m_q}$$

- Captures features of volume effect of chiral condensate $\langle \bar{\Psi}\,\Psi \rangle$. We compare with lattice, looking at the RGI combination $y = m_q\, (\langle \bar{\Psi}\,\Psi \rangle(L) - \langle \bar{\Psi}\,\Psi \rangle(L \to \infty))$



- We will study the phase diagram by monitoring $\langle \bar{\Psi}\,\Psi \rangle$, and look at the volume dependence of the chiral symmetry restoration line.

Boundary conditions

- The details of the phase diagram depend on the parameters.

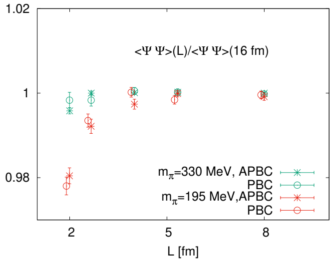
  H. Kohyama, D. Kimura, T. Inagaki, NP B 896 (2015) 682.

- This is true with any model study of the phase diagram.
- In particular, our set II has a first order region, while set I does not. They are chosen to study the volume effect in the no-critical region and the critical region cases.
- In any finite volume system, the boundary condition plays an important role.
- Various previous works with QCD-like models have looked at the effect of the finite volume system.

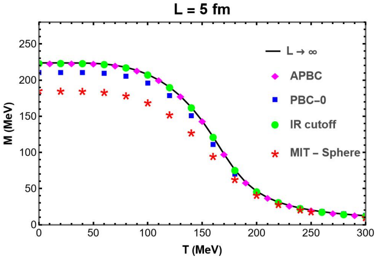
  Bhattacharyya et al. (PRD, 2013; PRC, 2015); Klein, Phys. Rep. (2017); Bernhardt, et al., PLB 841 (2023) 137908; Kovacs, et al., 2307.10301; ...

- Periodic or antiperiodic boundary condition is not a physical boundary condition for a finite volume system like the QGP fireball; they attempt to mimick the infinite volume system in a finite size.

Different Boundary conditions



Lattice results for $\langle \bar{\Psi}\,\Psi \rangle$



NJL, $M \;=\; m_q \;-\; 2G\,\langle \bar{\Psi}\,\Psi \rangle$

Very little volume effect seen for PBC and APBC.

Perfect b.c. for getting infinite volume results, but does not resemble the QGP fireball.

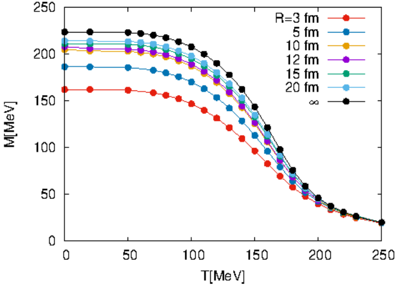
Realistic b.c. and geometry for the QGP fireball is complicated.

We take the MIT b.c., $\vec{n} \cdot \bar{\Psi}\,\vec{\gamma}\,\Psi \;=\; 0$ and spherical geometry, $\vec{n} \equiv \hat{r}$

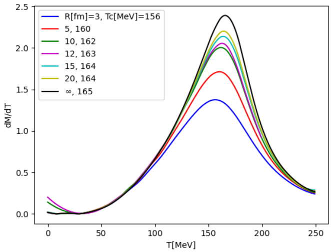
MIT b.c. for cylindrical and spherical geometry have been considered before for the study of rotating QGP system.

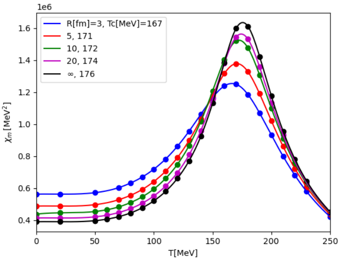
Chernodub & Gongyo, JHEP 01 (2017) 136; Zhang, Luo & Zong, PRD 101 (2020) 074306.

Results at $\mu=0$

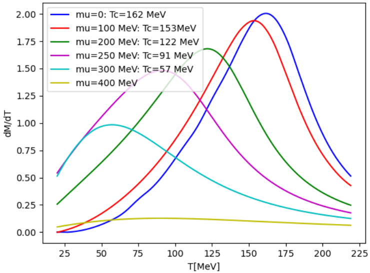


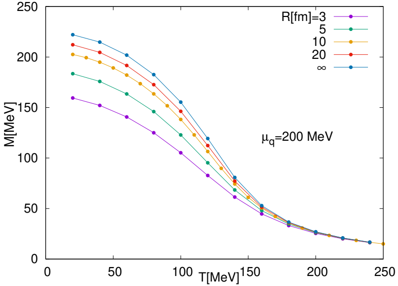
Crossover temperature can be defined by the maximum of $\frac{dM}{dT}$ or the susceptibility $\chi_{\bar{\Psi}\Psi}$.
Mild shift of the crossover temperature with volume.
$\chi_{\bar{\Psi}\Psi}$ peak gets broader with decreasing volume.





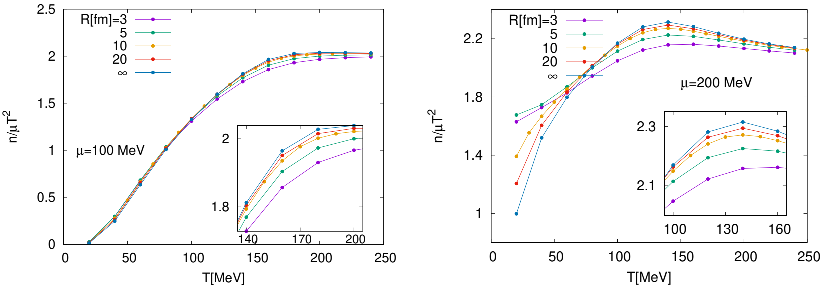
Results at finite μ_B





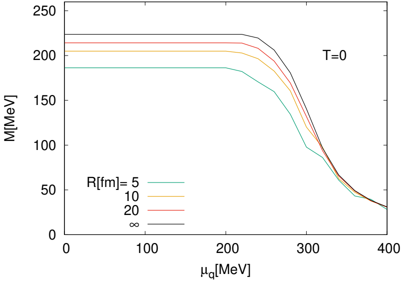
- As μ increases, the chiral symmetry restoration transition happens at lower temperatures.
- The volume dependence of the transition temperature continues to be mild.
- For set I, no indication of a discontinuous transition anywhere.

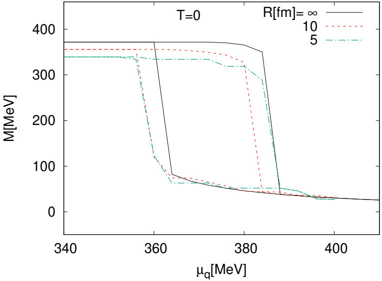
Number density



- While the constituent mass M is not an observable, the number density $n = \frac{\partial P}{\partial \mu}$ is.
- n shows a clear volume dependence for small systems, though the volume dependence is mild for $R \geq 10$ fm.

Crossover (Set I) vs a first order transition (set II)

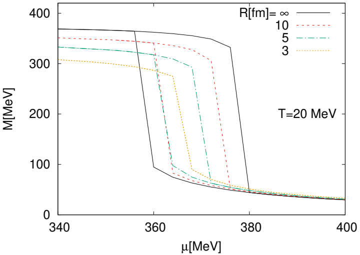


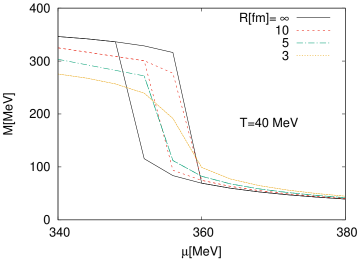


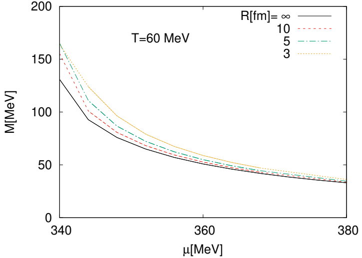
- Set I: no discontinuous transition at T=0
- Chiral symmetry restoration at $\mu_q \sim 300$ MeV
- Volume dependence mild.

- Set II: discontinuous transition at T=0
- Hysteresis region similar for M and n.
- Transition region not shifted much with volume.

First order region and volume

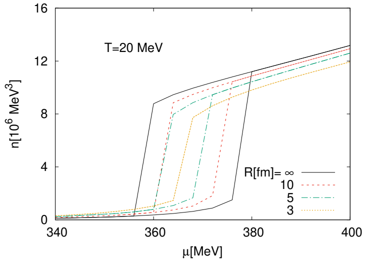


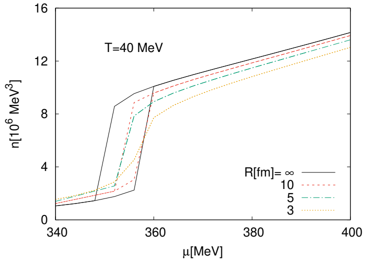


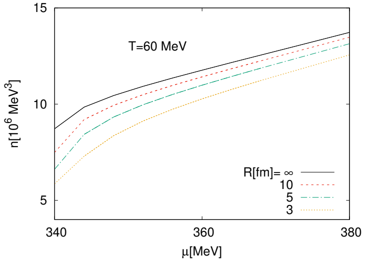


As temperature increases, the hysteresis region shrinks.
Volume hierarchy: the hysteresis region disappears earlier at the smaller volumes, indicating critical region at a lower temperature.

Hysteresis in number density

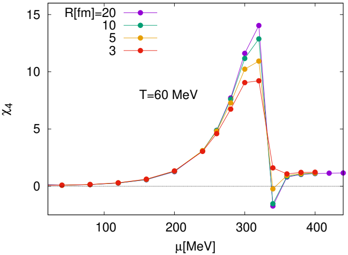


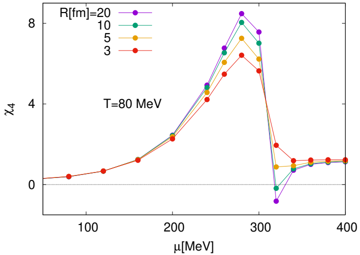




The number density shows very similar behavior.
As expected in a first order transition, the discontinuity for different observables happen at the same region.
Volume hierarchy of critical point location.

χ_4

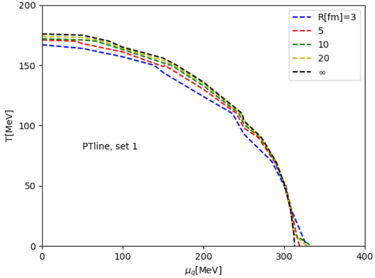


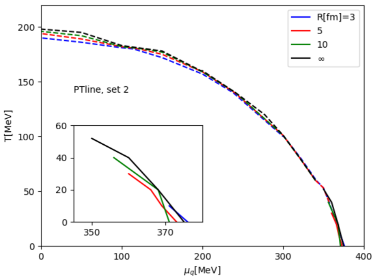


- The susceptibilities of the number density can be connected to the cumulants of the proton number distribution.
- Higher order susceptibilities show stronger critical behavior.
- In particular, the kurtosis κ, related to χ_4, is an important probe of the critical region.

  Stephanov, PRL 102(2009) 032301; S. Gupta, PoS CPOD 2009 (2009) 025.
- The peak and the dip of χ_4 show substantial volume dependence near the critical region.

R dependence of the transition line





- With the spherical MIT boundary condition, the chiral symmetry restoration line is seen to move in (T, μ) plane.
- The movement is mild: in the crossover region, $\delta T \lesssim 10$ MeV, similar in size to the spread between the line for different markers.
- The first order region is seen to shrink with decreasing volume, pushing the critical point to lower temperatures.
- Relevant for critical point search in finite volume fireball.

Summary

- The fireball created in heavy ion collision experiments is moderate sized, in units of the relevant correlation length: important to get an estimate of the size of the finite volume effect before comparing with theoretical results obtained in the thermodynamic limit.
- Need to put in realistic boundary conditions to study FV effects.
- We investigate the FV effects using the NJL model; compare with a lattice calculation for a.p.b.c.
- We use spherical MIT boundary conditions, which sets the component of the quark number current to zero along normal to the boundary, and study FV effects by looking at systems of radii in the range 3-20 fm.
- In particular, it is found that the critical region gets pushed to lower temperatures at smaller volumes, shrinking the first order region.
- The nonlinear susceptibilities of the number current are also studied.
- This study is for static medium. The time evolution of the system also gets embroiled with the FV effect, as it restricts the growth of the correlation length in the critical regime. Including this dynamics may therefore reduce the FV effect.