Charge distributions inside strange baryons in impact parameter space and transverse coordinate space

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Overview

- *Introduction:Internal Structure of the Hadrons*
- *Distribution functions*
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Introduction

- Quantum Chromodynamics (QCD) provides a fundamental description of hadronic structure and dynamics in terms of their elementary quark and gluon degrees of freedom.
- **Internal Structure: The knowledge of internal structure provides a basis for understanding more complex, strongly interacting matter.**
- Knowledge has been rather limited because of **confinement** and it is still a big challenge to perform the calculations from the first principles of QCD.

Quantum chromodynamics (QCD): Present Theory of Strong Interactions

- At high energies, **(α_s is small)**, QCD can be used perturbatively.
- At low energies, **(α_s becomes large)**, one has to use other methods such as effective Lagrangian models to describe physics.
- Wide range of applications ranging from the dynamics and structure of hadrons and nuclei to the properties and phases of hadronic matter at the earliest stages of the universe.
- New experimental tools are continually being developed to probe the nonperturbative structure of the theory, for example the hard diffractive reactions, semi-inclusive reactions, deeply virtual Compton scattering etc..
- Many fundamental questions have not been resolved. **The most challenging nonperturbative problem in QCD is to determine the structure and spectrum of hadrons in terms of their quark and gluon degrees of freedom.**

Fundamental Questions

- How are the static observable related to each other and how do they emerge?
- How are the sea quarks and their spins, distributed in space and momentum inside the nucleon?
- Role of orbital angular momentum of the quarks and gluons in the non-perturbative regime of QCD.
- The role played by non-valence flavors in understanding the nucleon internal structure.
- How do the quarks and gluons interact with a nuclear medium?

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin).
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)
  "Proton spin crisis".
- Confirmed by the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments by SMC, E142-3 and HERMES experiments.
- Provides evidence that the valence quarks of proton carry only a small fraction of its spin suggesting that they should be surrounded by an indistinct sea of quark-antiquark pairs.

Internal Structure of the Hadrons

- QCD closes the number of voids in our knowledge of particle physics to give us a detailed picture of such complex structures.
- **A lot of theoretical as well as experimental research work has been done to reveal the mysterious behavior of quarks inside the nucleons** which are the members of octet baryon with spin-parity quantum number as $J^P = (\frac{1}{2})^+$.
- Other members in the octet baryon, like **isospin partners of Σ and Ξ have been ignored.**

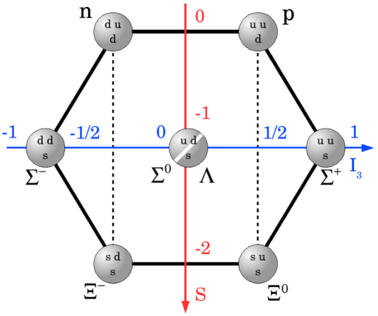


Figure 1: Octet of Baryons.

Limitations in measurements of other members of octet baryon

- Presence of **strange quark(s).**
- Small life span of these strange baryons.
- Measurements demand collision energy significantly above the hadronic scale to allow transfer of energy and momentum.

Instant form v/s Front form

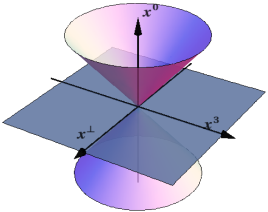


Figure 2: The instant form

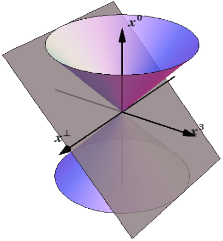


Figure 3: The front form

- All measurements are made at fixed t i.e. at $x^0 = 0$.
- All measurements are made at fixed light-cone time x^+ i.e. at $x^+ = x^0 + x^3 = 0$.

- Energy-momentum dispersion relation:

In the instant form,

$$p^0 = \sqrt{\vec{p}^2 + m^2}.$$

In the front form,

$$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}.$$

No square-root for the Hamiltonian in light front form. Therefore, simplifes the dynamical structure.

- Instant-form vacuum is infinitely complex.
- Simple vacuum structure vacuum expectation value is zero as all the massive fluctuations in the ground state are absent.

Light-front provides the wavefunctions (LFWFs) required to describe the structure and dynamics of hadrons in terms of their constituents (quarks and gluons).

- S. J. Brodsky, G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).

Light-Front Coordinates

- A generic four vector x^μ in light-cone coordinates is describe as $x^\mu = (x^-, x^+, x_\perp)$.
- $x^+ = x^0 + x^3$ is called as light-front time.
- $x^- = x^0 - x^3$ is called as light-front longitudinal space variable.
- $x^\perp = (x^1, x^2)$ is the transverse variable.
- Similarly, we can define the longitudinal momentum $k^+ = k^0 + k^3$ and light-front energy $k^- = k^0 - k^3$.

Light-Front in QCD

- Light Front QCD (LFQCD) is an *ab initio* approach to study the strongly interacting system. It is similar to perturbative and lattice QCD and is directly connected to the QCD Lagrangian.
- It is a Hamiltonian method, formulated in Minkowinski space rather than Euclidean space.
- The theory is quantized at fixed light-cone time $\tau = t + z/c$ rather than ordinary time t.

Distribution functions

(Mathematical tool to unfold the internal structure of hadrons)

Parton Distribution Functions (PDFs)
Generalized Parton Distributions (GPDs)
Transverse Momentum-Dependent Parton Distributions (TMDs)

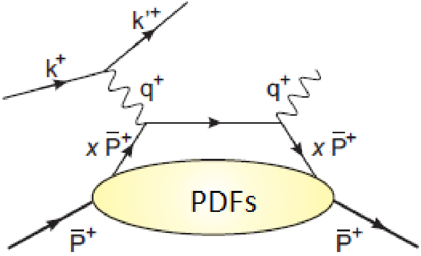
Parton Distribution Functions (PDFs)

To understand the structure of the hadron in terms of quarks and gluons, different categories of parton distributions are present.

- PDFs were introduced by Feynman in 1969.
- PDFs are the basic ingredient to understand the internal hadron structure.
- From parton densities one can extract the distribution of longitudinal momentum carried by the quarks, antiquarks and gluons and their polarizations.
- PDF $f(x)$ imparts information about the probability of finding a parton carrying a longitudinal momentum fraction x inside the hadron.

- Partonic structure is probed in scattering processes such as Deep Inelastic Scattering (DIS).
- The quark-quark correlation to evaluate proton PDFs are defined as

$$F^{[\gamma^+]}(x) = \frac{1}{2}\int \frac{dz^-}{4\pi} e^{ik^+z^-/2} \langle P|\bar{\Psi}(0)\gamma^+\Psi(z^-)|P\rangle\Big|_{z^+=\mathbf{z}_\perp=0}.$$



How partons are distributed in the plane transverse to the direction in which the hadron is moving, or how important their orbital angular momentum is in making up the total spin of a nucleon?

This missing information is compensated in Generalized Parton Distributions (GPDs). The GPDs are physical observables which can provide deep insight about the internal structure of the nucleon and more generally, in non-perturbative QCD.

Generalized Parton Distributions (GPDs)

- GPDs provide a 3-D picture of the partonic nucleon structure. Encode information on the distribution of partons both in the transverse plane and longitudinal direction.
- GPDs can be accessed through deep exclusive processes such as DVCS or DVMP. DVCS reaction $\gamma^* + p \to \gamma + p$ has extraordinary senstivity to fundamental features of the proton's structure.
- GPDs are much richer in content about the hadron structure than ordinary parton distributions.
- GPDs allow us to access partonic configurations with a given longitudinal momentum fraction, but also at specific location (transverse) inside the hadron.

- GPDs depends on three variables x, ζ, t.
  - x is the fraction of momentum transfer.
  - ζ gives the longitudinal momentum transfer.
  - t is the square of the momentum transfer in the process.
- Several experiments such as H1 collaboration, ZEUS collaboration and fixed target experiments at HERMES have finished taking data on DVCS. In the forward limit of zero momentum transfer, the GPDs reduce to ordinary parton distributions.
- One can define the correlation to evaluate unpolarized GPD in proton $F^{[\Gamma]}(x, \zeta = 0, t)$ as

$$F^{[\Gamma]}(x, 0, t) = \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ixP^+ z^-/2} \langle P_f | \bar{\Psi}(0) \Gamma \Psi(z) | P_i \rangle \Big|_{z^+ = \mathbf{z}_\perp = 0}.$$

- GPDs have **deep pockets** as it is a source to study the intrinsic properties of a baryon.
- Correlator corresponding to the unpolarized baryon with zero skewness

$$\frac{1}{2}\int \frac{dy^-}{2\pi}\, e^{ixP^+y^-} \left\langle P' \middle| \bar{\psi}\left(\frac{-y}{2}\right)\gamma^+ \psi\left(\frac{y}{2}\right) \middle| P \right\rangle \Big|_{y^+=0,\mathbf{y}_\perp=\mathbf{0}}$$
$$= \frac{1}{2\bar{P}^+}\bar{u}(P')\Big[H_X^q(x,0,\mathbf{\Delta}_\perp)\,\gamma^+ + E_X^q(x,0,\mathbf{\Delta}_\perp)\,\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}\Big]u(P).$$

- These $H_X^q(x,0,\mathbf{\Delta}_\perp)$ and $E_X^q(x,0,\mathbf{\Delta}_\perp)$ can be used to evaluate charge distribution and magnetization densities respectively.

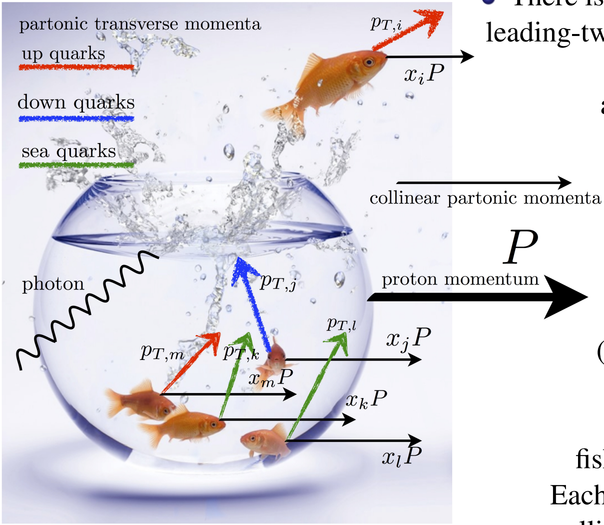
Transverse Momentum-Dependent Parton Distributions (TMDs)

To get the information of hadron structure in momentum space, transverse momentum-dependent parton distributions (TMDs) were introduced.

- TMDs describe the probability to find a parton with longitudinal momentum fraction x and transverse momentum with respect to the direction of the parent hadron momentum in a hadron.
- TMDs $f(x, \vec{k}_\perp)$, are function of longitudinal momentum fraction carried by the active quark $x = \frac{k^+}{P^+}$ and the quark transverse momentum $\vec{k}_\perp$.
- TMDs are also of particular importance because they give rise to single spin asymmetries (SSAs).
- TMDs represent three-dimensional hadron picture in *momentum space*.

- They can be measured in a variety of reactions in lepton-proton and proton-proton collisions as semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan production where a final-state particle is observed with a transverse momentum.
- The quark-quark correlation to evaluate quark TMDs in proton is given by

$$\Phi^{[\Gamma]}(x, \mathbf{k}_\perp) = \frac{1}{2}\int \frac{dz^-}{2\pi}\frac{d^2\mathbf{z}_\perp}{(2\pi)^2} e^{ik.z/2}\langle P|\bar{\Psi}(0)\Gamma\Psi(z)|P\rangle\Big|_{z^+=0}.$$



-Image courtesy: A. Signori

- There is one quark TMD at leading-twist in case of kaon, while 8 quark and gluon TMDs at the leading twist in case of nucleon (spin-1/2).

- In the figure, partons (quarks and gluons) are like fishes confined inside a fishbowl (the proton). Each parton has its own collinear and transverse velocity, indicated by black and colored arrow respectively.

Quark-Scalar Diquark Model

- Adopted a QCD inspired quark-scalar diquark model of a hadron representing a simplistic panorama of an active quark q and a spectator of a diquark n.
- Based on one-loop quantum fluctuations of the Yukawa theory, the two particle Fock state has two possible spin combinations.
- For each Fock state in *momentum space*, light-front wave function is given by

For $J^z = +\frac{1}{2}$,

$$\psi^{\uparrow X}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \left(M_X + \frac{m_q}{x}\right)\varphi_X ,$$

$$\psi^{\uparrow X}_{-\frac{1}{2}}(x, \mathbf{k}_\perp) = -\frac{(k^1 + \iota k^2)}{x}\varphi_X .$$

For $J^z = +\frac{1}{2}$,

$$\psi^{\downarrow X}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \frac{(k^1 - \iota k^2)}{x}\varphi_X ,$$

$$\psi^{\downarrow X}_{-\frac{1}{2}}(x, \mathbf{k}_\perp) = \left(M_X + \frac{m_q}{x}\right)\varphi_X .$$

The scalar part φ has a form

$$\varphi_X = \varphi_X(x, \mathbf{k}_\perp) = \frac{\frac{g}{\sqrt{1-x}}}{M_X^2 - \frac{\mathbf{k}_\perp^2 + m_q^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda_n^2}{1-x}} ,$$

with M_X, m_q and λ_n as a mass of a baryon X, an active quark and a spectator diquark.

Transverse Coordinate Space

By employing the Fourier transformation, one can relate the light-front wave functions in transverse coordinate space and in momentum space as

$$\psi^X(x,\mathbf{k}_\perp) = \int d^2\mathbf{r}_\perp e^{-\iota \mathbf{k}_\perp\cdot\mathbf{r}_\perp}\tilde{\psi}^X(x,\mathbf{r}_\perp), \qquad \tilde{\psi}^X(x,\mathbf{r}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} e^{-\iota \mathbf{k}_\perp\cdot\mathbf{r}_\perp}\psi^X(x,\mathbf{k}_\perp).$$

From these equations, we can write

$$\int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\psi^{X*}(x,\mathbf{k}_\perp)\psi^X(x,\mathbf{k}_\perp) = \int d^2\mathbf{r}_\perp\tilde{\psi}^{X*}(x,\mathbf{r}_\perp)\tilde{\psi}^X(x,\mathbf{r}_\perp).$$

For each Fock state in *transverse coordinate space*, wave function is given by

$$\tilde{\psi}^{\uparrow X}_{+\frac{1}{2}}(x,\mathbf{r}_\perp) = (x\,M_X + m_q)\frac{\mathbf{r}_\perp}{\sqrt{\mathcal{M}_X}}\frac{gM_X^2}{4\pi}(1-x)^{\frac{3}{2}}K_1\Big(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\Big),$$

$$\tilde{\psi}^{\uparrow X}_{-\frac{1}{2}}(x,\mathbf{r}_\perp) = -\iota(r^1+\iota r^2)\frac{gM_X^2}{4\pi}(1-x)^{\frac{3}{2}}K_0\Big(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\Big).$$

$$\tilde{\psi}^{\downarrow X}_{+\frac{1}{2}}(x,\mathbf{r}_\perp) = \iota(r^1-\iota r^2)\frac{gM_X^2}{4\pi}(1-x)^{\frac{3}{2}}K_0\Big(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\Big),$$

$$\tilde{\psi}^{\downarrow X}_{-\frac{1}{2}}(x,\mathbf{r}_\perp) = (x\,M_X + m_q)\frac{\mathbf{r}_\perp}{\sqrt{\mathcal{M}_X}}\frac{gM_X^2}{4\pi}(1-x)^{\frac{3}{2}}K_1\Big(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\Big).$$

Input Parameters

Particle (X)	Σ^+	Ξ^o
Mass, M_X (GeV)	1.189	1.314

Table 1: Masses of members of octet baryons used in the present calculations

Quark content	u/d	s	uu/ud	us/ds	ss
Mass (GeV)	0.33	0.48	0.80	0.95	1.10

Table 2: Masses of quarks and their combinations used in the present calculations [1]

K_p represents the Bessel function of the second kind of order p in wave functions of transverse coordinate space.

[1]For the sake of simplicity, we have considered $m_u = m_d$ and have represented the contrast of these light quarks with strange quark, s. All the plots presented in this paper are in the units of $\frac{g^2}{4\pi}$.

Charge Distributions In Impact Parameter Space

- Charge distribution **portrait a spatial distribution of an active quark charge in space.**
- Burkardt showed that one can elicit a Dirac form factor $F_{1X}(\mathbf{\Delta}_\perp)$ from GPD $H_X^q(x, 0, \mathbf{\Delta}_\perp)$.
- Fourier transform of Dirac form factor will interpret the charge distribution as a function of a transverse distance in the infinite momentum frame

$$\rho_X^q(\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} F_{1X}(\mathbf{\Delta}_\perp).$$

- Miller introduced a relation

$$\rho_X^q(x, b_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H_X^q(x, 0, \mathbf{\Delta}_\perp) = \frac{1}{(1-x)^2}\, P_X^q\left(x, \frac{\mathbf{b}_\perp}{-1+x}\right).$$

- The explicit expression for an active quark charge is jotted down as

$$\rho_X^q(x, \mathbf{b}_\perp) = \frac{g^2 M_X^4}{(4\pi)^2}\, \frac{\mathbf{b}_\perp^2}{1-x}\left[\frac{(x\, M_X + m_q)^2}{\mathcal{M}_X}\, K_1^2\left(\frac{\mathbf{b}_\perp}{-1+x}\sqrt{\mathcal{M}_X}\right) + K_0^2\left(\frac{\mathbf{b}_\perp}{-1+x}\sqrt{\mathcal{M}_X}\right)\right].$$

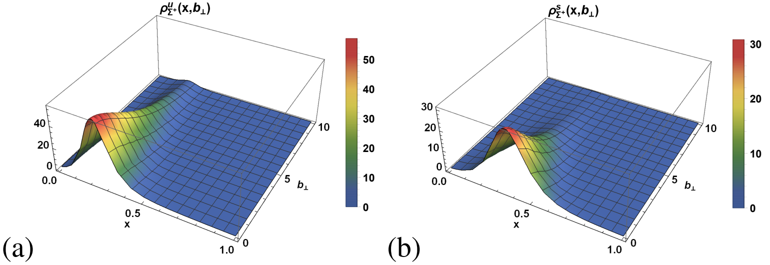


Figure 4: Charge distributions in impact parameter space for Σ^+ with single s content. The left and right distribution corresponds to u and s quarks sequentially.

Distributions portray that

- Peaks are intense at $|\mathbf{b}_\perp| = 0$.
- Intensity of peak for an active s quark is intensified at higher value of x than an active u quark.
- with increase in $|\mathbf{b}_\perp|$, amplitude of peaks decreases with narrowing down of peak over the range of x.
- The pace of decrement of charge distribution with $|\mathbf{b}_\perp|$ is faster for an active s quark than u quark.

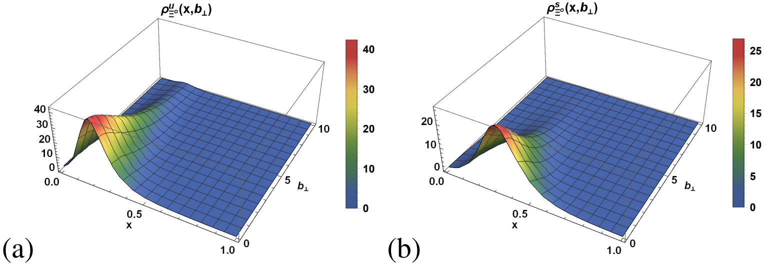


Figure 5: Charge distributions in impact parameter space for Ξ^o with double s content. The left and right distribution corresponds to u and s quarks sequentially.

Distributions portray that

- Peaks are intense at $|\mathbf{b}_\perp| = 0$.
- Intensity of peak for an active s quark is intensified at higher value of x than an active u quark.
- with increase in $|\mathbf{b}_\perp|$, amplitude of peaks decreases with narrowing down of peak over the range of x.
- The pace of decrement of charge distribution with $|\mathbf{b}_\perp|$ is faster for an active s quark than u quark.

Difference in the distributions of both the baryons

On moving from $\Sigma^+(uus)$ to $\Xi^o(uss)$

- Amplitude
  - The amplitude for both u and s quarks decreases.
  - Shifting of peak towards lower value of x is clearly visible in impact parameter space than transverse coordinate space.
- With increment in $|\mathbf{r}_\perp|$
  - The pace of decrement of charge distribution becomes faster for both u and s quark.

  It implies that **charge is more concentrated towards center of a baryon Ξ^o having double s content** than Σ^+ baryon with single s content

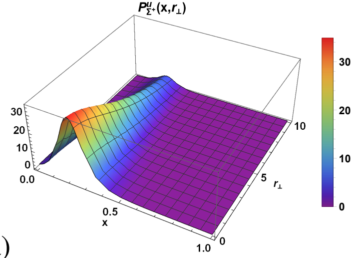
Charge Distributions In Transverse Coordinate Space

- By making the use of wave functions in transverse coordinate space, the overlap form of charge distribution with **transverse coordinate** $(x, \mathbf{r}_\perp)$ is given by

$$P_X^q(x,\mathbf{r}_\perp) = \left[\tilde{\psi}^{\uparrow X*}_{+\frac{1}{2}}(x,\mathbf{r}_\perp)\,\tilde{\psi}^{\uparrow X}_{+\frac{1}{2}}(x,\mathbf{r}_\perp) + \tilde{\psi}^{\uparrow X*}_{-\frac{1}{2}}(x,\mathbf{r}_\perp)\,\tilde{\psi}^{\uparrow X}_{-\frac{1}{2}}(x,\mathbf{r}_\perp)\right].$$

- The explicit expression for charge distribution comes out to be

$$P_X^q(x,\mathbf{r}_\perp) = \frac{g^2 M_X^4}{(4\pi)^2}(1-x)^3\mathbf{r}_\perp^2\left[\frac{(x\,M_X + m_q)^2}{\mathcal{M}_X}K_1^2\left(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\right) + K_0^2\left(\mathbf{r}_\perp\sqrt{\mathcal{M}_X}\right)\right].$$



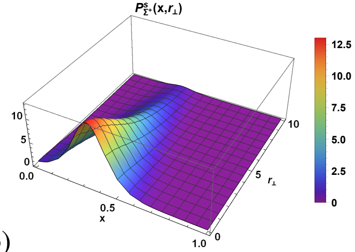


Figure 6: Charge distributions in coordinate space for Σ^+ with single s content. The left and right distribution corresponds to u and s quarks sequentially.

Distributions portray that

- Peaks are intense at $|\mathbf{r}_\perp| = 0$.
- Intensity of peak for an active s quark is intensified at higher value of x than an active u quark.
- with increase in $|\mathbf{r}_\perp|$, amplitude of peaks decreases with narrowing down of peak over the range of x.
- The pace of decrement of charge distribution with $|\mathbf{r}_\perp|$ is faster for an active s quark than u quark.

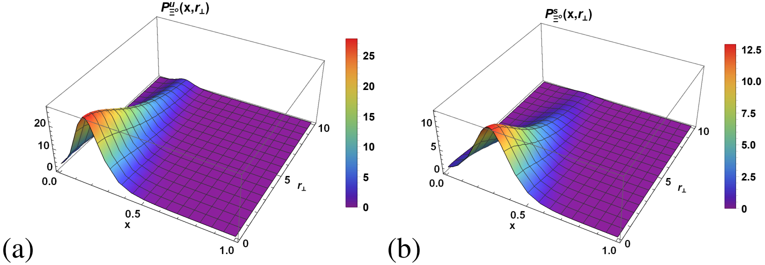


Figure 7: Charge distributions in coordinate space for Ξ^o with double s content. The left and right distribution corresponds to u and s quarks sequentially.

Distributions portray that

- Peaks are intense at $|\mathbf{r}_\perp| = 0$.
- Intensity of peak for an active s quark is intensified at higher value of x than an active u quark.
- with increase in $|\mathbf{r}_\perp|$, amplitude of peaks decreases with narrowing down of peak over the range of x.
- The pace of decrement of charge distribution with $|\mathbf{r}_\perp|$ is faster for an active s quark than u quark.

Difference in the distributions of both the baryons:

On moving from $\Sigma^+(uus)$ to $\Xi^o(uss)$

- Amplitude
  - The amplitude for u quark decreases significantly.
  - No such predominant variation observed in the amplitude for s quark.
- With increment in $|\mathbf{r}_\perp|$
  - The pace of decrement of charge distribution becomes faster for both u and s quark.

    It implies that **charge is more concentrated towards center of a baryon Ξ^o having double s content** than Σ^+ baryon with single s content

Juxtapose of charge distributions in both spaces

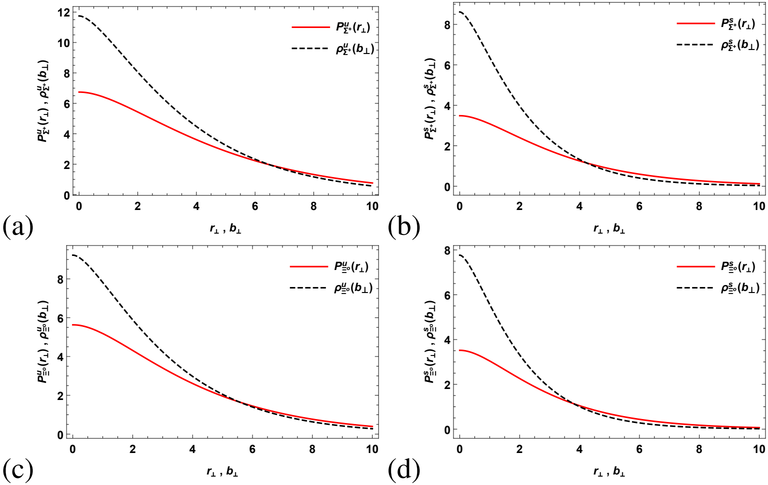


Figure 8: Charge distributions in coordinate and impact parameter space for Σ^+ (Row-I) and Ξ^o (Row-II) with double s content. The left and right distribution corresponds to u and s quarks sequentially.

Contrast behavior of charge distributions in both spaces

- Amplitude
  Predominant contrast lies at small values of $|\mathbf{r}_\perp|$ and $|\mathbf{b}_\perp|$.
  (representing more amplitude of charge distribution in impact parameter space.)
  Subsidiary contrast lies at large values of $|\mathbf{r}_\perp|$ and $|\mathbf{b}_\perp|$.
  (representing more spread of charge distribution in transverse coordinate space.)
- Variation with $|\mathbf{r}_\perp|$ and $|\mathbf{b}_\perp|$
  Very slow decrease with $|\mathbf{r}_\perp|$ than $|\mathbf{b}_\perp|$ has been observed

Causes of this contrasting behavior

- In impact parameter space, skewness is fixed as zero which in turn fixes the longitudinal momentum fraction giving pure access to impact parameter space.
- There is no such fixation in transverse coordinate space giving contribution from longitudinal momentum fraction too.
- Presence of an extra factor of $\frac{1}{-1+x}$ in impact parameter space.

Summary

Understanding the structure of the hadrons will help to resolve the most challenging problems facing subatomic physics which include

- Presence of intrinsic strange and charm quarks in the light baryons and the other non-constituent quark contributions in the other baryons.
- The phenomena of chiral symmetry breaking will have far-reaching implications in terms of the elementary degrees of freedom of the composite particles.
- Lattice calculations and future experiments at EIC will not only have the possibility to illuminate the complicated issue of spin structure but also impose significant and decisive restraints in different kinematic regions.

Thank You!