Thermal effects on weak-interaction nuclear reactions under supernova conditions

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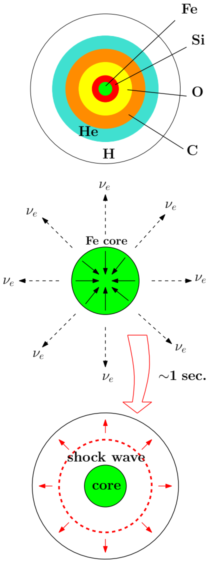
Core-collapse supernova



- Massive stars ($M \geq 10 M_\odot$) at the end of their life have an onion like structure.
- When $M_{Core} \approx M_{Ch} = 1.44(2Y_e)^2 M_\odot$ the iron core starts to collapse ($Y_e = n_e/n_N$ is the electron fraction).
- Weak-interaction processes are **crucial** for supernova explosion mechanism:
  1. electron capture

     $$e^- + A(N,Z) \rightarrow A(Z-1,N+1) + \nu_e$$

     reduces the electron gas pressure and determine M_{Core}.
  2. ν-nucleus reactions are important at $\rho \gtrsim 10^{11}\ \mathrm{g/cm^3}$:

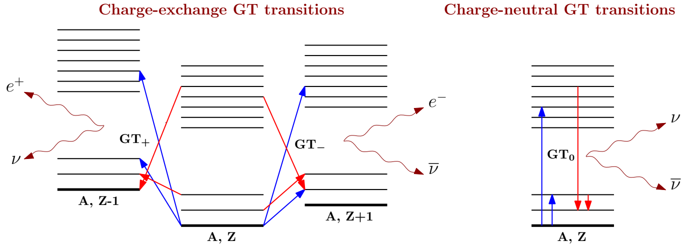
     $$\nu + A(Z,N) \rightarrow A(Z,N) + \nu',$$

     $$\nu + A(Z,N) \rightarrow A(Z+1,N-1) + e^-$$

     trap neutrinos inside the core and affect the energy transport.

Hot nuclei in stellar interior

In the supernova environment nuclear weak-interaction reactions take place at finite temperatures $T = 0.1 \div 5$ MeV ($0.86\,\mathrm{MeV} = 10^{10}$ K). Nuclear excited states are thermally populated according to Boltzmann distribution $p_i(T) \sim \exp(-E_i/T)$



In supernova $E_{e,\nu} \leq 30$ MeV and **Gamow-Teller** (1^+) transitions dominate the nuclear weak-interaction processes:

- $e^- + A(N,Z) \rightarrow A(Z-1, N+1) + \nu_e \qquad (GT_+ = \sum_i \sigma_i t_i^+)$;
- $\nu_e + A(Z,N) \rightarrow A(Z+1, N-1) + e^- \qquad (GT_- = \sum_i \sigma_i t_i^-)$;
- $\nu + A(Z,N) \rightarrow A(Z,N) + \nu' \qquad (GT_0 = \sum_i \sigma_i t_i^0)$.

Shell-Model calculations at $T \neq 0$

$$\sigma(T, E_l) = Z(T)^{-1} \sum_i e^{-E_i/T} \sigma_i(E_l), \quad \lambda(T) = Z(T)^{-1} \sum_i e^{-E_i/T} \lambda_i$$

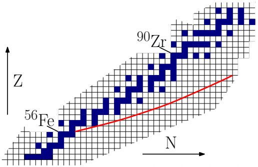
K. Langanke, G. Martinez-Pinedo, ADNDT 79 (2001) 1

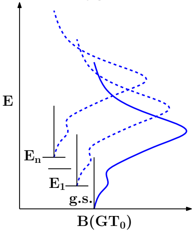
Shortcomings of SM calculations at $T \neq 0$:

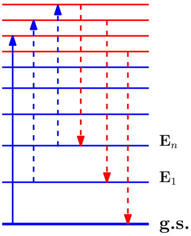
- Brink-Axel hypothesis is applied;
- the method of "back-resonances" is used;
- detailed balance principle is violated, i.e.

$$S(T, -E) \neq S(T, E) \exp(-E/T);$$

- the contribution of low- and negative-energy GT transitions is underestimated;
- to date SM calculations are limited by the iron-group nuclei (A=40-65)




Brink hypothesis


Back resonances

Statistical approach and superoperators in the Liouville space

Cross-section for semi-leptonic reaction on hot nucleus

$$\sigma(\varepsilon_l,T)=\sum_{if}p_i(T)\sigma_{if}(\varepsilon_l)=\frac{2G_F^2}{\hbar^4c^4}\frac{\varepsilon_l}{p_l}\int\limits_{-\infty}^{\varepsilon_l-m_{l'}c^2}dE\,\varepsilon_{l'}p_{l'}\int_{-1}^{1}d(\cos\theta)\Big\{\sum_{J\geq0}\eta_J^{CL}(E,T)+\sum_{J\geq1}\eta_J^{T}(E,T)\Big\},$$

where $\eta_J^{CL}(E,T)$ и $\eta_J^{T}(E,T)$ is a linear combination of spectral densities $S_{A,B}(E,T)$ of correlation functions for multipole operators of weak nuclear current

$$S_{A,B}(E,T)=\int\frac{dt}{2\pi}e^{iEt}\langle\langle A(t)B(0)\rangle\rangle,\qquad (A,B=M_J,\,L_J,\,T_J^{el},\,T_J^{mag}).$$

The explicit form of $M_J,\,L_J,\,T_J^{el},\,T_J^{mag}$ is obtained from the Donelly-Walecka theory.

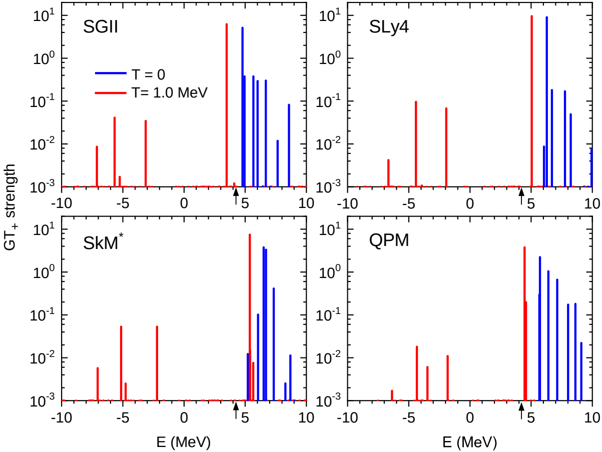
Superoperators and calculation of spectral densities

Statistical average is given by $\langle\langle A(t)B(0)\rangle\rangle=\langle 0(T)|A(t)B(0)|0(T)\rangle$, and the thermal Hamiltonian $\mathcal{H}=H(\boldsymbol{a}^\dagger,\boldsymbol{a})-H(\boldsymbol{a}^\dagger,\boldsymbol{a})$ determines spectral properties of hot nuclei:

$$S_{A,B}(E,T)=\sum_k\Big\{\langle\mathcal{O}_k|B|0(T)\rangle\langle\mathcal{O}_k|A|0(T)\rangle^*\delta(E-\mathcal{E}_k)+\langle\widetilde{\mathcal{O}}_k|B|0(T)\rangle\langle\widetilde{\mathcal{O}}_k|A|0(T)\rangle^*\delta(E+\mathcal{E}_k)\Big\},$$

where $|0(T)\rangle$ $(\mathcal{H}|0(T)\rangle=0)$ is the thermal vacuum, $\mathcal{H}|\mathcal{O}_k\rangle=+\mathcal{E}_k|\mathcal{O}_k\rangle$ and $\mathcal{H}|\widetilde{\mathcal{O}}_k\rangle=-\mathcal{E}_k|\widetilde{\mathcal{O}}_k\rangle$.

GT$_+$ strength distribution in ^{56}Fe at $T \neq 0$

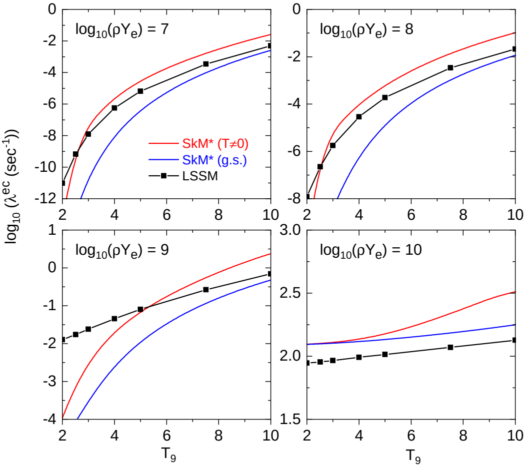


The arrows indicate the zero-temperature EC threshold:

$Q = M(^{56}\mathrm{Mn}) - M(^{56}\mathrm{Fe}) = 4.2$ MeV.

A. A. Dzhioev et al, PRC100 (2019) 0258101

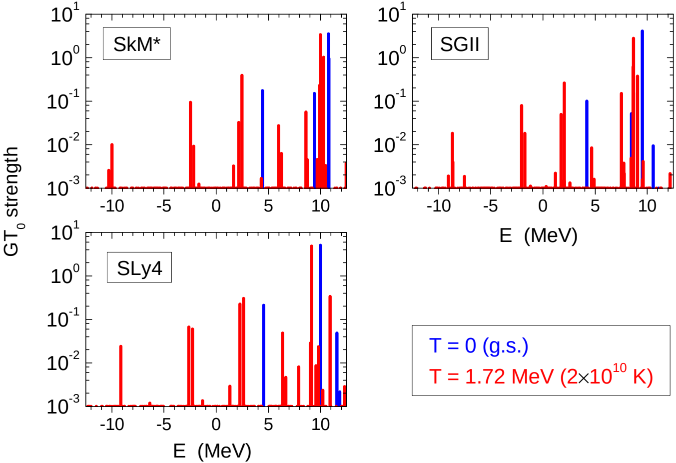
Electron capture rates for ^{56}Fe at $T \neq 0$



$T_9 = T/10^9$ Kelvin ($1T_9 \approx 0.086$ MeV), ρY_e is the electron gas density

A. A. Dzhioev et al, PRC100 (2019) 0258101

GT_0 distribution in ^{56}Fe at $T \neq 0$

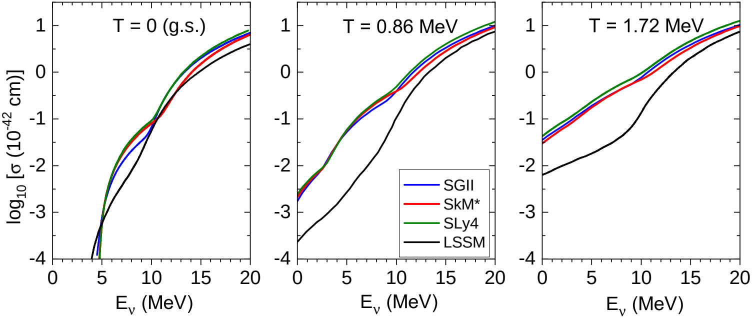


Detailed balance: $S_{\mathrm{GT}_0}(-E,T) = S_{\mathrm{GT}_0}(E,T)\exp\left(-\frac{E}{T}\right)$

A. A. Dzhioev et al, PRC94 (2016) 015805

Neutrino inelastic scattering cross-sections for ^{56}Fe at $T \neq 0$

$\nu +^{56}\mathrm{Fe} \rightarrow \nu' +^{56}\mathrm{Fe}$



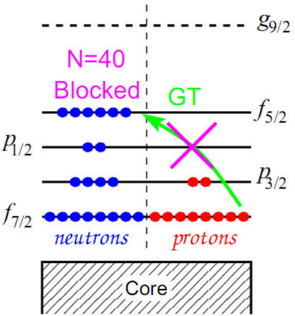
$$\text{TQRPA}: \sigma(E_\nu, T) = \sigma_\uparrow(E_\nu, T) + \sigma_\downarrow(E_\nu, T)$$

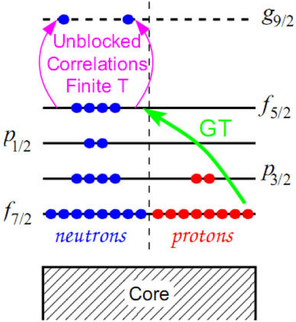
$$\text{LSSM}: \sigma(E_\nu, T) = \sigma_{g.s}(E_\nu) + \sigma_\downarrow(E_\nu, T)$$

A. A. Dzhioev et al, PRC94 (2016) 015805

Electron capture on neutron-rich nuclei

Neutron-rich nuclei ($N > 40, \quad Z < 40$)

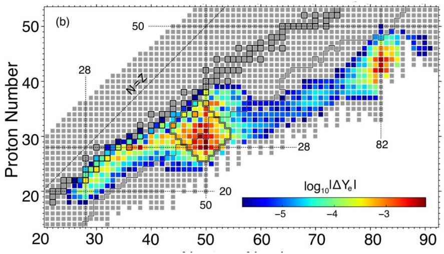




Unblocking mechanisms for GT$_+$ transitions: configurational mixing and thermal effects.

Electron capture by $N = 50$ neutron-rich nuclei

Ch. Sullivan et al, The Astrophysical Journal 816 (2016)



Top 500 electron-capturing nuclei with the largest absolute change to the electron fraction up to neutrino trapping.

R. Titus et al, PRC 100 (2019)

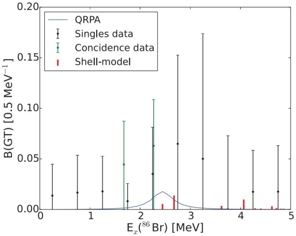


FIG. 5. Gamow-Teller strength distribution extracted from the ^{86}Kr(t, ^{3}He) data and comparison with shell-model and QRPA calculations, as described in the text.

J.C. Zamora et al, PRC100(R), 2019

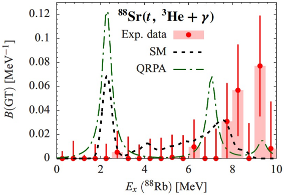
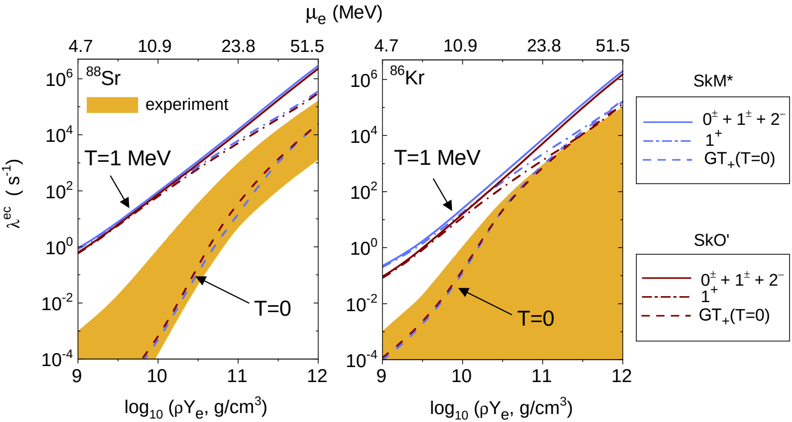


FIG. 2. B(GT) distribution extracted from MDA for $E_x <$ 10 MeV. The error bars denote only the statistical uncertainties.

Electron capture by $N = 50$ neutron-rich nuclei



Electron capture rates at $T = 1\,\text{MeV}$ (10^{10} K=0.86 MeV)

A. A. Dzhioev et al, PRC101 (2020) 025805

Conclusion

1. A thermodynamically consistent statistical approach to calculate cross-sections and rates of weak-interaction reactions with hot nuclei under supernova conditions is developed. The approach is based on the superoperator formalism.
2. An analysis of the temperature dependence of the GT strength function in ^{56}Fe and in a number of neutron-rich nuclei with $N \approx 50$ shows that the refusal to use the Brink-Axel hypothesis as well as the consistent implementation of the detailed balance principle noticeably increase the contribution of thermally excited nuclear states to the rates and cross sections of weak processes in comparison with the shell-model calculations. In particular, the enhanced contribution of de-excitation processes accelerates the temperature-induced growth of low-energy cross sections and rates.
3. It is found that temperature-induced weakening of pairing correlations drastically increases the temperature dependence of the energy and strength of unblocked Gamow-Teller $p \to n$ transitions in neutron-rich nuclei. It was shown that thermal effects are the main unblocking mechanism of low-energy Gamow-Teller transitions. For this reason the process of electron capture by nuclei in stellar matter is not stopped at neutron-rich nuclei with N=50.