

India-JINR workshop on elementary particle and  
nuclear physics, and condensed matter research  
Dubna, October 16-19, 2023

## **A new mechanism of incomplete fusion of nuclei**

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Dubna, October 18, 2023

# Content

- Introduction
- New mechanism of the incomplete fusion as a channel of the quasifission mechanism.
- Role of the angular momentum distribution in the incomplete fusion mechanism.
- Excitation energy of dinuclear system.
- Description of the measured data obtained in incomplete fusion reactions.

# The motivation and aim of the talk.

- The authors of the recent experiments concluded that “none of the theoretical models is able to explain satisfactorily the incomplete fusion reaction dynamics at lower energies below 10 MeV/nucleon” .

[ Avinash Agarwal *et al.* PHYSICAL REVIEW C 103, 034602 (2021)]

## The aims of this work are

- To calculate the yield of evaporation residues formed in the incomplete fusion reactions in the framework of the dinuclear system model.
- To explore a role of physical quantities determining the cross section of the incomplete fusion processes.

The citation from the report of the reviewer of the paper by A.K. Nasirov et al., Physics Letter B 842, (2023) 137976.

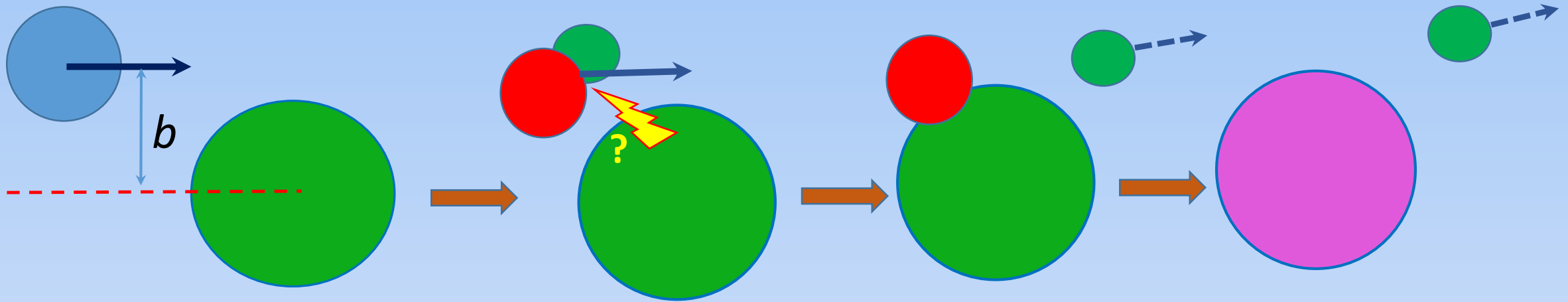
"A new dynamical mechanism of incomplete fusion in heavy-ion collision"

“Understanding such (incomplete fusion) experiments has renewed interest in the formation of superheavy elements.

The authors show that their model description, without additional adjustment, reproduces the experimental data qualitatively. This and the interpretation of this specific exit channel provide an interesting insight into the experiment and might be interesting for future studies.”

This is a result of the fruitful cooperation of the nuclear physicists of India, Russia and Uzbekistan in JINR.

# The popular mechanism of incomplete fusion in heavy-ion collisions since 1961 from the first experiment:



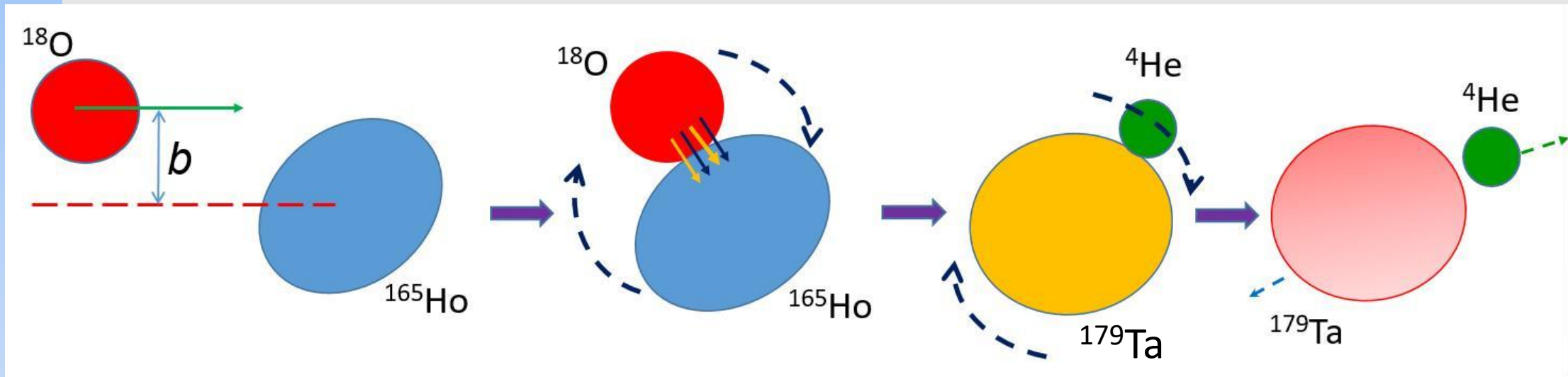
[7] T. Udagawa, T. Tamura, Phys. Rev. Lett. 45 (1980) 1311.

[8] J. Wilczyński, K. Siwek-Wilczyńska, J. van Driel, S. Gonggrijp, D.C.J.M. Hageman, R.V.F. Janssens, J. Łukasiak, R.H. Siemssen, Phys. Rev. Lett. 45 (1980) 606.

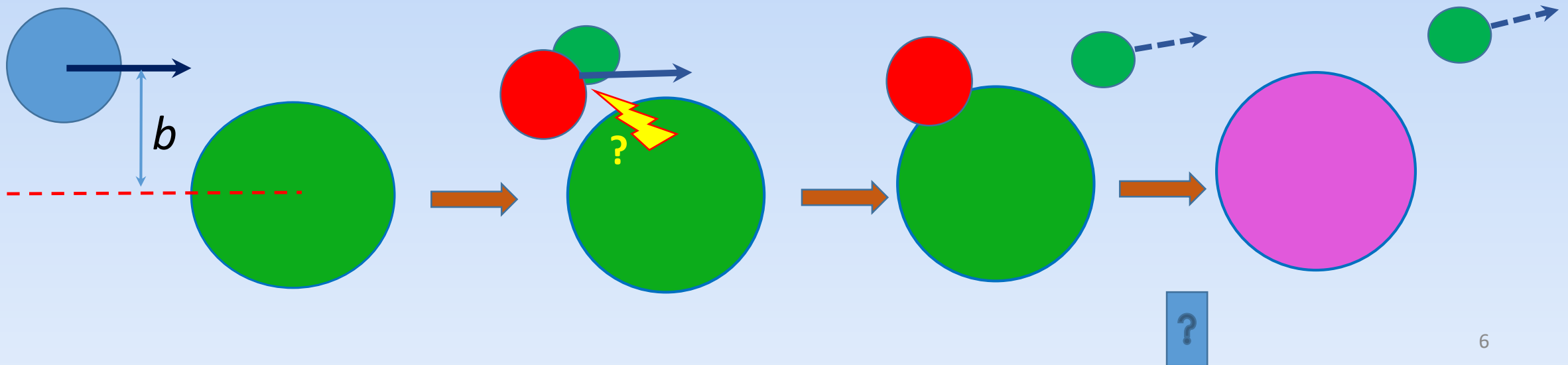
[9] J. Wilczynski, K. Siwek-Wilczynska, J. Van Driel, S. Gonggrijp, D. Hageman, R. Janssens, J. Lukasiak, R. Siemssen, S. Van Der Werf, Nucl. Phys. A 373 (1982) 109.

[10] R. Van den Bossche, A. Diaz-Torres, Phys. Rev. C 100 (2019) 044604.

# A new mechanism of incomplete fusion in heavy-ion collisions

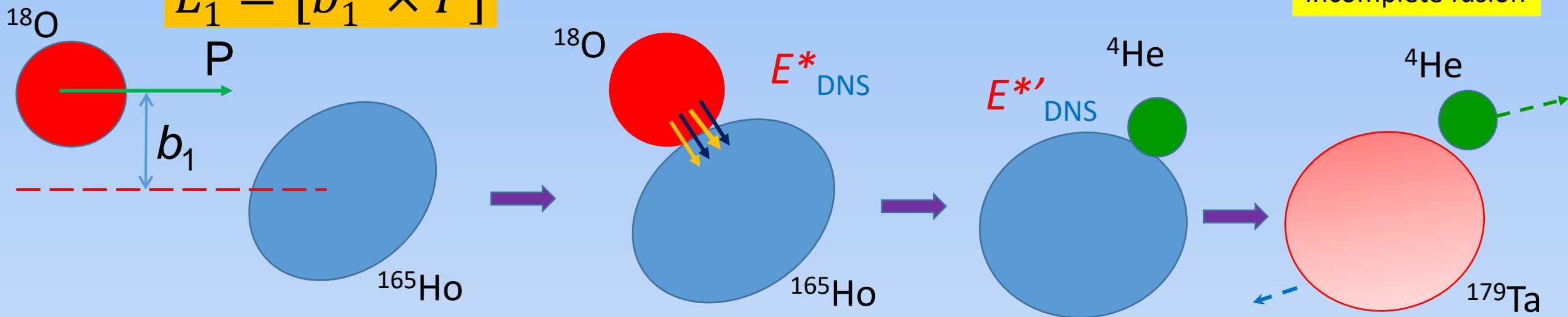


The popular mechanism of incomplete fusion in heavy-ion collisions since 1961 from the first experiment



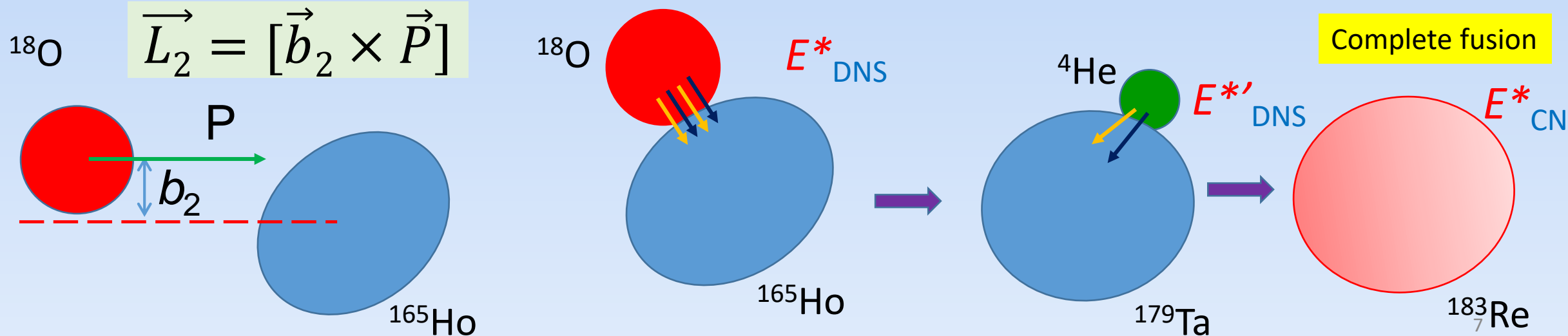
# Role of the orbital angular momentum in the reaction mechanism.

$$\vec{L}_1 = [\vec{b}_1 \times \vec{P}]$$



$$b_1 > b_2$$

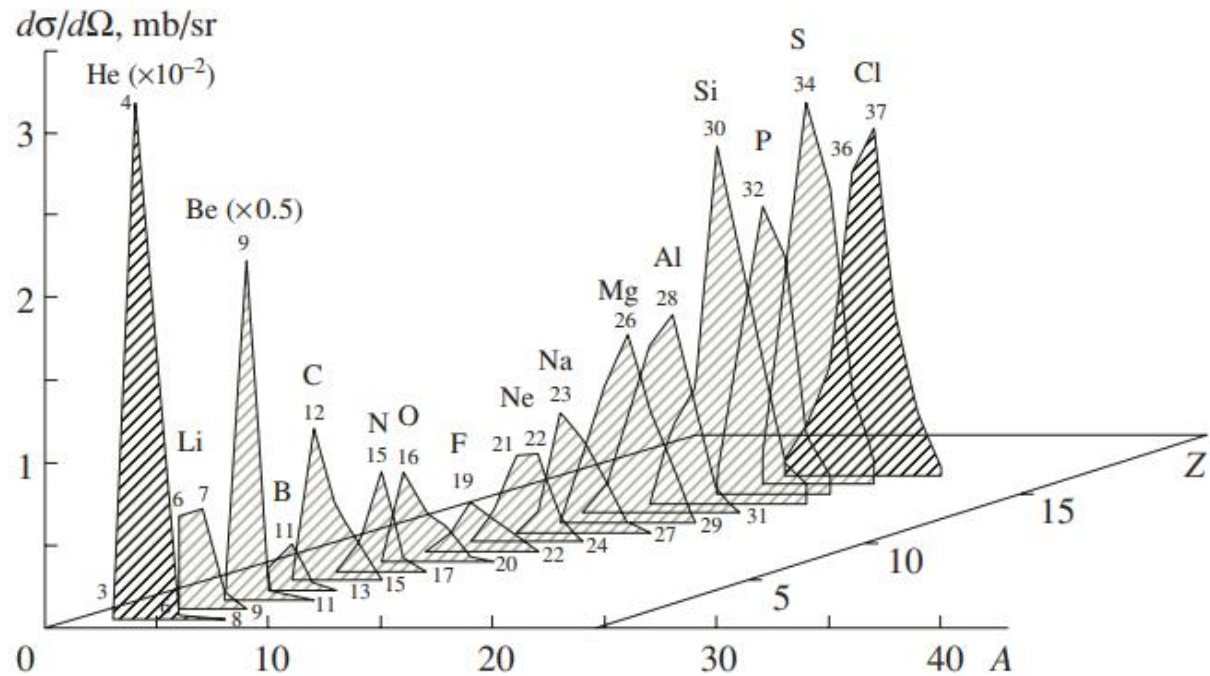
$$\vec{L}_2 = [\vec{b}_2 \times \vec{P}]$$



The role of the centrifugal forces was discussed by Prof. Vadim Volkov in his paper **Centrifugal fragmentation of a dinuclear system in the process of its evolution toward a compound nucleus** [V. V. Volkov](#) *Physics of Atomic Nuclei* **volume 70**, pages 2046–2053 (2007)

2048

VOLKOV

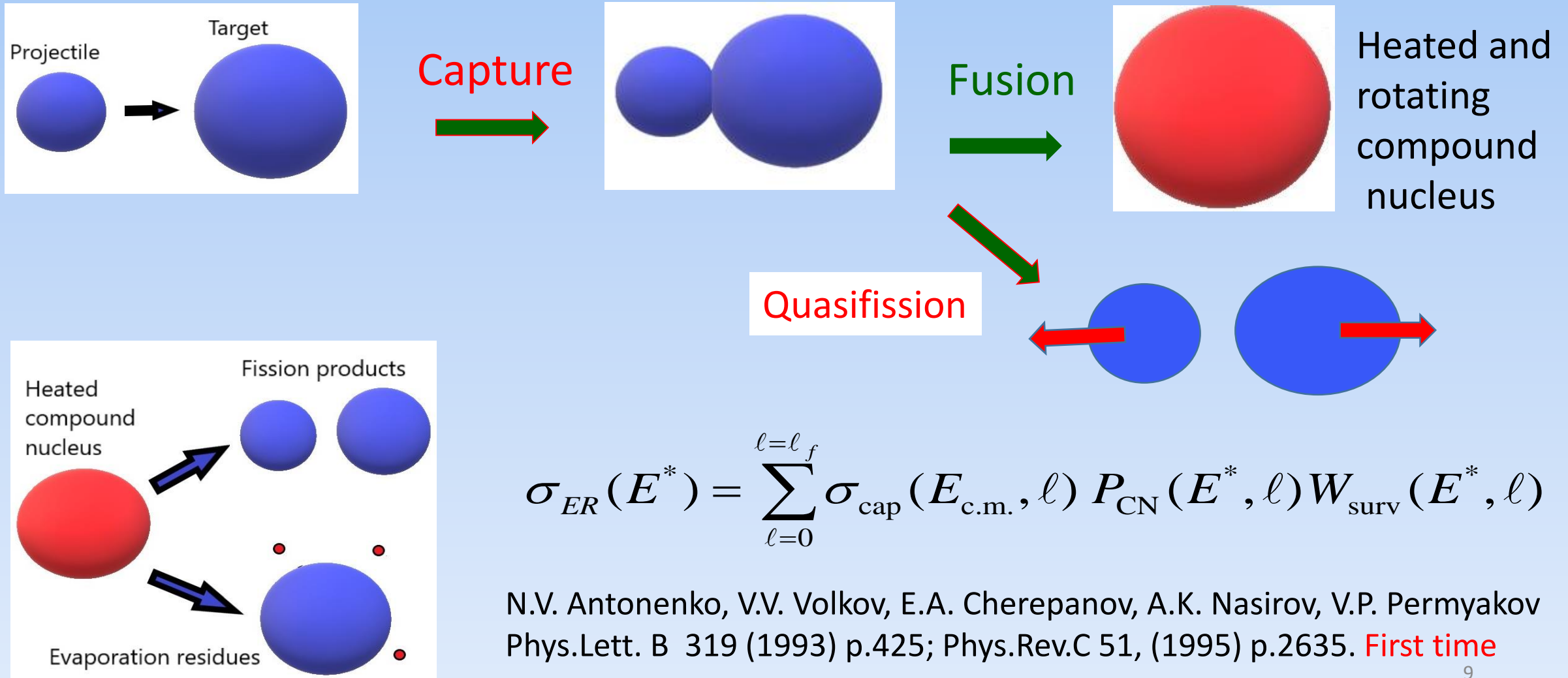


**Fig. 2.** Differential cross sections for the  $^{nat}\text{Ag} + ^{40}\text{Ar}$  (285 MeV) deep-inelastic-transfer reaction leading to the fission of the compound nucleus  $^{140}\text{Ag}$  into isotopes of elements whose charge number  $Z$  ranges from 2 to 17 [10, 11].

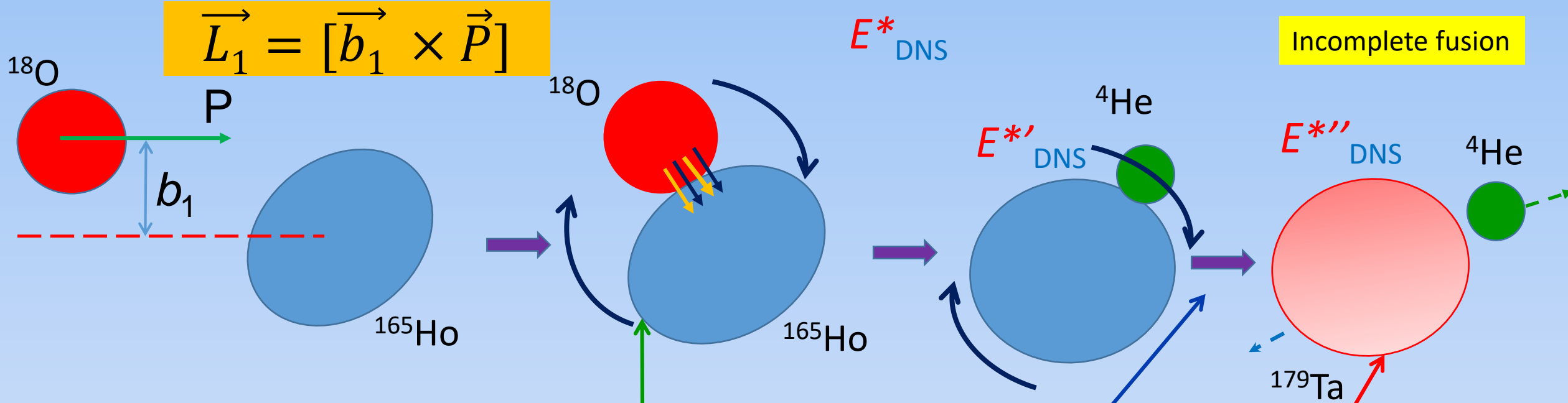




Competition between quasifission and formation of the compound nucleus is the other reason causing decreasing of the probability of synthesis of superheavy elements. The quasifission is dominant in cold fusion processes.



# Role of the orbital angular momentum in the reaction mechanism.



Calculation of the cross section of incomplete fusion

$$\sigma_{ER}^{(ICF)}(E_{\text{Lab}}, Z) = \sum_{l=0}^{l_d} \sigma_{cap}(E_{\text{Lab}}, l) Y_Z(E_{\text{Lab}}, l) W_{sur}(E_{\text{Lab}}, l, Z)$$

Experimental setup to observe evaporation residues which are formed in complete and incomplete fusion reactions.

Experiment was performed in Inter University Accelerator Centre in New Delhi .

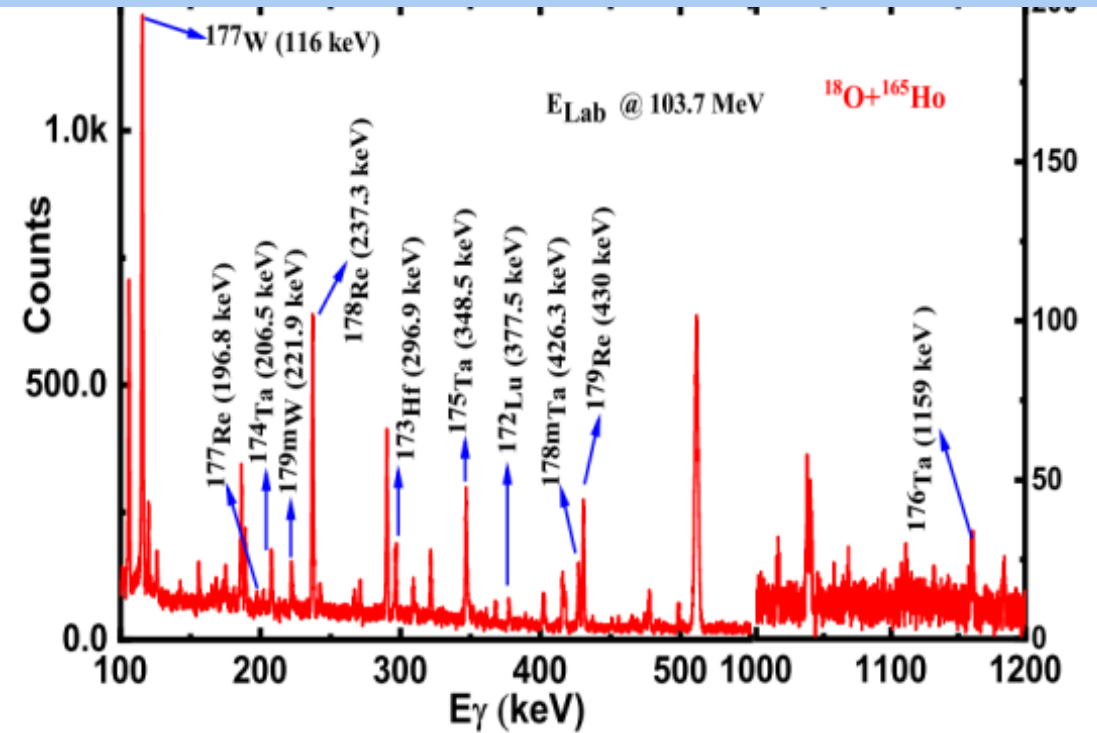
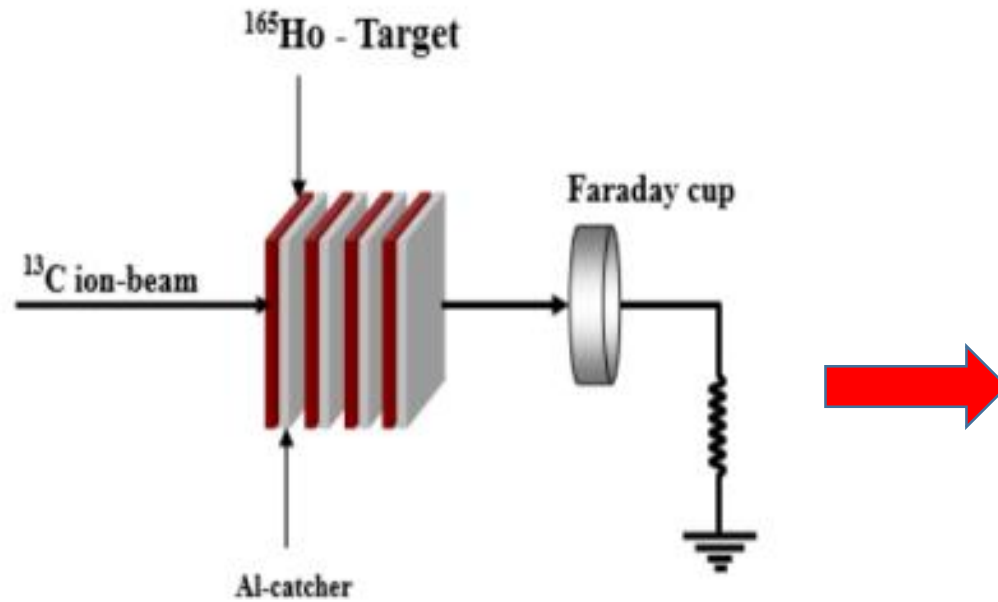
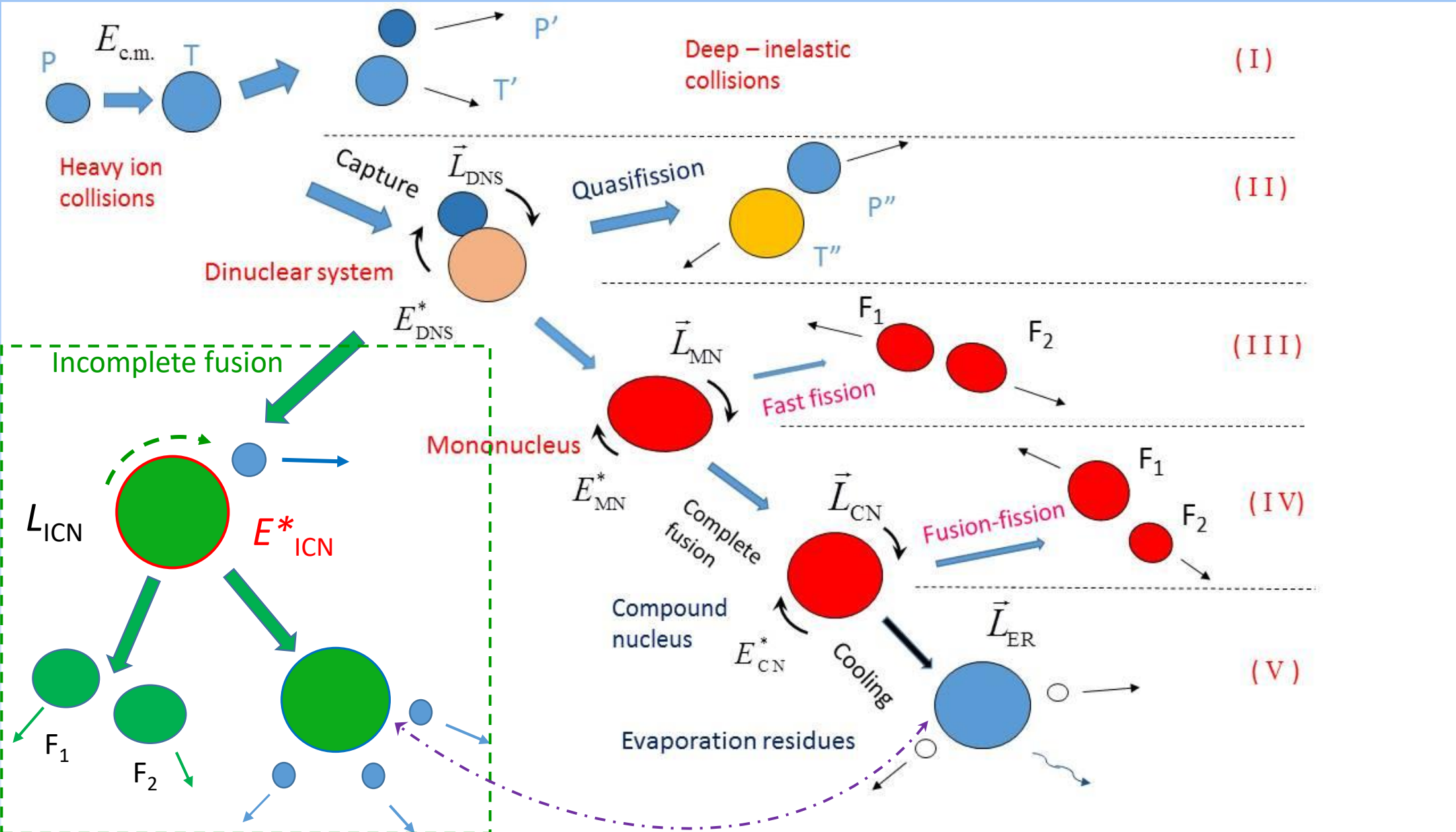


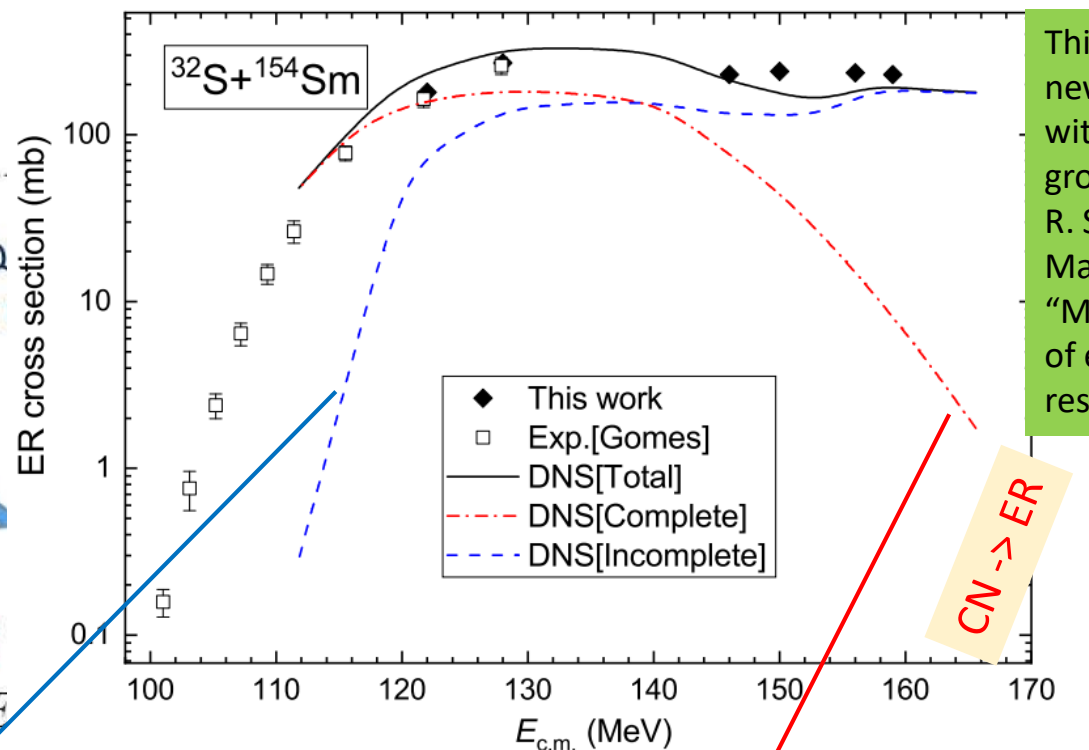
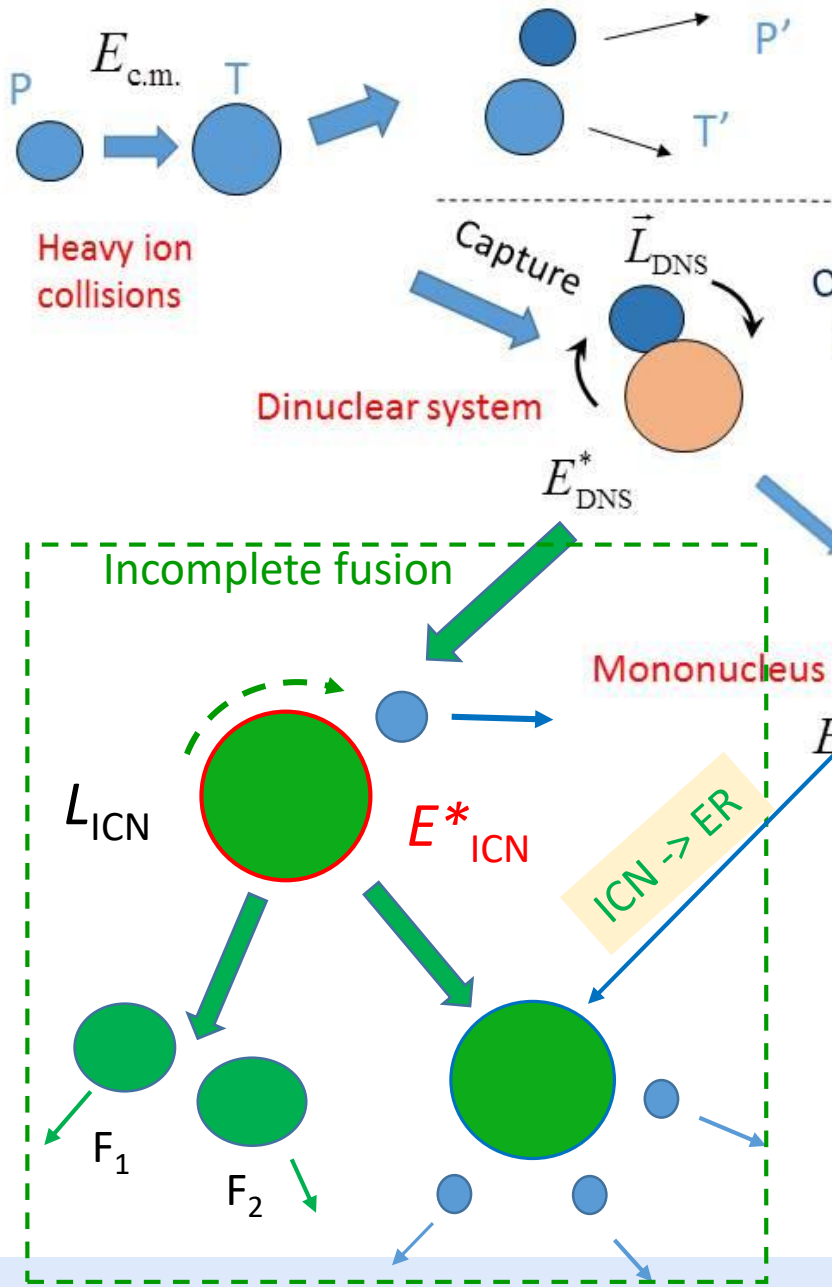
Figure 2.6: (Color online.) A schematic diagram of target-catcher foil arrangement used for the study of excitation function measurements.

A. Agarwal *et al.*, Phys. Rev.C **105**, 034609 (2022).  
D. Singh *et al.*, Phys. Rev.C **79**, 054601 (2009).

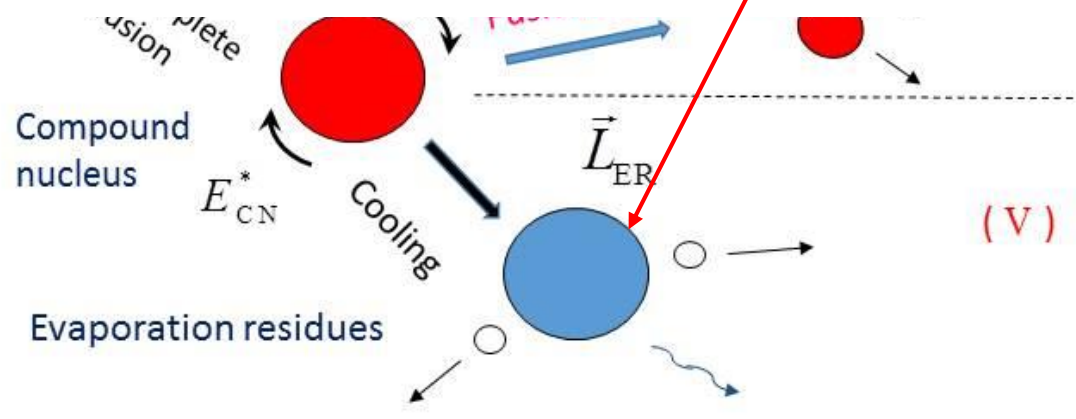
# Main reaction channels of the heavy ion collisions



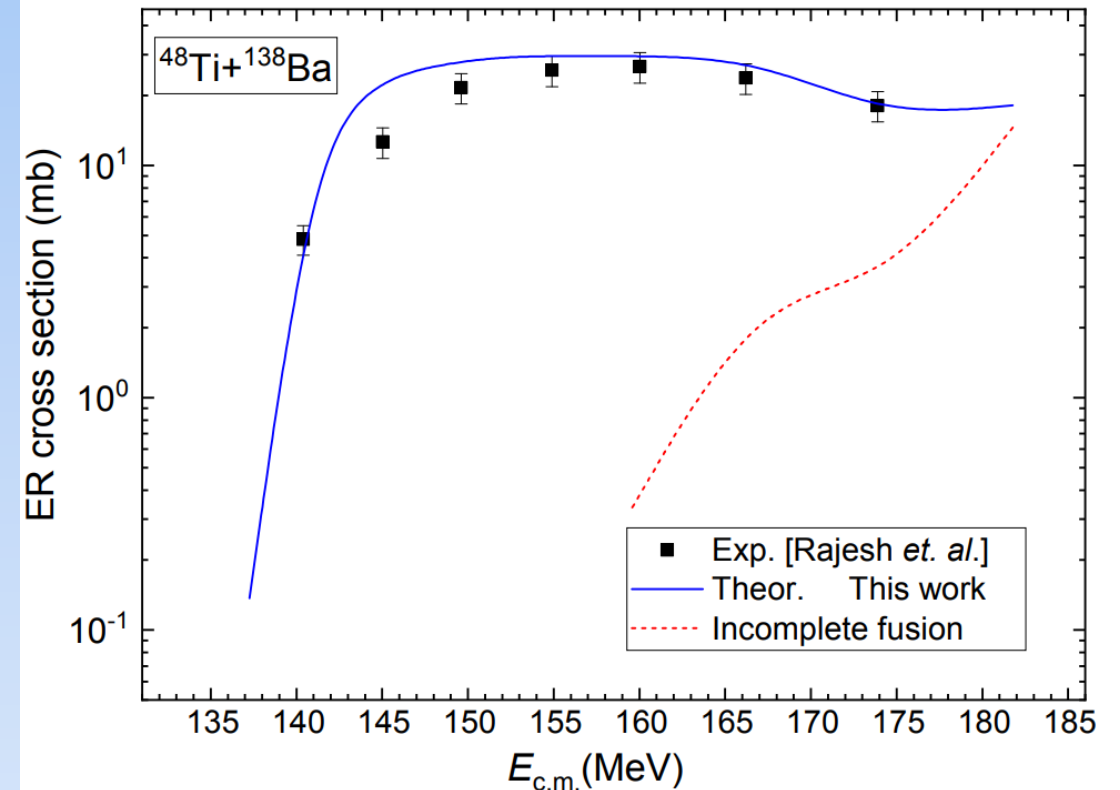
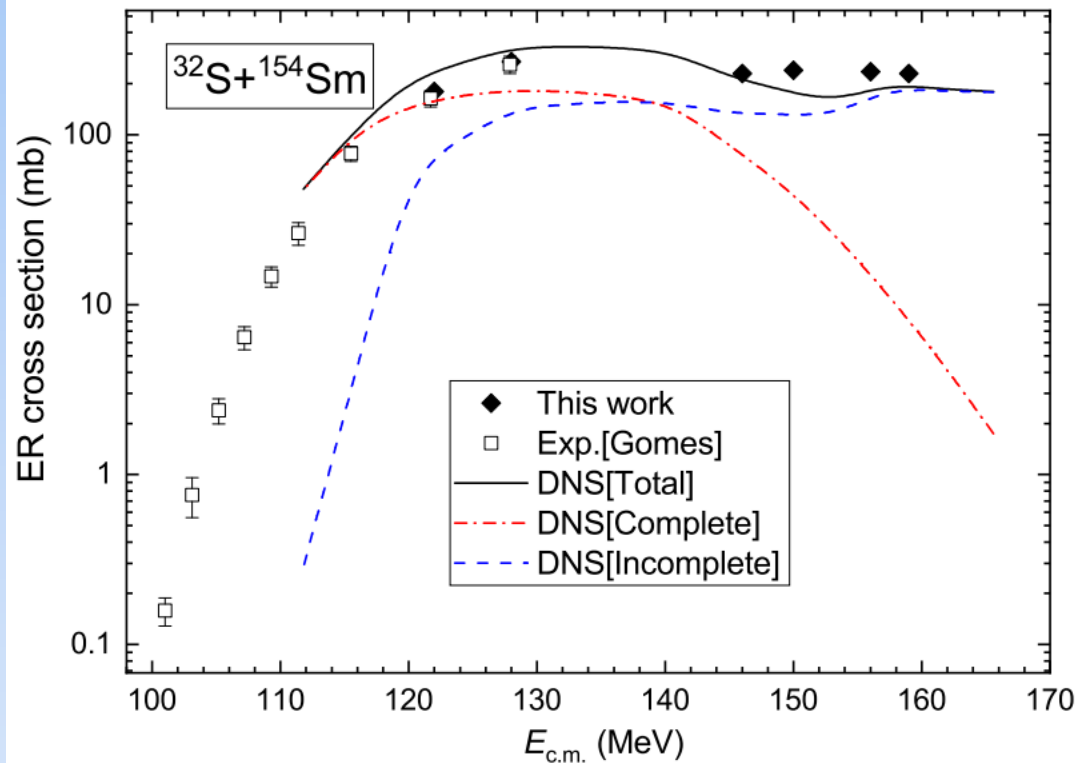
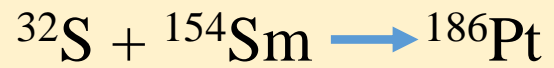
# Two mechanisms of formation of the evaporation residues



This Fig. from new work with Indian group R. Sariyal, I. Mazumdar, ... "Measurements of evaporation residue ..."

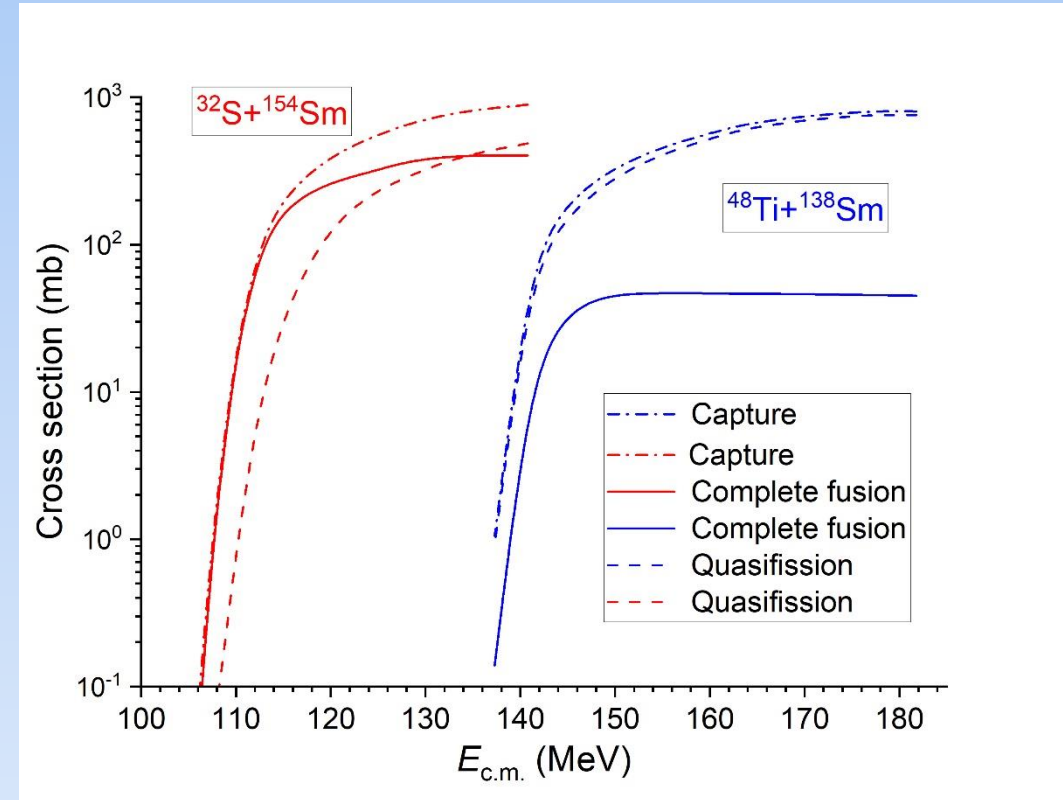
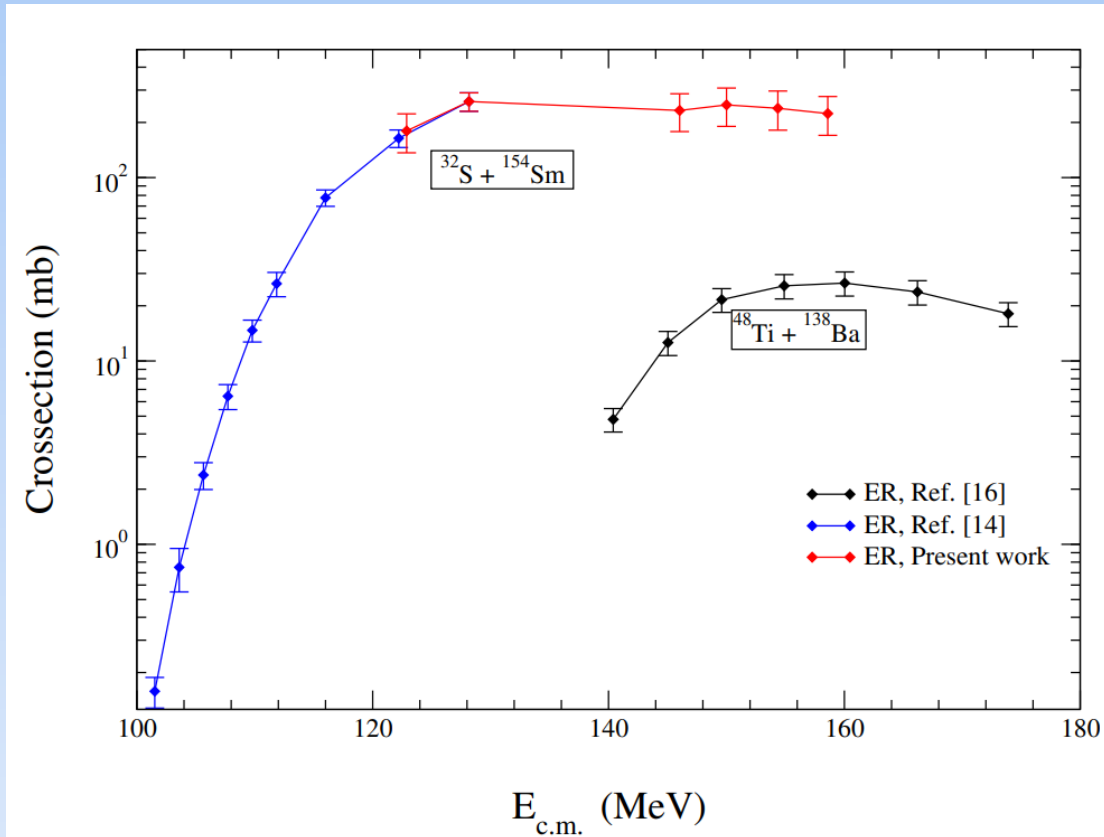


# Dependence of incomplete fusion contribution on the entrance channel

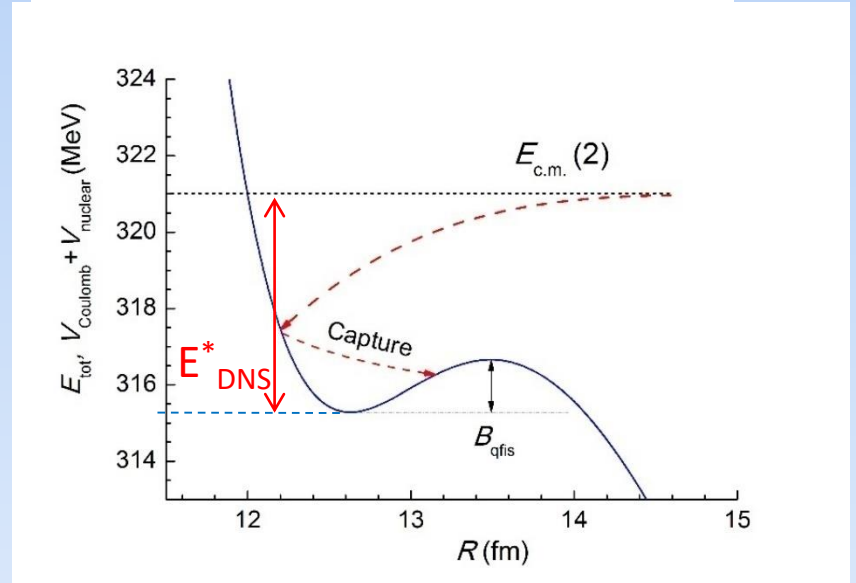
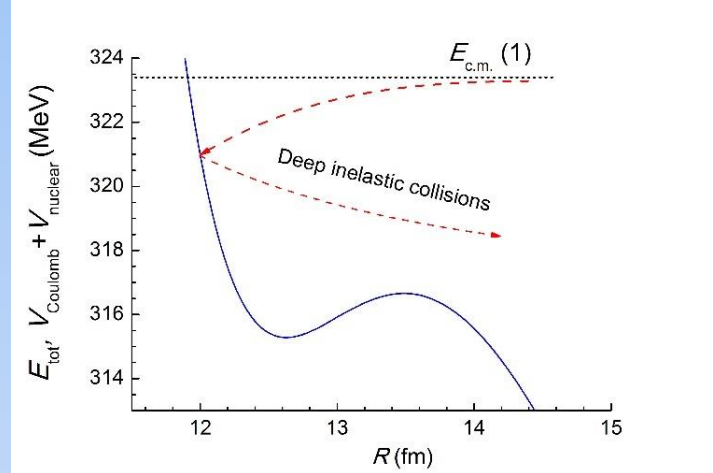
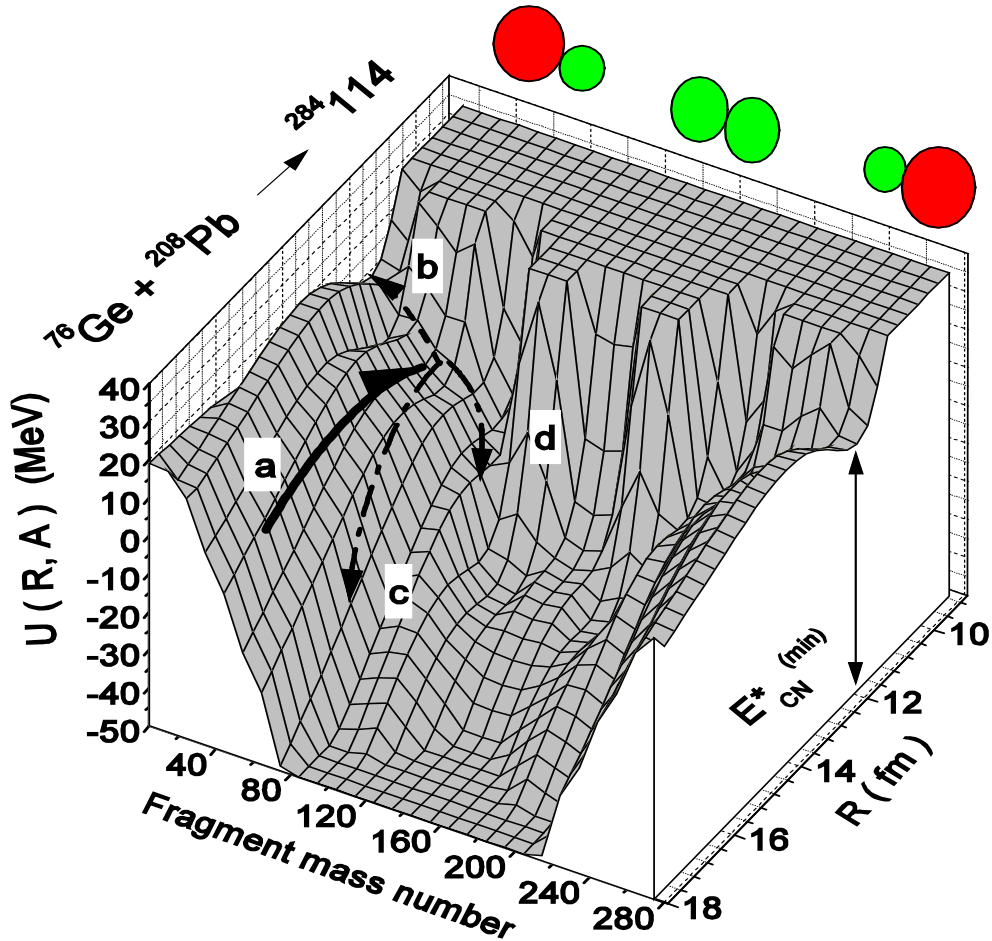


These figures from our new work with Indian group R. Sariyal, I. Mazumdar, ..., S. Nath, ... A. K. Nasirov, B.M. Kayumov  
“Measurements of evaporation residue cross-sections and evaporation residue-gated  $\gamma$ -ray fold distributions for  $^{32}\text{S} + ^{154}\text{Sm}$  system”

# Comparison of the cross sections of the capture, complete fusion, quasifission cross sections



# Competition of different reaction mechanisms in complete fusion of colliding nuclei at synthesis of $^{272}\text{Ds}$



$$U_{\text{tot}} = B_1 + B_2 + V_{\text{int}}(R) - B_{\text{CN}}$$

A. K. Nasirov *et al*,  
 Eur. Phys. Jour. A **55**, (2019) 29.



# Calculation of the competition between complete fusion and quasifission:

$$P_{cn}(E_{DNS}, L)$$

$$P_{CN}(E_{DNS}^*, \ell) = \sum_{Z_{sym}}^{Z_{max}} Y_Z(E_{DNS}^*, \ell) P_{CN}^{(Z)}(E_{DNS}^*, \ell)$$

where

$$P_{CN}^{(Z)}(E_{DNS}^*, \ell) = \frac{\rho(E_{DNS}^*(Z) - B_{fus}^*(Z), \ell)}{\rho(E_{DNS}^*(Z) - B_{fus}^*(Z), \ell) + \rho(E_{DNS}^*(Z) - B_{qf}^*(Z), \ell) + \rho(E_{DNS}^*(Z) - B_{sym}^*(Z), \ell)}$$

$$\begin{aligned} \frac{\partial}{\partial t} Y_Z(E_Z^*, \ell, t) &= \Delta_{Z+1}^{(-)} Y_{Z+1}(E_Z^*, \ell, t) + \Delta_{Z-1}^{(+)} Y_{Z-1}(E_Z^*, \ell, t) \\ &\quad - (\Delta_Z^{(-)} + \Delta_Z^{(+)} + \Lambda_Z^{qf}) Y_Z(E_Z^*, \ell, t) \end{aligned}$$

for  $Z = 2, 3, \dots, Z_{tot} - 2$

Nasirov A.K. et al. *Nuclear Physics A* 759 (2005) 342–369

Fazio G. et al, *Modern Phys. Lett. A* 20 (2005) p.391

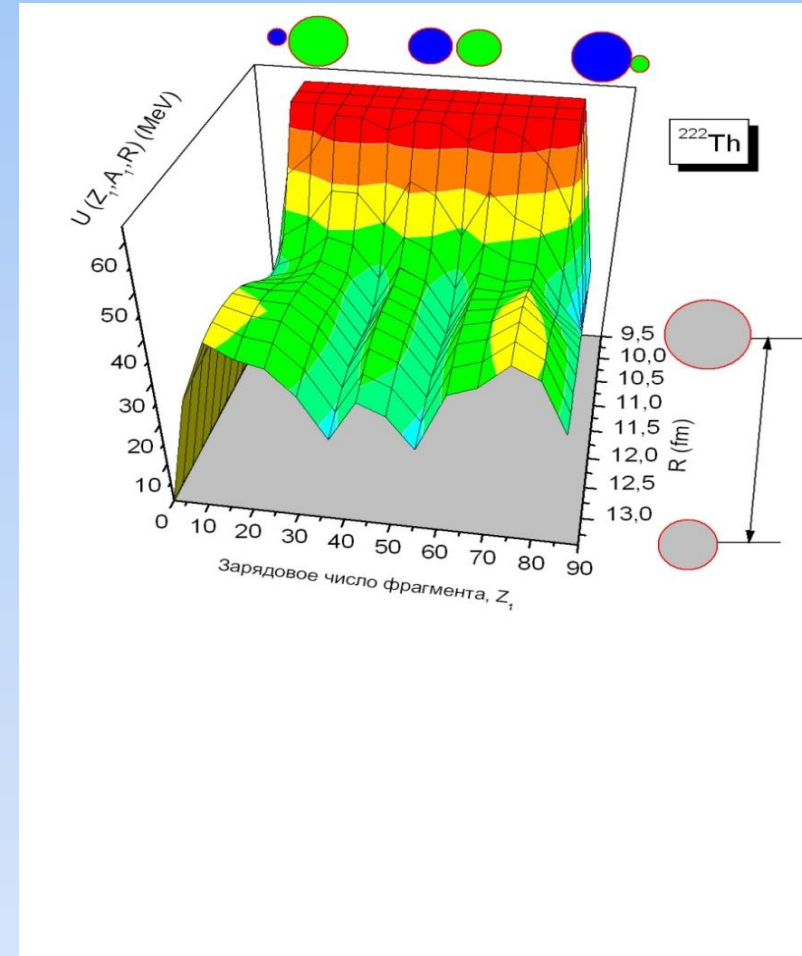
# Calculation of the charge distributions.

$$\frac{\partial}{\partial t} Y_Z(E_Z^*, \ell, t) = \Delta_{Z+1}^{(-)} Y_{Z+1}(E_Z^*, \ell, t) + \Delta_{Z-1}^{(+)} Y_{Z-1}(E_Z^*, \ell, t) - (\Delta_Z^{(-)} + \Delta_Z^{(+)} + \Lambda_Z^{gf}) Y_Z(E_Z^*, \ell, t)$$

for  $Z = 2, 3, \dots, Z_{\text{tot}} - 2$

$$\Delta_Z^{(\pm)} = \frac{1}{\Delta t} \sum_{P_z, T_z} |g_{P_z T_z}|^2 n_{T_z(P_z)}(t) (1 - n_{T_z(P_z)}(t)) W_{P_z T_z}(\Delta t) / (\varepsilon_{P_z} - \varepsilon_{T_z})^2$$

$$W_{P_z T_z}(\Delta t) = (1 + e^{-2(\Gamma_{P_z} + \Gamma_{T_z})\Delta t/\hbar} - 2 \cdot e^{-2(\Gamma_{P_z} + \Gamma_{T_z})\Delta t/\hbar} \cos((\varepsilon_{P_z} - \varepsilon_{T_z})\Delta t/\hbar))$$



Details of calculation of the matrix elements are presented in papers:

G.G. Adamian, et al. Phys. Rev. C **53**, (1996) p.871-879

R.V. Jolos et al., Eur. Phys. J. A **8**, 115–124 (2000)

Nucleon transfer coefficients for evolution of the charge asymmetry of dinuclear system

$$\Delta_Z^{(\pm)} = \frac{1}{\Delta t} \sum_{P_z, T_z} |g_{P_z T_z}|^2 n_{T_z(P_z)}(t) (1 - n_{T_z(P_z)}(t)) W_{P_z T_z}(\Delta t) / (\varepsilon_{P_z} - \varepsilon_{T_z})^2$$

$$W_{P_z T_z}(\Delta t) = (1 + e^{-2(\Gamma_{P_z} + \Gamma_{T_z})\Delta t/\hbar} - 2 \cdot e^{-2(\Gamma_{P_z} + \Gamma_{T_z})\Delta t/\hbar} \cos((\varepsilon_{P_z} - \varepsilon_{T_z})\Delta t/\hbar))$$

$$\frac{1}{\tau_i^{(\alpha)}} = \frac{\sqrt{2}\pi}{32\hbar\varepsilon_{F_K}^{(\alpha)}} \left[ (f_K - g)^2 + \frac{1}{2}(f_K + g)^2 \right] \times \left[ (\pi T_K)^2 + (\tilde{\varepsilon}_i - \lambda_K^{(\alpha)})^2 \right] \left[ 1 + \exp\left(\frac{\lambda_K^{(\alpha)} - \tilde{\varepsilon}_i}{T_K}\right) \right]^{-1}, \quad (\text{A.1})$$

where

$$\Gamma_i = \hbar/\tau_i \quad T_K(t) = 3.46 \sqrt{\frac{E_K^*(t)}{\langle A_K(t) \rangle}} \quad (\text{A.2})$$

Classical equations of the radial and tangential motions with the kinetic coefficients which are calculated microscopically

$$\frac{d(\mu(R)\dot{R})}{dt} + \gamma_R(R)\dot{R}(t) = -\frac{\partial V(R)}{\partial R}$$

$$\frac{dL}{dt} = \gamma_\theta(R)\dot{R}(t) \left[ \dot{\theta}R(t) - \dot{\theta}_1 R_{1eff} - \dot{\theta}_2 R_{2eff} \right]$$

$$L_0 = J_R \dot{\theta}_R + J_1 \dot{\theta}_1 + J_2 \dot{\theta}_2$$

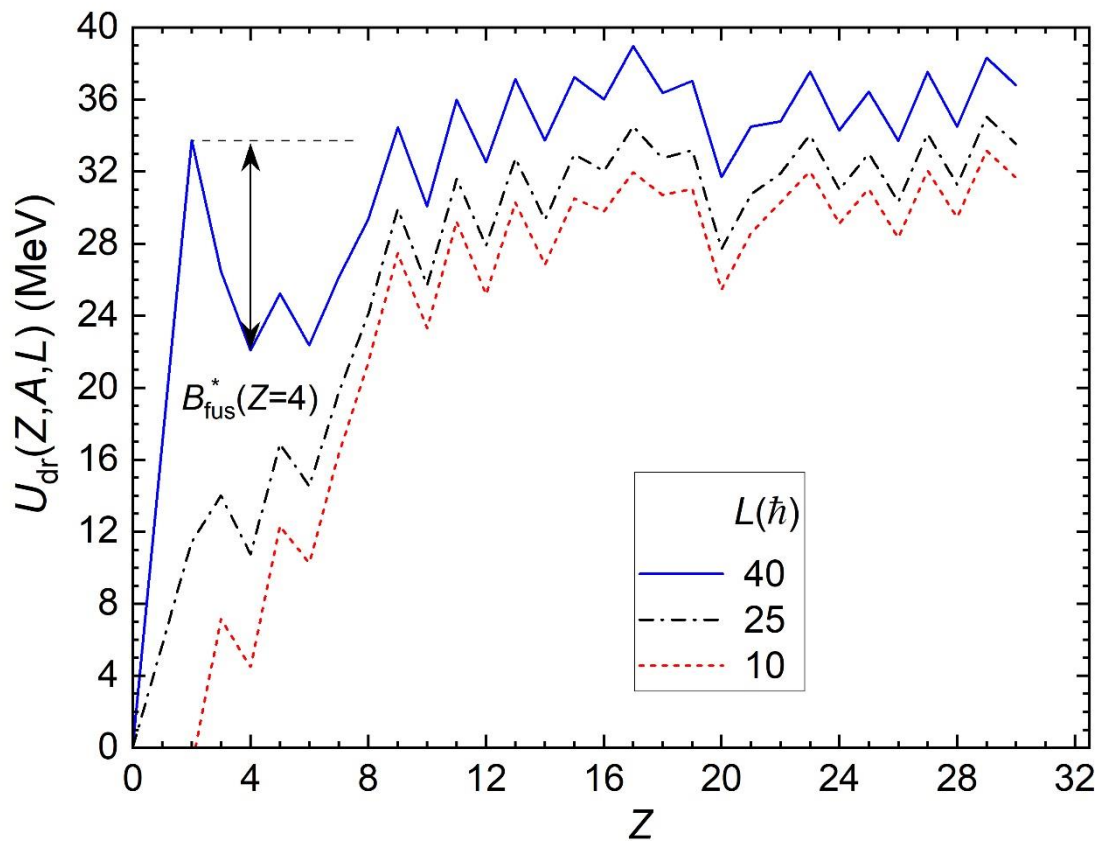
$$E_{rot} = \frac{J_R \dot{\theta}^2}{2} + \frac{J_1 \dot{\theta}_1^2}{2} + \frac{J_2 \dot{\theta}_2^2}{2}$$

A.K. Nasirov *et al.*, Nucl. Phys. A 946 (2016) 89.

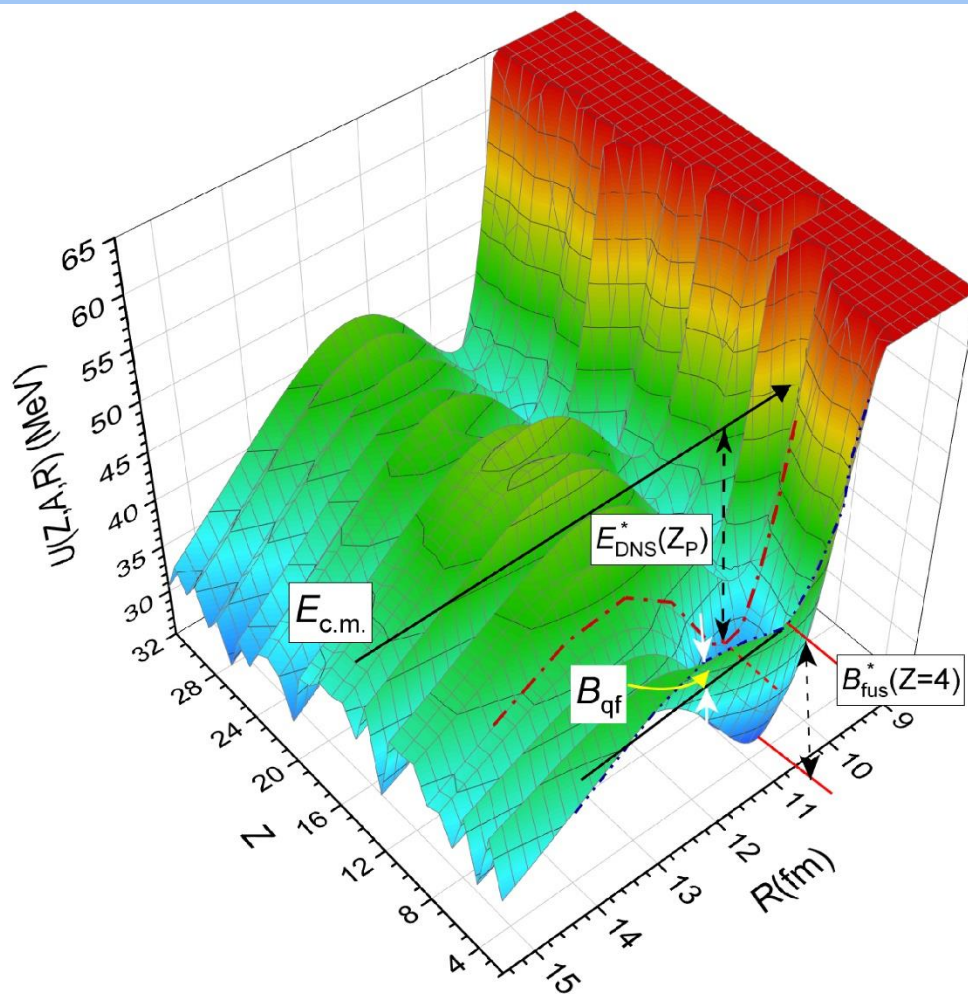
B.M. Kayumov *et al.*, Phys. Rev. C 105 (2022) 014618.

A.K. Nasirov *et al.* Eur. Phys. J. A 49 (2013) 147.

# $^{16}\text{O} + ^{130}\text{Te}$

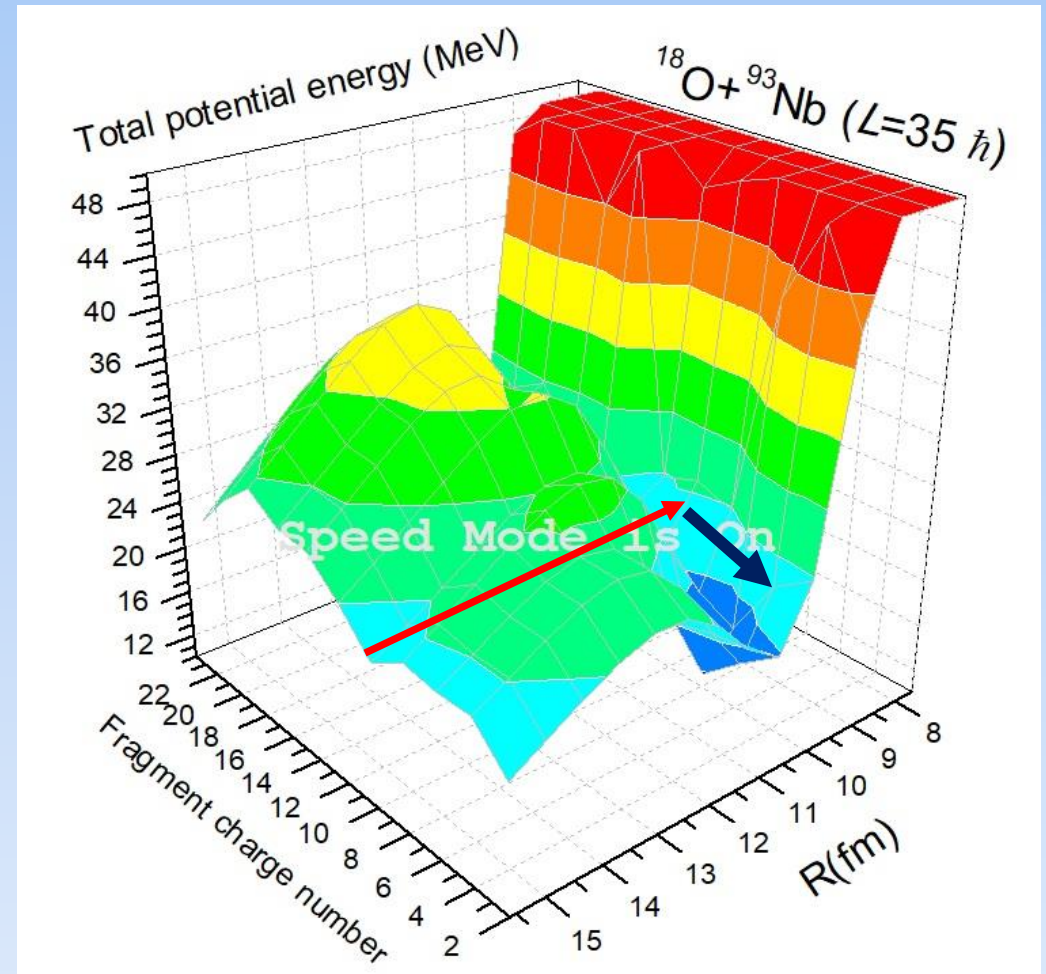
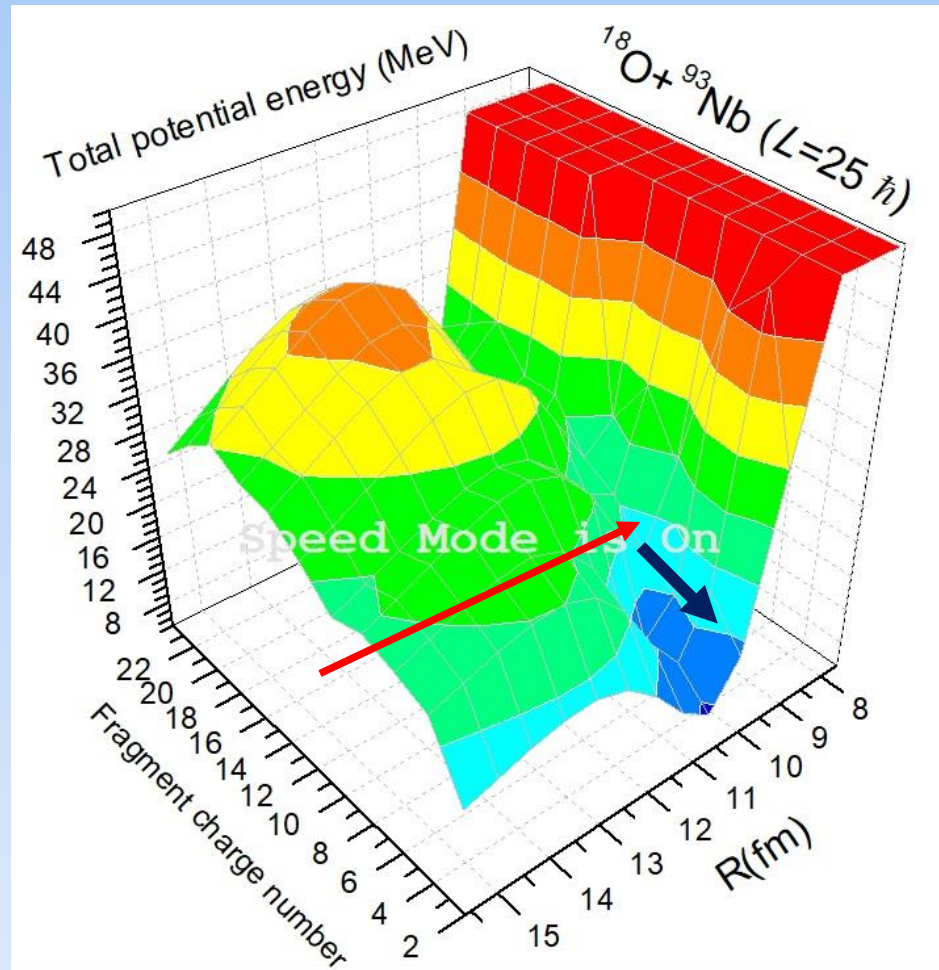


Driving potential

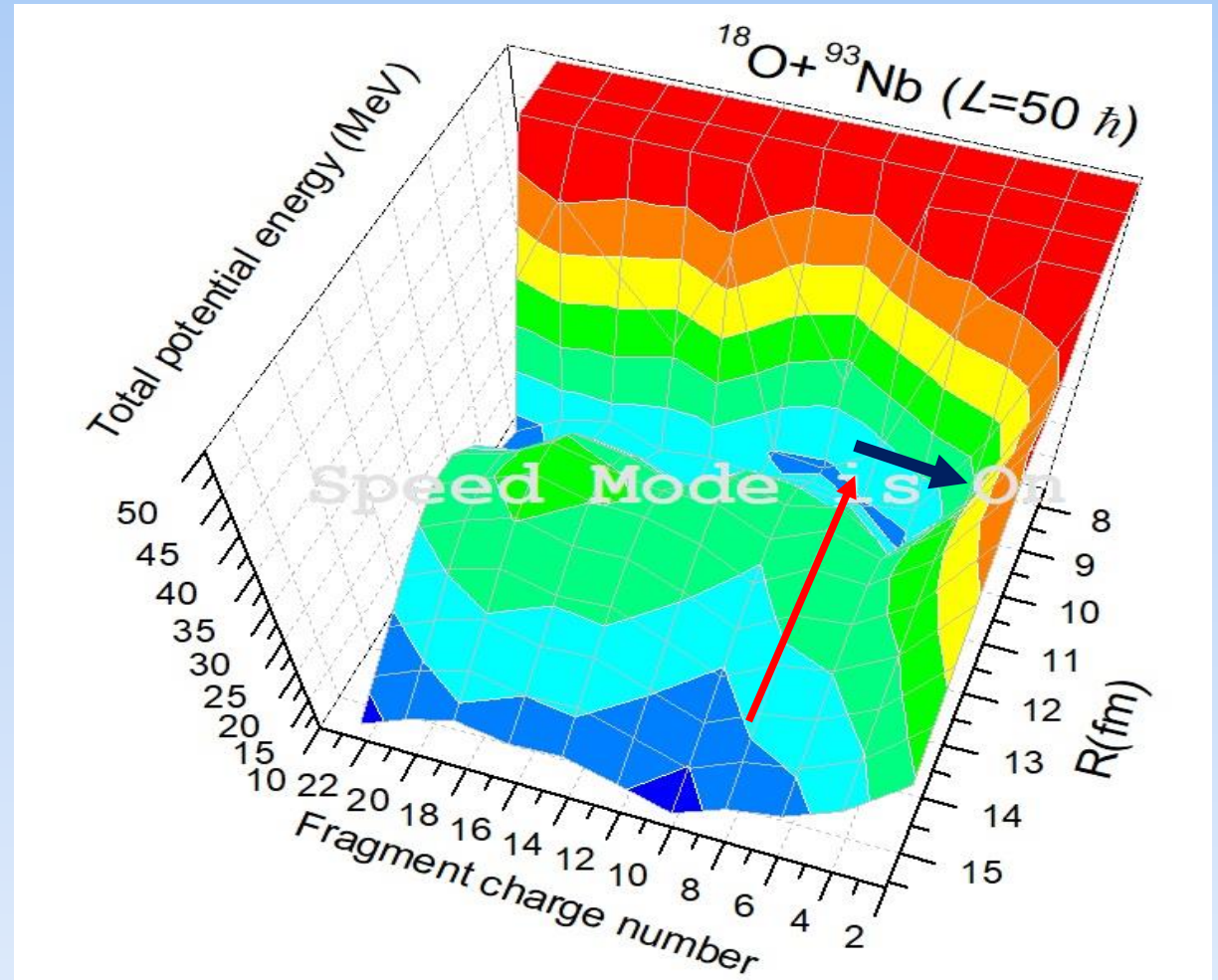
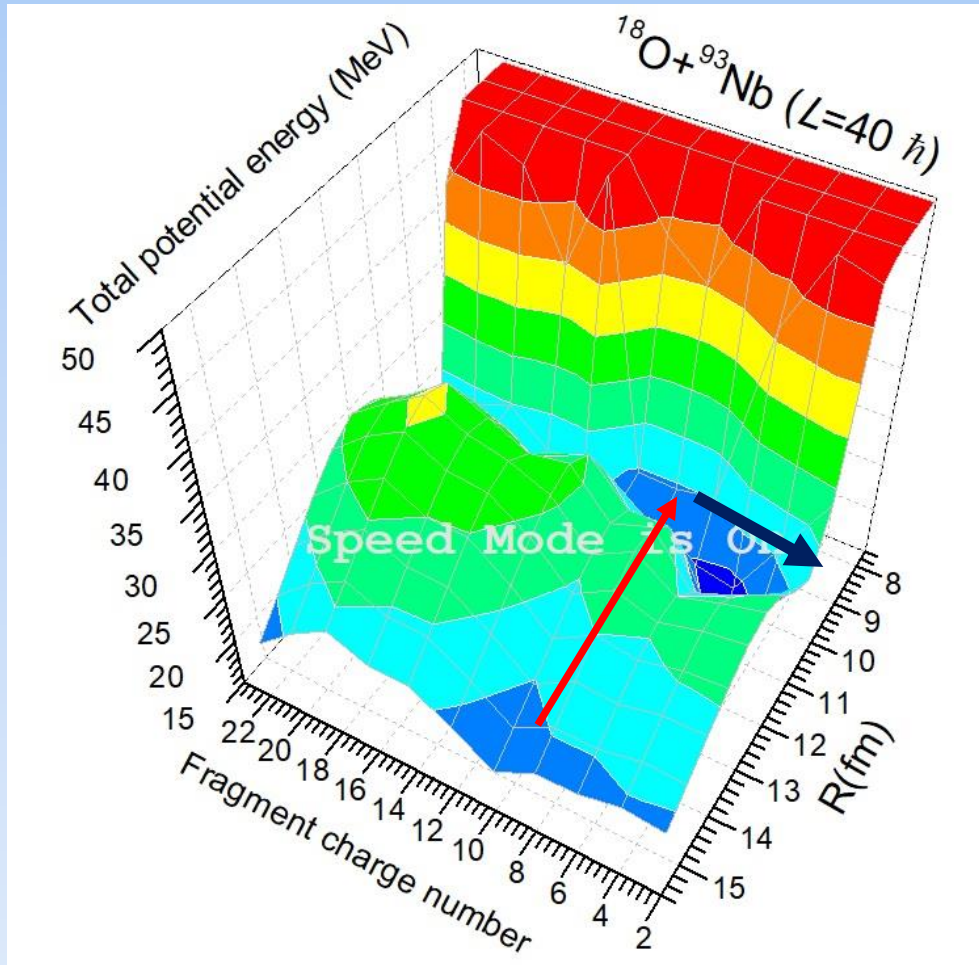


Potential energy surface

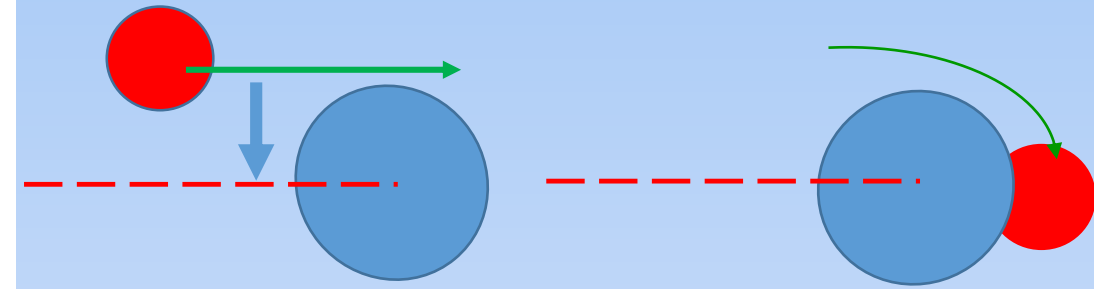
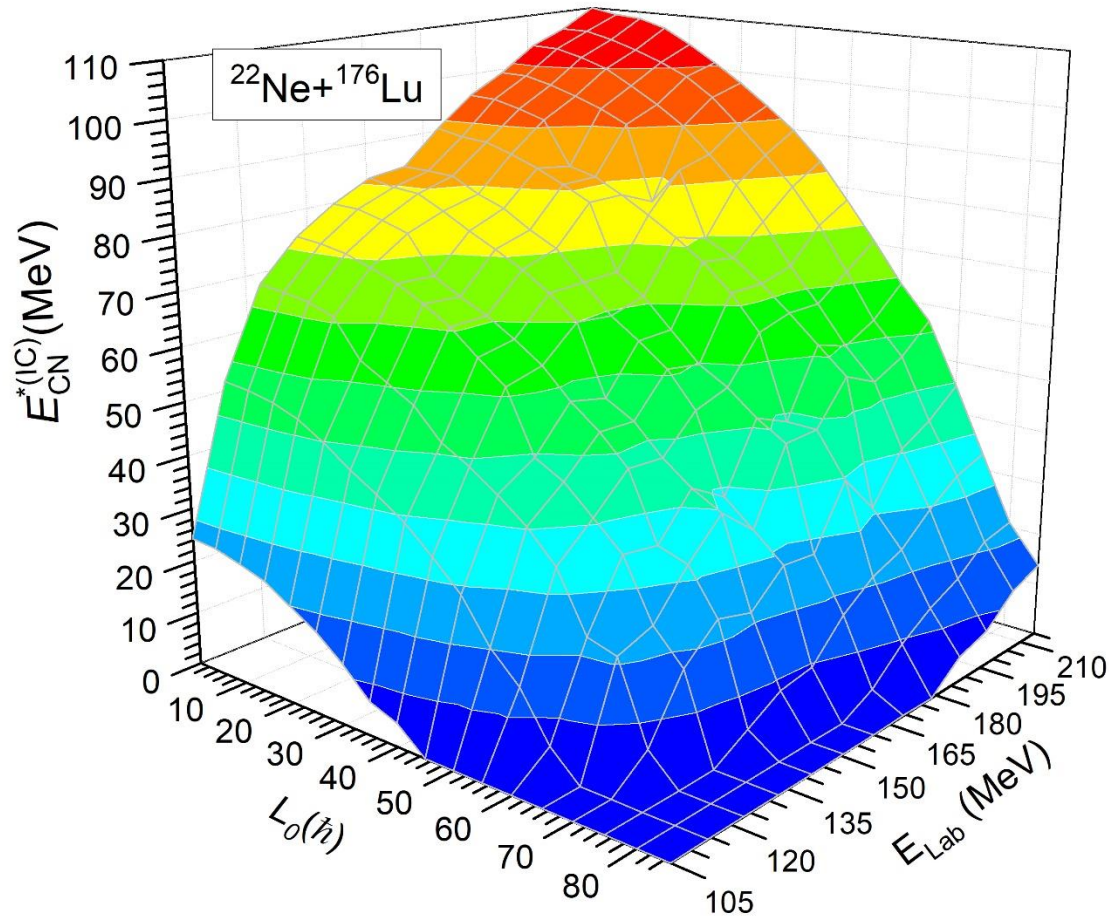
# Appearance of the hindrance to complete fusion



# Appearance of the hindrance to complete fusion



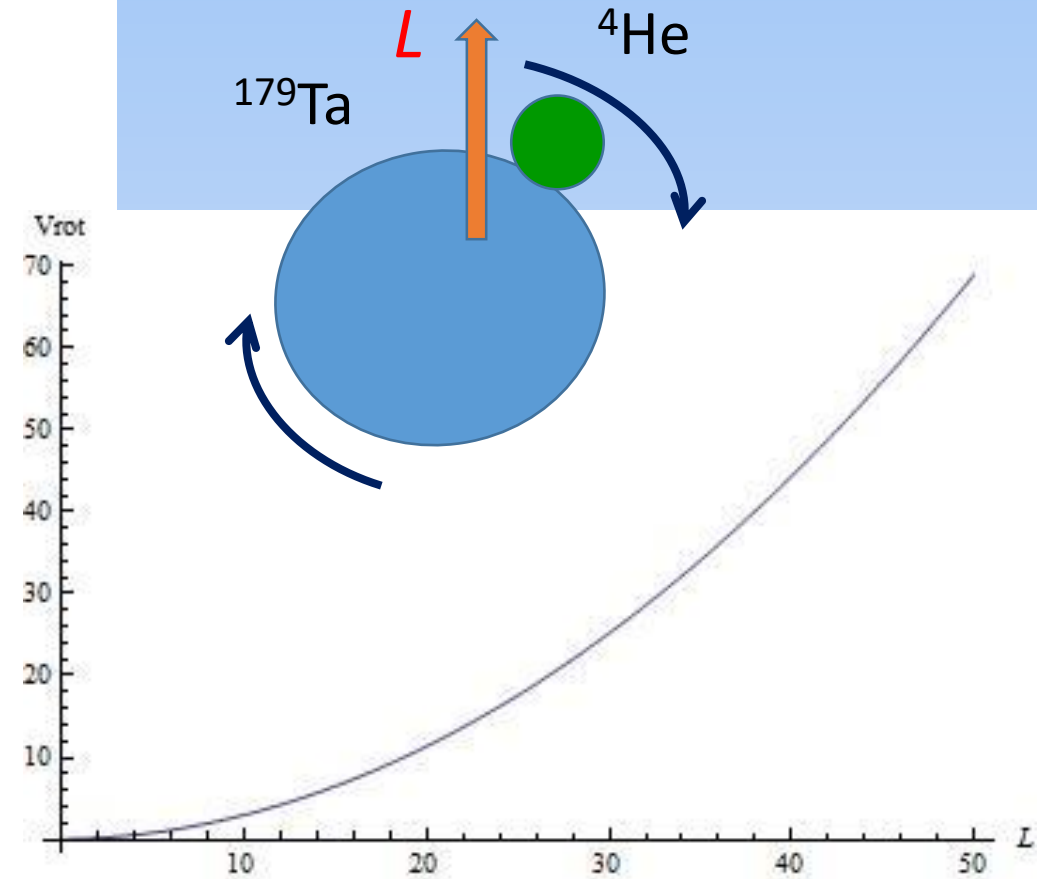
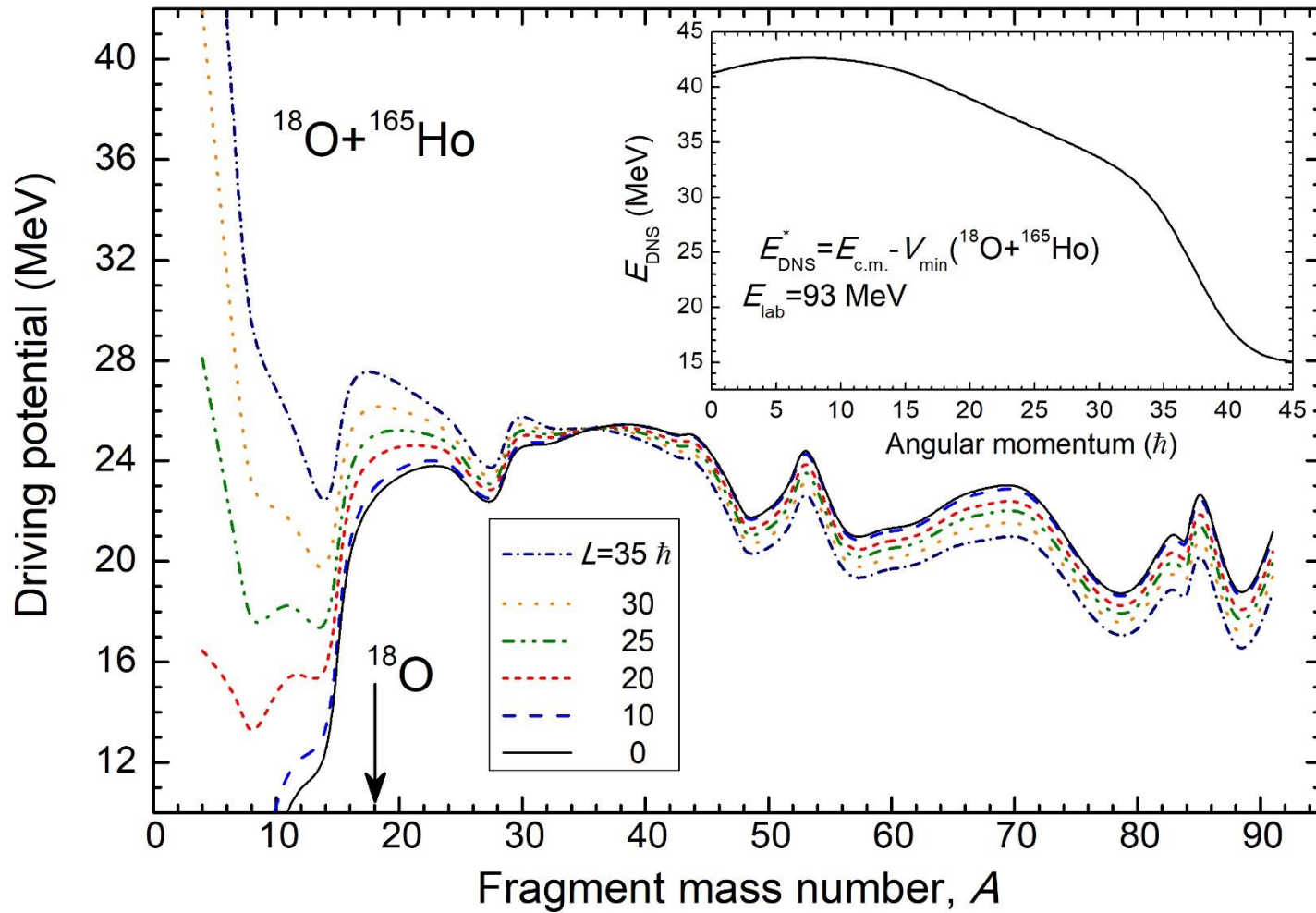
# The excitation energy of dinuclear system as a function of the beam energy and orbital angular momentum



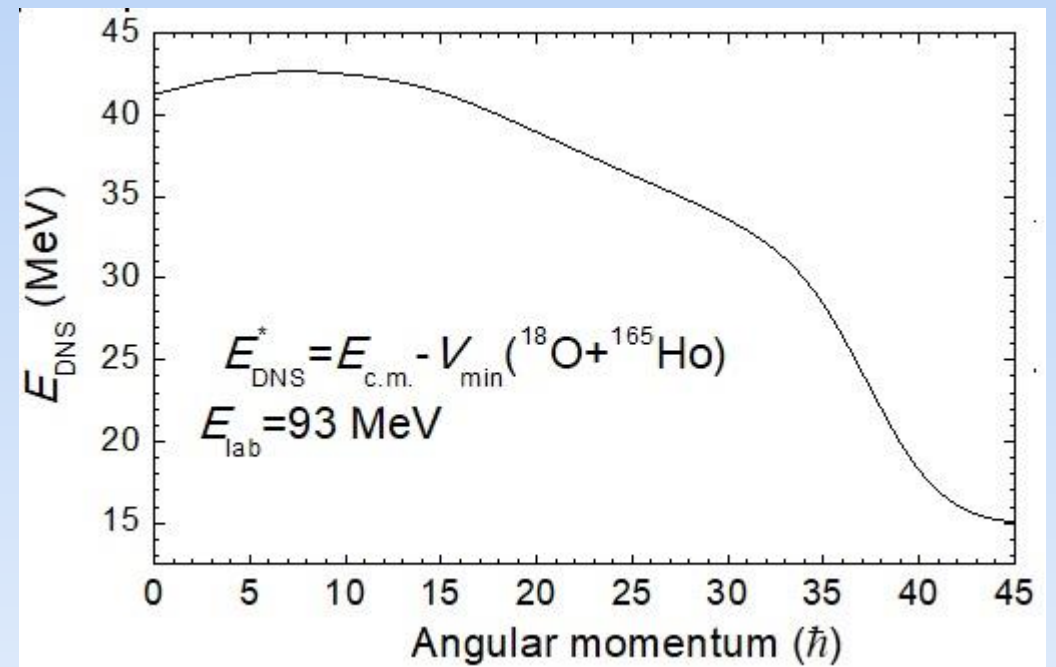
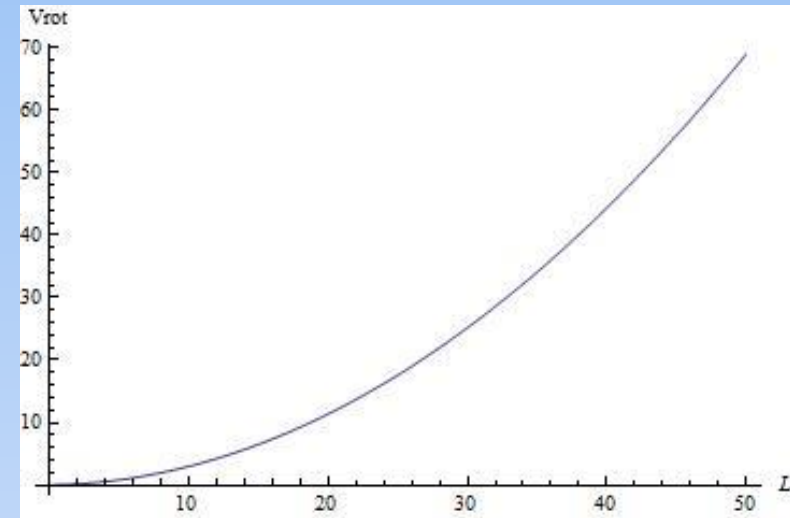
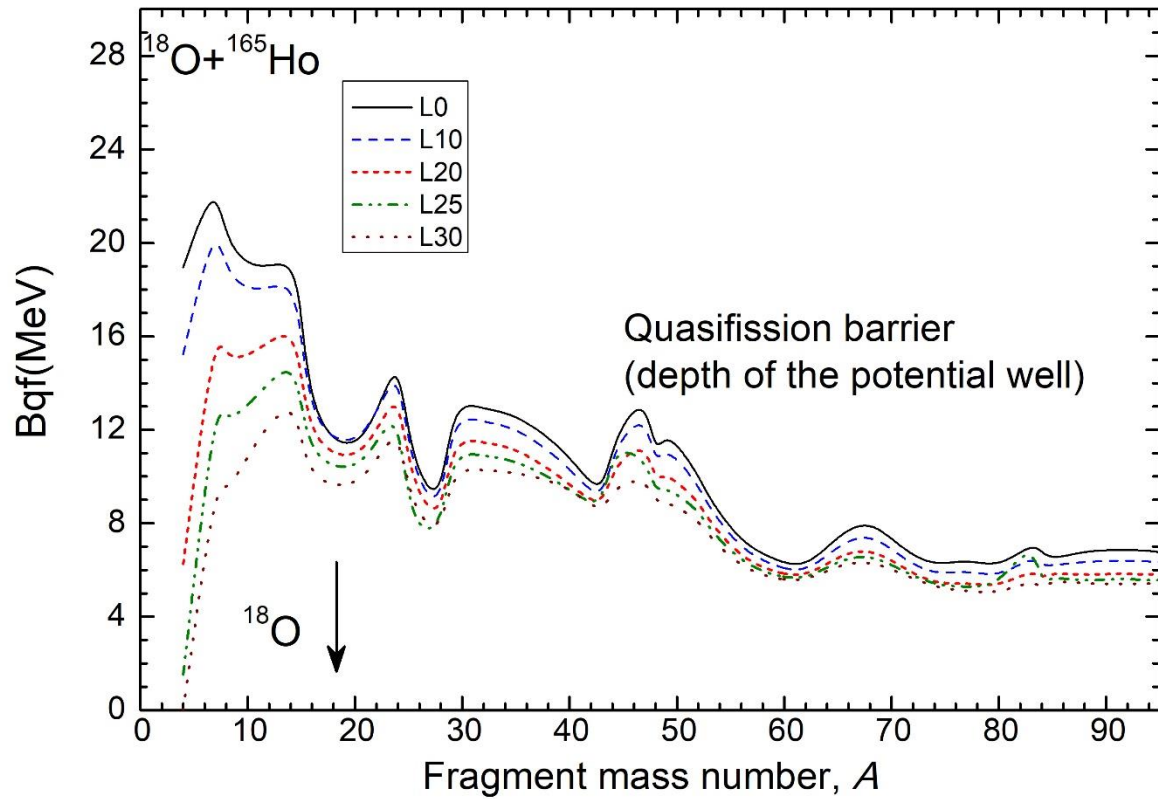
$$L_0 = [b \times P]$$



Driving potential of dinuclear system for the reaction  $^{18}\text{O}+^{165}\text{Ho}$  and dependence of its excitation energy on the orbital angular momentum  $L$ .

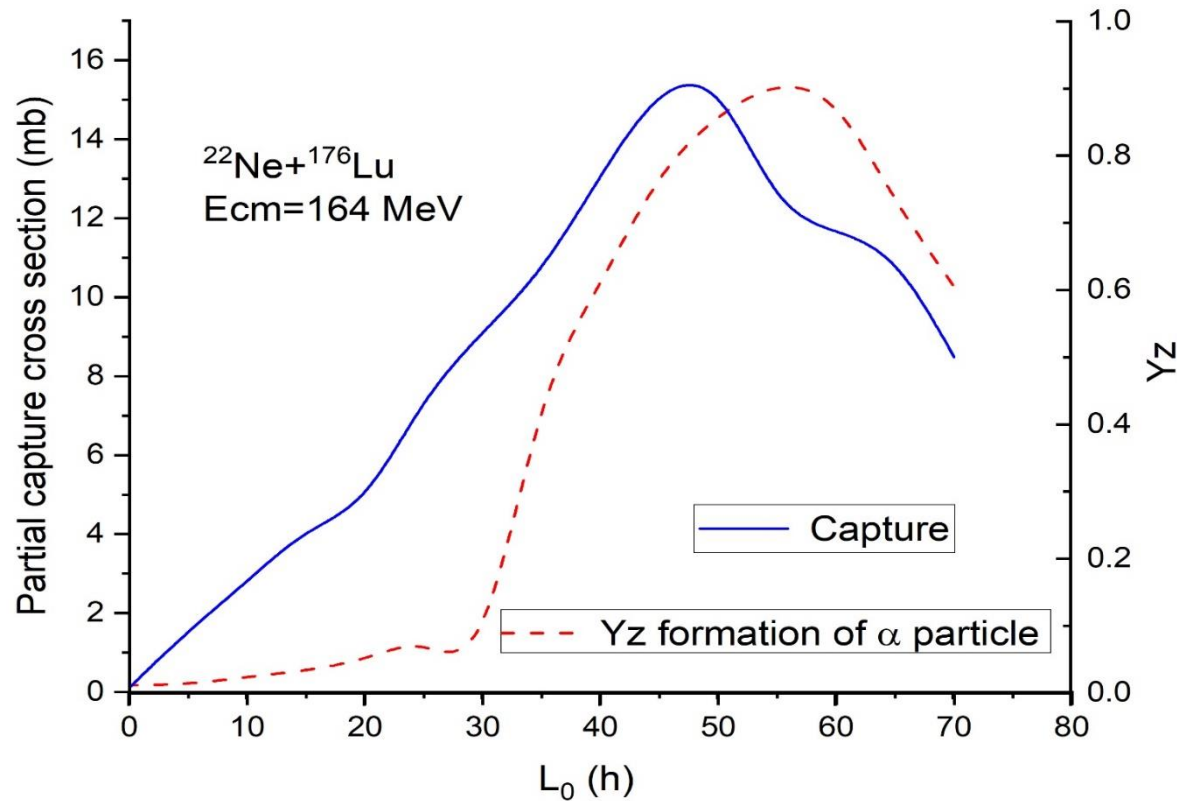


# Rotation energy of dinuclear system consisting of alpha particle and $^{179}\text{Ta}$



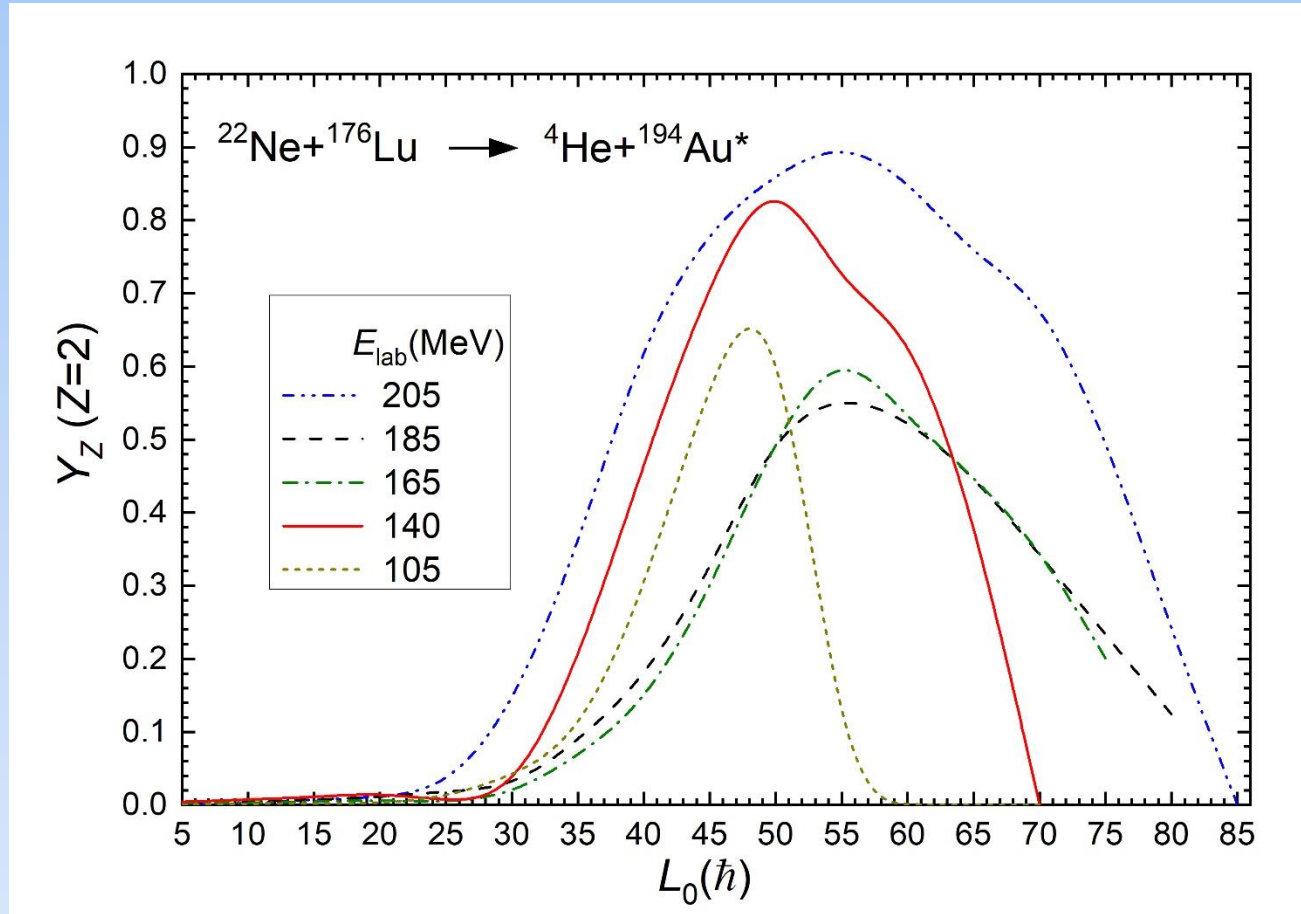
# Calculation of the cross section of incomplete fusion

$$\sigma_{icf}(E_{Lab}, Z) = \sum_{l=0}^{l_d} \sigma_{cap}(E_{Lab}, l) Y_z(E_{Lab}, l) W_{sur}(E_{Lab}, l, Z)$$



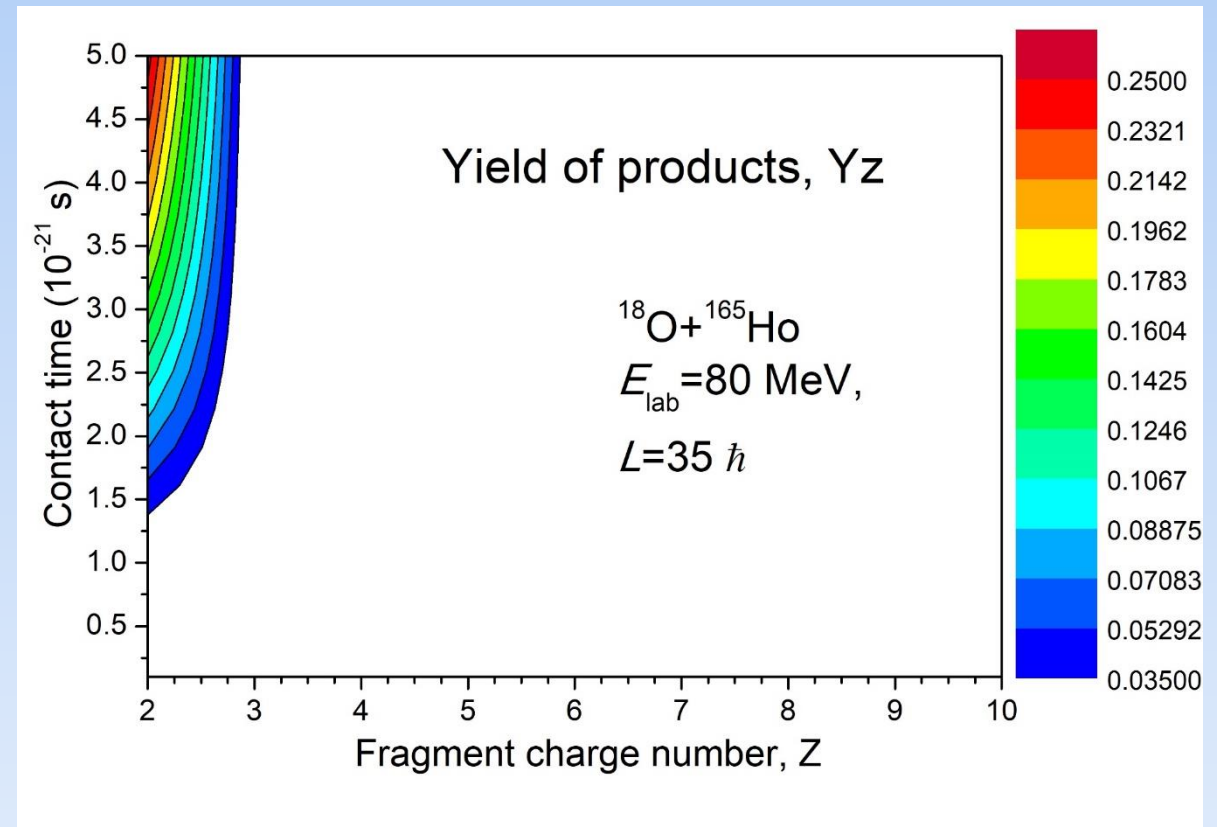
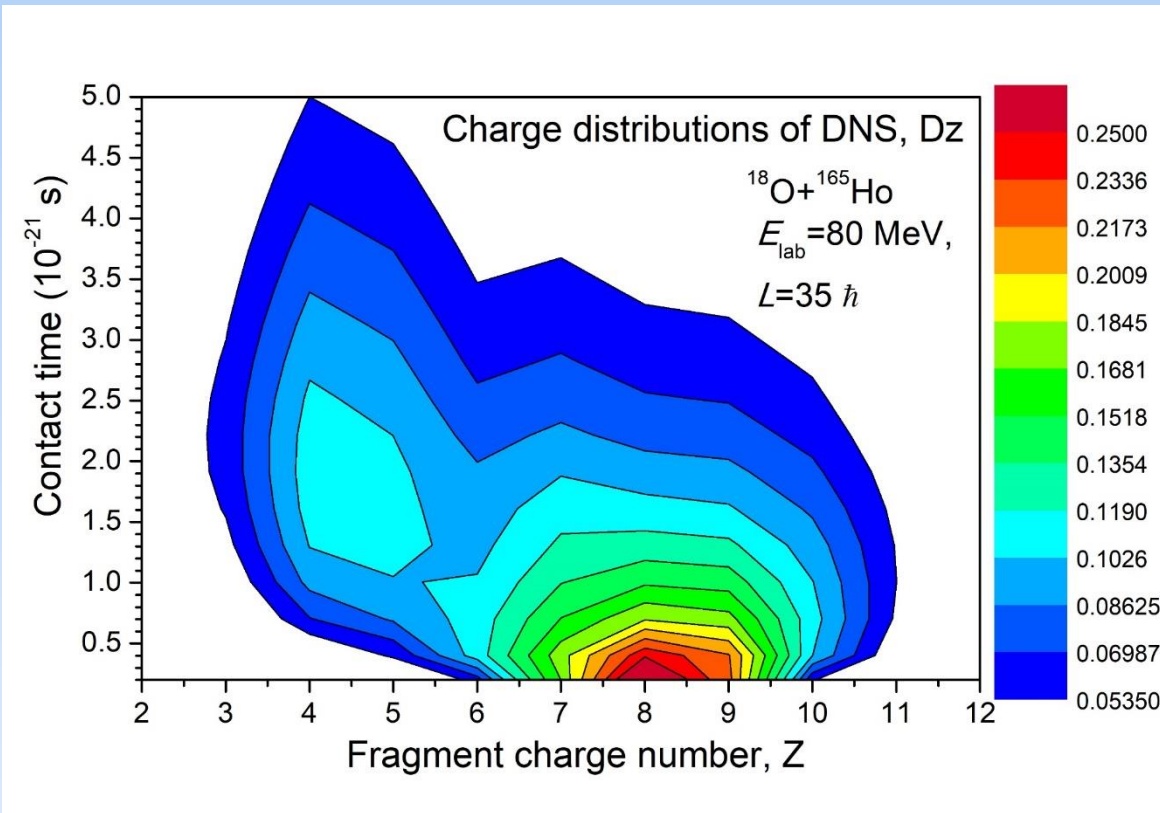
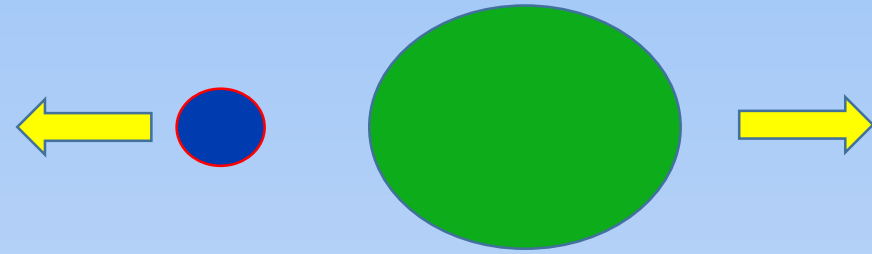
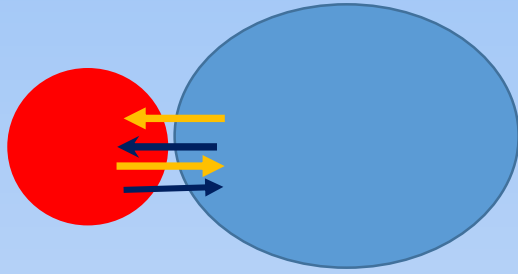
$$\sigma_{ICF}(E_{c.m.}, L) = \sigma_{\text{Capture}}(L) Y_z(L)$$

# Partial cross section of the incomplete fusion

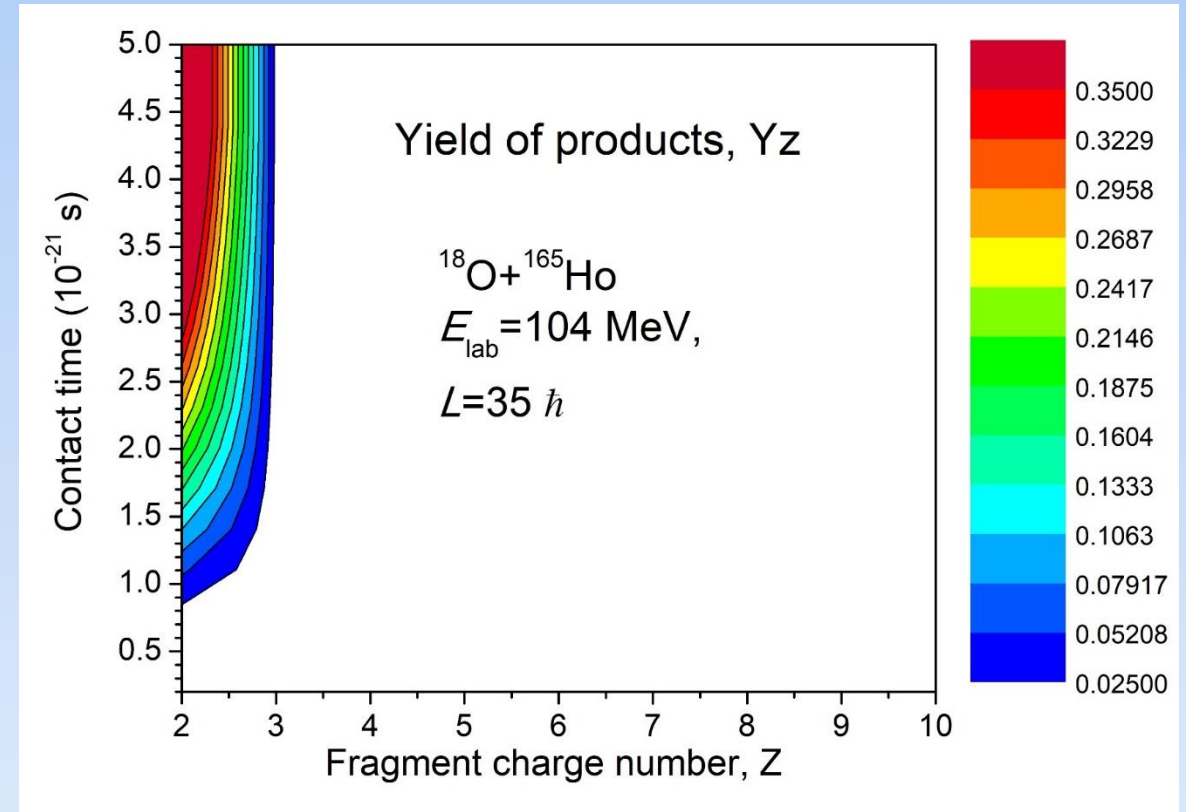
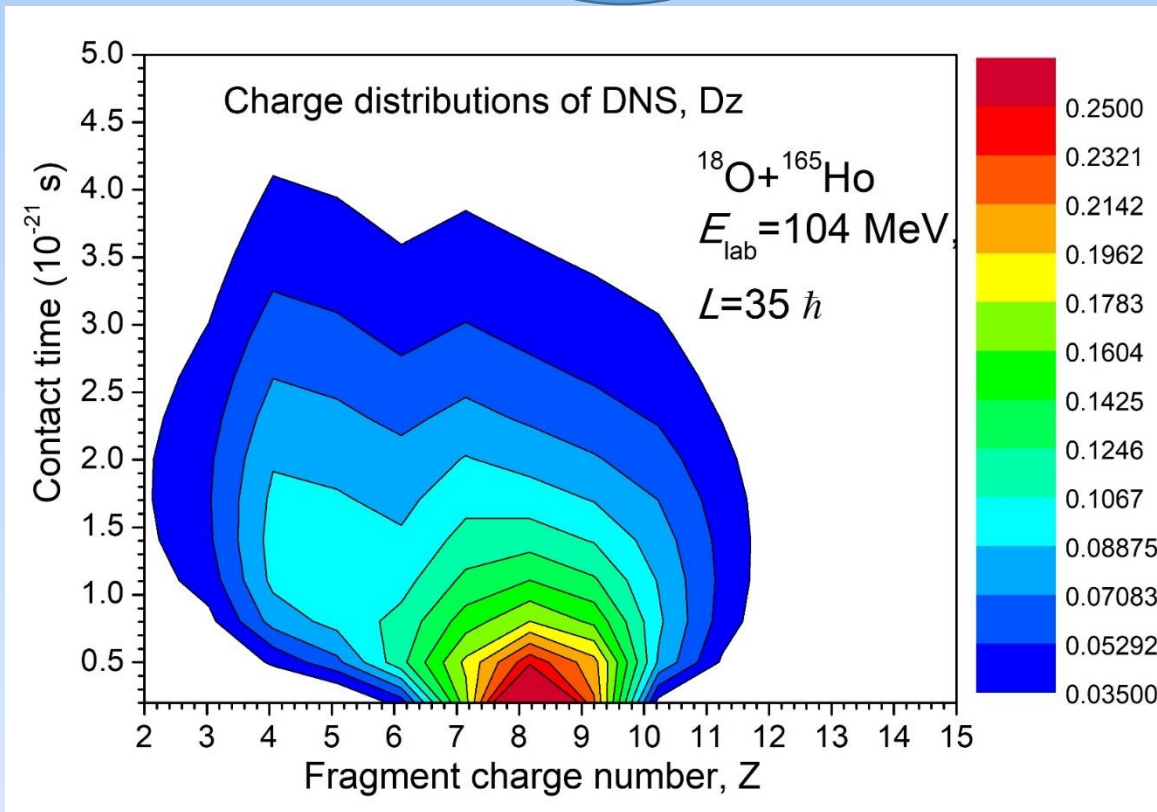
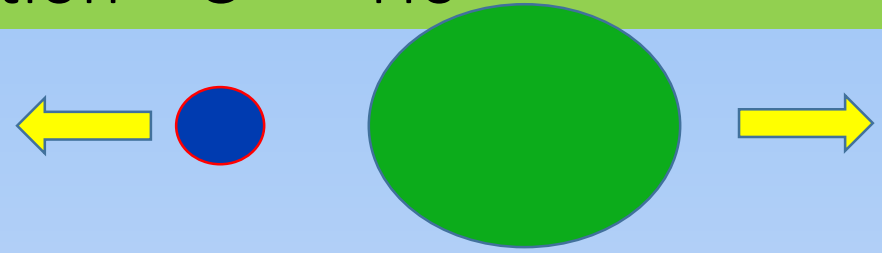
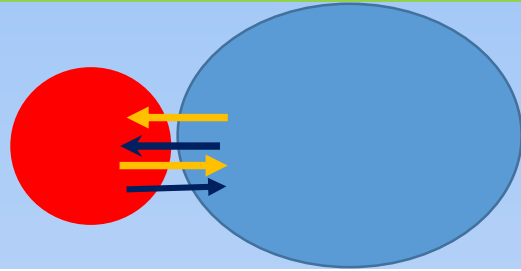


$$\sigma_{\text{ICF}}(E_{\text{c.m.}}, L) = \sigma_{\text{Capture}}(L) Y_Z(L)$$

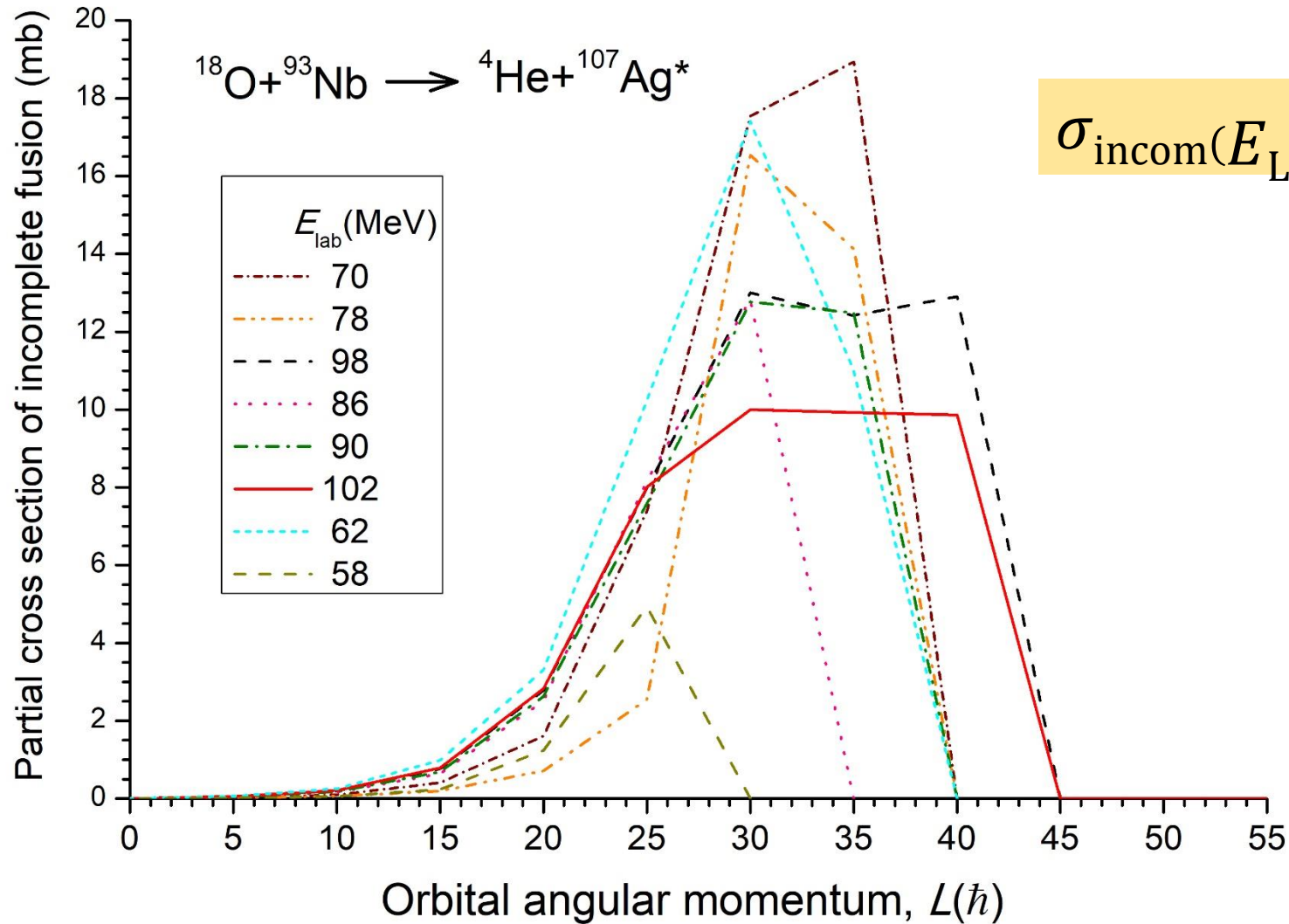
# Solutions of the transport master equations for the evolution and decay dinuclear system formed in reaction $^{18}\text{O}+^{165}\text{Ho}$



# Solutions of the transport master equations for the evolution and decay dinuclear system formed in reaction $^{18}\text{O}+^{165}\text{Ho}$

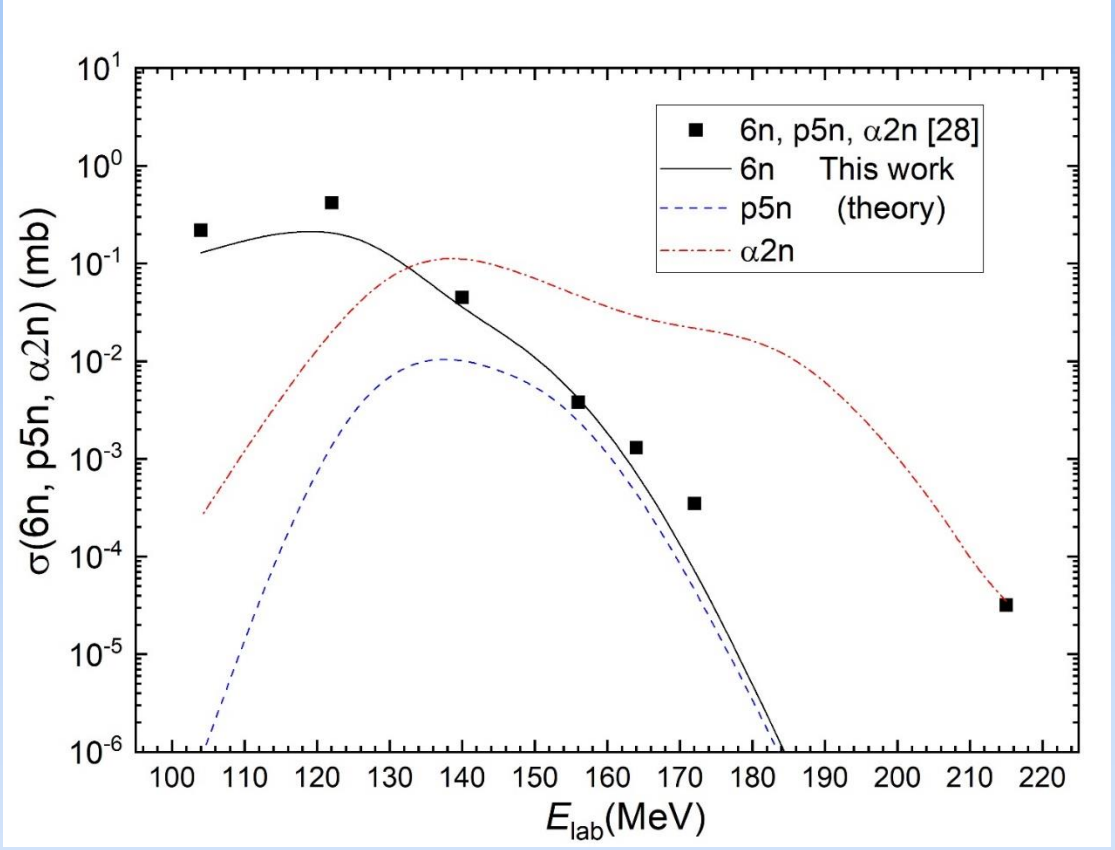
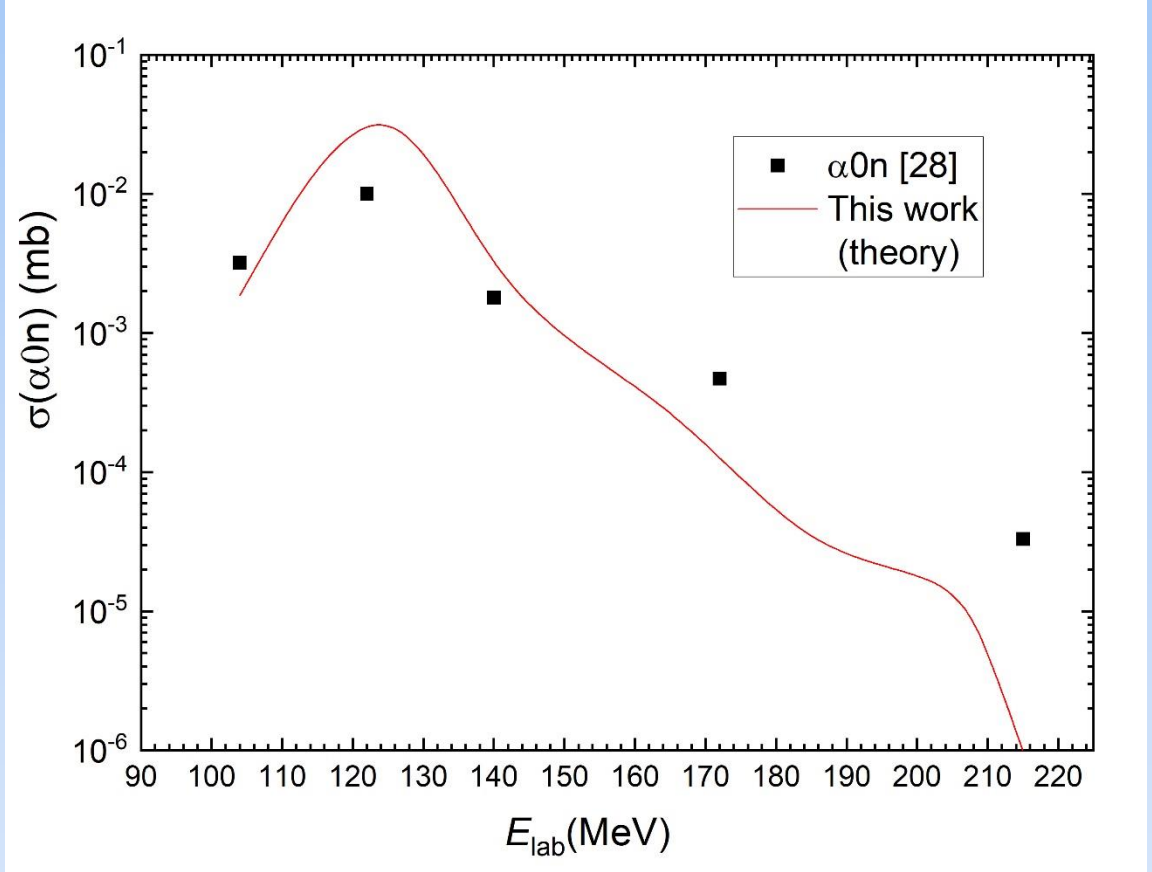


# Partial cross sections of incomplete fusion in the $^{18}\text{O}+^{165}\text{Ho}$ reactions



$$\sigma_{\text{incom}}(E_{\text{Lab}}, l) = \sigma_{\text{cap}}(E_{\text{Lab}}, l) Y_z(E_{\text{Lab}}, l)$$

Experimental data obtained from the paper by H. Bruchertseifert et al., Soviet Journal of Nuclear Physics; v. 33(6); p. 778-782 (1981). The conjugate nucleus  $^{194}\text{Au}$  has enough small excitation energy  $E^*_{ICF}(L)$ .



$$\sigma_{ER}^{(ICF)}(E_{Lab}, Z) = \sum_{l=0}^{l_d} \sigma_{ICF}(E_{Lab}, l) W_{sur}(E_{Lab}, l, Z)$$

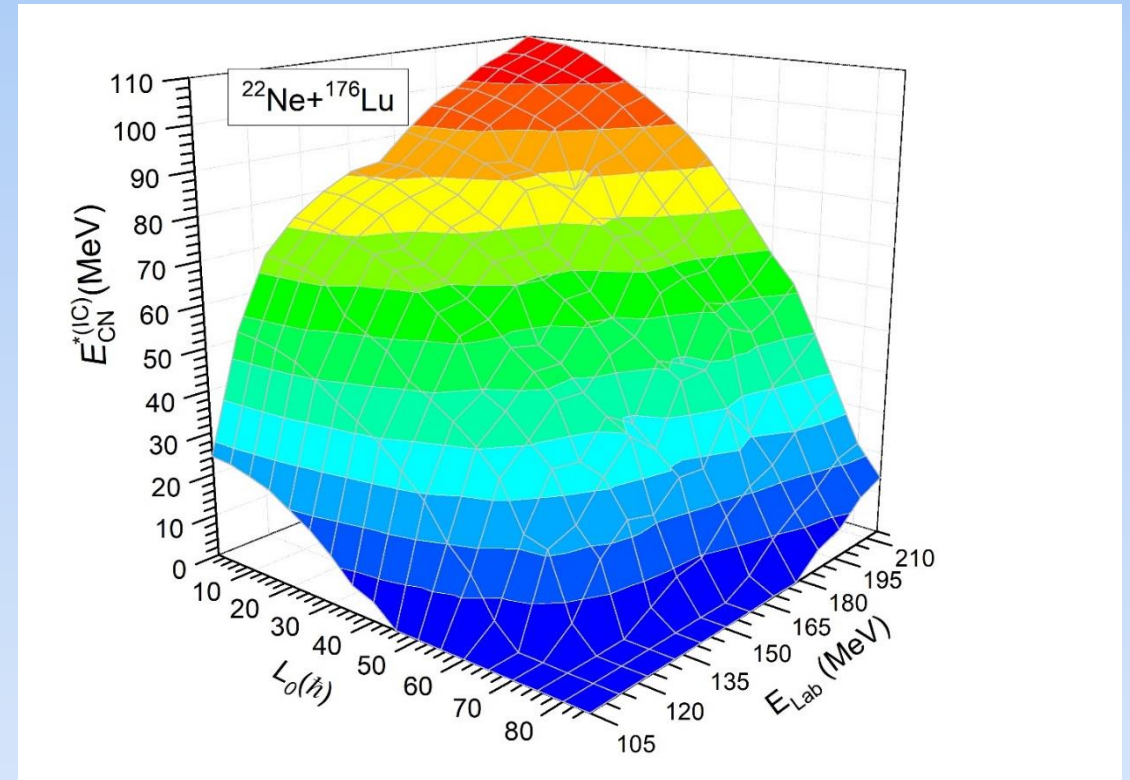
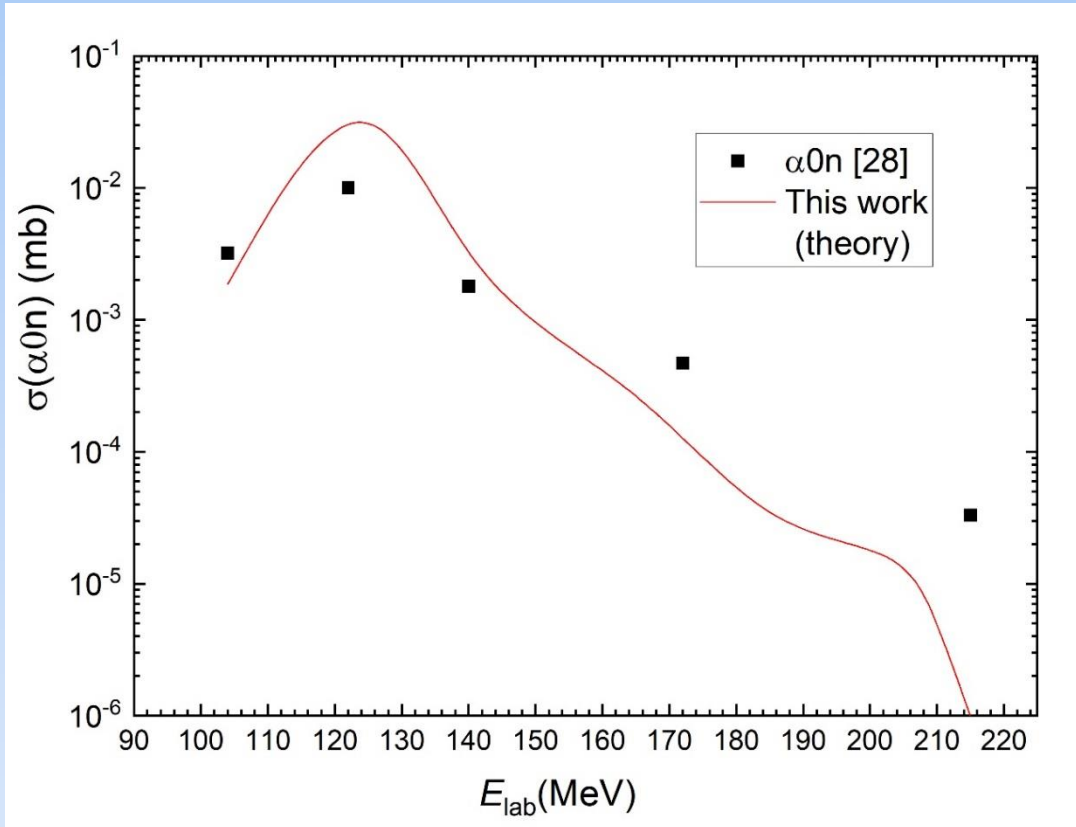
The survival probability  $W_{sur}$  has been calculated by B.M. Kayumov: H. Lü, A. Marchix, et al, Comput. Phys. Com. 200 (2016) 381.

Code KEWPIE2

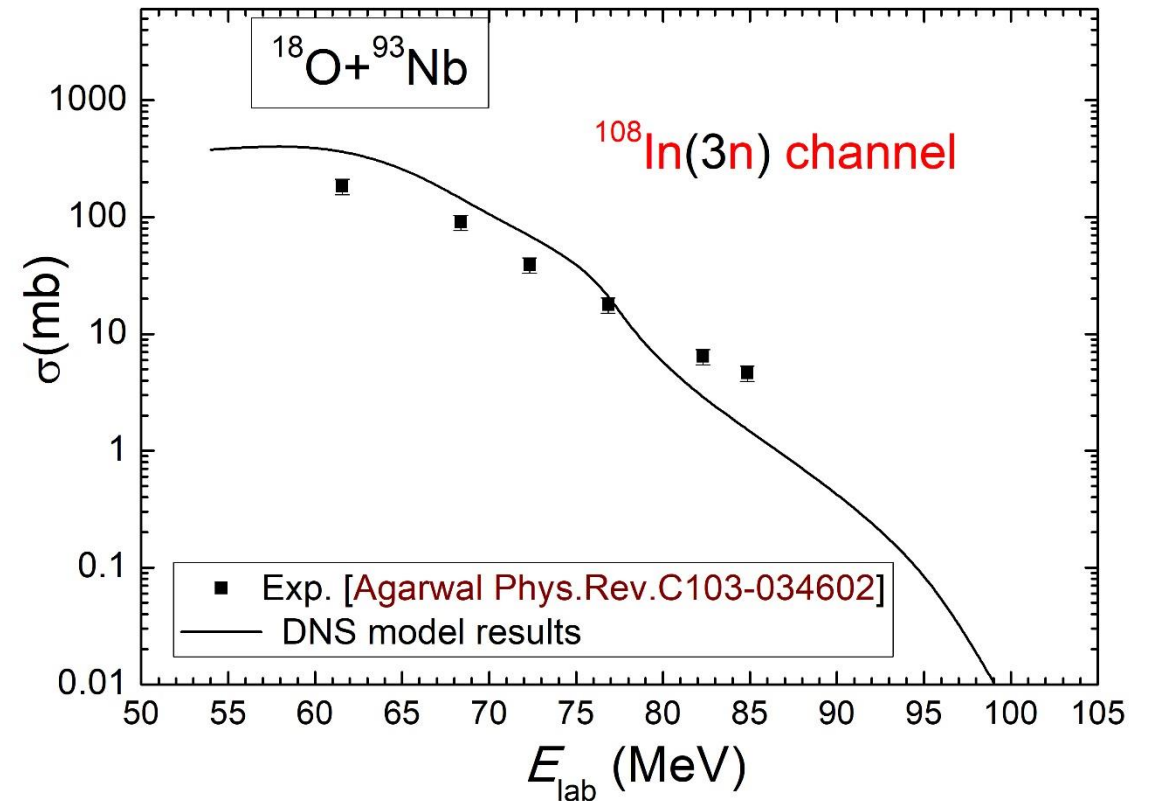
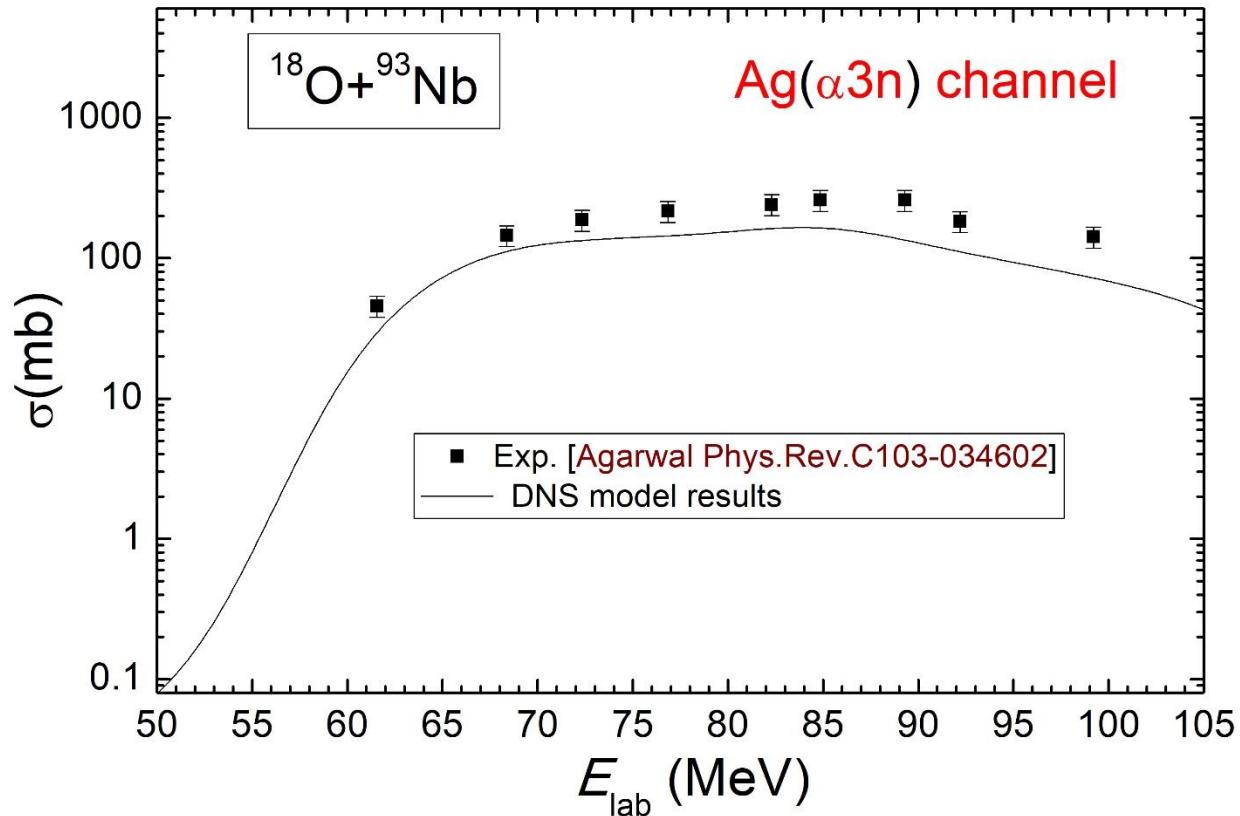


# Explanation of the formation of the cold conjugate nucleus in the incomplete fusion.

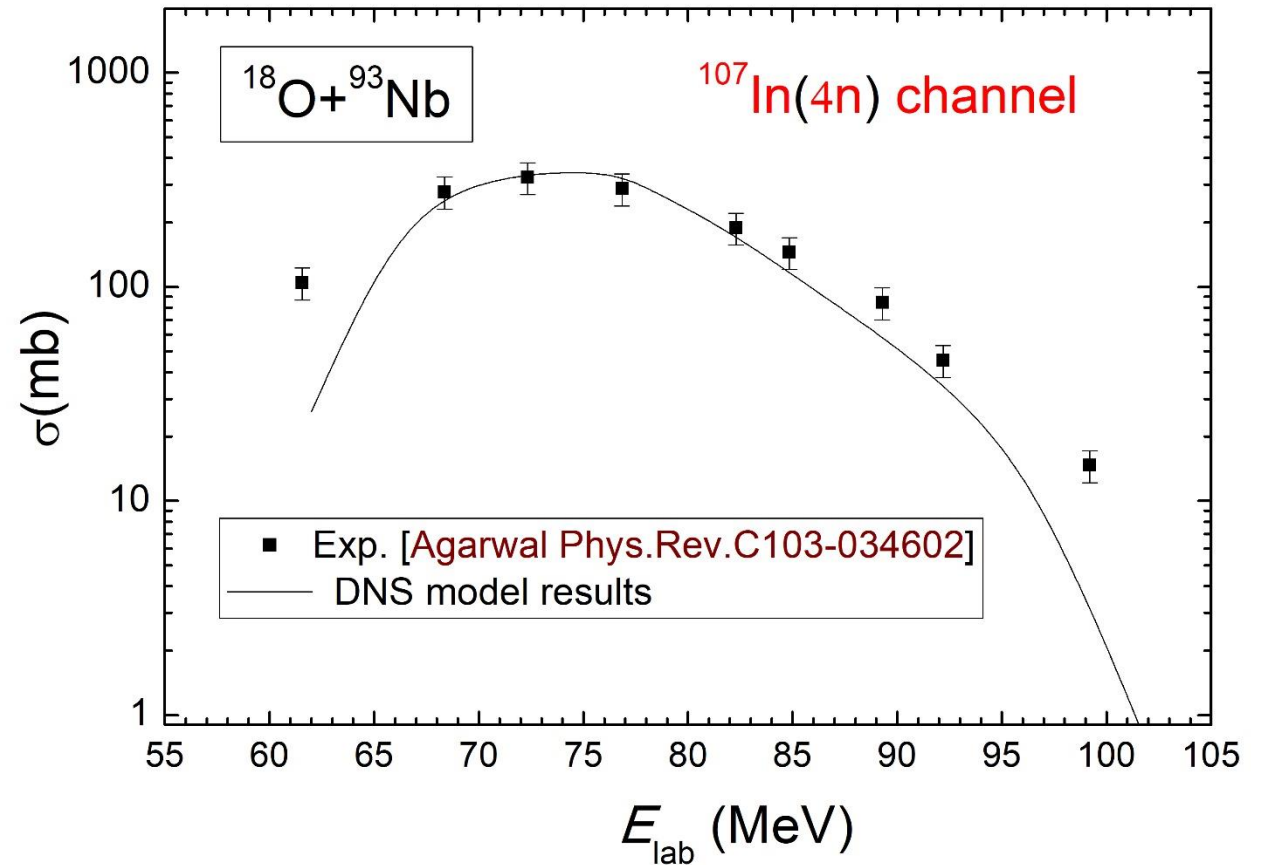
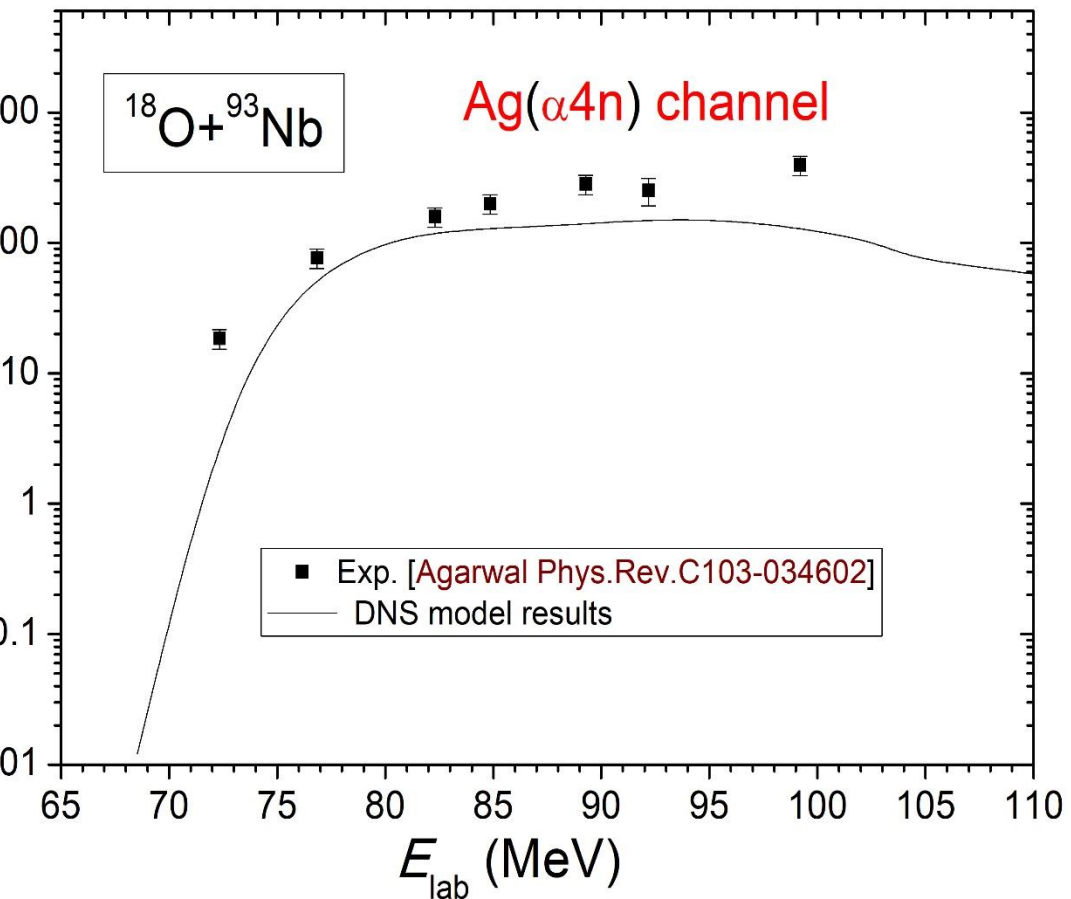
This work has been published in A. K. Nasirov, B.M. Kayumov, O. K. Ganiev, G.A. Yuldasheva  
Phys. Lett. B 482, 137976 (2023).



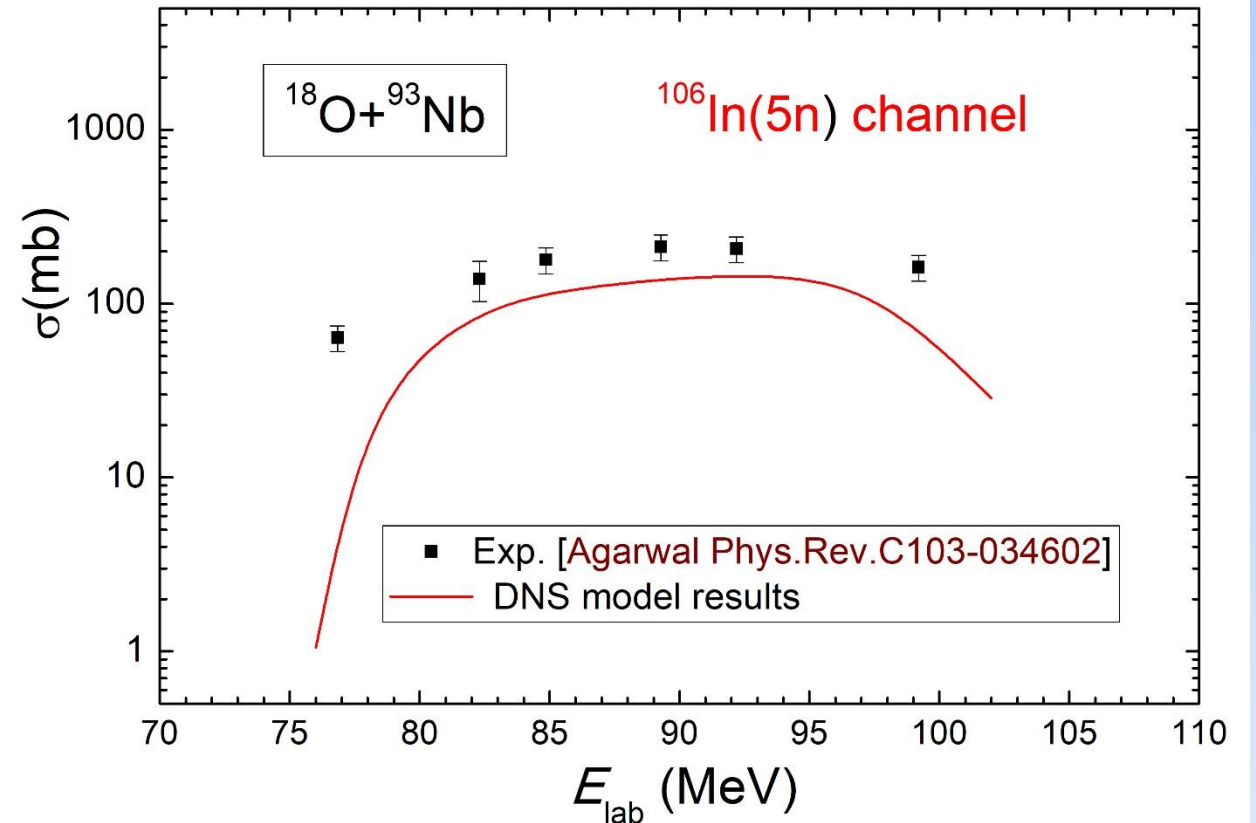
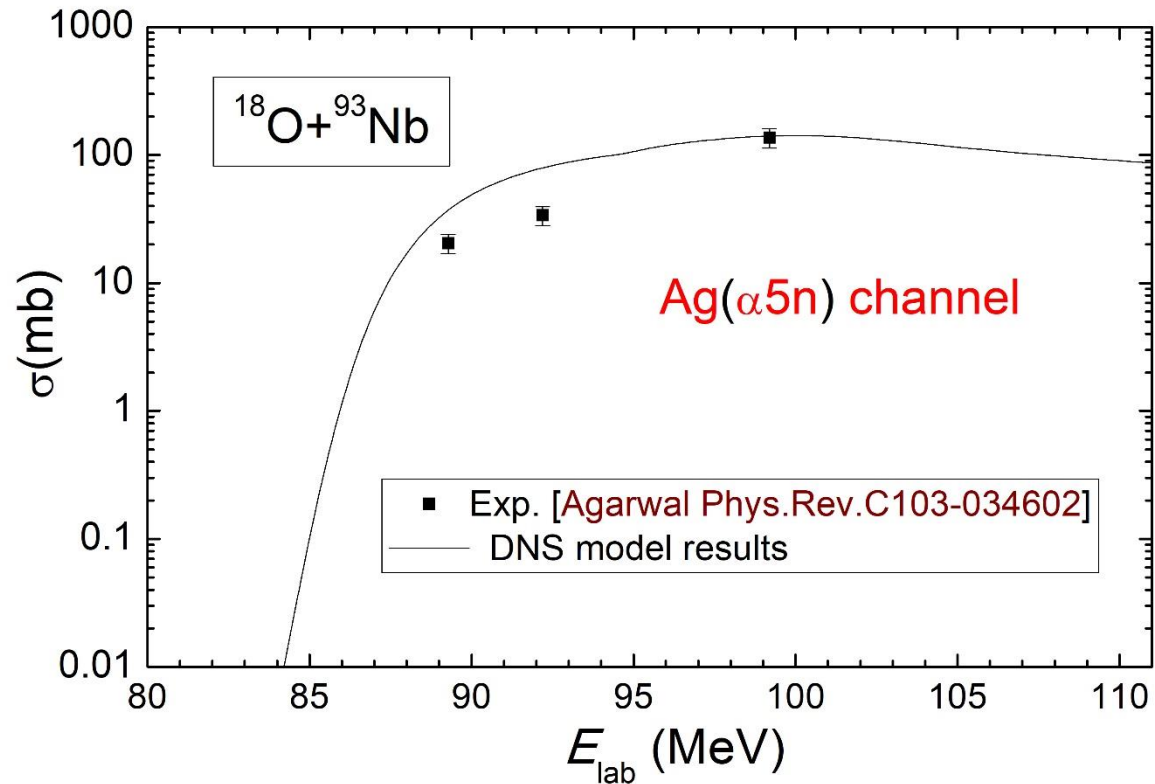
Comparison of the experimental and theoretical results the formation of the evaporation residues accompanied by emission of 3 neutrons in the incomplete fusion and complete fusion reactions.



Comparison of the experimental and theoretical results the formation of the evaporation residues accompanied by emission of 4 neutrons in the incomplete fusion and complete fusion reactions.



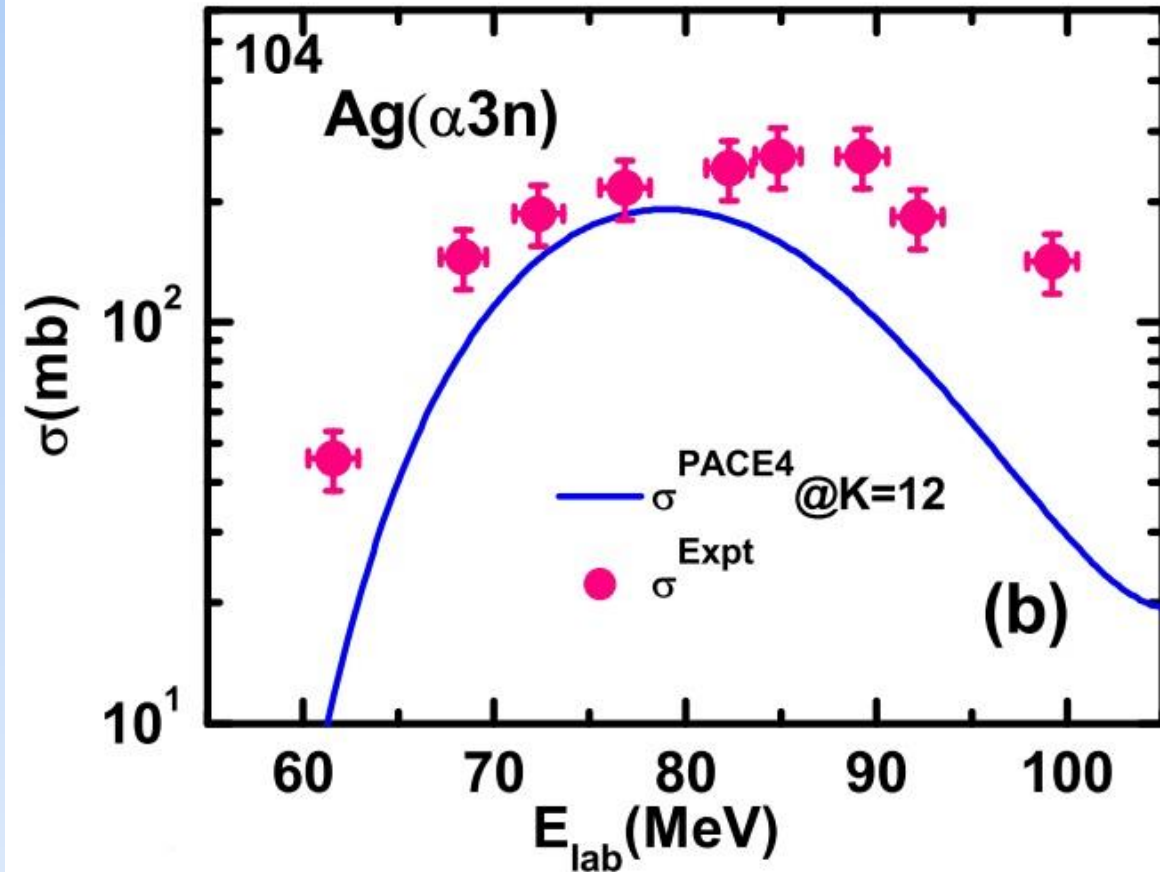
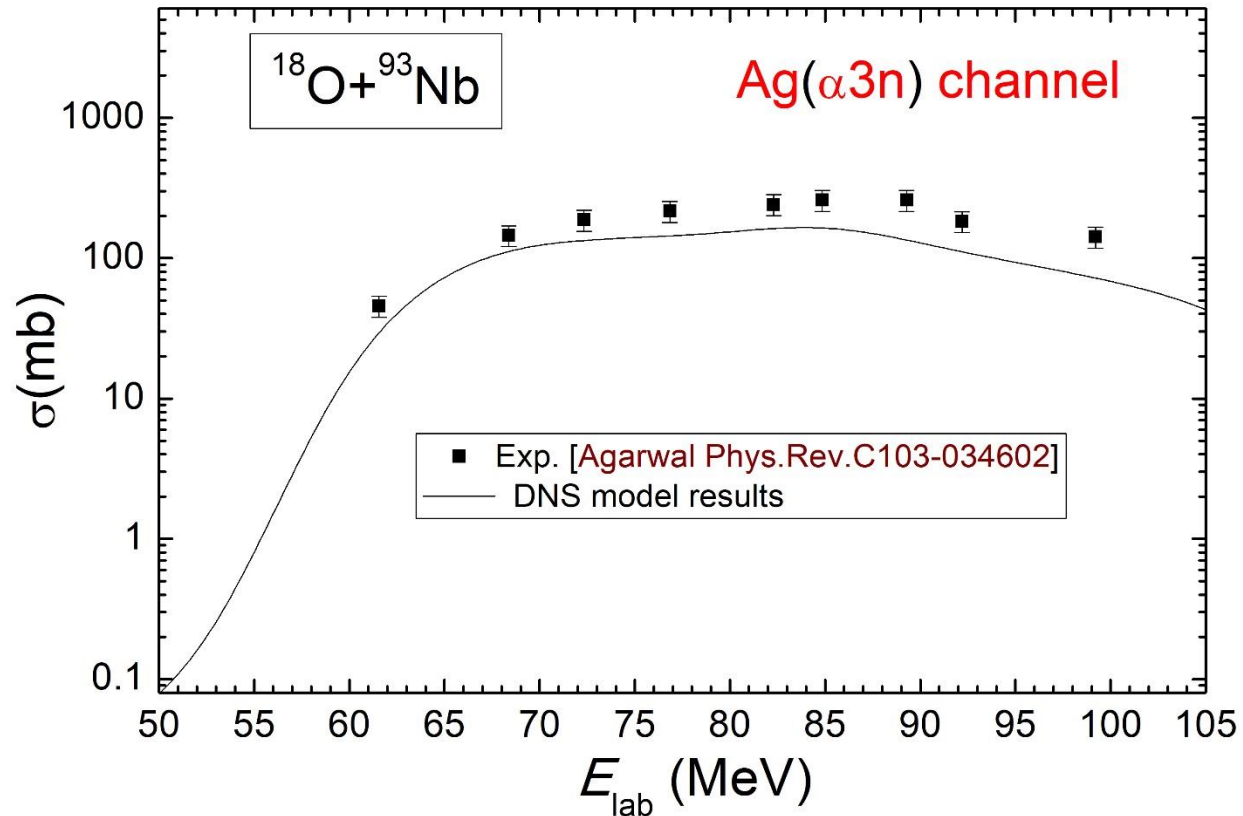
Comparison of the experimental and theoretical results the formation of the evaporation residues accompanied by emission of 5 neutrons in the incomplete fusion and complete fusion reactions.



# Comparison of the results by dinuclear system model and PACE4 code.

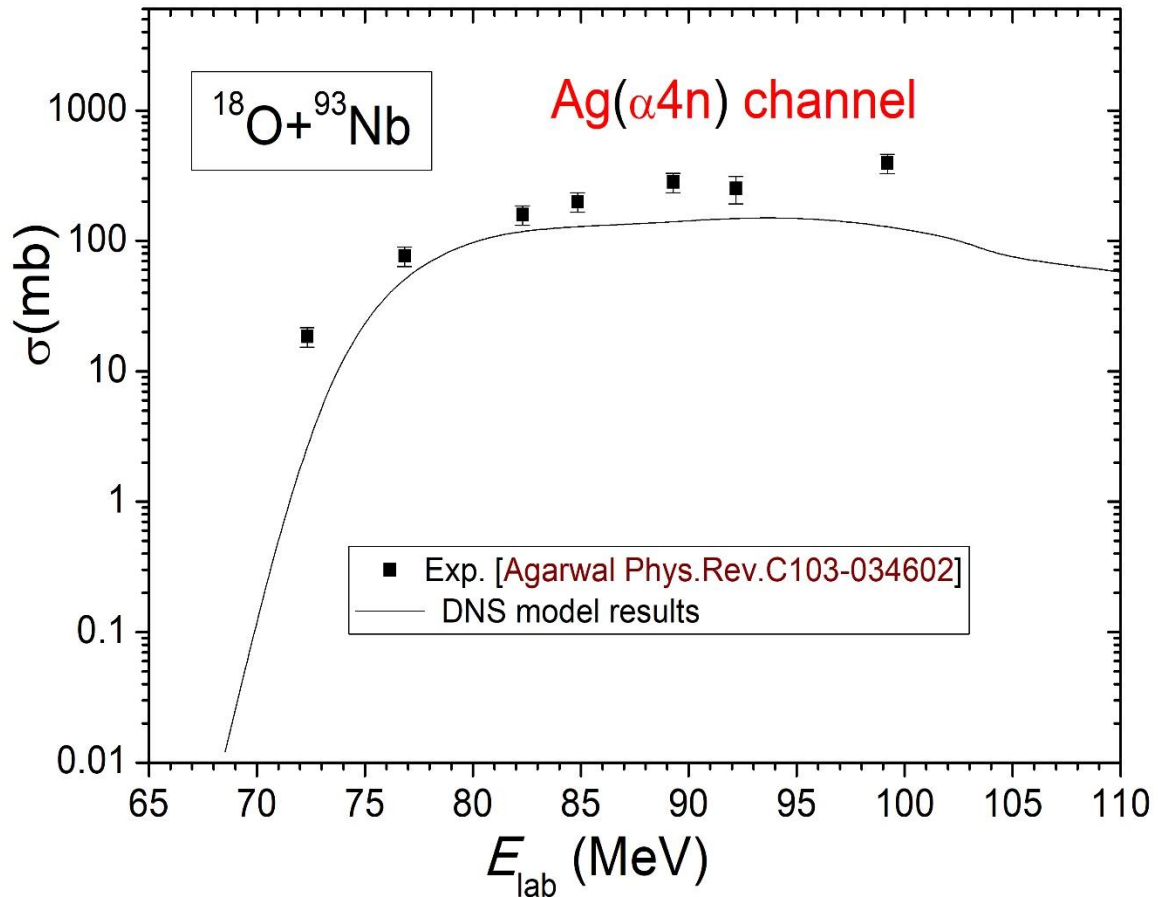
Theoretical result of this work

Avinash Agarwal, Phys.Rev.C103-034602]

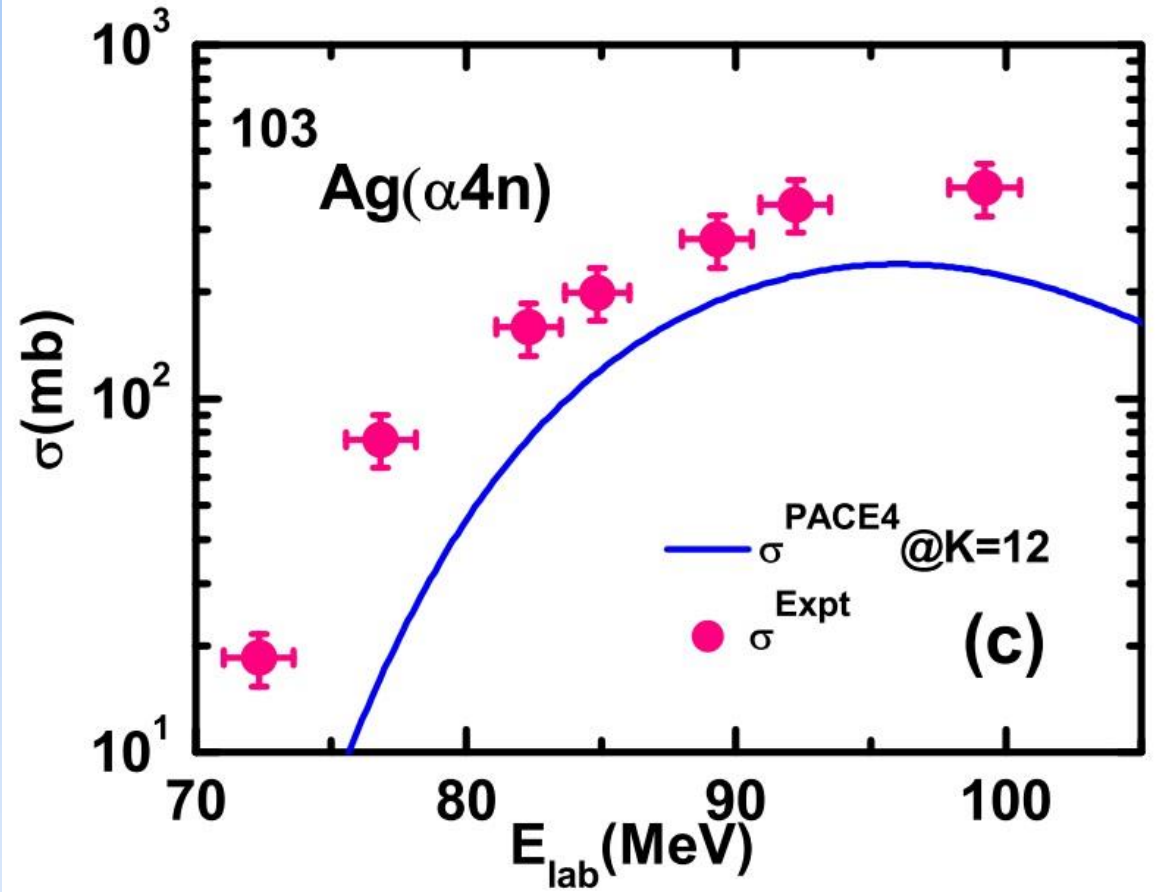


# Comparison of the results by dinuclear system model and PACE4 code.

Theoretical result of this work

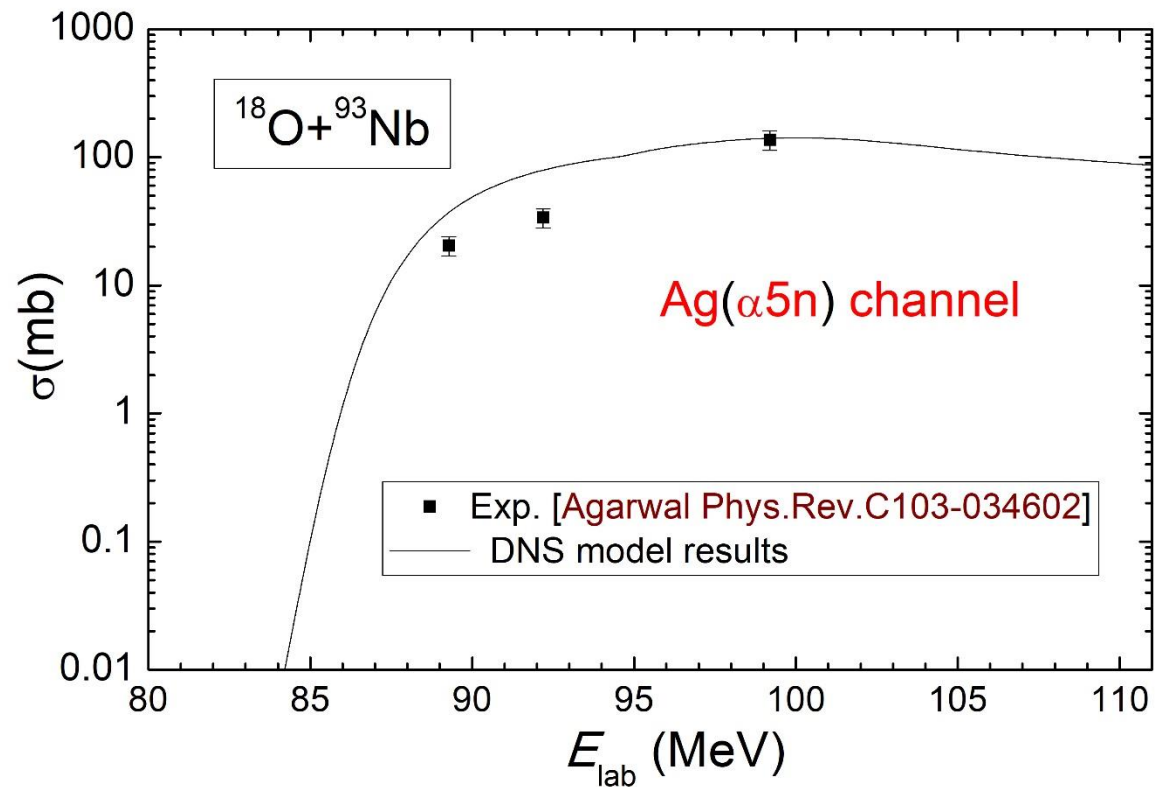


Avinash Agarwal, Phys.Rev.C103-034602]

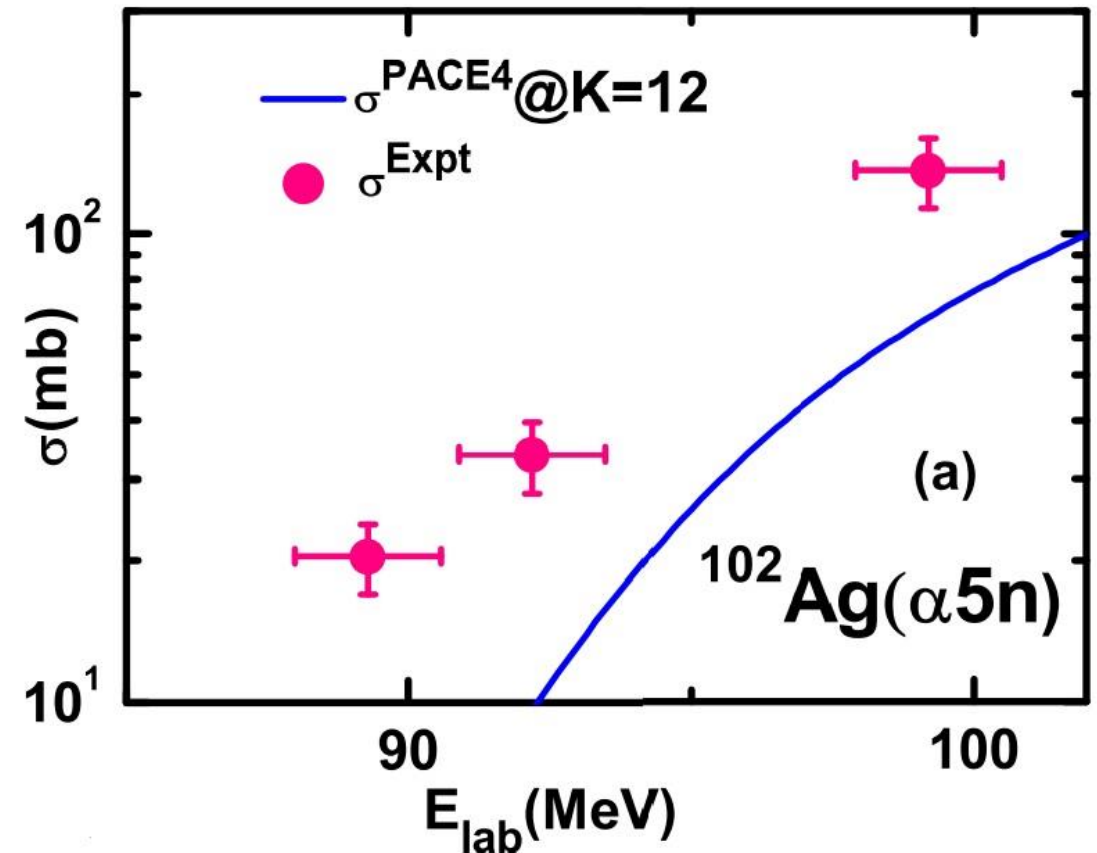


# Comparison of the results by dinuclear system model and PACE4 code.

Theoretical result of this work



Avinash Agarwal, Phys.Rev.C103-034602]



# Conclusion

We can conclude that the incomplete fusion is a quasifission in the region of the very asymmetric masses. Projectile nucleus transforms into  $\alpha$ -particle during evolution of the dinuclear system formed as the capture.  $\alpha$ -particle could not be transformed to the partner nucleus due to hindrance caused by the centrifugal force and this force throws away  $\alpha$ -particle.

The difference between the evaporation residue cross sections in complete fusion and incomplete fusion is explained by the strong increase of the intrinsic fusion barrier at large angular momentum of the dinuclear system. Therefore, contribution from the evaporation residues formed in the incomplete fusion increases in the large values of the beam energy.



**THANK YOU VERY MUCH FOR YOUR ATTENTION !**