Gluon distributions in the proton in a light-front spectator model

Poonam Choudhary
IIT Kanpur

Work done in collaboration with

Dipankar Chakrabarti, Bheemsehan Gurjar, Raj Kishore, Tanmay Maji,
Chandan Mondal, Asmita Mukherjee

Talk Based on: **Phys. Rev. D 108, 014009 (2023)**

India-JINR Worskhop 2023

October 17, 2023

Motivation: Why study gluons?

- The nucleons are fundamental building blocks of the visible universe.
- They are further composed of quarks and gluons.
- How are the quarks and gluon distributed and contributing in the total mass, spin and momentum of the proton ?
- Gluons are mediator of the strong interaction and their structure is less explored as compared to quarks.

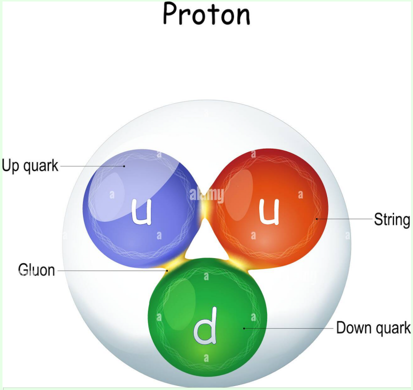


Figure: Source: Wiki

Light front dynamics

- Light-front dynamics describes how a relativistic system changes along a light-front direction.
- In light front field theory, the new variables are defined such as $x^+ = x^0 + x^3$ and $x^- = x^0 - x^3$.
- x^+ and x^- are called light-front time and longitudinal space variables respectively.
- Transverse variable $x^\perp = (x^1, x^2)$.
- The energy dispersion relation $k^2 = k^+k^- - k_\perp^2 = m^2$ does not involve a square root structure.

$$H_{LF}|k\rangle = k^-|k\rangle \longrightarrow k^- = \frac{k_\perp^2 + m^2}{[k^+]} .$$

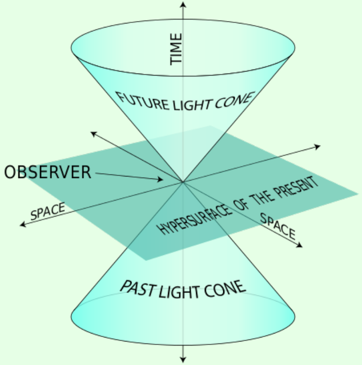


Figure: Source: Wiki

Gluon light front spectator model

• In this simplified model, we describe the proton as a composite state of one active gluon and a spin-$\frac{1}{2}$ spectator.

$$|P;\uparrow(\downarrow)\rangle = \int \frac{\mathrm{d}^2\mathbf{p}_\perp \mathrm{d}x}{16\pi^3\sqrt{x(1-x)}} \times$$

$$\Big[\psi^{\uparrow(\downarrow)}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|+1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle + \psi^{\uparrow(\downarrow)}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|+1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle$$

$$+\psi^{\uparrow(\downarrow)}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|-1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle + \psi^{\uparrow(\downarrow)}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|-1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle\Big],$$

For the choice of proton momentum $P = (P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp)$,

where $p = (xP^+, \frac{p^2+\mathbf{p}_\perp^2}{xP^+}, \mathbf{p}_\perp)$ and $P_X = ((1-x)P^+, P_X^-, -\mathbf{p}_\perp)$ momentum of the active gluon and spectator respectively.

Light front wave functions(LFWFs)

The LFWFs for the Fock-state expansion for a proton with $J_z = +1/2$

$$\begin{aligned}
\psi^{\uparrow}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\frac{(-p^1_\perp + ip^2_\perp)}{x(1-x)}\varphi(x,\mathbf{p}^2_\perp), \\
\psi^{\uparrow}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\left(M - \frac{M_X}{(1-x)}\right)\varphi(x,\mathbf{p}^2_\perp), \\
\psi^{\uparrow}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\frac{(p^1_\perp + ip^2_\perp)}{x}\varphi(x,\mathbf{p}^2_\perp), \\
\psi^{\uparrow}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp) &= 0
\end{aligned}$$

for the proton with $J_z = -1/2$ have the form

$$\begin{aligned}
\psi^{\downarrow}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp) &= 0, \\
\psi^{\downarrow}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\frac{(-p^1_\perp + ip^2_\perp)}{x}\varphi(x,\mathbf{p}^2_\perp), \\
\psi^{\downarrow}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\left(M - \frac{M_X}{(1-x)}\right)\varphi(x,\mathbf{p}^2_\perp), \\
\psi^{\downarrow}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\frac{(p^1_\perp + ip^2_\perp)}{x(1-x)}\varphi(x,\mathbf{p}^2_\perp)
\end{aligned}$$

$$\varphi(x,\mathbf{p}^2_\perp) = N_g\frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}x^b(1-x)^a\exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2x^2}\mathbf{p}^2_\perp\right]$$

Gluon Transverse momentum distributions (TMDs)

- TMDs provide three dimensional picture of gluon distribution in momentum space.
- The unintegrated gluon correlation function for leading twist gluon TMDs in the SIDIS process is given by the following relation:

$$\Phi^{g[ij]}(x,\mathbf{p}_\perp;S) = \frac{1}{xP^+}\int \frac{d\xi^-}{2\pi}\,\frac{d^2\xi_\perp}{(2\pi)^2}\,e^{ik\cdot\xi}\left\langle P;S\right| F_a^{+j}(0)\;\mathcal{W}_{+\infty,ab}(0;\xi)\,F_b^{+i}(\xi)\left|P;S\right\rangle\Big|_{\xi^+=0^+},$$

$$\Phi^g(x,\mathbf{p}_\perp;S) \quad = \quad \delta_T^{ij}\,\Phi^{g[ij]}(x,\mathbf{p}_\perp;S) = f_1^g(x,\mathbf{p}_\perp^2) - \frac{\epsilon_\perp^{ij}\,\mathbf{p}_\perp^i\,S_\perp^j}{M}\,f_{1T}^{\perp g}(x,\mathbf{p}_\perp^2)\,,$$

$$\tilde{\Phi}^g(x,\mathbf{p}_\perp;S) \quad = \quad i\epsilon_T^{ij}\,\Phi^{g[ij]}(x,\mathbf{p}_\perp;S) = \lambda\,g_{1L}^g(x,\mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp\cdot\mathbf{S}_\perp}{M}\,g_{1T}^g(x,\mathbf{p}_\perp^2)\,,$$

$$\begin{aligned}\Phi_T^{g,\,ij}(x,\mathbf{p}_\perp;S) = -\hat{\boldsymbol{S}}\,\Phi^{g[ij]}(x,\mathbf{p}_\perp;S) = &-\frac{\hat{\mathbf{S}}\mathbf{p}_\perp^i\,\mathbf{p}_\perp^j}{2M^2}\,h_1^{\perp g}(x,\mathbf{p}_\perp^2) + \frac{\lambda\,\hat{\mathbf{S}}\mathbf{p}_\perp^i\,\epsilon_\perp^{jk}\,\mathbf{p}_\perp^k}{2M^2}\,h_{1L}^{\perp g}(x,\mathbf{p}_\perp^2)\\ &+\frac{\hat{\mathbf{S}}\mathbf{p}_\perp^i\,\epsilon_\perp^{jk}\,S_\perp^k}{2M}\left(h_{1T}^g(x,\mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp^2}{2M^2}\,h_{1T}^{\perp g}(x,\mathbf{p}_\perp^2)\right)\\ &+\frac{\hat{\mathbf{S}}\mathbf{p}_\perp^i\,\epsilon_\perp^{jk}\left(2p_\perp^k\,\mathbf{p}_\perp.\mathbf{S}_\perp - S_\perp^k\,\mathbf{p}_\perp^2\right)}{4M^3}\,h_{1T}^{\perp g}(x,\mathbf{p}_\perp^2).\end{aligned}$$

T-even TMDs

- The unpolarized TMD $f_1^g(x, \mathbf{p}_\perp^2)$

$$f_1^g(x, \mathbf{p}_\perp^2) = \frac{1}{16\pi^3} \sum_{\lambda_g \lambda'_g \lambda_X} (\epsilon_1^{\lambda'_g *} \epsilon_1^{\lambda_g} + \epsilon_2^{\lambda_g' *} \epsilon_2^{\lambda_g}) \psi^{\uparrow *}_{\lambda'_g \lambda_X}(x, \mathbf{p}_\perp^2) \psi^{\uparrow}_{\lambda_g \lambda_X}(x, \mathbf{p}_\perp^2)$$

$$= \frac{1}{16\pi^3} \Big[|\psi^{\uparrow}_{+1+1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{+1-1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{-1+1/2}(x, \mathbf{p}_\perp^2)|^2 \Big].$$

$$f_1^g(x, \mathbf{p}_\perp^2) = N_g^2 \frac{2}{\pi \kappa^2} \frac{\log[1/(1-x)]}{x} x^{2b} (1-x)^{2a} \Big[A(x) + \mathbf{p}_\perp^2 B(x) \Big] \exp[-C(x) \mathbf{p}_\perp^2],$$

where $A(x)$, $B(x)$ and $C(x)$ are given by

$$A(x) = \left(M - \frac{M_X}{(1-x)} \right)^2, \quad B(x) = \frac{1 + (1-x)^2}{x^2 (1-x)^2} \quad \text{and} \quad C(x) = \frac{\log[1/(1-x)]}{\kappa^2 x^2}.$$

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

T-even TMDs

• The gluon helicity TMD $g_{1L}^g(x, \mathbf{p}_\perp^2)$ describes the distribution of a circularly polarised gluon in a longitudinally polarised proton.

$$g_{1L}^g(x, \mathbf{p}_\perp^2) = N_g^2 \frac{2}{\pi\kappa^2} \frac{\log[1/(1-x)]}{x} x^{2b}(1-x)^{2a} \left[A(x) + \mathbf{p}_\perp^2 \tilde{B}(x)\right] \exp[-C(x)\mathbf{p}_\perp^2],$$

• The worm-gear gluon TMD $g_{1T}^g(x, \mathbf{p}_\perp^2)$ is defined as the distribution of a circularly polarised gluon in a transversely polarized proton.

$$g_{1T}^g(x, \mathbf{p}_\perp^2) = -\frac{4M}{\pi\kappa^2} N_g^2 \left(M(1-x) - M_X\right) \log[1/(1-x)] x^{2b-2}(1-x)^{2a-1} \exp[-C(x)\mathbf{p}_\perp^2]$$

• The Boer-Mulders gluon TMD $h_1^{\perp g}(x, \mathbf{p}_\perp^2)$ describes a linearly polarized gluon inside an unpolarized proton.

$$h_1^{\perp g}(x, \mathbf{p}_\perp^2) = \frac{8M^2}{\pi\kappa^2} N_g^2 \log[1/(1-x)] x^{2b-3}(1-x)^{2a-1} \exp[-C(x)\mathbf{p}_\perp^2]$$

where

$$\tilde{B}(x) = \frac{1-(1-x)^2}{x^2(1-x)^2}.$$

Fixing model parameters

- We fixed our model parameters by fitting the unpolarized gluon PDF, $f_1^g(x)$ with the NNPDF3.0nlo global analysis.
- The model scale $Q_0 = 2$ GeV. Also $N_g = 2.088$, $M_X = 0.985$ GeV.

$$\langle x \rangle_g = \int_{0.001}^{1} dx x f_1^g(x) = 0.416^{+0.048}_{-0.041},$$

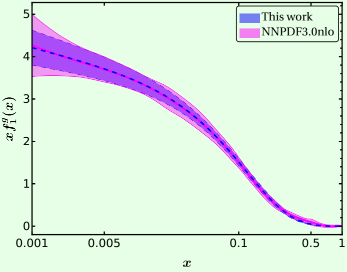
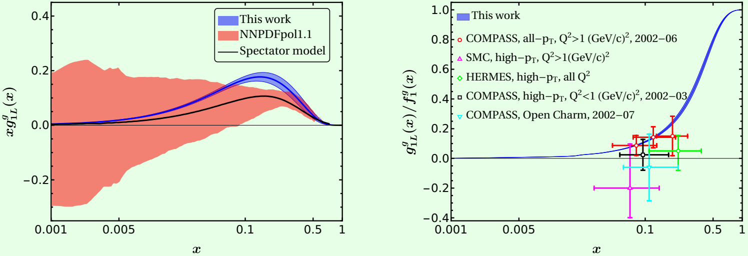


Figure: Fitted in the region $0.001 \leq x \leq 1$ at $Q_0 = 2$ GeV

Parameter	Central Value	1σ-Error band	2σ-Error band
a	3.88	$\pm$ 0.1020	$\pm$ 0.2232
b	-0.53	$\pm$ 0.0035	$\pm$ 0.0071

	This work	Bacchetta	Ma-Lu	Pion model	Lattice
$\langle x \rangle_g$	0.416	0.424	0.411	0.409	0.427

Helicity prediction and comparison

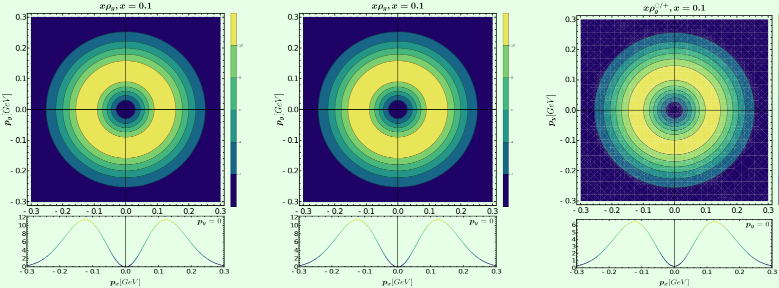


Gluon helicity	Central Value	our predictions
$\Delta G = \int_{0.05}^{0.3} dx \Delta g(x)$	0.20 [PHENIX-2008]	$0.28^{+0.047}_{-0.037}$
$\Delta G = \int_{0.05}^{0.2} dx \Delta g(x)$	0.23(6) [NNPDF]	$0.22^{+0.033}_{-0.024}$
$\Delta G = \int_{0.05}^{1} dx \Delta g(x)$	0.19(6) [RHIC]	$0.326^{+0.066}_{-0.050}$

Gluon density

• The unpolarized gluon density describes the probability density of finding the unpolarized gluons at given x and $\mathbf{p}_\perp$.

• The "Boer-Mulders" density, which shows the probability density of finding the linearly polarized gluons with x and $\mathbf{p}_\perp$.

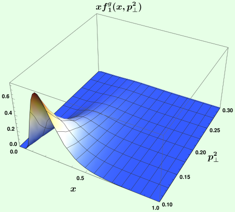


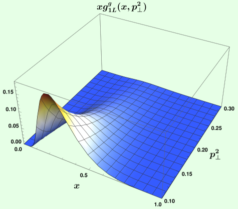
• Helicity density describes the probability density of circularly polarized gluons at particular x and $\mathbf{p}_\perp$ inside the longitudinally polarized proton.

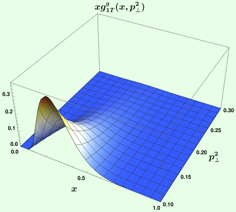
Relation among TMDs

$$f_1^g(x, \mathbf{p}_\perp^2) > 0, \qquad f_1^g(x, \mathbf{p}_\perp^2) \geq |g_{1L}^g(x, \mathbf{p}_\perp^2)|$$

$$f_1^g(x, \mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|}{M} |g_{1T}^g(x, \mathbf{p}_\perp^2)|,$$







• Connection between the square of an unpolarized TMD and a combination of squares of three polarised TMDs.

$$[f_1^g(x, \mathbf{p}_\perp^2)]^2 = [g_{1L}^g(x, \mathbf{p}_\perp^2)]^2 + \left[\frac{|\mathbf{p}_\perp|}{M} g_{1T}^g(x, \mathbf{p}_\perp^2)\right]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2} h_1^{\perp g}(x, \mathbf{p}_\perp^2)\right]^2$$

Conclusion and future work

• We discussed a light-front spectator model with the light-front wave functions modeled from the soft-wall holographic AdS/QCD prediction for two-body bound states.

• We fixed our model parameters by fitting the unpolarized gluon PDF, $f_1^g(x)$ with the NNPDF3.0nlo global analysis.

• The helicity PDF and other T-even TMDs are calculated as predictions of the model.

• TMDs satisfy all the model independent relations among them and are shown to satisfy the positivity bound.

• Using this model, we can predict the gluon GPDs and OAM of the gluon as well.