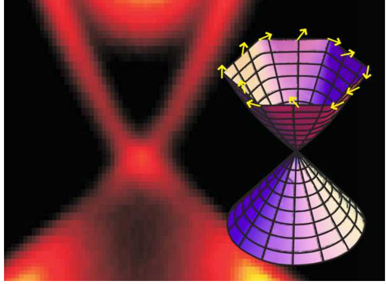

Non-linear Hall Effect

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***18th Oct.* 2023, India-JINR Meet**

Drude Conductivity

Response of conduction electrons in an electric field (E)

Equation of motion: $m\frac{d\vec{v}}{dt} = e\vec{E} - m\vec{v}/\tau$ τ relaxation time

Acceleration by E

Scattering term

Steady-state drift velocity: $\vec{v} = \frac{e\vec{E}\tau}{m}$ $\left(\frac{dv}{dt} = 0\right)$

Electric current density: $\vec{J} = ne\vec{v} = \frac{ne^2\tau}{m}\vec{E}$ Ohm's law $B = 0$

Drude conductivity

In matrix form $\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{ne^2\tau}{m} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$ (Electrons in 2D Plane)

What happens if we apply static magnetic field $B \neq 0$

$+e(v \times B)$

$$\sigma \sim \begin{pmatrix} 1 & \cancel{0} \\ \cancel{0} & 1 \end{pmatrix}$$

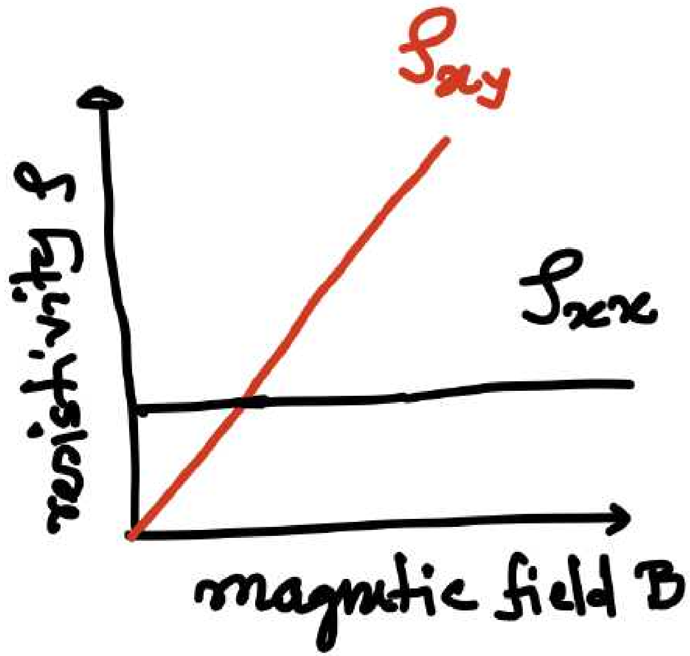
Remark: Transverse current in the presence of both electric and magnetic field

Off-diagonal conductivites → Hall conductivities (Classical)

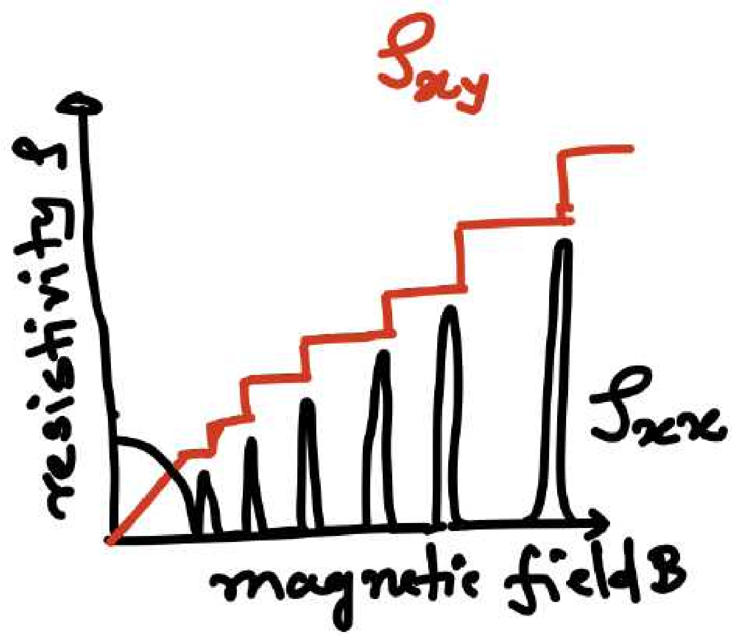
Hall Conductivity

Hall conductivity $\cancel{0} = \sigma_{xy} \sim \frac{1}{B}$ Hall resistivity $\rho_{xy} \sim B$

Classical



Quantum



$$\sigma_{xy} = n\frac{e^2}{h}$$

Quantum Hall Conductivity $\sigma_{xy} = n\frac{e^2}{h}$

$$n \sim \int_k f_0 \Omega_c$$

Berry Curvature

Key point: Time reversal symmtery is **broken** (since B is not invariant under the transformation $t \rightarrow -t$)

Remarks

$B \neq 0$ Transverse current **NON-Zero**

$B = 0$ Transverse current **ZERO**

Question: Is there any counterexample where transverse current is non Zero even with $B = 0$

Indeed YES

What is our goal?

$$J \sim \sigma E + \chi E^2$$

Zero for B=0

NON-Zero and significant

Non-linear Hall effect

Liang Fu

Inti Sodemann

Hall Conductivity $\sigma_{ab} \sim \int_k f_0 \Omega_c$ =0 T symmtery ✓

$j \sim E$

Hall-like conductivity $D_{ab} = \int_k f_0 \partial_a \Omega_b$ T symmtery ✓ Inversion symmetry ✗

$(j \sim E^2)$

1st order moment of the Berry curvature → (Berry curvature dipole)

Fermi surface quantity

PRL 115, 216806 (2015)

Non-linear Hall effect

The electric current density

$$j_a = e \int_k f_0(k) v_a$$

where $v_a = \partial_a \epsilon(k) + \epsilon_{abc} \Omega_b \dot{k}_c$

Group velocity — anomalous velocity

The change in momentum

$$\dot{k}_c = -e E_c(t) \qquad E_c(t) = \mathrm{Re}(\varepsilon_c e^{i\omega t})$$

In relaxation time approximation, the distribution of electrons

$$-e E_a \tau \partial_a f + \tau \partial_t f = f_0 - f \qquad f_0 \text{ equilibrium distribution}$$

Expansion of f up to second order eventually gives distribution

$$j_a^0 = \chi_{abc} \varepsilon_b \varepsilon_c^* \qquad j_a^{2\omega} = \chi_{abc} \varepsilon_b \varepsilon_c$$

where $$\chi_{abc} \sim \epsilon_{adc} \int_k f_0 (\partial_b \Omega_d)$$

(Berry curvature dipole)

Non-zero Berry curvature dipole

- Spin orbit coupled linear Dirac Hamiltonian with a tilt

(PRL 115, 216806, 2015)

$$H(k) = \sigma_y k_x - \sigma_x k_y + \alpha k_y$$

Tilting term

- Dirac Hamiltonian without SOC but warping term

(PRL 123, 196403 2019)

$$H(k) = \sigma_y k_x + \sigma_x k_y + (\alpha k_x^2 - \beta k_y^2) + \gamma k_x k_y$$

LETTER

https://doi.org/10.1038/s41586-018-0807-6

Observation of the nonlinear Hall effect under time-reversal-symmetric conditions

Qiong Ma[1,13], Su-Yang Xu[1,13], Huitao Shen[1,13], David MacNeill[1], Valla Fatemi[1], Tay-Rong Chang[2], Andrés M. Mier Valdivia[1], Sanfeng Wu[1], Zongzheng Du[3,4,5], Chuang-Han Hsu[6,7], Shiang Fang[8], Quinn D. Gibson[9], Kenji Watanabe[10], Takashi Taniguchi[10], Robert J. Cava[9], Efthimios Kaxiras[8,11], Hai-Zhou Lu[3,4], Hsin Lin[12], Liang Fu[1], Nuh Gedik[1]* & Pablo Jarillo-Herrero[1]*

Nature 565, 337 (2019)

LETTERS

https://doi.org/10.1038/s41563-019-0294-7

nature materials

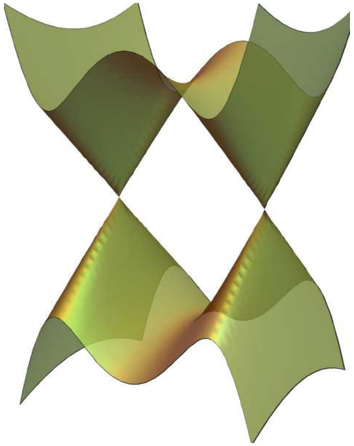
Nonlinear anomalous Hall effect in few-layer WTe_2

Kaifei Kang[1,5], Tingxin Li[1,5], Egon Sohn[2,3,5], Jie Shan[1,2,4]* and Kin Fai Mak[1,2,4]*

Nature Materials, 18, 324 (2019)

We identify

2D Dirac Hamiltonian



$$H(k)=(\alpha k_x^2-\delta_0)\sigma_x+v_F k_y\sigma_y$$

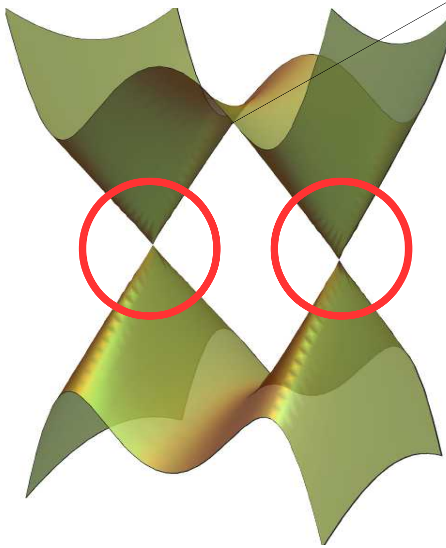
No tilting of Dirac cones

No warping term

Model Hamiltonian

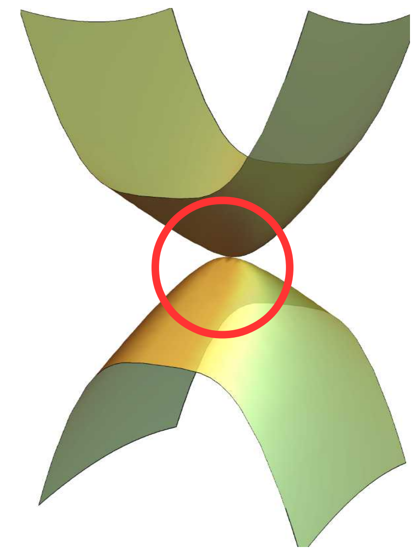
$\delta_0 > 0$

Saddle point



$H(k) = \sigma . k$ $\quad$ $H(k) = \sigma . k$

$\delta_0 = 0$

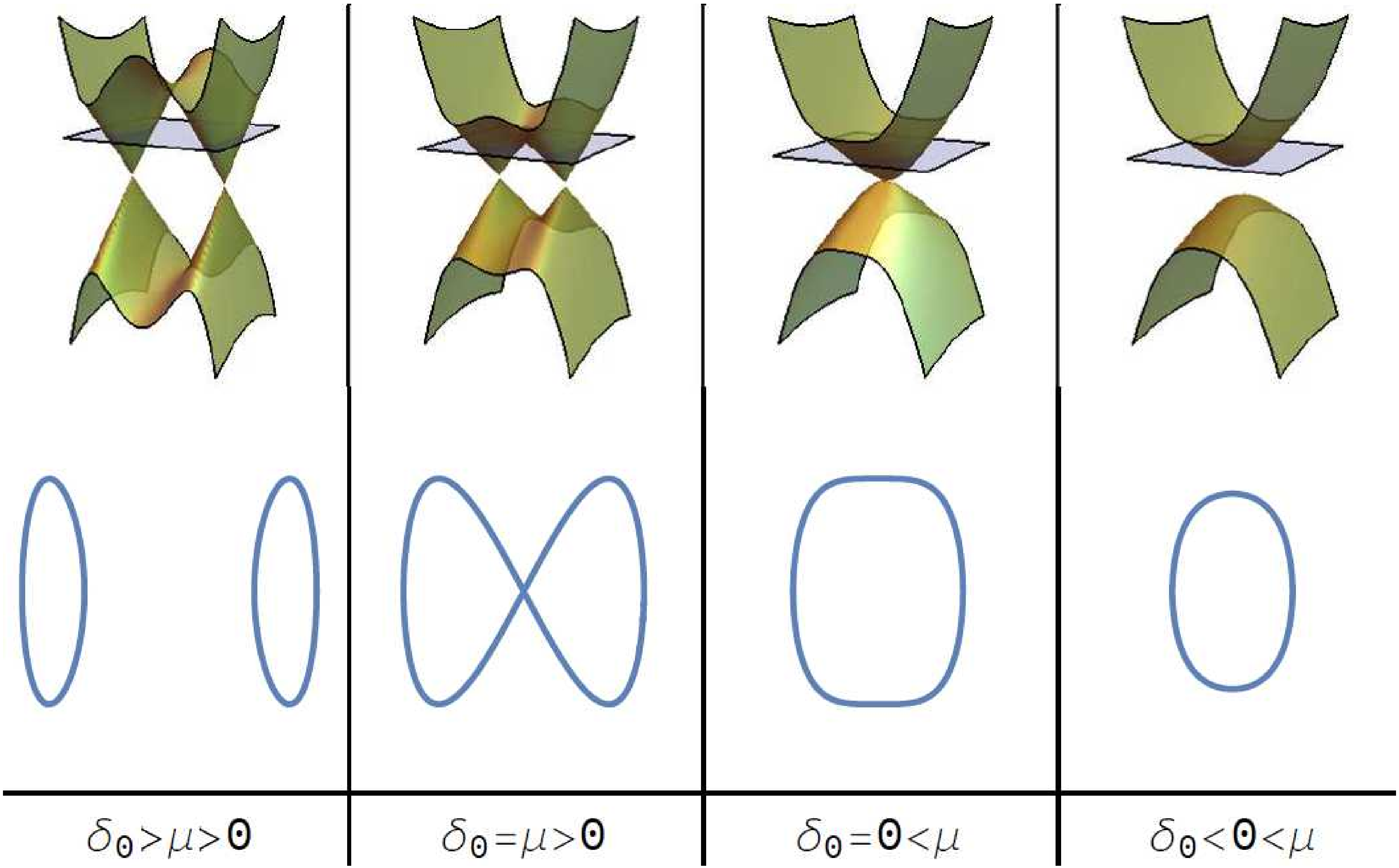


$$H(k) = (\alpha k_x^2 - \delta_0)\sigma_x + v_F k_y \sigma_y$$

$$H(k) = \alpha k_x^2 \sigma_x + v_F k_y \sigma_y$$

Spinless Fermions

Fermi Surface topology



Effective Symmetries

$$H(k) = (\alpha k_x^2 - \delta_0)\sigma_x + v_F k_y \sigma_y$$ Spinless Fermions

Time reversal $T = K$ $T^{-1} H(k) T = H(-k)$

Inversion $P = \sigma_y$ $P^{-1} H(k) P = -H(-k)$

Mirror symmtery $M_x : (x, y) \to (-x, y)$ $= \sigma_0$ $M_y : (x, y) \to (x, -y)$ $= \sigma_x$

Symmetry breaking perturbation

$$\delta H(k) = m \sigma_z$$

Breaks P and M_y

Berry Curvature

$$H(k) = \vec{d}(k).\vec{\sigma}$$

Energy dispersion $E_k = \pm|d(k)|$

$$\Omega = \vec{d}(k).\frac{(\partial_{k_x}\vec{d}(k)\times\partial_{k_y}\vec{d}(k))}{d(k)^3}$$

$$\Omega(k) = \frac{\alpha\beta k_x}{E_k^{\;3}}$$

For isotropic case:

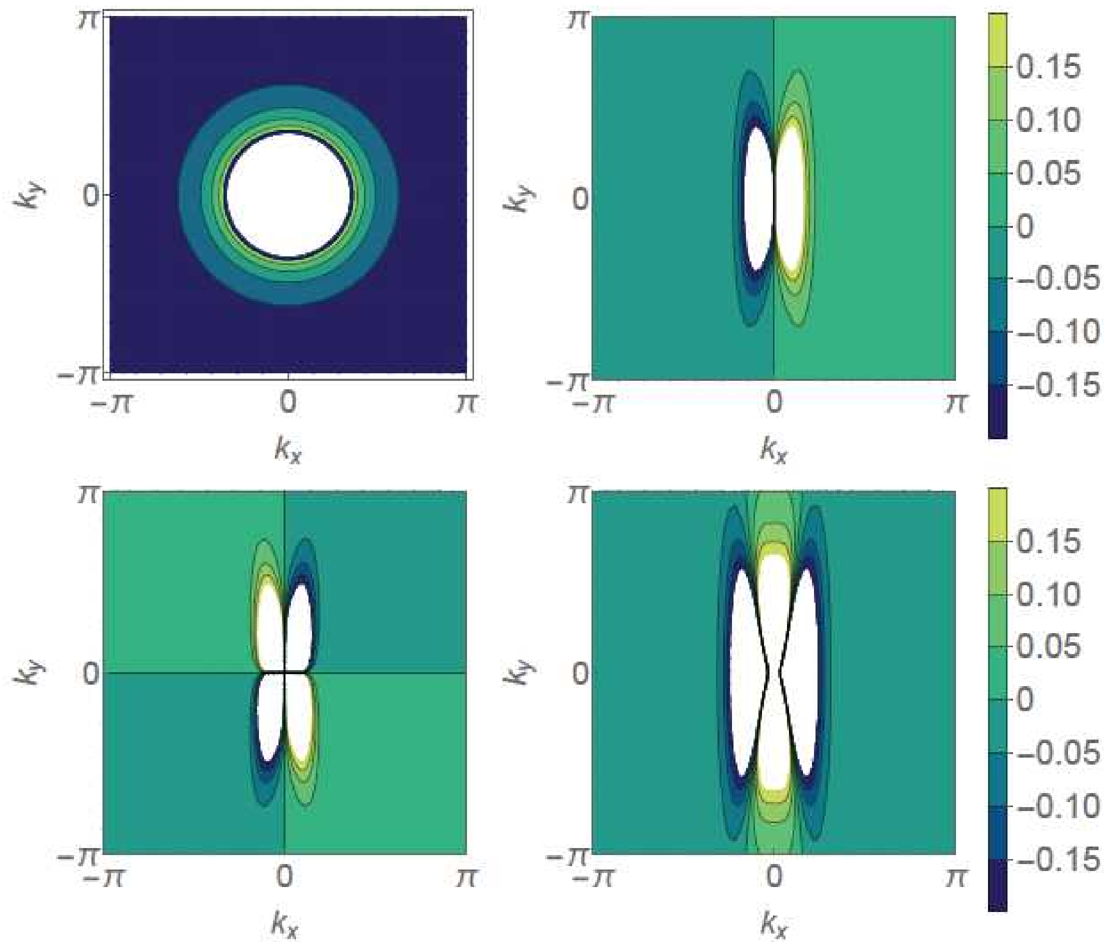
$$\Omega(k) \sim \frac{1}{E_k^{\;3}}$$

This momentum dependence of Berry curvature plays an important role!!

Berry Curvature

isotropic case

$\Omega(k_x, k_y) = -\Omega(-k_x, -k_y)$ TR Symmetry



$\partial_{k_y}\Omega(k_x, k_y) = -\partial_{k_y}\Omega(k_x, -k_y)$ $\partial_{k_x}\Omega(k_x, k_y) = \partial_{k_x}\Omega(-k_x, k_y)$

Berry Curvature Dipole

$$D_{x(y)} = \int_k f_0 \partial_{x(y)} \Omega_z$$

$$D_x = 2\sqrt{m_0}\,\alpha \frac{\sqrt{\mu^2 - m_0^2}}{\mu^3} I(\mu, \delta_0)$$

$$D_y = 0$$

with $$I(\mu, 0) \simeq 3.5\left(\mu^2 - m_0^2\right)^{1/4}$$

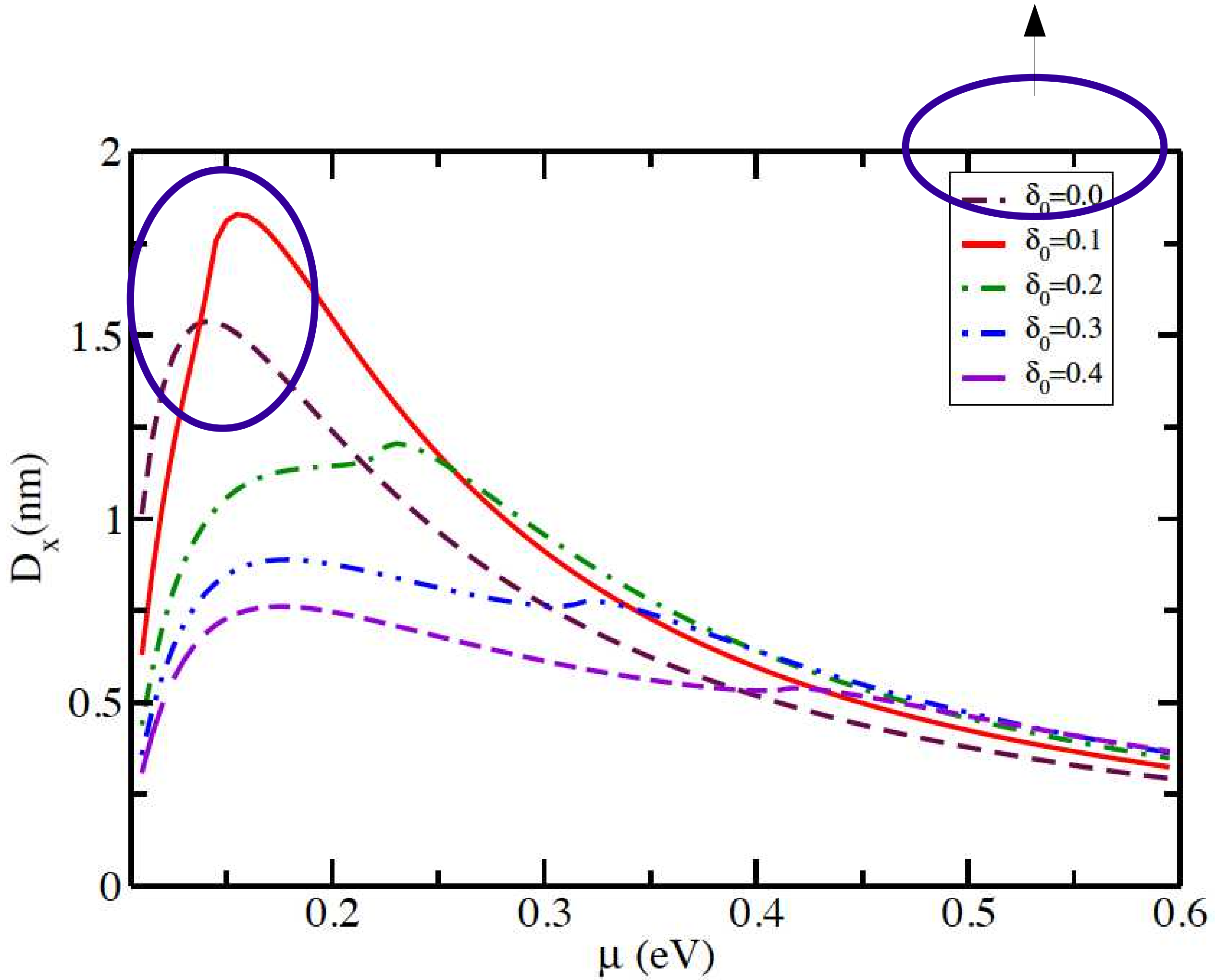
$$H(k) = (\alpha k_x^2 - \delta_0)\sigma_x + v_F k_y \sigma_y$$

D_x independent of v_F

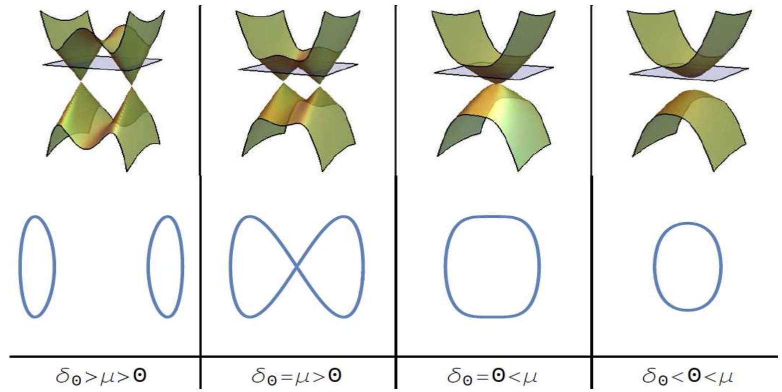
Remark: This fact may guide us to identify materials with low effective masses for sizable Berry curvature dipole

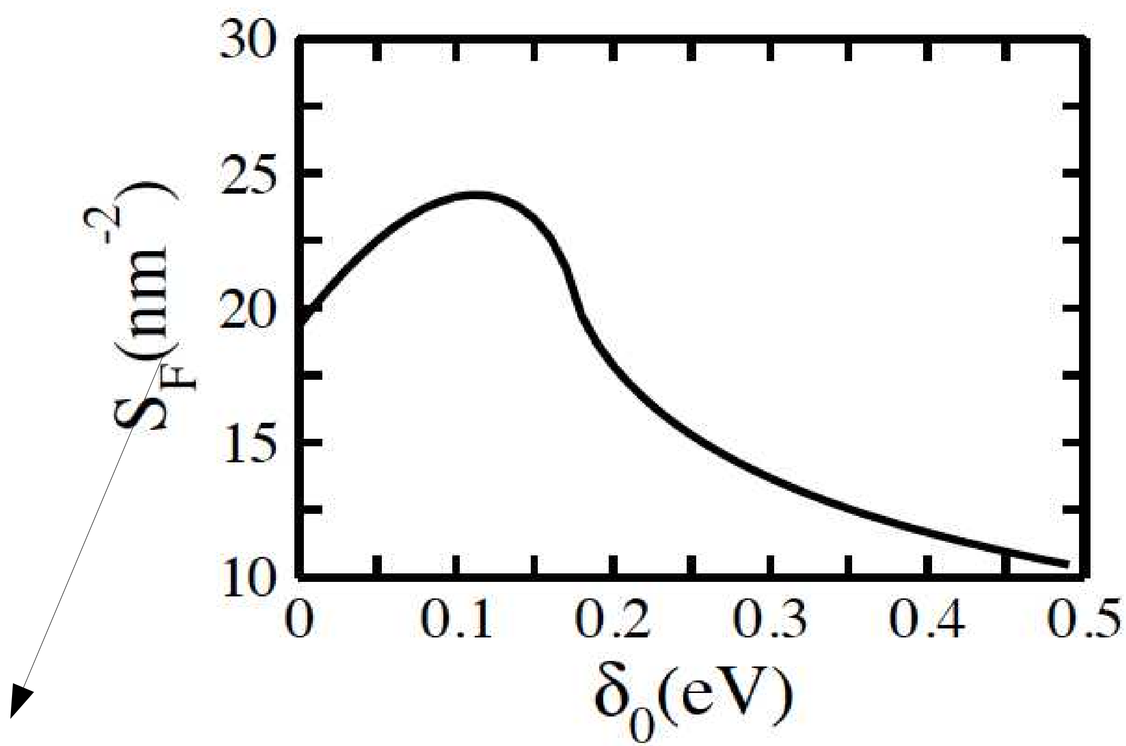
Berry Curvature Dipole

Semi-Dirac point



Samal, Nandy and **KS**,
Phys. Rev. B (L), 103, 201202 (2022)





Area of Fermi Surface

Berry Curvature Dipole

Why $D_x \neq 0$ $D_y = 0$

This is because M_y is broken

$$D_{x(y)} = \int_k f_0 \partial_{x(y)} \Omega_z$$

At low temperature $\int_k f_0 \longrightarrow \int_{\mu < E_k} d^2k$ =0 (due to isotropy)

Thus tilting of Dirac dispersion or warping term lead to $D_{x(y)} \neq 0$

Material	m/m_e	m_0 (eV)	μ (eV)	D_x(nm)
$(TiO_2)_5/(VO_2)_3$	13.6	0.2	0.25	0.27
α-$(BEDT\text{-}TTF)_2I_3$	3.1	0.1	0.15	0.86
Photonic crystals	1.2×10^{-3}	1.0	1.5	13.0

Helicity dependent photocurrent

Origin: Berry Curvature

The current density arising from the anomalous velocity

$$J = 2e\int_k (\dot{k} \times \Omega) f_{\text{noneq}}$$

f_{noneq} → Nonequilibrium distribution function

Key Remark   Anomalous velocity leads to *helicity dependent* photocurrent if

$$\Omega(k) \sim k_x$$

(PRL 105, 026805 (2010))

$$J \sim \chi\left(E_y E_x^* + E_y^* E_x\right)$$

$$+\chi\left(E_y E_x^* - E_y^* E_x\right)$$

Linear Photogalvanic effect

circular photogalvanic effect

Changes sign with the helicity of light

$$\chi \sim e D_x / \hbar$$

$$J \sim \chi \left(E_y E_x^* + E_y^* E_x \right)$$

$$+ \chi \left(E_y E_x^* - E_y^* E_x \right)$$

Linear Photogalvanic effect

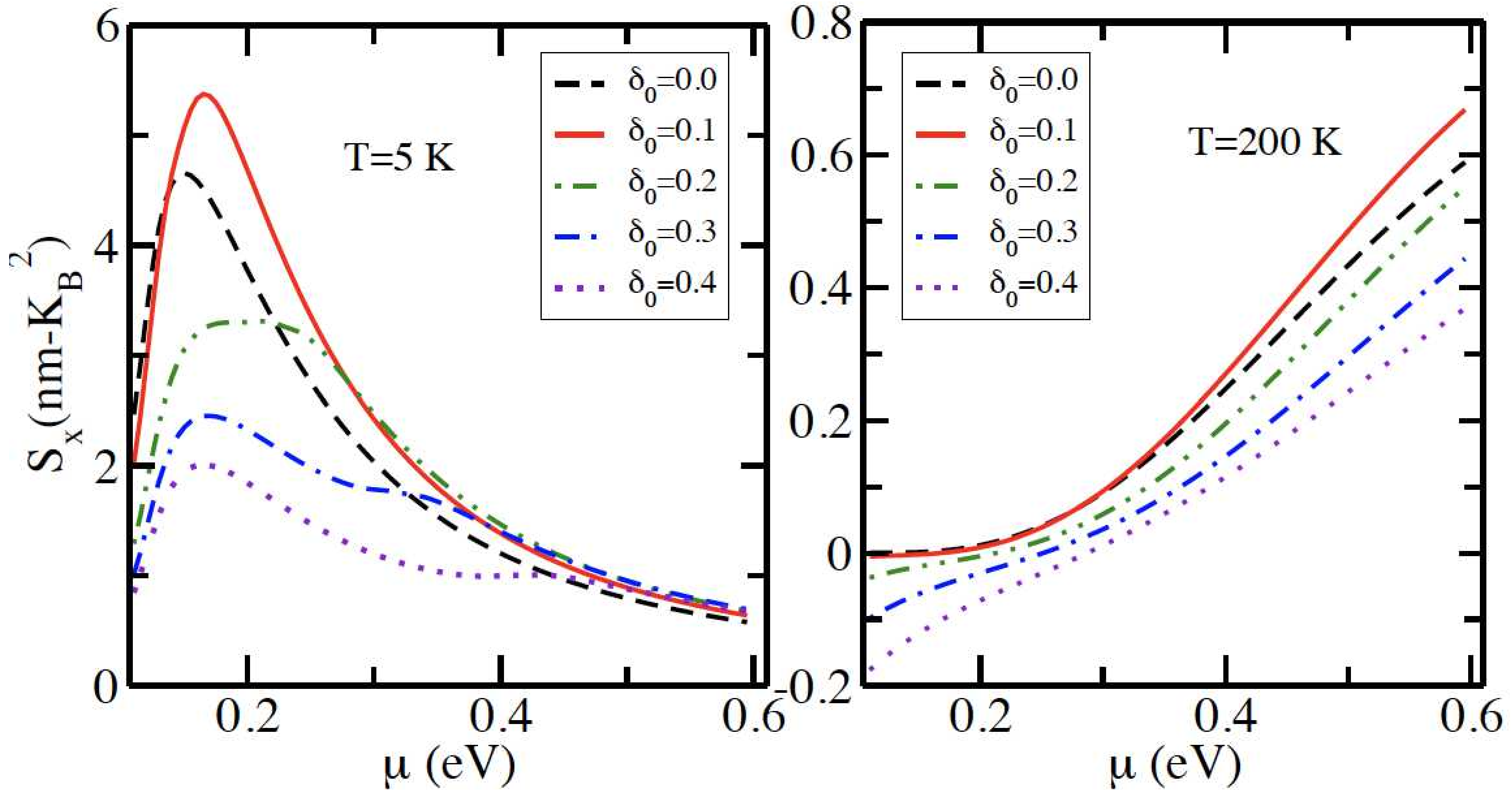
circular photogalvanic effect

Changes sign with the helicity of light

$$\chi \sim e D_x / \hbar$$

Non-linear Nernst effect

$$S_i = \int_k \frac{(E_k - \mu)^2}{T^2} v_i \Omega(k) f_0'$$



Conclusions

We have identified a simple platform to have subtantial Berry curvature dipole

For the particular system discussed shows Berry curvature dipole dominantly depends on the effective mass of the quasiparticle. This may guide us to identity materials with large dipole moment.

The present model naturally can host helicity dependent photocurrent

Q: Can we have significant non-linear Hall response even in the presence of a magnetic field?

Thank You