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Magic numbers in heavy nuclei

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Abstract:-

- The nuclear shell model plays an important role in the pre-formation of alpha particles. It shows the existence of magic numbers, which predicts the nucleus to form a strongly bound closed shell. When the protons number (Z) or the neutrons number (A-Z) is equal to 2,8,20,28,50,82 or 126, which are known as the "Magic Numbers".
- We have developed a model which we named as S-potential by smoothening the potential well inside the nucleus to the top of the coulomb barrier at the outer side of the potential well.
- We have studied the alpha decay of some even-even heavy nuclei and the half-lives obtained were found to be in a very good agreement with the experimental data available. The pre-formation factor and the penetration probability were determined by modifying the Gamow's theory of alpha decay by varying the potential.
- We have studied the microscopic structure properties for $^{202-226}Ra_{88}$ and $^{210-232}Th_{90}$. It was observed that at N=126 the pre-formation factor and penetration probability decreases with increasing N which shows that the closed shell effects plays an important role in the alpha formation process. The S-potential was found to be a better model for obtaining the accurate half-lives of heavy nuclei.

INTRODUCTION:-

- To get a better understanding of nuclear structure and its properties, the detailed study of alpha emission becomes promising. The first theoretical explanation of the α-decay was given by George Gamow in the year 1928[1].The spontaneous emission of nuclei with higher A values compared to α-particles and without neutron emission was proposed by Sandulescu et al[2].
- In this present work the square well model is modified by smoothening the value of the potential inside the nucleus which we will call as S-potential or S-Model (S stands for smoothened potential) for a Gamow like theory i.e. square well potential in one dimension. A simple formula for half-life was derived using the Schrödinger equation in one dimension.
- The $T_{1/2}$were calculated and was found to be in good match with the experimental $T_{1/2}$ available[3]. An exceptional case occurs when the proton number (Z) or the neutron number (A-Z) is equal to 2,8,20,28,50,82 or 126, which are known as the "Magic Numbers". Let us take an example of lead; $^{208}Pb_{82}$ which shows Z=82 and A-Z is 126 which is termed as doubly magic numbers and it forms a strong bound closed shell. We have taken nuclei like, $^{202\text{-}226}Ra_{88}$, $^{210\text{-}232}Th_{90}$,and had calculated the penetration probability and correspondingly their half-life[(3)] .
- An outline was done on the existence of magic numbers and how it is related to half-lives of the nuclei. The result was found to be outstanding and it shows our S-potential to be an authentic tool to detect $T_{1/2}$ with more accuracy. The shell model of the nucleus assumes that the energy structure(energy level of the nucleons) of nucleus is similar to that of an atom. According to this model, the proton and neutron are grouped in shells in the nucleus. The numbers of each shell is limited by Pauli Exclusion Principle.

MODEL

- A particle partially bound within a potential well has a certain probability upon each encounter with the barrier of appearing as a free particle on the other side. Classically a particle cannot overcome a barrier but it tunnels according to quantum mechanics. In spherical symmetric potential there is a discontinuous jump of the potential which tends to be not physical because the force there becomes infinite.
- The detailed analysis of the spherical symmetric potential was studied previously[4].For obtaining a better outcome; a modification was made in the model by smoothening the value of the potential inside the nucleus which we termed as S-potential or S-Model[5].The modified model is sketched and shown in Fig.1 in next slide.

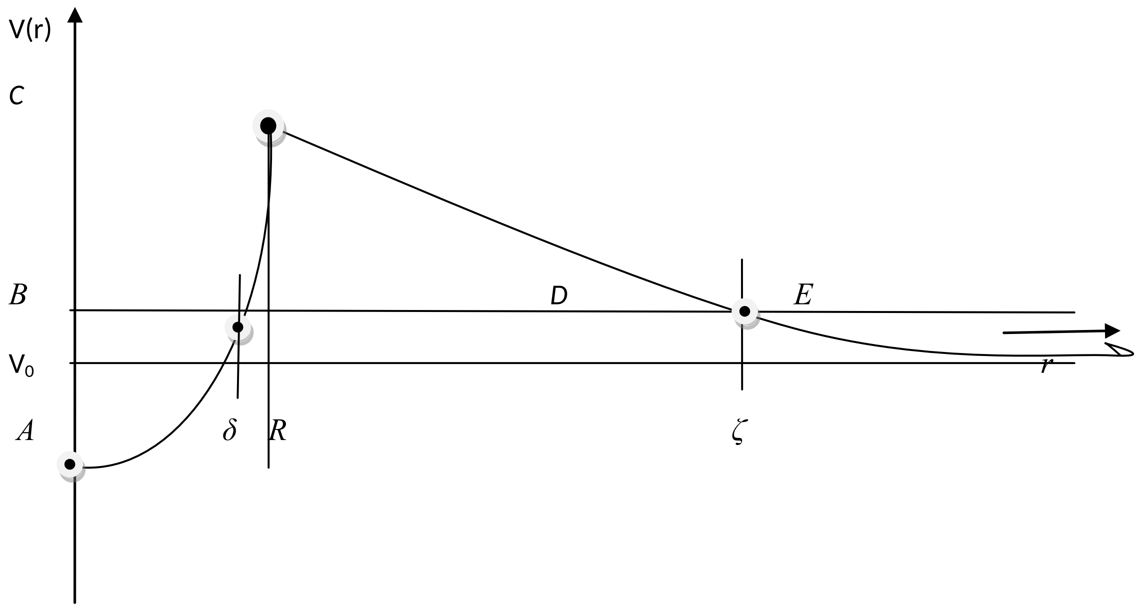


Figure 1: S-Model on the shape of V (r); the inside potential has been assumed to be of the (spherical) harmonic kind.

- The force is absent on the emitted cluster particle when it is present in the range $0 < r < \delta$ followed by a force of attraction present in the range of $\delta < r < R$.
- The coulomb repulsive force becomes active outside the barrier where $r \to R$. This approach would be closer to the actual physical process and shall give more accurate results.

Mathematical Calculations:-

The potential function is now:

$$V(r) = \frac{1}{2} m_c \omega r^2 \qquad \delta \leq r \leq R$$

$$= \frac{2 z_1 z_2 e^2}{r} \qquad R \leq r \leq \zeta$$

e Gamow’s factor “G”, calculated to be:

$$G = \sqrt{\frac{2m_c}{\hbar^2}} \left[\int_{\delta}^{R} dr \sqrt{\frac{1}{2} m_c \omega r^2 - E} + \int_{R}^{\zeta} dr \sqrt{\frac{2Ze^2}{r} - E} \right] \text{..........} \ (1)$$

$$G = \sqrt{\frac{2m_c}{\hbar^2}} \left\{ \frac{R}{2} \sqrt{\frac{1}{2} m_c \omega^2 R^2 - E} - \frac{\delta}{2} \sqrt{\frac{1}{2} m_c \omega^2 r^2 - E} + \frac{E}{\omega \sqrt{2 m_c}} \ln \left[\frac{\delta m_c \omega^2 + \omega \sqrt{2 m_c} \sqrt{\frac{1}{2} m_c \omega^2 \delta^2 - E}}{R m_c \omega^2 + \omega \sqrt{2 m_c} \sqrt{\frac{1}{2} m_c \omega^2 R^2 - E}} \right] \right\} +$$

$$\left\{ \frac{2 Z_1 Z_2 e^2 \pi^2}{h v_c} - \frac{8 \pi e}{h} \sqrt{\frac{2 \mu Z_1 Z_2 r_0}{2}} \right\} \text{...........} \ (2)$$

Equation (1) shows the estimated decay rate after solving the model using Gamow's theory of alpha decay. Then the second term appears to be the correction in decay rate with respect to Gamow's result. The penetration probability is given by P = e^{-2G}

By substituting the value of equation (2) in the above expression As decay constant; $\lambda = nP \ldots$ (3)

We can write that the decay constant by the equation,

$\lambda = nPP_0$....(4) where: n is the frequency of assault. P is the penetration probability. P_0 is known as the Pre-formation factor[12].

In general it is seen that the mass of the alpha particle is lighter.

$$\ln\lambda = \ln n - 2\sqrt{\frac{2m_c}{\hbar^2}}\left[\frac{R}{2}\sqrt{\frac{1}{2}m_c\omega^2R^2 - E} - \frac{\delta}{2}\sqrt{\frac{1}{2}m_c\omega^2 r^2 - E} + \frac{E}{\omega\sqrt{2m_c}}\ln\left(\frac{\delta m_c\omega^2 + \omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2\delta^2 - E}}{Rm_c\omega^2 + \omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2R^2 - E}}\right)\right.$$

$$\left. + \frac{2Z_1Z_2e^2\pi^2}{hv_c} - \frac{8\pi e}{h}\sqrt{\frac{2\mu Z_1Z_2r_0}{2}}\right] + \ln P_0 \ldots\ldots\ldots (5)$$

The above equation is the modified value of λ of emitted particles. $T_{1/2}$ is defined as the time taken by a nucleus to reduce to half of its original values.

It can be calculated by the formula: $log_{10}T_{1/2} = log_{10}0.693 - log\ \lambda$............ (6)

By using equations (5) and (6) the **decay constants** and **half lives** of different nuclei can be calculated respectively.

Tabulations and Graphical analysis

Table 1: Comparison chart of experimental Log $T_{1/2}$(sec) with the calculated Log $T_{1/2}$(sec) for S-Model for $^{202-226}Ra_{88}$ nuclei.

Nuclei	Daughter	Q_α (MeV)	V m/s	*n* *sec*$^{-1}$	λ *sec*$^{-1}$	Ln P	Ln P_0	Log $T_{1/2}$ S-Model sec	Log $T_{1/2}$ (exp) Sec
$^{202}Ra_{88}$	$^{198}Rn_{86}$	8.02	1.979×10^{7}	1.471×10^{21}	2.630×10^{2}	-49.102	3.735	-2.405	-2.58
$^{204}Ra_{88}$	$^{200}Rn_{86}$	7.636	1.931×10^{7}	1.378×10^{21}	1.174×10^{1}	-51.899	3.477	-1.066	-1.23
$^{206}Ra_{88}$	$^{202}Rn_{86}$	7.42	1.903×10^{7}	1.354×10^{21}	2.889×10^{0}	-53.587	3.761	-0.62	-0.62
$^{208}Ra_{88}$	$^{204}Rn_{86}$	7.27	1.884×10^{7}	1.336×10^{21}	4.907×10^{-1}	-54.729	3.129	0.356	0.15
$^{210}Ra_{88}$	$^{206}Rn_{86}$	7.16	1.869×10^{7}	1.321×10^{21}	1.909×10^{-1}	-55.538	3.018	0.730	0.56
$^{212}Ra_{88}$	$^{208}Rn_{86}$	7.03	1.852×10^{7}	1.305×10^{21}	4.900×10^{-2}	-56.618	2.758	1.318	1.15
$^{214}Ra_{88}$	$^{210}Rn_{86}$	7.27	1.880×10^{7}	1.320×10^{21}	2.759×10^{-1}	-52.374	2.451	0.511	0.40
$^{216}Ra_{88}$	$^{212}Rn_{86}$	9.53	2.156×10^{7}	1.509×10^{21}	3.809×10^{6}	-38.484	2.607	-6.76	-6.74
$^{218}Ra_{88}$	$^{214}Rn_{86}$	8.55	2.042×10^{7}	1.425×10^{21}	2.696×10^{4}	-43.990	3.693	-4.609	-4.59
$^{220}Ra_{88}$	$^{216}Rn_{86}$	7.60	1.925×10^{7}	1.339×10^{21}	3.809×10^{1}	-50.052	4.046	-1.76	-1.74
$^{222}Ra_{88}$	$^{218}Rn_{86}$	6.68	1.805×10^{7}	1.252×10^{21}	1.782×10^{-2}	-58.918	4.494	1.581	1.59
$^{224}Ra_{88}$	$^{220}Rn_{86}$	5.79	1.680×10^{7}	1.161×10^{21}	2.045×10^{-6}	67.822	5.066	5.231	5.53
$^{226}Ra_{88}$	$^{222}Rn_{86}$	4.87	1.541×10^{7}	1.062×10^{21}	1.290×10^{-11}	50.736	5.731	10.681	10.73

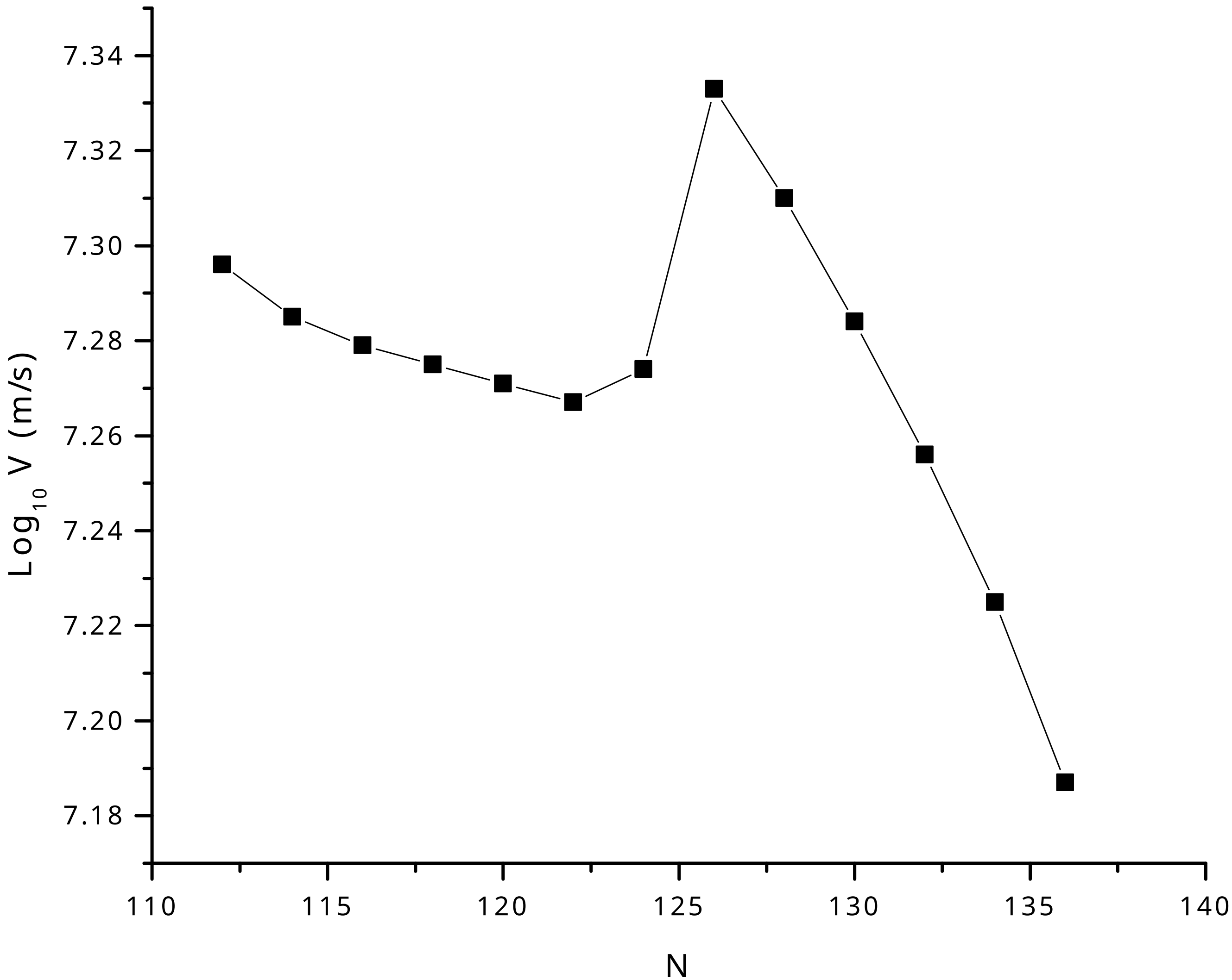


Figure 2: Variation of neutron number N With logarithmic values of velocity

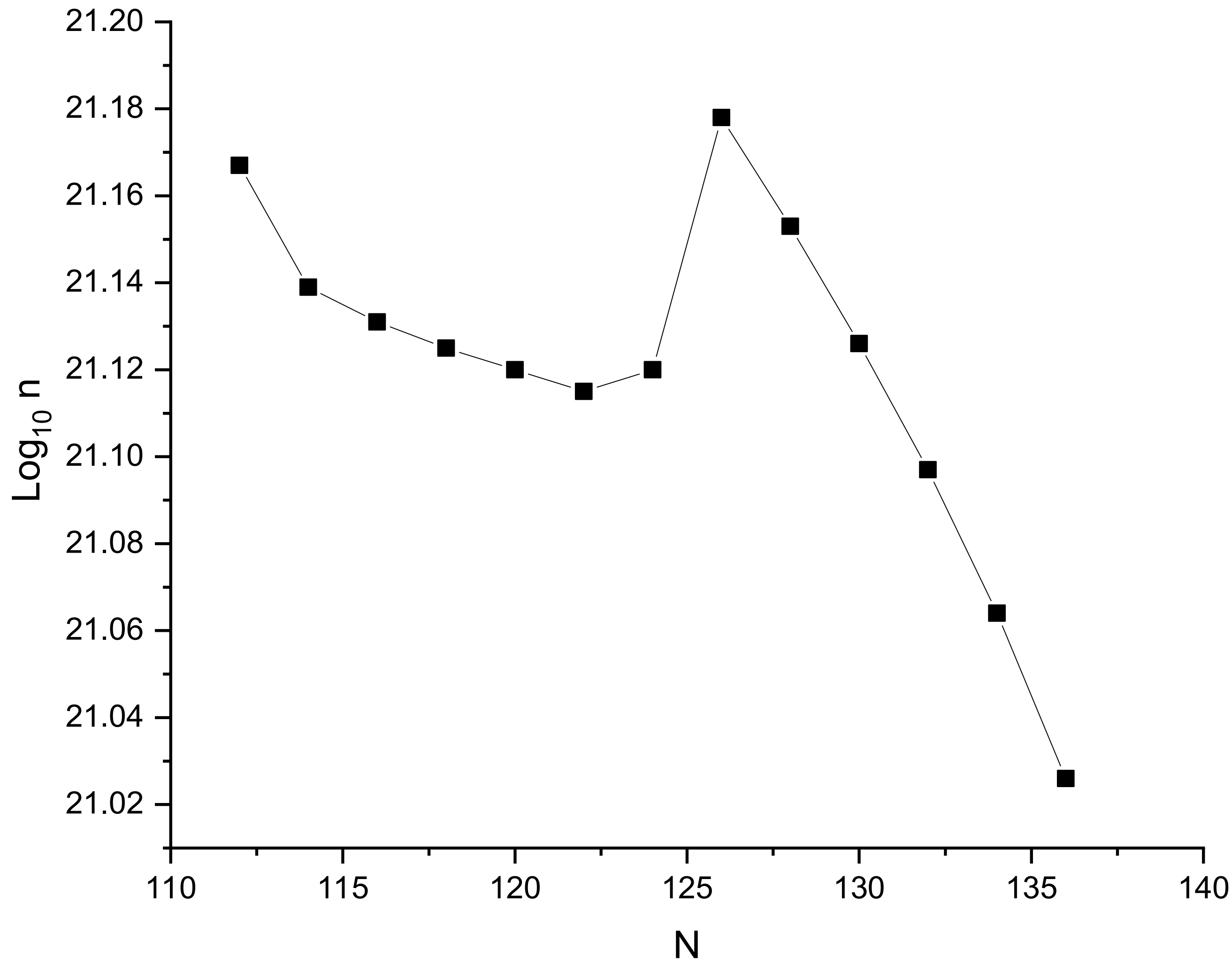


Figure 3: Variation of neutron number N with logarithmic value of frequency.

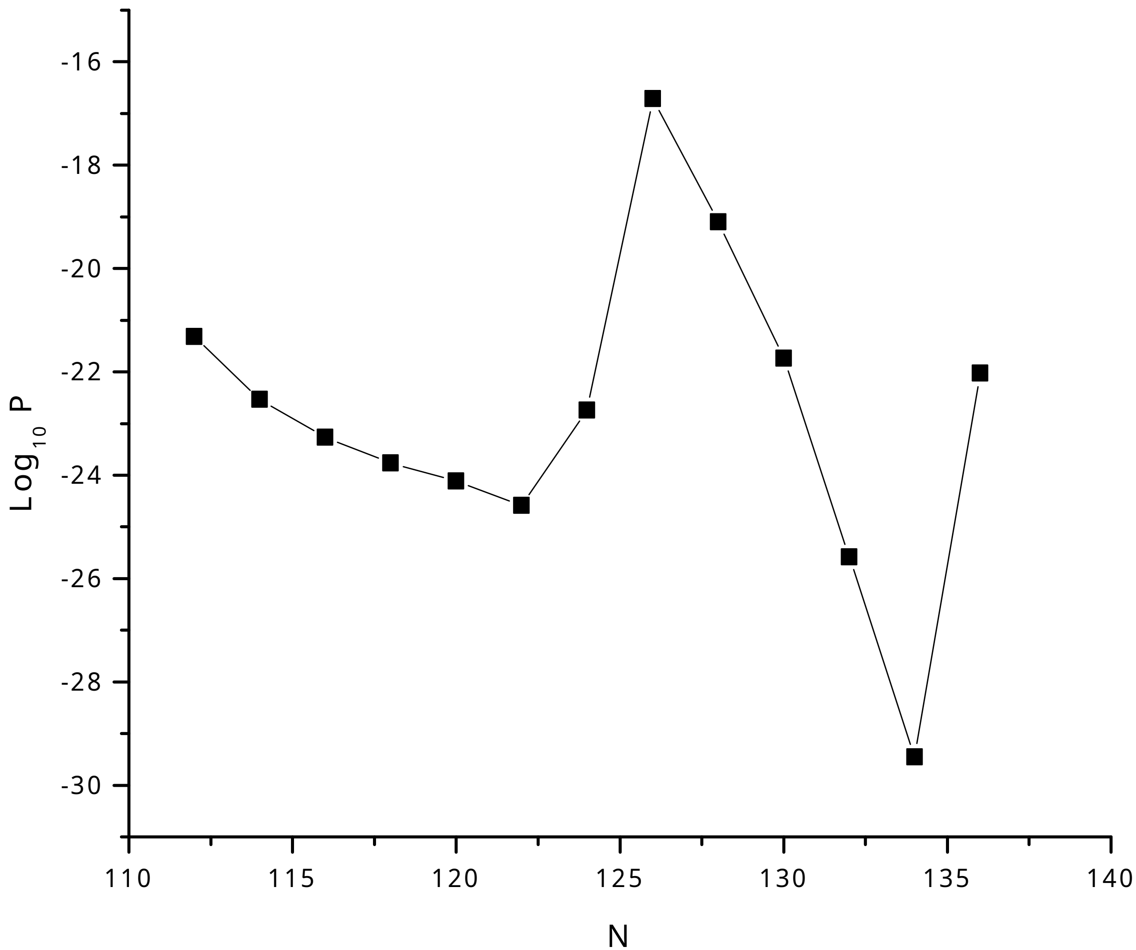


Figure 4: Dependence of penetration probability P with N

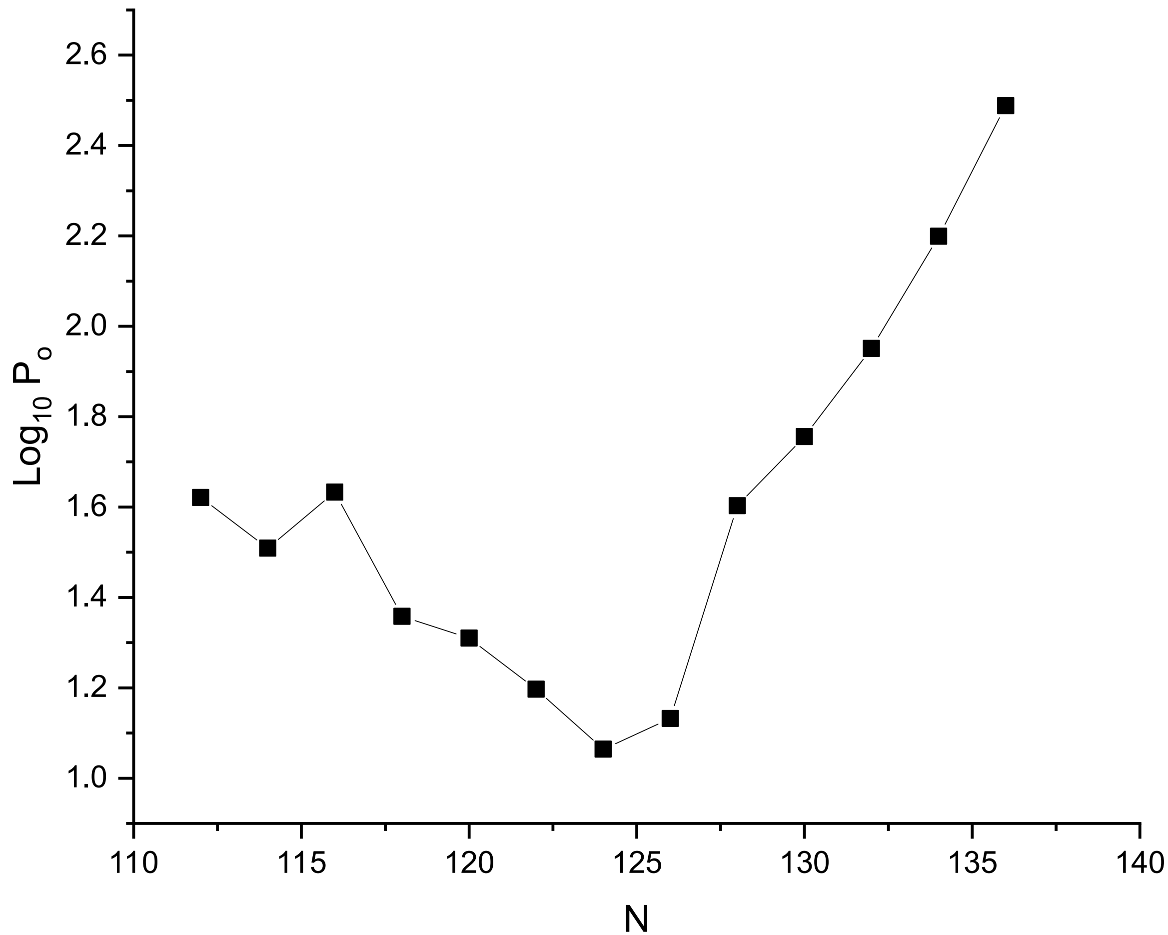


Figure 5: Variation of neutron number N With logarithmic values of Pre-formation factor of alpha particle.

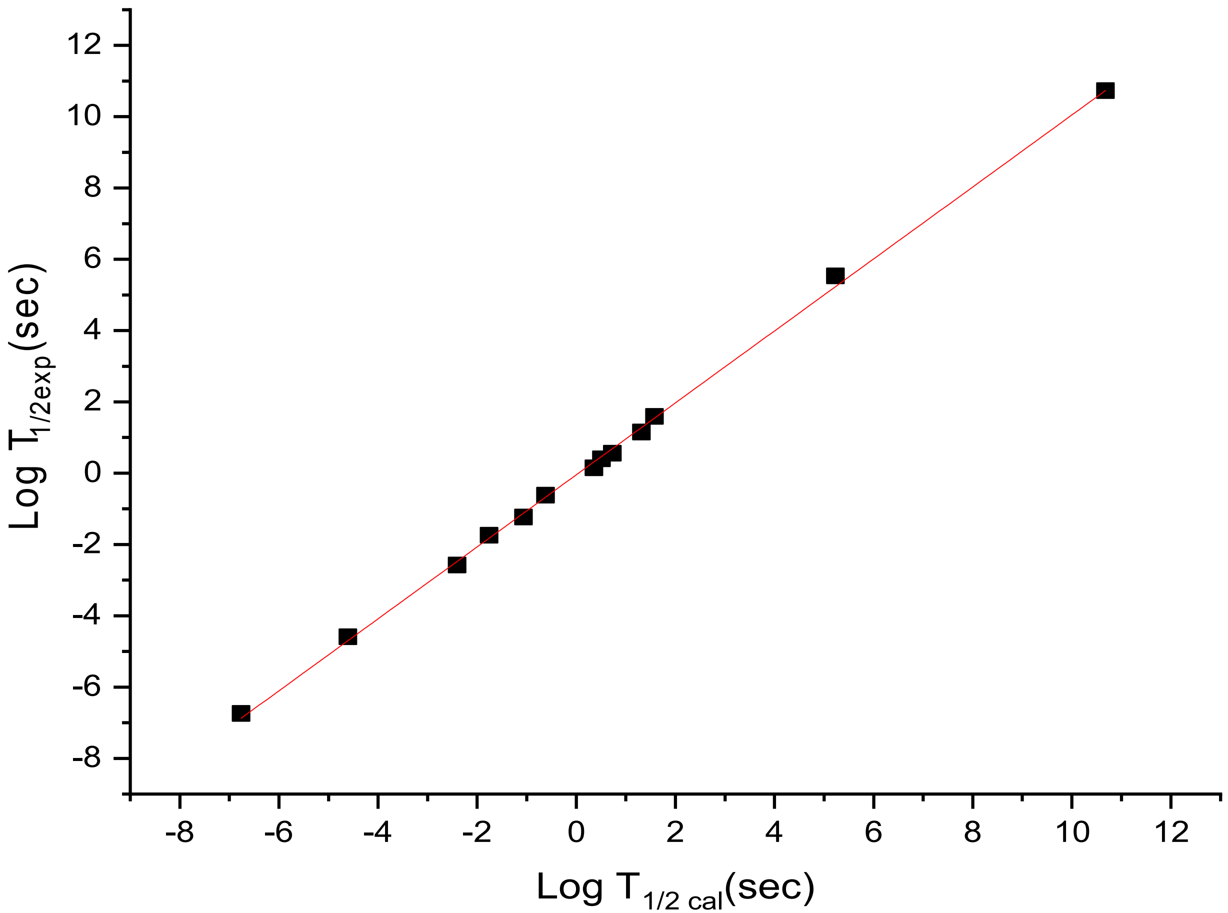
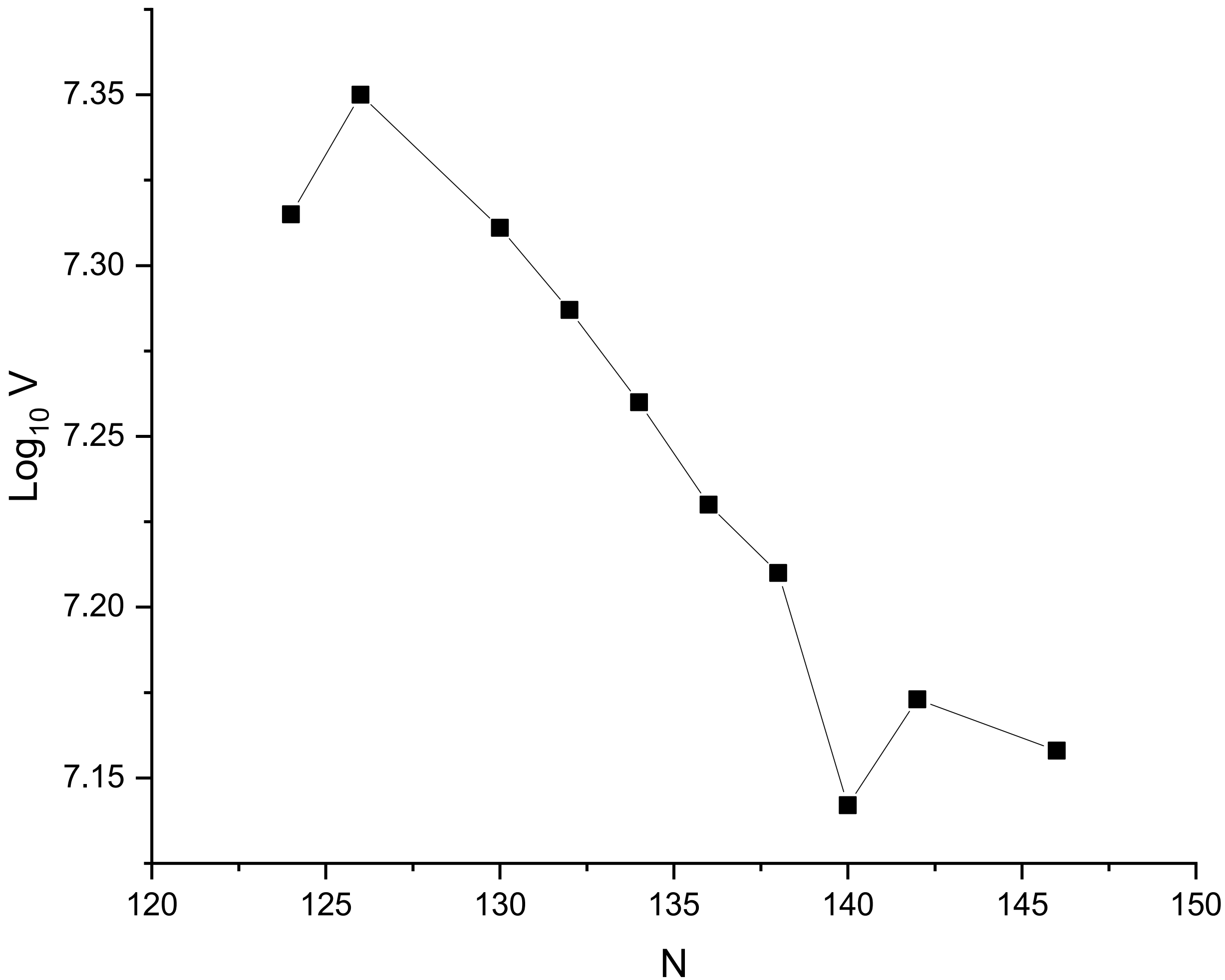


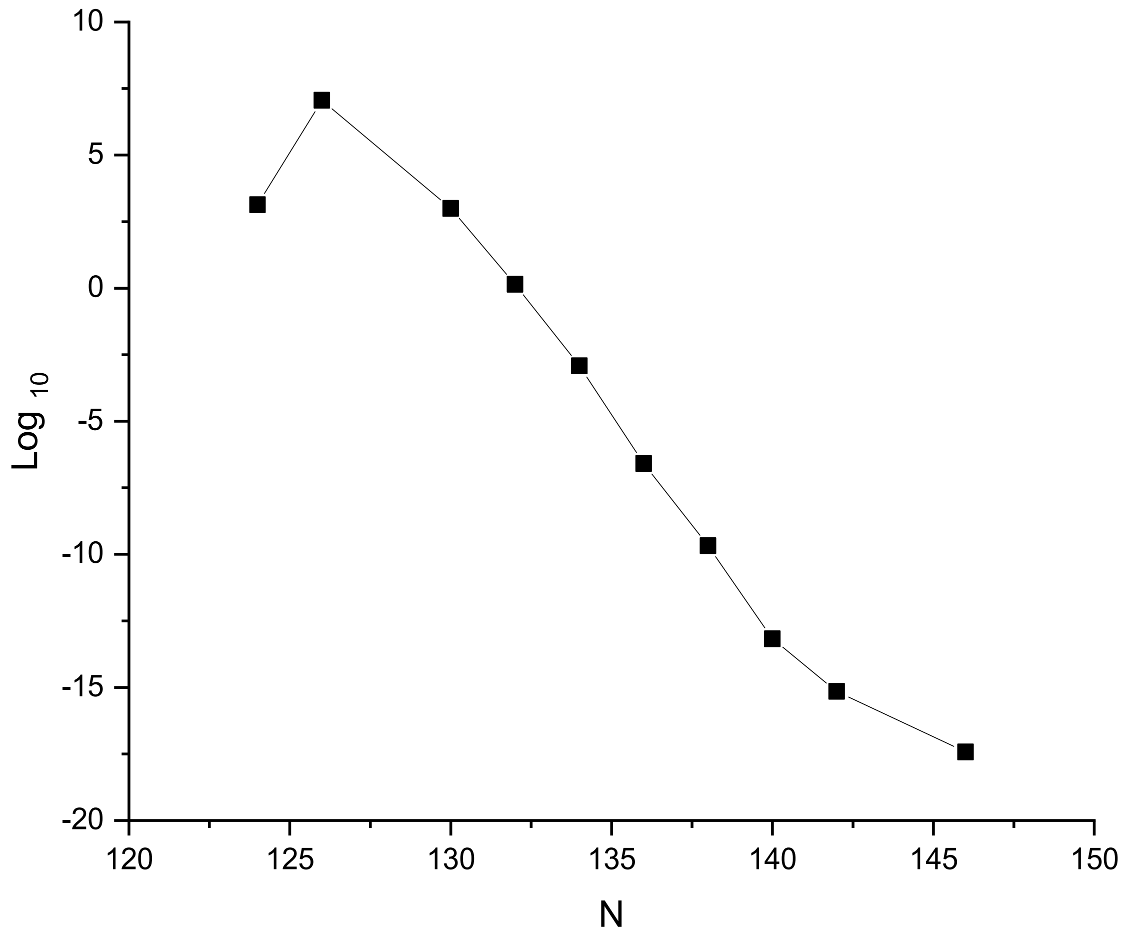
Figure 6: Linear relation between the Experimental and calculated $T_{1/2}$ of $^{202-226}Ra_{88}$ nuclei.

Table 3: Comparison chart of experimental Log $T_{1/2}$(sec)with the calculated Log $T_{1/2}$(sec)for S-Model for $^{218-238}U_{92}$ nuclei.

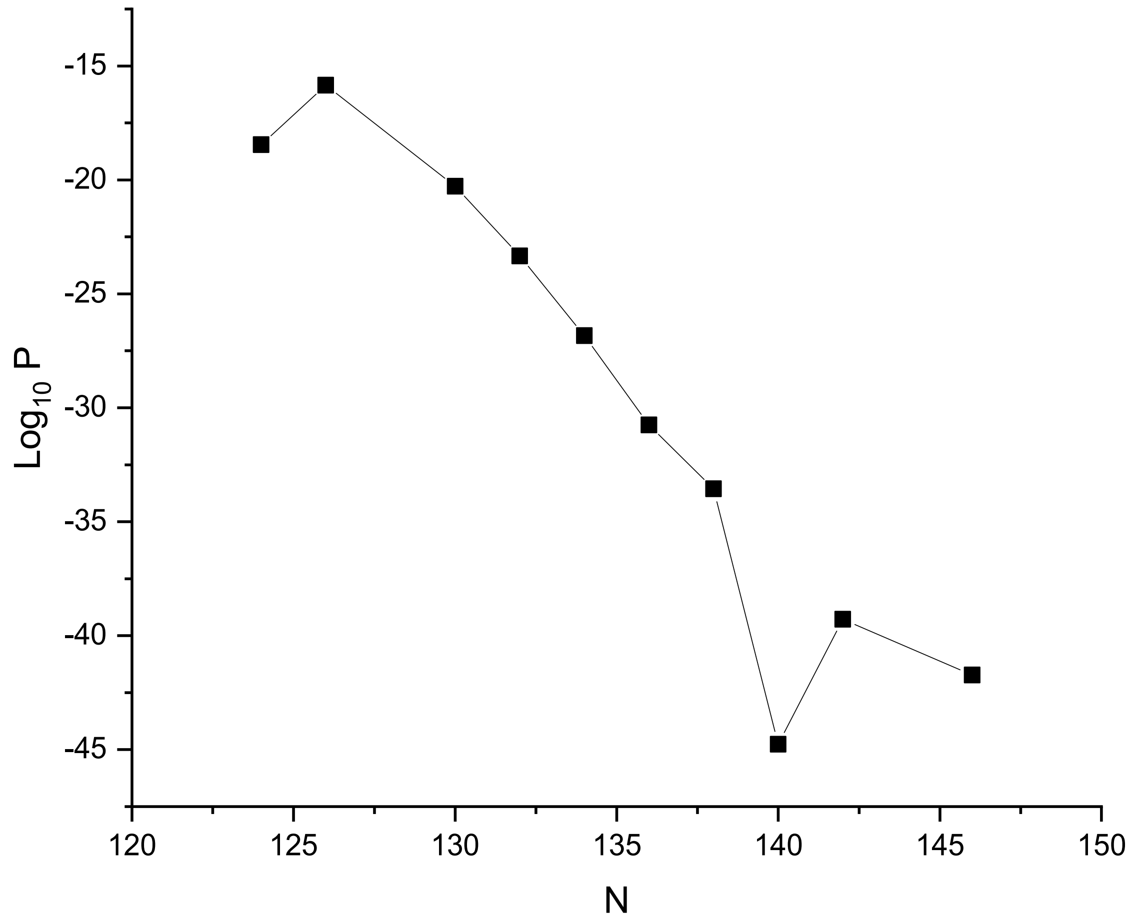
Nuclei	Daughter	$Q_{\alpha(MeV)}$	V (m/s)	*n* *sec*$^{-1}$	λ *sec*$^{-1}$	Ln P	Ln P_0	Log $T_{1/2}$ S-Model sec	Log $T_{1/2}$ (exp) Sec
$^{218}U_{92}$	$^{214}Th_{90}$	8.773	2.069×10^7	1.444×10^{21}	1.358×10^3	-42.492	-0.181	-3.30	-3.29
$^{220}U_{92}$	$^{216}Th_{90}$	10.30	2.241×10^7	1.559×10^{21}	1.156 × 107	-36.488	2.499	-7.387	-7.22
$^{224}U_{92}$	$^{220}Th_{90}$	8.620	2.050×10^7	1.417×10^{21}	9.904 × 102	-46.715	3.717	-3.434	-3.15
$^{226}U_{92}$	$^{222}Th_{90}$	7.701	1.937×10^7	1.340×10^{21}	1.386 × 100	-53.747	3.620	-0.313	-0.30
$^{228}U_{92}$	$^{224}Th_{90}$	6.80	1.820×10^7	1.251×10^{21}	1.205×10^{-3}	-61.802	4.686	2.750	2.76
$^{230}U_{92}$	$^{226}Th_{90}$	5.99	1.70×10^7	1.171×10^{21}	2.575×10^{-7}	-70.838	5.884	6.129	6.43
$^{232}U_{92}$	$^{228}Th_{90}$	5.41	1.624×10^7	1.109×10^{21}	2.093×10^{-10}	-77.268	5.364	9.54	9.52
$^{234}U_{92}$	$^{230}Th_{90}$	4.86	1.387×10^7	0.945×10^{21}	6.620×10^{-14}	-103.10	23.275	12.732	13.02
$^{236}U_{92}$	$^{232}Th_{90}$	4.57	1.492×10^7	1.013×10^{21}	7.093×10^{-16}	-90.471	6.050	14.73	14.99
$^{238}U_{92}$	$^{236}Th_{90}$	4.27	1.442×10^7	0.977×10^{21}	3.722×10^{-18}	-96.096	6.457	16.984	17.27



<u>Figure 7: Variation of neutron number N With logarithmic values of velocity.</u>



<u>Figure 8: Variation of neutron number N with logarithmic value of frequency.</u>



<u>Figure9: Dependence of penetration probability P with N</u>

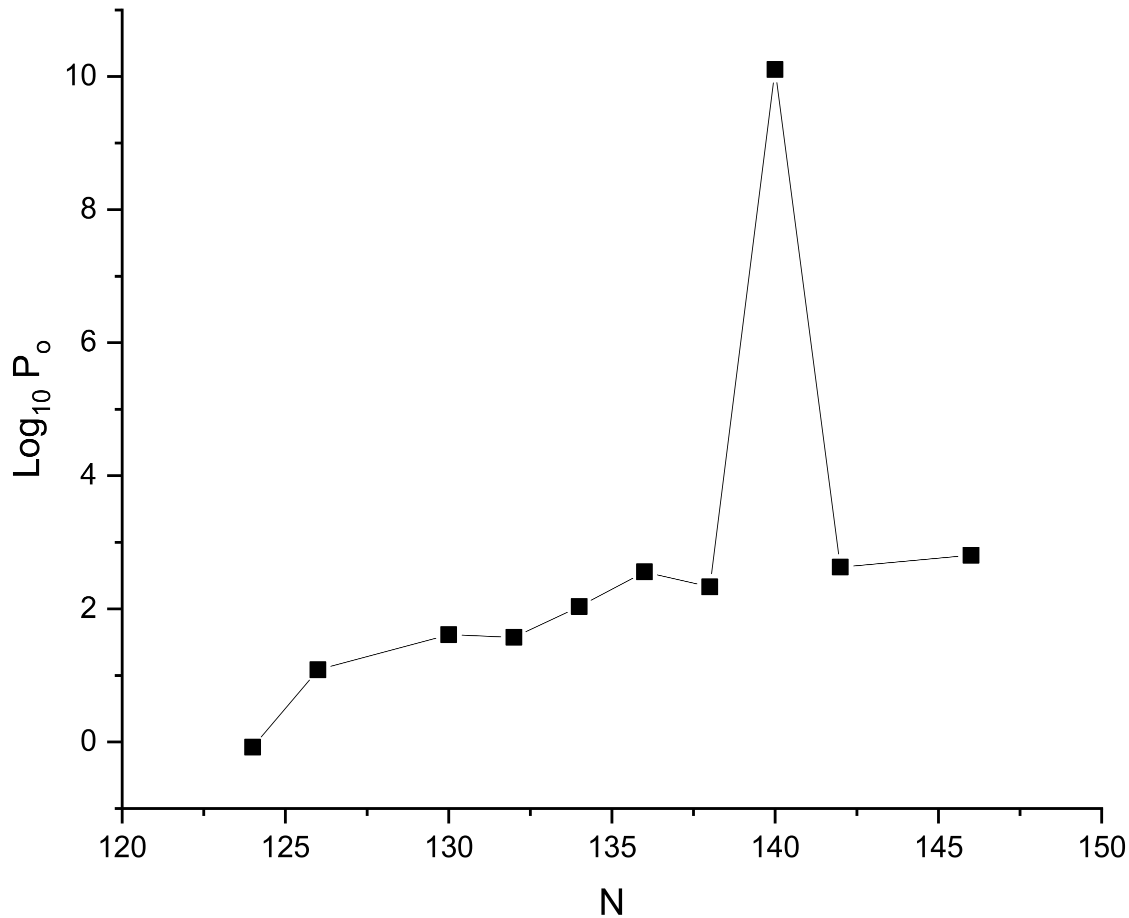


Figure 10: Variation of neutron number N With logarithmic values of Pre-formation factor of alpha particle.

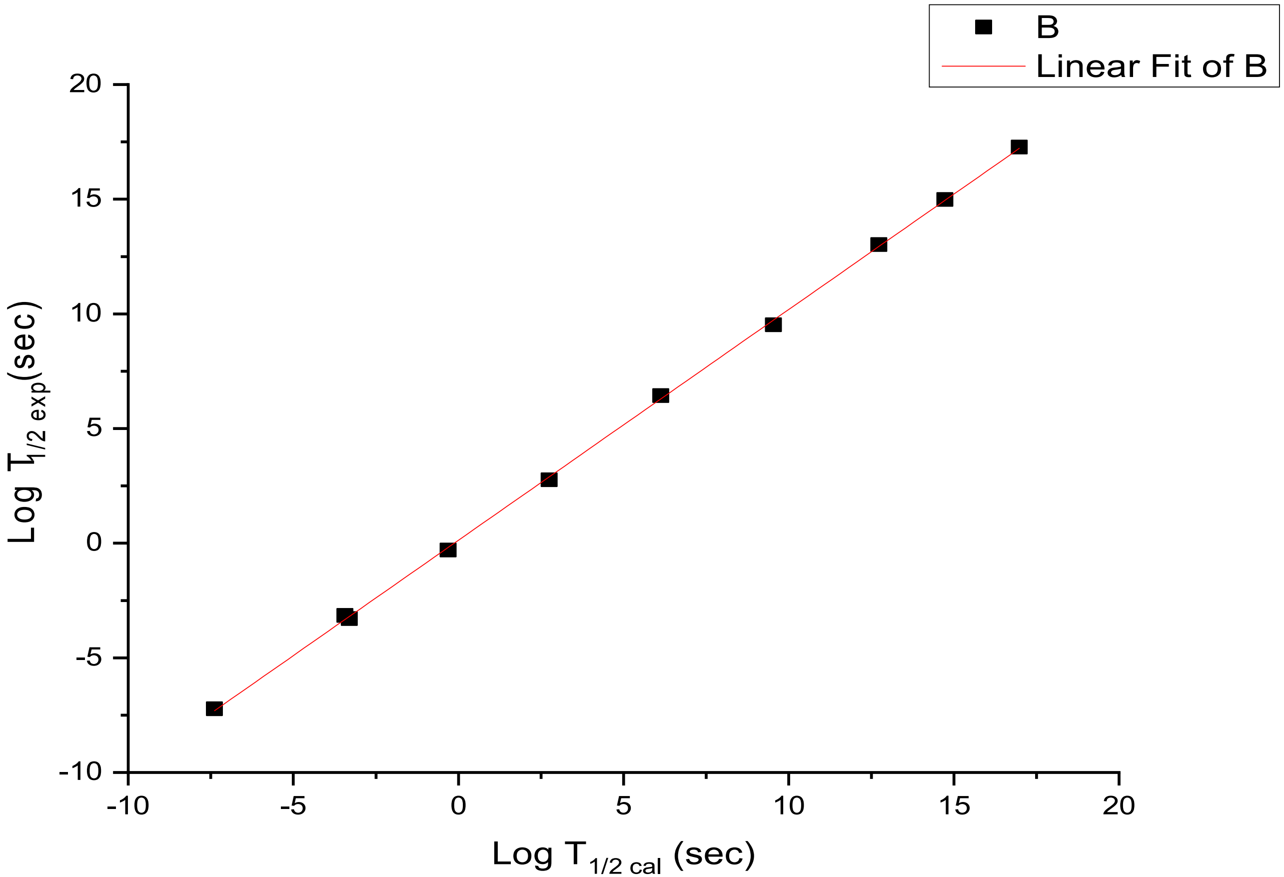


Figure 11: Linear relation between the Experimental and calculated $T_{1/2}$ of $^{218\text{-}238}U_{92}$

Results and Discussion

- We have already studied the detailed analysis of the simple spherical symmetric potential[4]. We have seen that in simple spherical symmetric potential which was used by Gamow there is a potential immediately outside becomes unphysical. Therefore the potential was smoothened by taking the term δ and ζ .
- We modified the potential which we call the S-potential. With the help of pre-formation factor and the other details the half lives have been calculated[17,18]. In the model given in Figure 1, we observe that the alpha particle moves with kinetic energy “E” (MeV) inside the nucleus. Table 1 & 2 shows parent and the emitted daughter nuclei, the kinetic energy” (MeV), logarithmic value of penetration probability, pre-formation factor and logarithmic values of $T_{1/2}$ by both experimental and calculated by S –Model.

Results and Discussion

- We have taken nuclei like $^{202\text{-}226}Ra_{88}$ and $^{218\text{-}238}U_{92}$ for the extensive study of their structural properties and the characteristics features of alpha decay. The nuclei radius was kept fixed at 1.2 fm throughout the calculations. Figure 2 and 7 shows the variation of neutron number (A-Z) with the logarithmic value $Log_{10}v$ (m/sec) of nuclei $^{202\text{-}226}Ra_{88}$ and $^{218\text{-}238}U_{92}$ respectively. Fig 2 the velocity starts decreasing as the neutron number increases, as N becomes 126 we can observe an increase in velocity and again a gradual fall can be noticed with increasing N whereas in Fig 7 we can observe increase in velocity at two points at N=126 and N=142 which shows the existence of magic number and sub magic number (N=142) and the stability of the U nuclei.

Results and Discussion

- Figure 3 and 8 show the variation of neutron number (A-Z) with the logarithmic value $Log_{10}n$ (sec^{-1}) of nuclei $^{202-226}Ra_{88}$and $^{218-238}U_{92}$ respectively. The shape of the curve depicts that $Log_{10}n$ goes on decreasing smoothly and at N=126 we can see an increase in assault frequency, and again an decrease can be noticed as the neutron number starts increasing which tells that frequency of alpha particle can also proof the existence of shell effect.

- Figure 4 and 9 shows the dependence of Penetration Probability with N. In fig 4 the $Log_{10}P$ decreases gradually with increasing N but at point N=126 an increase in $Log_{10}P$ can be noticed whereas in fig 9 an increase can be observed at N=126 & 142 which shows the existence of magic and sub-magic number (N=142 sub magic number).It also confirms that alpha decay $T_{1/2}$ is decided by penetration probability ‘P’.

Results and Discussion

- In order to study about the microscopic structural properties of Ra and U, a graph is plotted in between pre-formationP_0 factor with neutron number N as shown in figure 5 & 10 respectively. We can observe that in fig 5,$Log_{10}P_0$ decreases with increasing N till it reaches to the spherical closed shell N=126, at this point $Log_{10}P_0$ seems to be minimum and $Log_{10}P_0$ fastly increases with N whereas for U in fig 10, $Log_{10}P_0$ is minimum at N=126.As a result it can be said that the closed shell effects plays an important role in the alpha formation process. The half life calculation depends on the product of pre-formation and the penetration probability.
- Figure 6 & 11 shows the variation of experimental and calculated $Log_{10} T_{1/2}$(sec) for Ra and U nuclei. The graph shows linearity which exhibit the fact that experimental and calculated $Log_{10}T_{1/2}$(sec)are in very good agreement with each other and this makes S-potential valid.

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