XXV International Baldin Seminar
on High Energy Physics Problems
Relativistic Nuclear Physics & Quantum Chromodynamics

September 18 - 23, 2023, Dubna, Russia

On a signature of phase transition in heavy ion nuclear matter

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Contents

- Introduction
- z-Scaling (ideas, definitions,…)
- Properties of data z-presentation in pp and AuAu collisions at RHIC
- Fractal entropy and fractal cumulativity
- Anomaly of specific heat and fractal entropy as signatures of phase transition
- Summary

Motivation & Goals

Systematic analysis of inclusive cross sections of particle production in p+p, p+A and A+A collisions to search for general features of hadron and nucleus structure, constituent interaction and fragmentation process over a wide scale range.

z-Scaling is a tool in high energy physics

Development of z-scaling approach for description of processes with unpolarized and polarized particle production in inclusive reactions and verification of fundamental physical principles of self-similarity, locality, fractality, maximal entropy, etc.

Search for signatures of a phase transition in nuclear matter produced in heavy ion collisions at high energies.

Phys. Part. Nucl. 54, 640 (2023)

Principles & Symmetries

"Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter and define the fundamental forces in nature."

Leon M. Lederman

"…for every conservation law there must be a continuous symmetry…."

Emmy Nöether

The concepts of symmetry, of invariance, play a very large role and, it appears, an increasing role in physics.

Eugene P. Wigner

"Scaling" and "Universality" are concepts developed to understanding critical phenomena.

Harry E. Stanley, Grigory I. Barenblatt,…

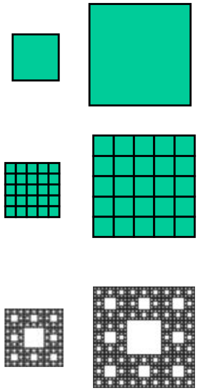
Principles & Symmetries

Discrete (C,P,T,..) and continuous symmetries correspond to fundamental principles (gauge, special, general and scale relativity, …) and conservation laws (charge,….) and vice versa.

- Principles are reflected as regularities in measurable observables and can be usually expressed as scaling in a suitable representation of data.
- z-Scaling of differential cross sections of inclusive particle production in p+p, p+A and A+A is used as a tool to search for and study of principles and symmetries that reflect properties of hadron interactions at constituent level.
- z-Scaling is based on the principles of *self-similarity, fractality, and locality.*

… Scaling means that systems near the critical points exhibiting self-similar properties are invariant under transformation of a scale. According to universality, quite different systems behave in a remarkably similar fashion near the respective critical points. Critical exponents are defined only by symmetry of interactions and dimension of the space.

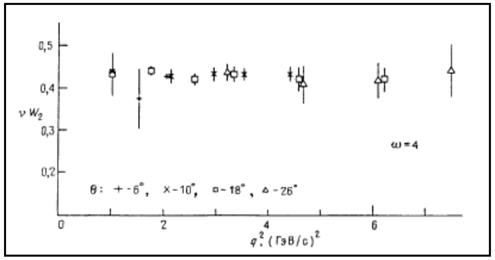
Self-similarity



- A self-similar object is exactly or approximately similar to a part of itself (i.e. the whole has the same shape as one or more of the parts).
- Self-similarity is a typical property of fractals.
- Scale invariance is an exact form of self-similarity where at any magnification there is a smaller piece of the object that is similar to the whole.

Description of a process in terms of
a scaling function and similarity parameter

Reynolds number	Mach number	Bjorken variable
$Re= \rho VD/\eta$	$Ma=v/c$	$x = -q^2/2(pq)$
laminar & turbulent flow	subsonic & supersonic wave	low x & high x



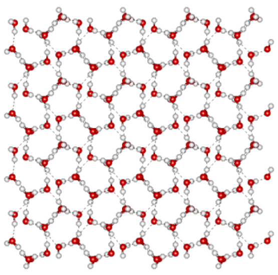
shock wave, explosion, confinement

Violation of a scaling is an indication of new phenomena

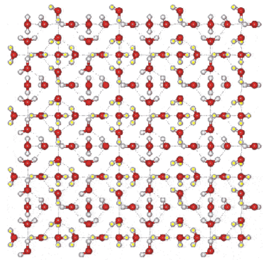
The phase diagram of water H_2O

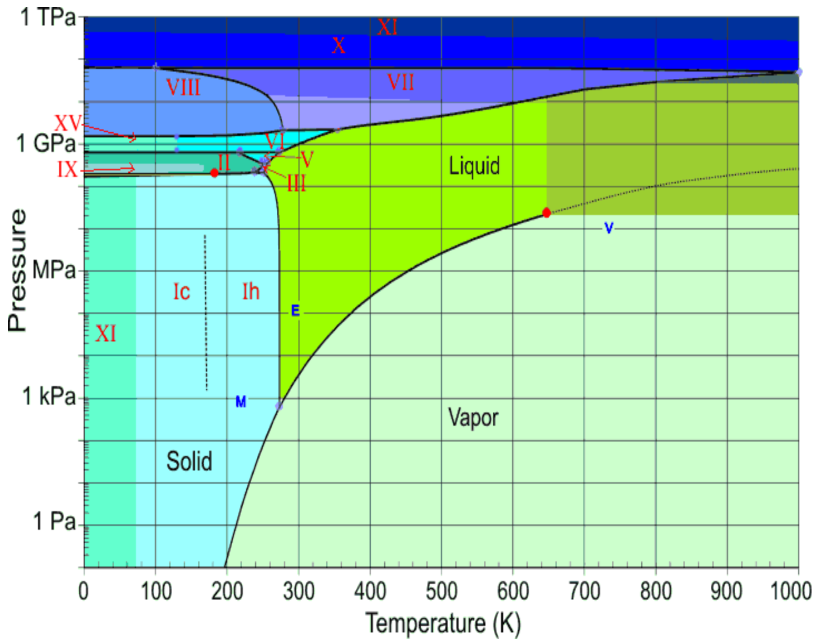
- Self-similarity as a symmetry principle is confirmed.
- The law of corresponding states, equation of state are found.
- Phase diagram – boundaries, triple and critical points,…, is established.
- Properties of phases are investigated.

Ice III

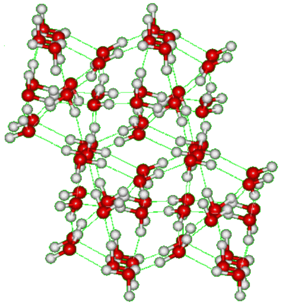


Ice VI





Ice XIII

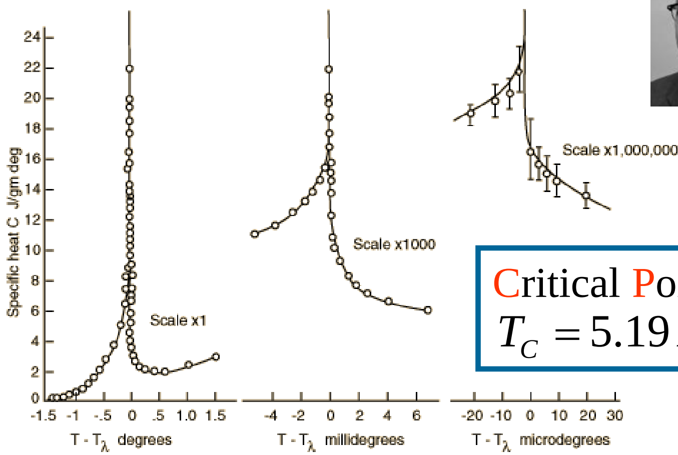


- Phases (ice I-XVIII, liquid, vapor)
- Phase boundaries
- Phase transitions
- Triple Point (17)
- Critical Point (1)

What one can say about phase diagram of nuclear matter ?

Singularity of specific heat near a Critical Point

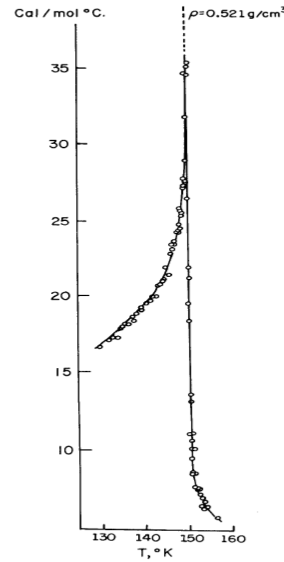
Specific heat of liquid ^4He
Superfluid transition

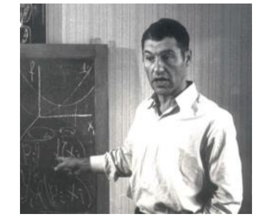


Critical Point
$T_C = 5.19\,K$

M. J. Buckingham and W. M. Fairbank, 1961
H.E. Stanley, 1971

Heat capacity of Ar





Critical Point
$T_C = 150.8\,K$

The isochoric heat capacity C_v of argon becomes infinite at the vapor-liquid critical point.

A.V. Voronel' et al.
Zh. Exp. Teor. Fiz. 43, 728 (1962).

- Near a critical point the singular part of thermodynamic potentials is a Generalized Homogeneous Function (GHF).
- The Helmholtz potential $F(\lambda^{a_\varepsilon}\varepsilon, \lambda^{a_V}V) = \lambda F(\varepsilon, V)$ is GHF of (ε, V).

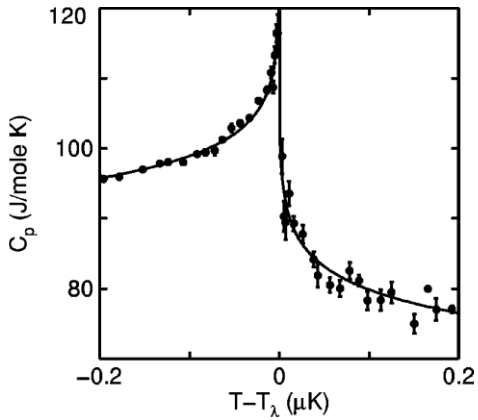
$$c_V \sim |\varepsilon|^{-\alpha} \qquad \varepsilon \equiv (T - T_c)/T_c \qquad c_V = -T(\partial^2 F/\partial T^2)|_V \qquad c_V = T\partial S/\partial T|_V$$

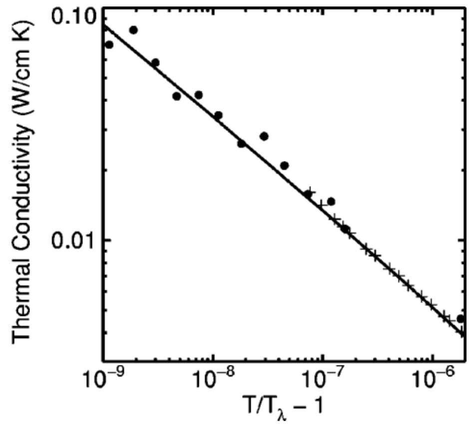
Critical exponents define the behavior of thermodynamic quantities nearby the Critical Point.

Singularity of specific heat c_p of liquid ^{4}He in cosmic space

Specific heat

Thermal conductivity





Specific heat and thermal conductivity vs. reduced temperature near the lambda point

$$t = \frac{T - T_\lambda}{T_\lambda}$$

- Density gradients cause substantial distortion of the singularity for reduced temperatures.
- Transition broadening associated with gravity and relaxation phenomena.

The experiment was performed in Earth orbit to reduce the rounding of the transition caused by gravitationally induced pressure gradients on Earth.

Critical exponent describing the specific-heat singularity was found to be $\alpha = -0.01276 \pm 0.0003$.

$$c_p = \frac{A^{\pm}}{\alpha} |t|^{-\alpha} + B^{\pm}$$

Expt. in space $|t|<10^{-10}$ (^{4}He , Lipa).
Expt. on Earth $|t|<10^{-7}$ (^{4}He , Fairbank).
Expt. on Earth $|t|<10^{-4}$ (Xe, Sengers,).

In space, the lambda transition is expected to be sharp to $|t|<10^{-12}$ in ideal conditions.

J. A. Lipa et al.,
"Specific heat of liquid helium in zero gravity very near the lambda point"
Phys. Rev. B **68**, 174518, (2003)

Phase transitions & Critical phenomena

- Critical phenomena reveal unusual characteristic behavior of substances in the vicinity of phase transition points.
- They are observed due to an increase in the characteristic sizes of different fluctuations.
- In these phenomena, the self-similarity of a system arises spontaneously.
- This scale property is characteristic of fractal structures.
- Second order transition is accompanied by a spontaneous symmetry breaking.

Signatures of critical phenomena:

- increase in compressibility (liquid-vapor equilibrium)
- increase in magnetic and dielectric susceptibility in the vicinity of the Curie points of ferromagnets and ferroelectrics
- anomaly in heat capacity at the point of transition of helium to the superfluid state
- slowing of the mutual diffusion of substances near the critical points of mixtures of stratifying liquids
- anomaly in the propagation of ultrasound (absorption of sound and an increase in its dispersion)
- anomalies in viscosity, thermal conductivity, slowdown in the establishment of thermal equilibrium, etc.

These anomalies are described by power laws with critical indices.
Strong fluctuations and infinite correlation radii
in such systems confirm self-similarity.

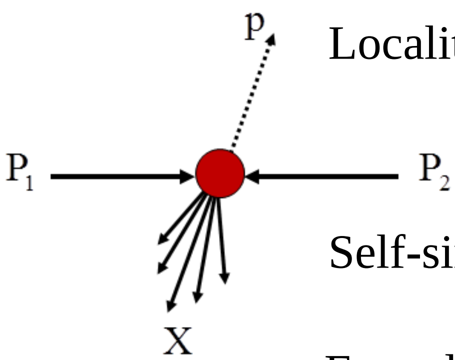
z-Scaling:
hypothesis, ideas, definitions,…

Basic principles:
locality, self-similarity, fractality,…

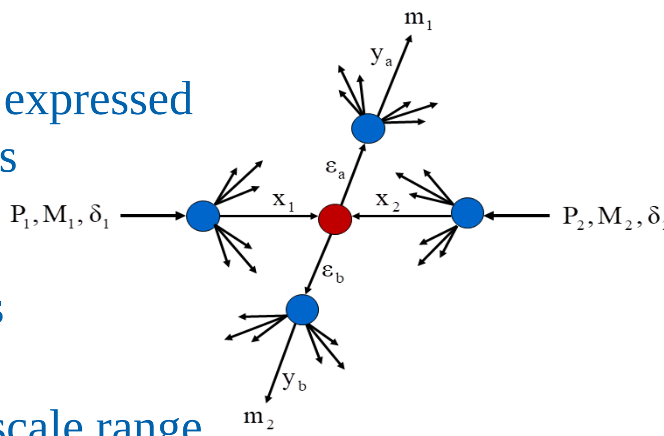
Int.J.Mod.Phys. A 27 (2012) 1250115
J.Mod.Phys. 3 (2012) 815
Int.J.Mod.Phys. A 32 (2017) 750029
Phys. Part. Nucl. 51 (2020) 141
Nucl.Phys. A 993 (2020) 121646
Nucl.Phys. A 1025 (2022) 122492

z-Scaling

Principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).



Self-similarity: interactions of the constituents are mutually similar.

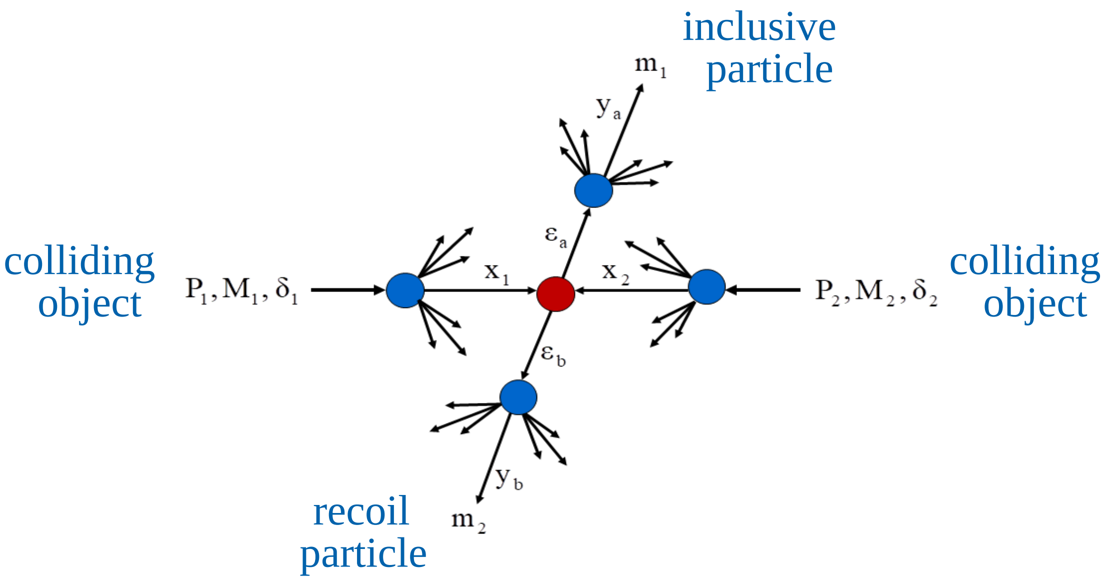
Fractality: self-similarity is valid over a wide scale range.

Hypothesis of z-scaling :

$s^{1/2}$, p_T, θ_{cms}	Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.	x_1, x_2, y_a, y_b $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$,c
$Ed^3\sigma/dp^3$	Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable z.	$\Psi(z)$

Locality

Collisions of colliding objects are expressed via interactions of their constituents



P_1, P_2, p – momenta of colliding and produced particles

M_1, M_2, m_1 – masses of colliding and produced particles

x_1, x_2 – momentum fractions of colliding particles carried by constituents

y_a, y_b – momentum fractions of scattered constituents carried by inclusive particle and its recoil

δ_1, δ_2 – fractal dimensions of colliding particles

$\varepsilon_a, \varepsilon_b$ – fractal dimensions of scattered constituents (fragmentation dimensions)

m_2 – mass of recoil particle

Elementary sub-process:

$$(x_1M_1) + (x_2M_2) \rightarrow (m_1/y_a) + (x_1M_1+x_2M_2+ m_2/y_b)$$

Momentum conservation law for sub-process

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

Mass of recoil system

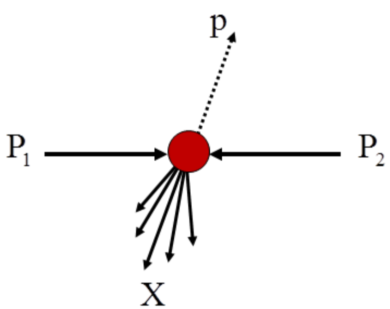
$$M_X= x_1M_1+x_2M_2+m_2/y_b$$

M.Tokarev, I.Zborovský
Yu.Panebratsev, G.Skoro
Phys.Rev.D54 5548 (1996)
Int.J.Mod.Phys.A16 1281 (2001)

Self-similarity

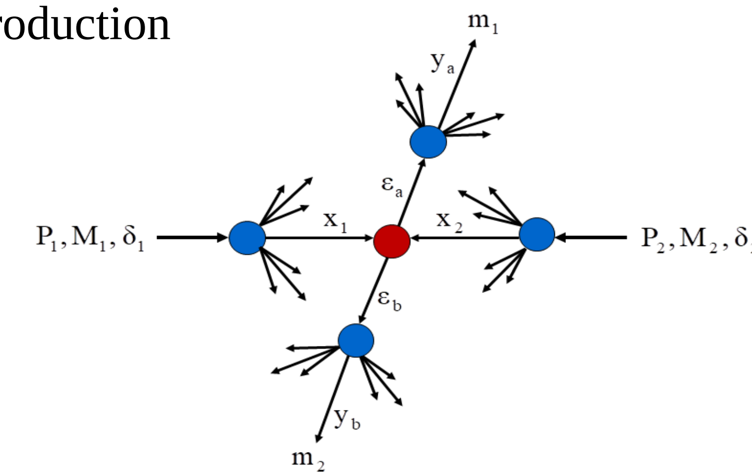
Interactions of constituents are mutually similar

The self-similarity parameter z is a dimensionless quantity, expressed through the dimensional values P_1, P_2, p, M_1, M_2, m_1, m_2, characterizing the process of inclusive particle production



$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



- Ω^{-1} – the minimal resolution at which a constituent sub-process can be singled out of the inclusive reaction
- $s_\perp^{1/2}$ – the transverse kinetic energy of the sub-process consumed on production of m_1 & m_2
- $dN_{ch}/d\eta|_0$ – the multiplicity density of charged particles at $\eta = 0$
- c – a parameter interpreted as a “specific heat” of created medium
- m_N an arbitrary constant (fixed at the value of nucleon mass)

Fractality

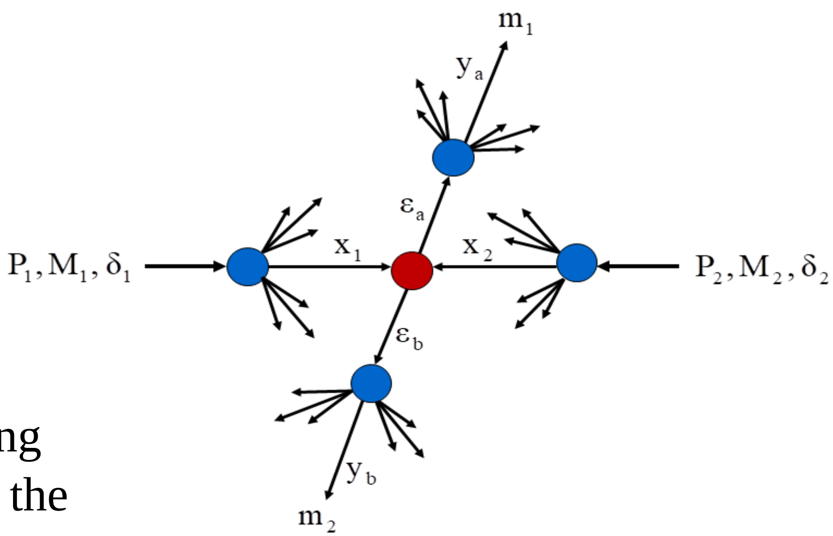
Self-similarity over a wide scale range

Fractal measure

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

$$0 < x_1, x_2 < 1 \qquad 0 < y_a, y_b < 1$$



Ω – relative number of configurations containing a sub-process with fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

δ_1, δ_2, ε_a, ε_b – parameters characterizing structure of the colliding objects and fragmentation process, respectively

$\Omega^{-1}(x_1, x_2, y_a, y_b)$ characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

The fractal measure z diverges as the resolution Ω^{-1} increases.

$$z(\Omega)\big|_{\Omega^{-1}\to\infty} \to \infty$$

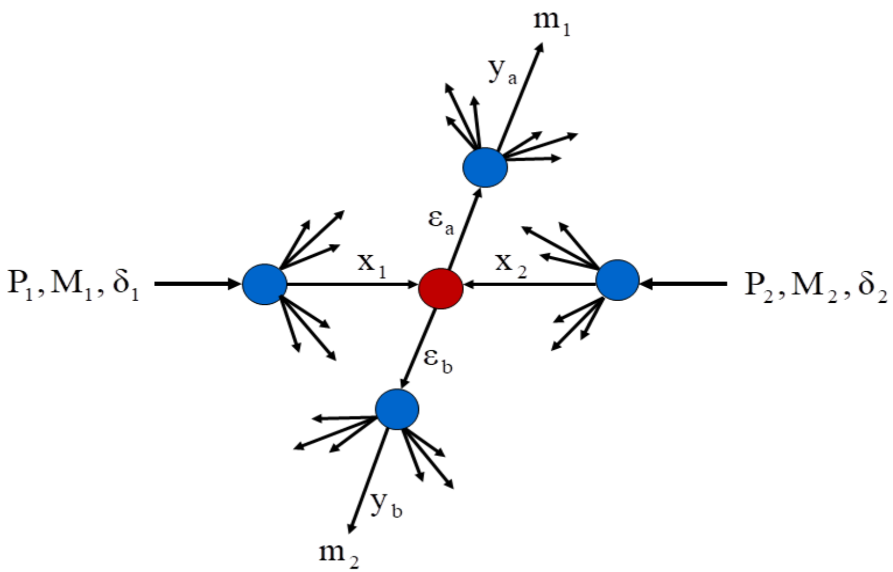
Momentum fractions x_1, x_2, y_a, y_b

Principle of minimal resolution: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to minimize the resolution Ω^{-1} of the fractal measure z with respect to all constituent sub-processes taking into account 4-momentum conservation law:

Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

$$\begin{cases} \partial\Omega/\partial x_1 \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial x_2 \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial y_b \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



Resolution of sub-process

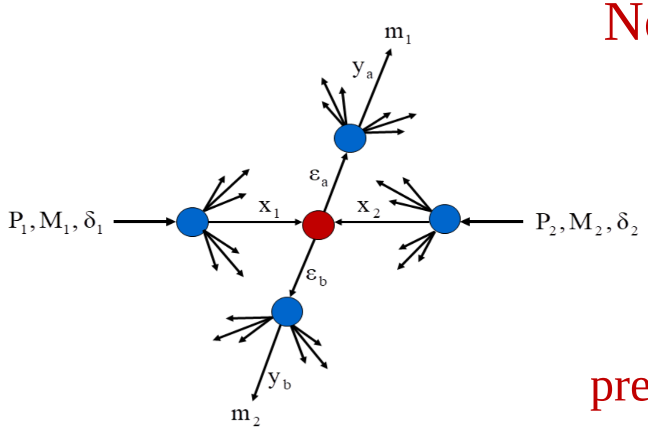
$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

Fractions x_1, x_2, y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.

Mass of recoil system

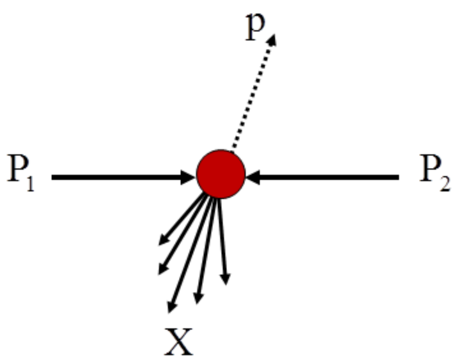
$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

Scaling function $\Psi(z)$



Normalization condition

$$\int_0^\infty \Psi(z)dz = 1$$



Scale transformation

$$z \to \alpha_F z \qquad \Psi(z) \to \alpha_F^{-1}\Psi(z)$$

preserves the normalization condition

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3} \quad \Longleftrightarrow \quad \int E\frac{d^3\sigma}{dp^3}dyd^2p_\perp = \sigma_{inel}\cdot <N>$$

- σ_{inel} – the inelastic cross section
- $<N>$ – the average multiplicity
- $dN/d\eta$ – the multiplicity density
- $J(z,\eta;p_T^2,y)$ – the Jacobian
- $Ed^3\sigma/dp^3$ – the inclusive cross section

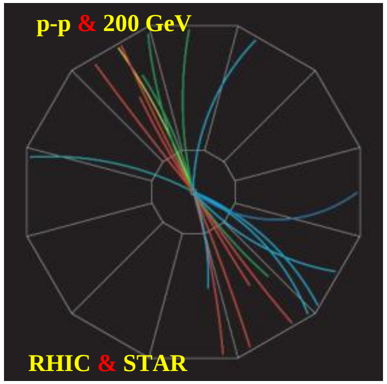
The scaling function $\Psi(z)$ is probability density to produce the inclusive particle with the corresponding z.

Properties of $\Psi(z)$ in pp collisions

- Energy independence of $\Psi(z)$ ($s^{1/2} > 20$ GeV)
- Angular independence of $\Psi(z)$ ($\theta_{cms}=3^0$-90^0)
- Multiplicity independence of $\Psi(z)$ ($dN_{ch}/d\eta=1.5$-26)
- Saturation of $\Psi(z)$ at low z ($z < 0.1$)
- Power law, $\Psi(z) \sim z^{-\beta}$, at high z ($z > 4$)
- Flavor independence of $\Psi(z)$ (π,K,φ,Λ,..,D,J/ψ,B,Υ,…, top)

These properties reflect self-similarity, locality, and fractality of hadron interactions at a constituent level.
It concerns the structure of the colliding objects, constituent interactions and fragmentation process.

Some results of data analysis for p+p collisions in z-scaling approach



Int. J. Mod. Phys. A 32, 1750029 (2017)
Phys. Part. Nucl. 51, 141 (2020)

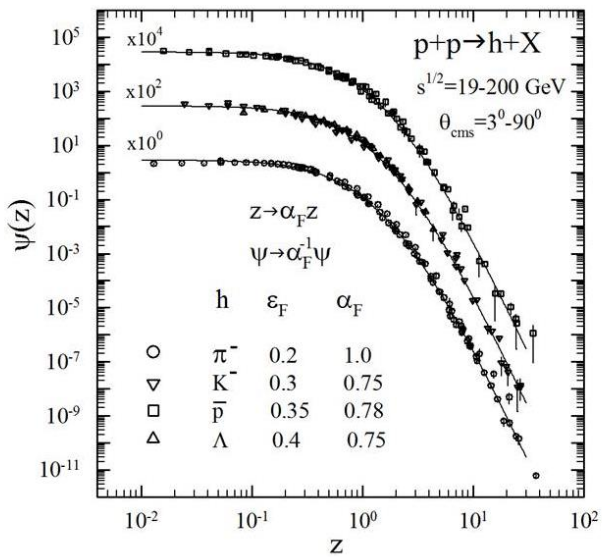
Self-similarity of strangeness production in p+p

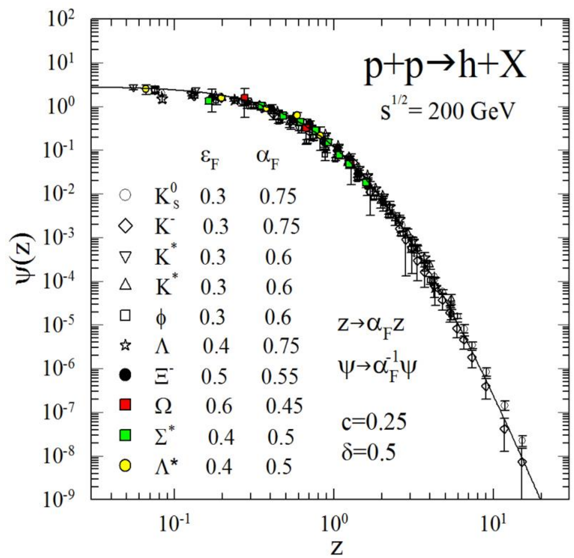
Universality: flavor independence of the scaling function

K_S^0, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

"Collapse" of data points onto a single curve

M.T.& I.Zborovský
Int. J. Mod. Phys.
A24,1417(2009)
A32,1750029 (2017)





STAR:
PRL 92 (2004) 092301
PRL 97 (2006) 132301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901
PRL 108 (2012) 072302

PHENIX:
PRC 75 (2007) 051902
PRD 83 (2011) 052004
PRC 90 (2014) 054905

Solid line for π^- meson is a reference frame

$\varepsilon_\pi = 0.2, \quad \alpha_\pi = 1$

- Energy independence
- Angular independence
- Flavor independence
- Saturation for $z < 0.1$

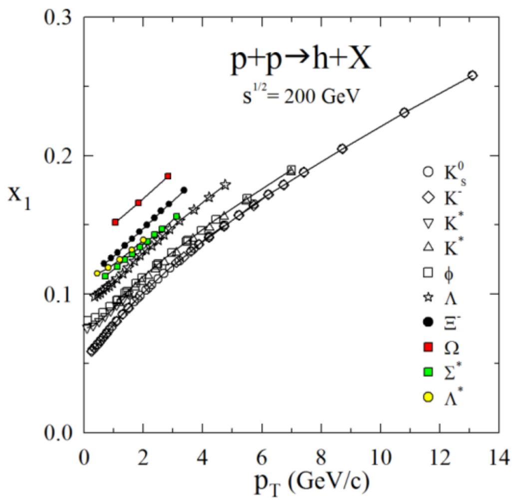
- Power law $\Psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

Self-similarity of strangeness production in p+p

K_S^0, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

Constituent sub-process in terms of

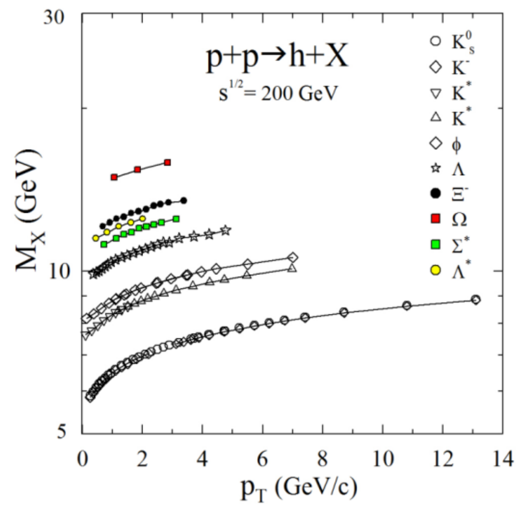
Momentum fraction



The more strangeness, the larger momentum fraction

$$x_1^{\Omega} > x_1^{\Xi} > x_1^{\Sigma} > x_1^{K}$$

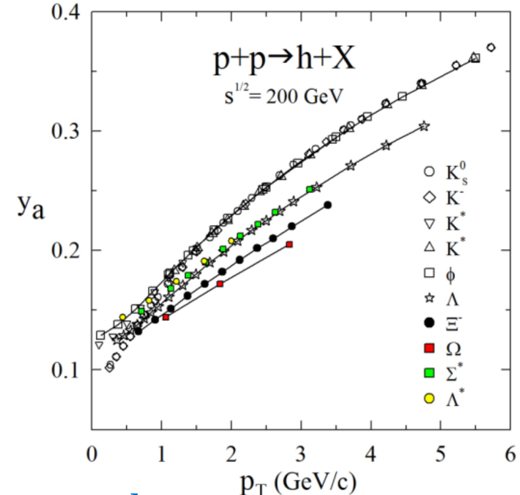
Recoil mass



The more strangeness, the larger recoil mass

$$M_X^{\Omega} > M_X^{\Xi} > M_X^{\Sigma} > M_X^{K}$$

Energy loss $\Delta E/E \sim (1-y_a)$



The more strangeness, the larger energy loss

$$\varepsilon_{\Omega} > \varepsilon_{\Xi} > \varepsilon_{\Sigma} > \varepsilon_{K}$$

Smooth behavior of x_1, y_a, M_X vs. p_T.

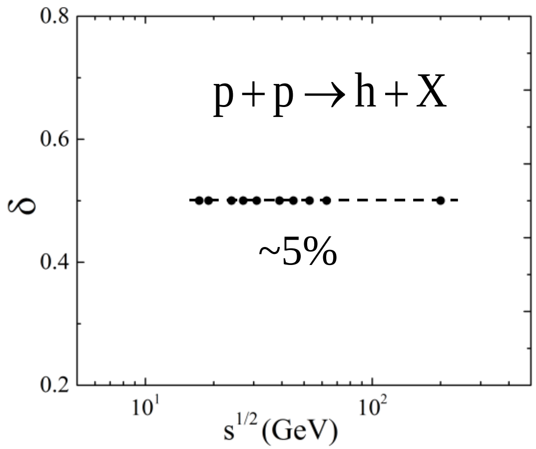
Self-similarity dictates the properties of constituent sub-process.

Model parameters: δ, ε_F , c for p+p

Parameters δ, ε_F , c are found from the scaling behavior of Ψ as a function of self-similarity variable z

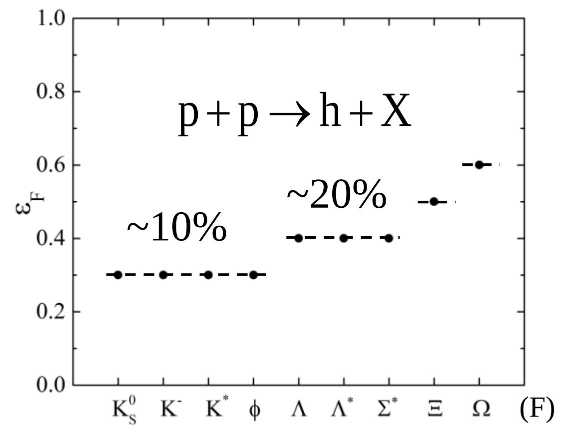
Proton fractal dimension

δ



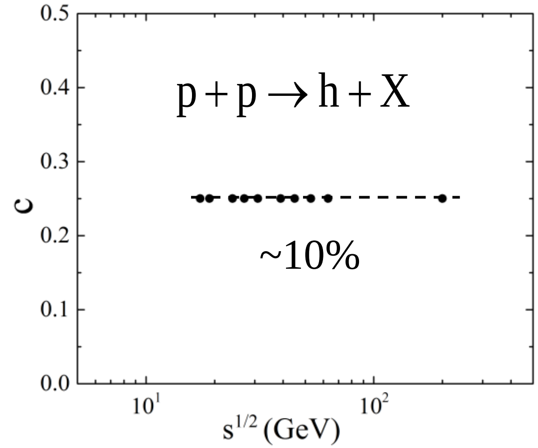
Fragmentation dimension

ε_F



"Specific heat"

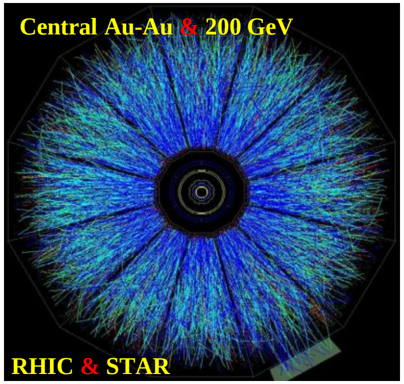
c



- δ, ε_F , c are independent of $\sqrt{s}$, p_T
- ε_F depends on flavor

- Self-similarity of proton sub-structure: δ = const
- Self-similarity of hadronization process: ε_F = const for F=const
- Constancy of "temperature" fluctuations: c = const

Some results of data analysis for A+A collisions in z-scaling approach



Nucl. Phys. A993 (2020) 121646
Nucl. Phys. A1025 (2022) 122492

Self-similarity of h^- production in Au+Au collisions

Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^{c_{AA}} m_N}$$

$$\Omega = (1-x_1)^{\delta_A}(1-x_2)^{\delta_A}(1-y_a)^{\varepsilon_{AA}}(1-y_b)^{\varepsilon_{AA}}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c_{AA} - "specific heat" of bulk matter
- δ_A - nucleus fractal dimension
- ε_{AA} - fragmentation dimension

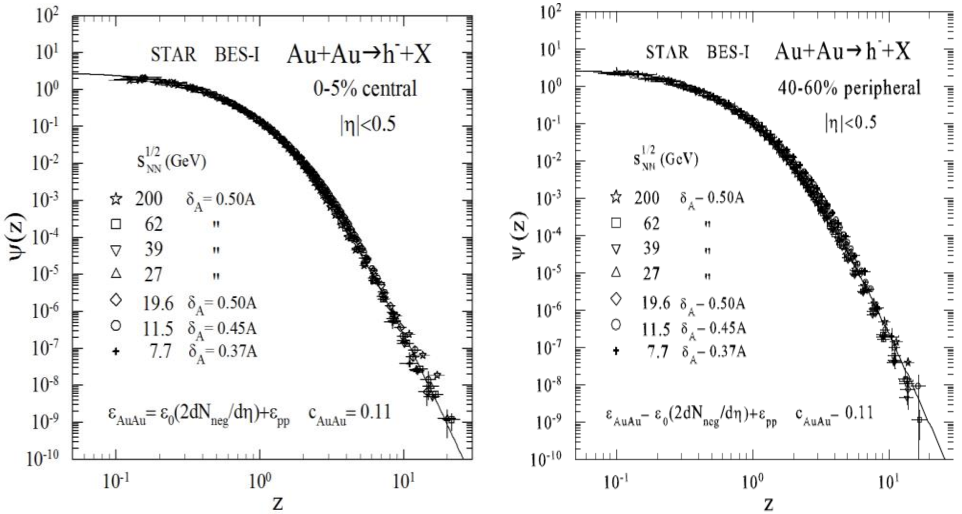
AA collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0 (2dN_{neg}^{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi A_1 A_2}{(dN/d\eta)\,\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

"Collapse" of data points onto a single curve

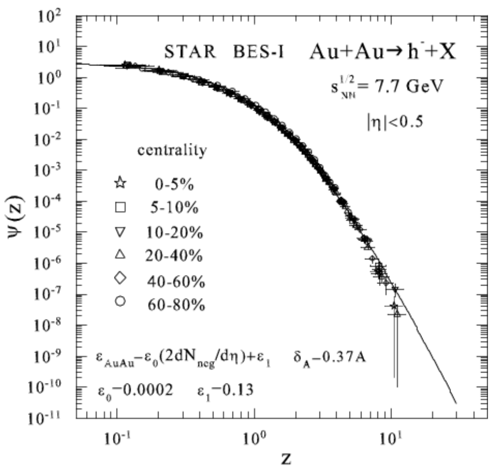


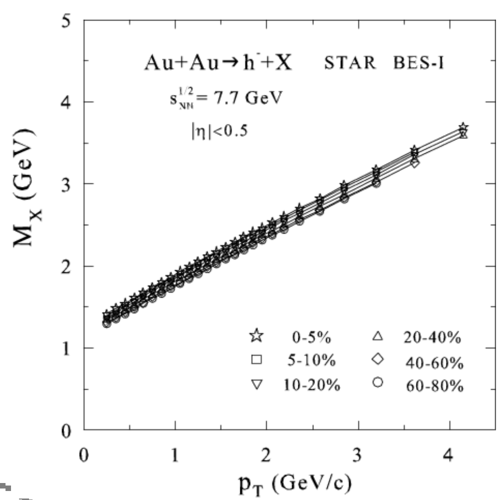
- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Dependence of ε_{AA} on multiplicity
- Power law at low- and high-z regions

Indication of the decrease of δ for $\sqrt{s_{NN}} < 19.6$ GeV

MT & I.Zborovsky, Nucl. Phys. A993 (2020) 121646

Scaling function, momentum fractions and recoil mass vs. centrality and p_T at $\sqrt{s_{NN}}$ = 7.7 GeV & $|\eta|<0.5$



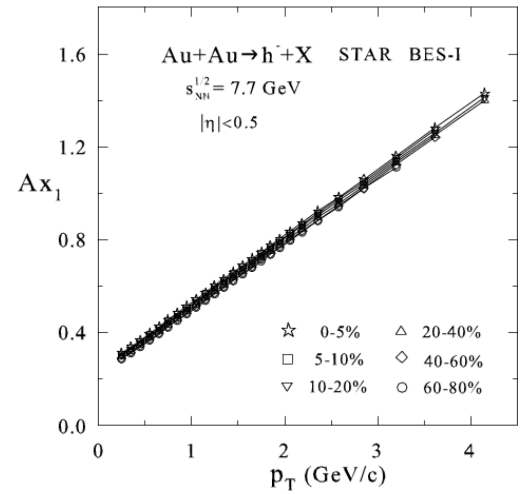


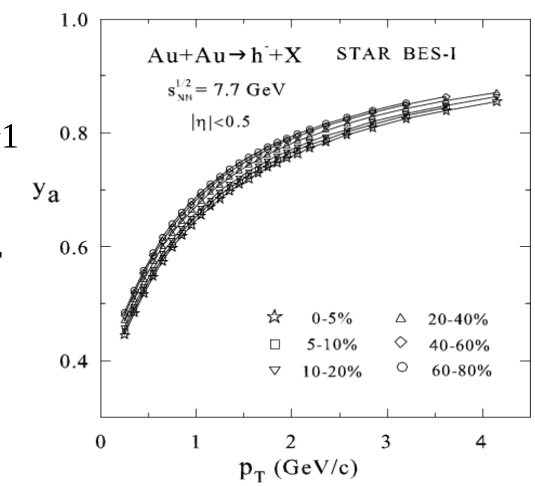
Au+Au→h^-+X

- Scaling behavior of Ψ(z)
- Weak dependence of Ax_1, y_a, M_X on centrality
- Cumulative region is reached
- Smooth dependence vs. variables

- Power behavior of Ψ(z) at z<0.4
- Power behavior of Ψ(z) at z>4
- Linear dependence of M_X and Ax_1 on p_T for all centralities
- Growth and flattening of y_a vs. p_T
- Decrease of $\delta_A=A\delta$ with $\sqrt{s_{NN}}$

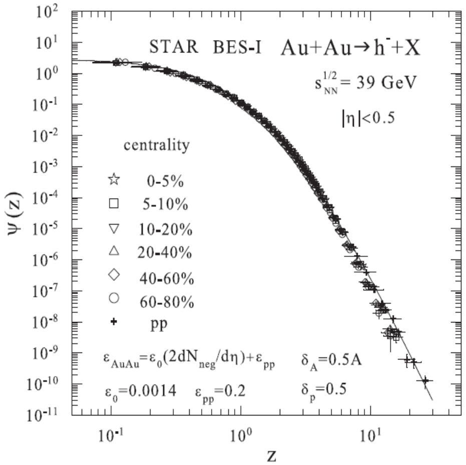
There are no found discontinuities in these dependences.



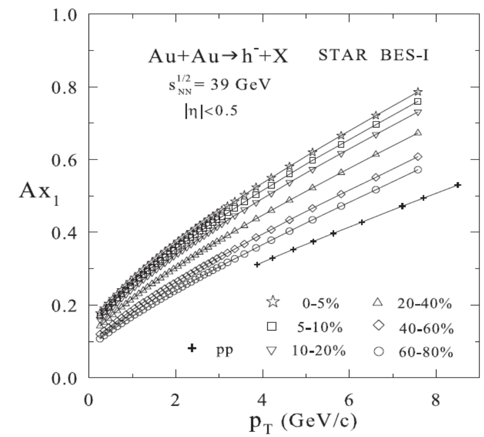


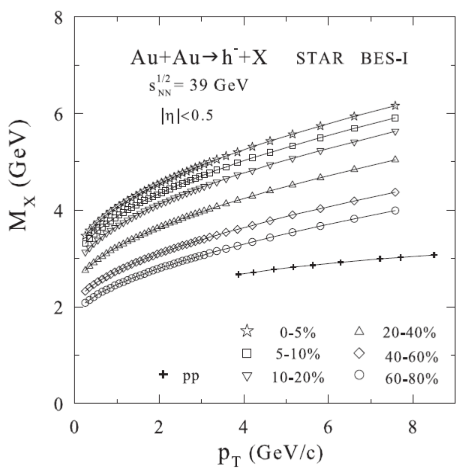
Scaling function, momentum fractions and recoil mass vs. centrality and p_T at $\sqrt{s_{NN}}$ = 39 GeV & $|\eta|<0.5$

$Au+Au \rightarrow h^- + X$

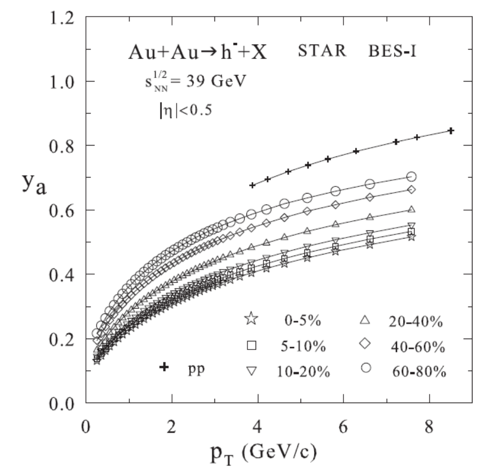


- Scaling behavior of $\Psi(z)$
- Strong dependence of Ax_1, y_a , M_X on centrality
- Cumulative region is not reached
- Smooth dependence vs. variables





- Power behavior of $\Psi(z)$ at $z<0.4$
- Power behavior of $\Psi(z)$ at $z>4$
- Growth of Ax_1, y_a , and M_X on p_T for all centralities
- Independence of $\delta_A=A\delta$ on $\sqrt{s_{NN}}$



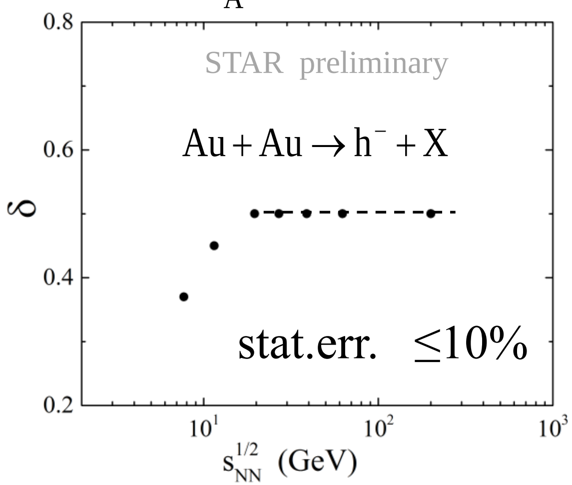
There are no found discontinuities in these dependences.

Model parameters: δ_A, ε_{AA}, c_{AA}

Parameters δ_A, ε_{AA}, c_{AA} are determined from the requirement of scaling behavior of Ψ as a function of self-similarity parameter z

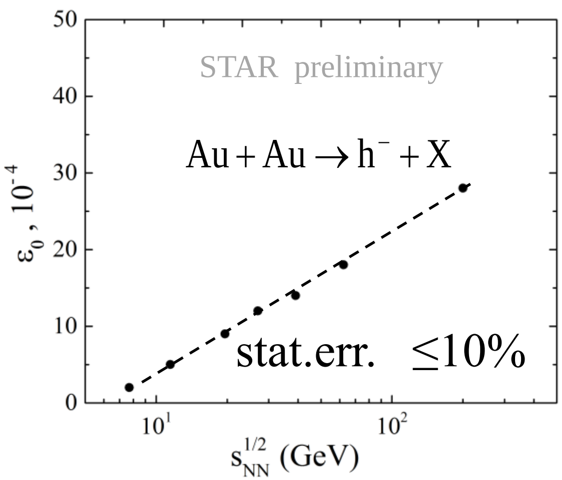
Nucleus fractal dimension

$$\delta_A = A \cdot \delta$$



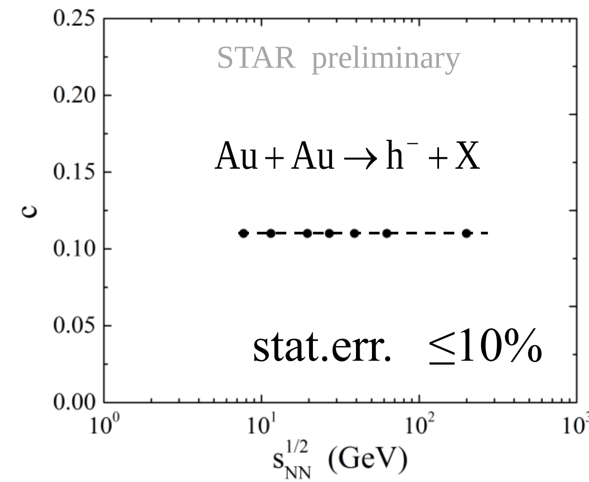
Fragmentation dimension

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$



"Specific heat"

$$c_{AA}$$



- δ_A decreases with energy for $\sqrt{s_{NN}} \leq 20$ GeV
- δ_A is independent of energy for $\sqrt{s_{NN}} \geq 20$ GeV
- ε_{AA} increases with energy
- c_{AA} is independent of energy

Search for discontinuity and correlations of the model parameters.

Self-similarity of K^0_S production in Au+Au

Self-similarity parameter

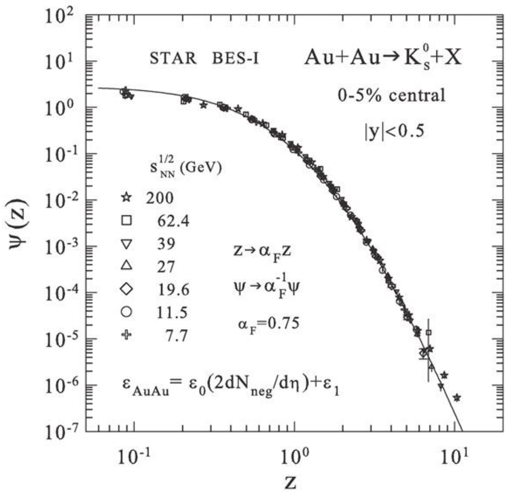
$$z = z_0 \Omega^{-1}$$

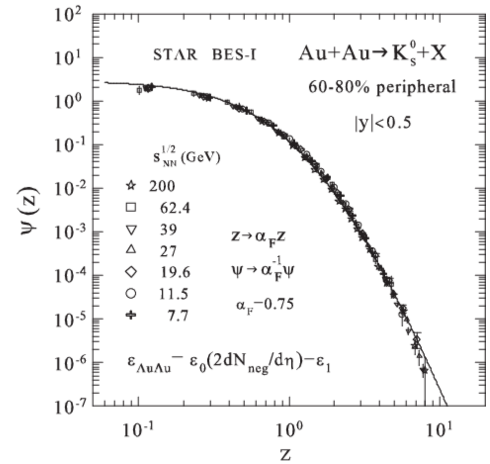
$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^{c_{AA}} m_N}$$

$$\Omega = (1-x_1)^{\delta_A}(1-x_2)^{\delta_A}(1-y_a)^{\varepsilon_{AA}}(1-y_b)^{\varepsilon_{AA}}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c_{AA} - "specific heat" of bulk matter
- δ_A - nucleus fractal dimension
- ε_{AA} - fragmentation dimension

"Collapse" of data points onto a single curve





AA collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0(2dN_{neg}^{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\,\sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Dependence of ε_{AA} on multiplicity
- Power law at low- and high-z regions

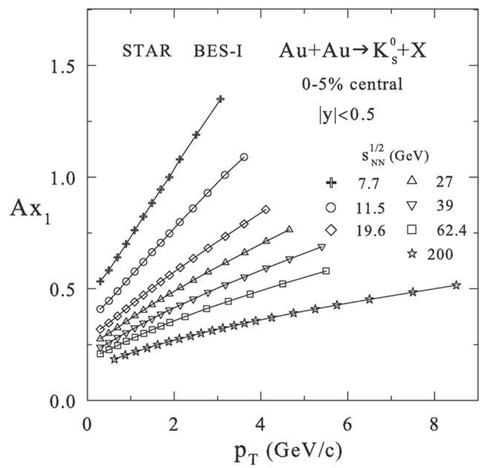
Indication of a decrease of δ for $\sqrt{s_{NN}}$ < 19.6 GeV

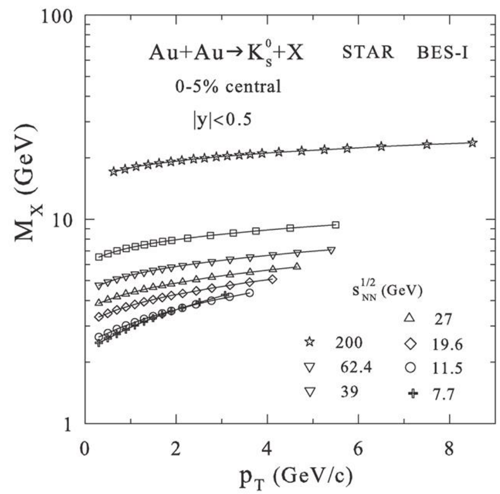
MT & I.Zborovsky, Nucl. Phys. A1025 (2022) 122492

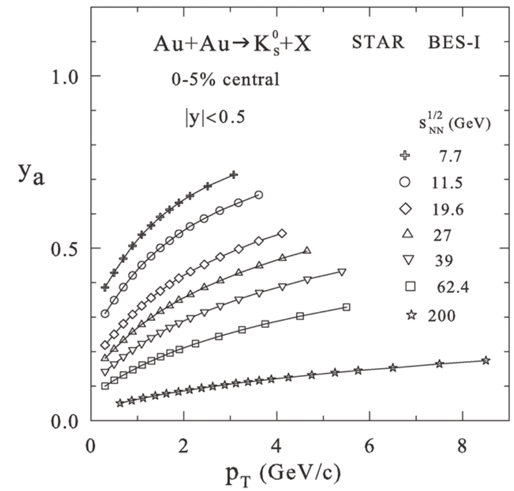
K^0_S production in central Au+Au @ 7.7-200 GeV

Constituent sub-process in terms of

Momentum fraction Ax_1 | Recoil mass M_X | Energy loss $\Delta E/E \sim (1-y_a)$







Momentum fraction
- increases with p_T
- decreases with $\sqrt{s_{NN}}$

Recoil mass
- increases with p_T
- increases with $\sqrt{s_{NN}}$

Energy loss
- decreases with p_T
- increases with $\sqrt{s_{NN}}$

Smooth behavior of x_1, y_a, M_X vs. p_T, centrality, collision energy

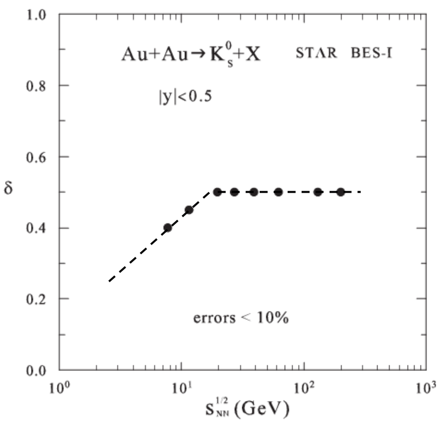
- High x_1 and p_T → compressed nuclear matter
- Large M_X → high density recoil system
- High y_a → small energy loss

Model parameters: δ_A, ε_{AA} , c_{AA} for Au+Au

Parameters δ_A, ε_{AA} , c_{AA} are found from
the scaling behavior of Ψ as a function of self-similarity variable z

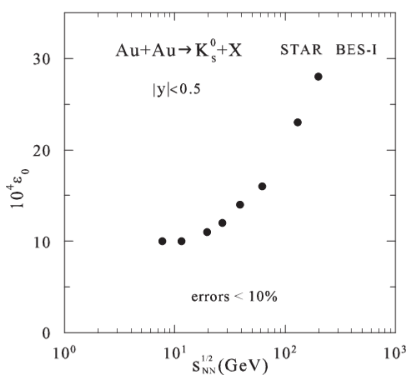
Nucleus fractal dimension

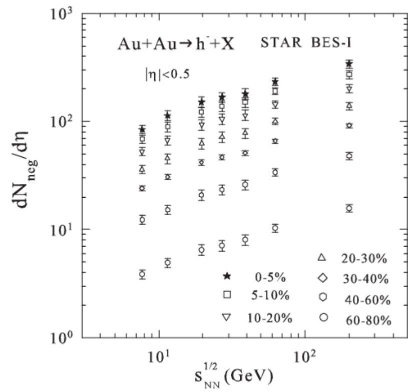
$\delta_A = A \cdot \delta$



Fragmentation dimension

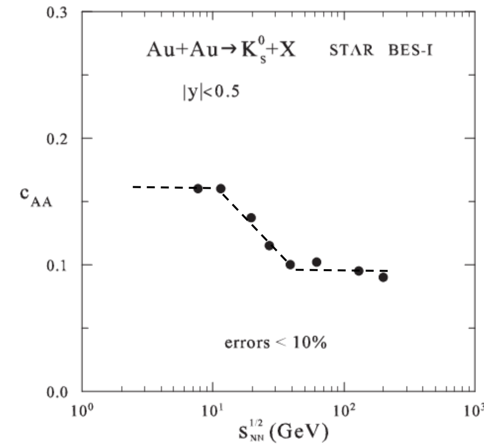
$\varepsilon_{AA} = \varepsilon_0(2dN^{AA}_{neg}/d\eta) + \varepsilon_{pp}$





"Specific heat"

c_{AA}



- δ_A, ε_{AA} , c_{AA} depend on $\sqrt{s_{NN}}$
- ε_{AA} depends on flavor and multiplicity

Decrease of resolution with energy : $\delta = 0$ for point-like object.
Increases of energy loss vs. energy, multiplicity : $\varepsilon_{AA} = 0$ no energy loss
Increase of temperature fluctuations with energy : decrease of specific heat c_{AA}
Strange meson K_S^0 is a sensitive probe to state of the nuclear matter.

Fractal entropy $S_{\delta,\varepsilon}$ for systems with structural constituents

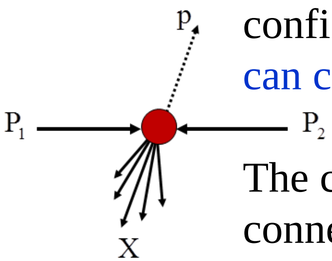
Physics 5 (2023) 537

Entropy of nuclear system produced in $A+A \rightarrow h+X$

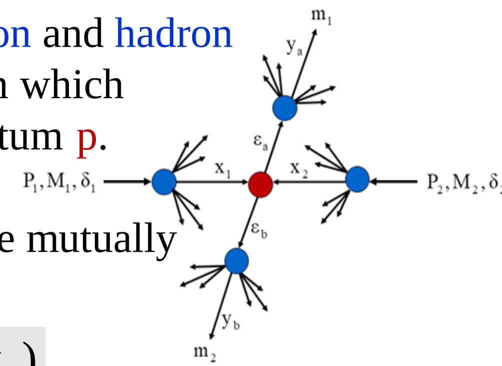
According to statistical physics, entropy of a system is given by a number Ws of its statistical states:

$$S = \ln W_S$$

The most likely configuration of the system is given by the maximal value of S.



For inclusive reactions, the quantity Ws is the number of all parton and hadron configurations in the initial and final states of the colliding system which can contribute to the production of inclusive particle with momentum p.



The configurations comprise all constituent configurations that are mutually connected by independent binary subprocesses:

$$(x_1M_1)+(x_2M_2) \rightarrow (m_a/y_a)+(x_1M_1+x_2M_2+m_b/y_b)$$

The subprocesses corresponding to the production of the inclusive particle with the 4-momentum p are subject to the momentum conservation law:

$$(x_1P_1+x_2P_2-p/y_a)^2=(x_1M_1+x_2M_2+m_b/y_b)^2$$

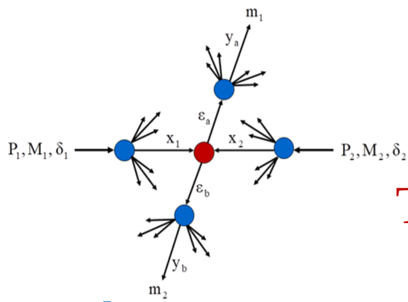
The underlying subprocess, which defines the variable z, is singled out from the corresponding subprocesses by the principle of maximal entropy S.

Self-similarity variable z & Fractal entropy $S_{\delta,\varepsilon}$

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{ch}/d\eta|_0)^c m_N}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$



Statistical entropy

$$S = \ln W_S$$

Thermodynamical entropy for ideal gas

$$S = c_V \ln T + R \ln V + S_0$$

The quantity W_S is the number of all parton and hadron configurations in the initial and final states of the colliding system which can contribute to the production of inclusive particle with momentum p

Fractal entropy for independent processes

$$z = \frac{\sqrt{s_\perp}}{W}$$

$$W_S = W \cdot W_0 = (dN_{ch}/d\eta|_0)^c \cdot \Omega \cdot W_0$$

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln(V_{\delta,\varepsilon}) + \ln W_0$$

Entropy $S_{\delta,\varepsilon}$ for systems with structural constituents

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + \ln W_0$$

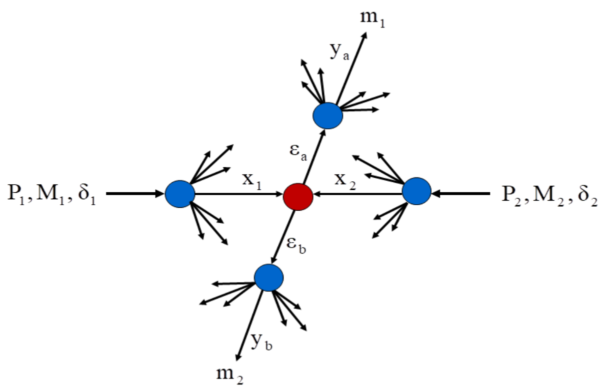
- $dN_{ch}/d\eta|_0$ characterizes “temperature” of the colliding system.
- c has meaning of a “specific heat” of the produced medium.
- Fractional exponents $\delta_1,\delta_2,\varepsilon_a,\varepsilon_b$ are fractal dimensions in the space of $\{x_1,x_2,y_a,y_b\}$.
- $V_{\delta,\varepsilon} = \Omega$ is fractal volume in the space of momentum fraction.

Fractal entropy $S_{\delta,\varepsilon}$ and momentum fractions x_1, x_2, y_a, y_b

Principle of maximal entropy: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to maximize the entropy $S_{\delta,\varepsilon}$ with respect to all constituent sub-processes taking into account 4-momentum conservation law.

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$

$$\begin{cases} \partial S_{\delta,\varepsilon} / \partial x_1 |_{y_a = y_a(x_1,x_2,y_b)} = 0 \\ \partial S_{\delta,\varepsilon} / \partial x_2 |_{y_a = y_a(x_1,x_2,y_b)} = 0 \\ \partial S_{\delta,\varepsilon} / \partial y_b |_{y_a = y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



Momentum conservation law

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2$$

Mass of the recoil system

$$M_X = x_1 M_1 + x_2 M_2 + m_2/y_b$$

Fractions x_1, x_2, y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.

Maximal entropy $S_{\delta,\varepsilon}$ $\Leftrightarrow$ minimal resolution Ω^{-1} of the fractal measure z.

Maximum entropy principle & New conservation law

Principle of maximal entropy:

The momentum fractions x_1, x_2, y_a, y_b are determined in a way to maximize the entropy $S_{\delta,\varepsilon}$ with a kinematic constraint (momentum conservation law).

Maximum of $S_{\delta,\varepsilon}$

$$\begin{cases} \partial\Omega/\partial x_1 = 0 & \partial\Omega/\partial y_a = 0 \\ \partial\Omega/\partial x_2 = 0 & \partial\Omega/\partial y_b = 0 \end{cases}$$

Momentum conservation law

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2$$

Mass of recoil system

$$M_X = x_1 M_1 + x_2 M_2 + m_2/y_b$$

Resolution w.r.t. constituent sub-processes

$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

Equivalence of minimal resolution and maximal entropy principle

Conservation law

$$\delta_1 \frac{x_1}{1-x_1} + \delta_2 \frac{x_2}{1-x_2} = \varepsilon_a \frac{y_a}{1-y_a} + \varepsilon_b \frac{y_b}{1-y_b}$$

for arbitrary $P_1, P_2, p, \delta_1, \delta_2, \varepsilon_a, \varepsilon_b$!!!

The conservation law corresponds to maximum of fractal entropy $S_{\delta,\varepsilon}$

I.Zborovsky & MT
Int. J. Mod. Phys. A 33, 1850057 (2018)
ICHEP 2020, Prague, July 28 –August 6

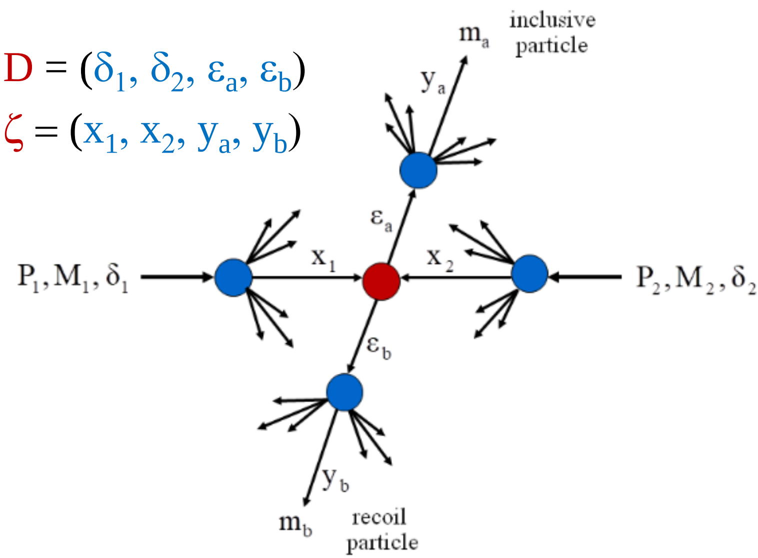
Conservation law for fractal cumulativity C(D, ζ)

"Fractal cumulativity"

$$C(D,\zeta) = D \cdot \frac{\zeta}{1-\zeta}$$

$D = (\delta_1, \delta_2, \varepsilon_a, \varepsilon_b)$

$\zeta = (x_1, x_2, y_a, y_b)$



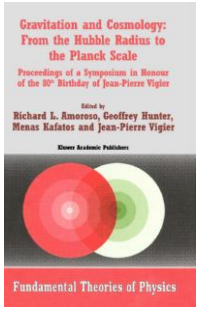
"The fractal cumulativity before a constituent interaction is equal to the fractal cumulativity after a constituent interaction for any binary constituent sub-process"

$$\sum_i^{in} C(D_i, \zeta_i) = \sum_j^{out} C(D_j, \zeta_j)$$

We assume that

- every physical particle is a structural one
- particle's constituents possess a fractal-like structure
- fragmentation is a fractal-like process
- compactness of the fractal structures is governed by the Heisenberg uncertainty principle

Fractal cumulativity C(D, ζ) is a property of a fractal-like object (or fractal-like process) with fractal dimension D to form a local compact "structural aggregate" - a FRACTALON, which carries the fraction ζ of momentum of its parent fractal.



BOHM & VIGIER: IDEAS AS A BASIS FOR A FRACTAL UNIVERSE

C.Ciubotariu, V.Stancu & C.Ciubotariu

"… the universality of fractal structure of spacetime at small and large scales areas…"

"… a quantum mechanical particle (corpuscule) moving on fractal paths may be one or a small cluster of stochastic elements constituting the particle... "

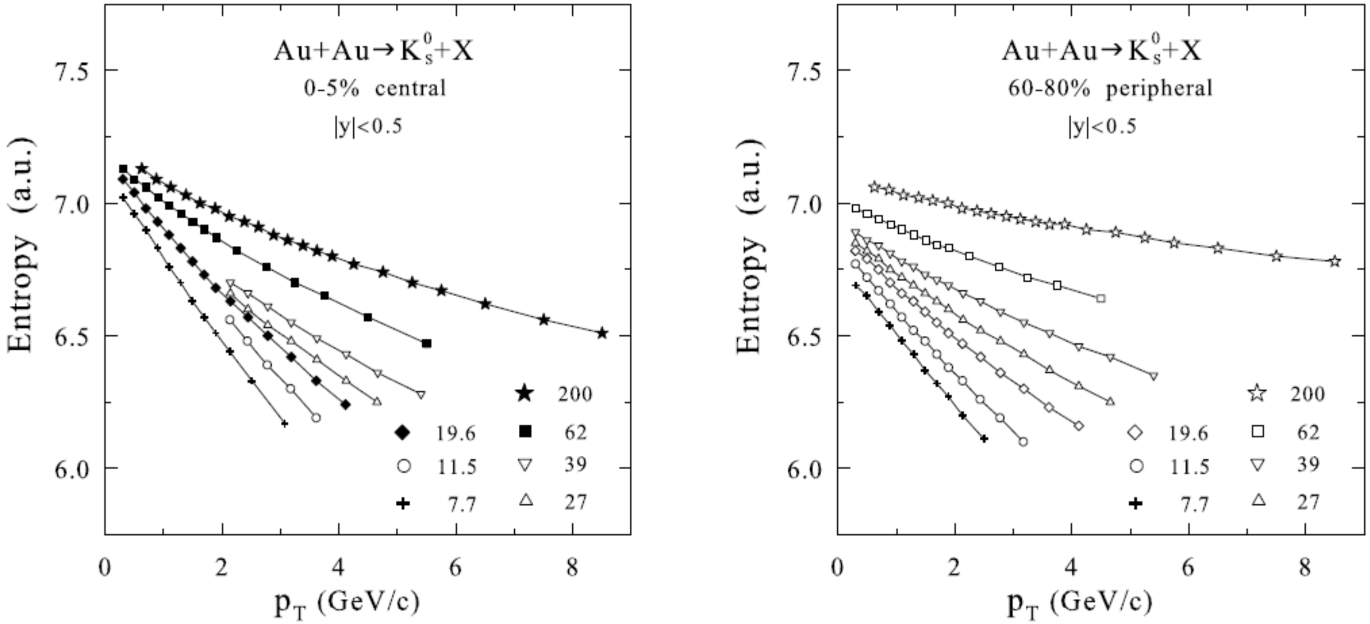
" … fractalon is a free particle conned to move on the fractal trajectory"

R.L. Amoroso et al (eds.),
Gravitation and Cosmology:From the Hubble Radius to the Planck Scale, 85-94.
© 2002 Kluwer Academic Publishers. Printed in the Netherlands.

The notion of "FRACTALON" in z-scaling approach
is applied for description of particle production
in collisions of hadrons and nuclei
at high energy and small scales.

Fractal entropy $S_{\delta,\varepsilon}$ vs. $\sqrt{s_{NN}}$, centrality, p_T

$$S_{\delta,\varepsilon} = c \cdot \ln\left(dN_{ch}/d\eta|_0\right) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$



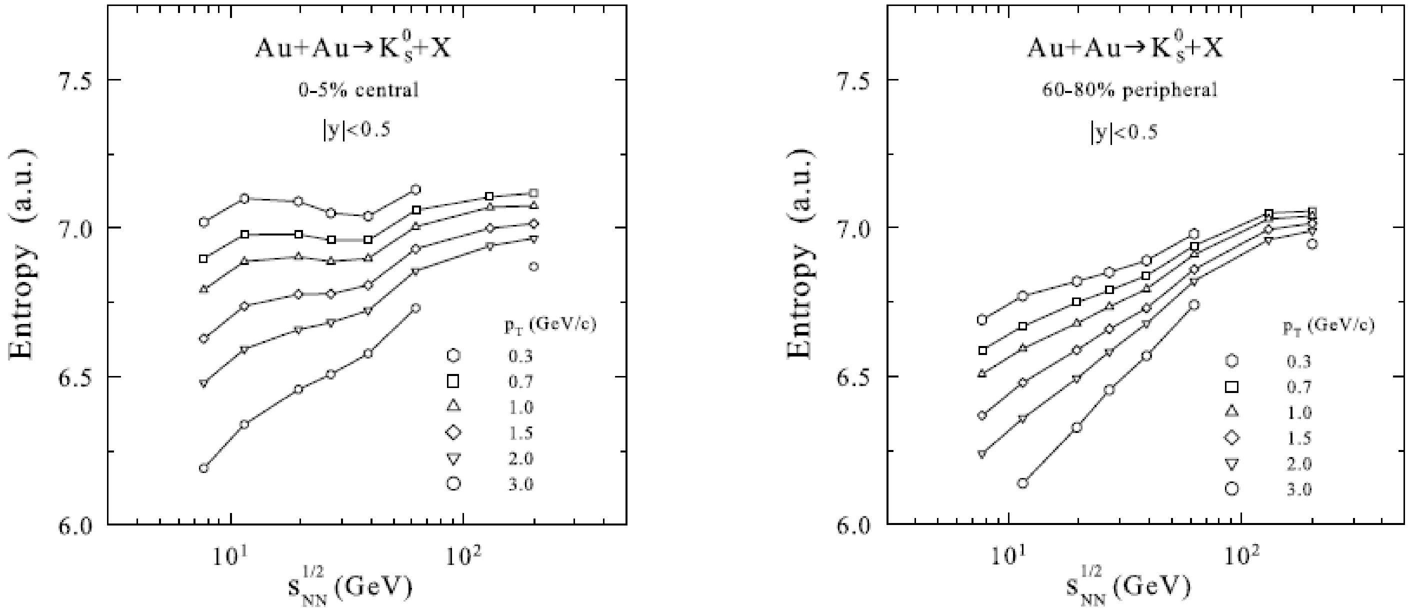
Fractal entropy $S_{\delta,\varepsilon}$

- is a smooth function of collision energy and p_T
- decreases as p_T increases
- increases with collision energy
- in central collisions is larger than in peripheral ones at low $p_T < 1$ GeV/c
- increases with multiplicity density $dN_{ch}/d\eta|_0$
- decreases with increasing resolution Ω^{-1}.

Fractal entropy $S_{\delta,\varepsilon}$ vs. $\sqrt{s_{NN}}$, centrality, p_T

Anomaly of $S_{\delta,\varepsilon}$ in the region $\sqrt{s_{NN}}$ = 11.5–39 GeV at low p_T

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$



- The entropy reaches a local maximum at the energy $\sqrt{s_{NN}}$=11.5 -19.6 GeV and p_T=0.3 GeV/c.
- An abrupt fall of $S_{\delta,\varepsilon}$ is seen at $\sqrt{s_{NN}}$=27 -39 GeV with a gradual increase at higher energies.
- Anomalous behavior of $S_{\delta,\varepsilon}$ is also visible at p_T=0.7 and 1.0 GeV/c in the same energy range.
- Monotonic growth of $S_{\delta,\varepsilon}$ is observed for all p_T in the peripheral collisions for all $\sqrt{s_{NN}}$.

Signatures of Phase Transition and Critical Point

- Discontinuity or abrupt change of the model parameters:
  "specific heat"- c, fractal dimensions – δ, ε

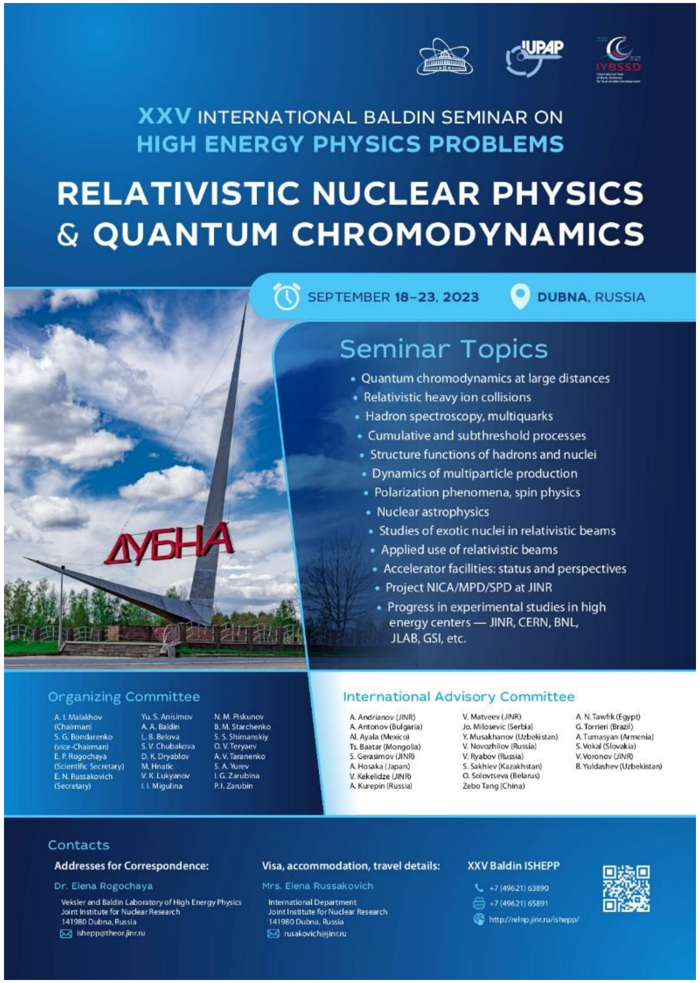
- Enhancement of c-δ-ε correlations

- Anomalous behavior of the fractal entropy $S_{\delta,\varepsilon}$

- Energy loss is a contamination factor leading to smearing of the phase transition signatures

Summary

- Some results of STAR data analysis on transverse momentum inclusive spectra of hadrons produced in p+p and Au+Au collisions at RHIC in the z-scaling approach were given.
- Self-similarity of hadron production in p+p and Au+Au collisions over a wide kinematic and centrality range was found.
- Properties of data z-presentation and dependence of the model parameters - fractal dimensions and "specific heat", on collision energy and centrality were discussed.
- Universality of Ψ vs. z and smooth behavior of x_1 , y_a , M_X vs. p_T , centrality, and collision energy were observed.
- Fractal entropy introduced in z-scaling approach was discussed.
- Conservation law of fractal cumulativity was formulated.
- Anomaly of "specific heat" c_{AA} in the range $\sqrt{s_{NN}}$ = 11-39 GeV was found.
- Anomaly of fractal entropy $S_{\delta,\varepsilon}$ in the range $\sqrt{s_{NN}}$= 27 -39 GeV was found.
- Signatures of phase transition and critical point of nuclear matter produced in heavy ion collisions were discussed.



Thank you for attention !