

Production of η_c mesons at high energy in proton-proton collisions

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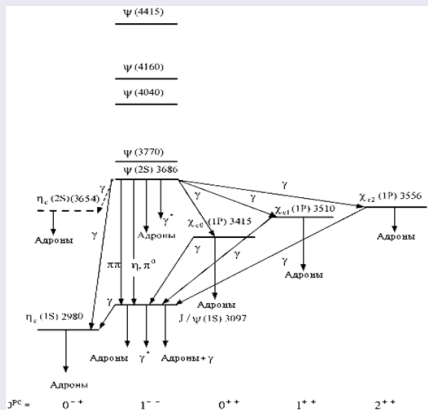
**The XXVth International Baldin Seminar on High Energy Physics
Problems "Relativistic Nuclear Physics and Quantum
Chromodynamics"**

Outline

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Introduction

$$\eta_c = c\bar{c}[^1S_0], M(\eta_c) = 2.981 \text{ GeV}, \Gamma = 29.7 \text{ MeV}$$



$$Br(\eta_c \rightarrow p\bar{p}) = 1.4 \times 10^{-3}$$

$$Br(\eta_c \rightarrow \Lambda\bar{\Lambda}) = 9.4 \times 10^{-4}$$

$$Br(\eta_c \rightarrow K\bar{K}\pi) = 7.2 \times 10^{-2}$$

$$Br(\eta_c \rightarrow \gamma\gamma) = 1.78 \times 10^{-4}$$

Factorization approaches: CPM, TMD and GPM

Hard (factorization) scale $\mu_F \sim M$

Intrinsic parton transverse momentum $\langle q_T^2 \rangle \sim 1 \text{ GeV}^2$

- **Collinear parton model:** $q_{1,2T} \ll p_T$ and $\mu_F = M_T \geq M$

$$\sigma(pp \rightarrow \eta_c X) = \int dx_1 \int dx_2 f_g(x_1, \mu_F) f_g(x_2, \mu_F) \hat{\sigma}(g + g \rightarrow \eta_c + g)$$

- **TMD PM** by Collins, Soper, Stermann: $q_{1,2T} \sim p_T$ and $p_T \ll \mu_F$

$$\begin{aligned} \sigma(pp \rightarrow \eta_c X) = & \int dx_1 d^2 q_{1T} \int dx_2 d^2 q_{2T} F_g(x_1, q_{1T}, \mu_F, \mu_Y) \times \\ & \times F_g(x_2, q_{2T}, \mu_F, \mu_Y) \hat{\sigma}(g + g \rightarrow \eta_c) \end{aligned}$$

- **Generalized parton model:** $q_{1,2T} \sim p_T$ and $p_T \sim \mu_F$

$$\begin{aligned} \sigma(pp \rightarrow \eta_c X) = & \int dx_1 d^2 q_{1T} \int dx_2 d^2 q_{2T} F_g(x_1, q_{1T}, \mu_F) \times \\ & \times F_g(x_2, q_{2T}, \mu_F) \hat{\sigma}(g + g \rightarrow \eta_c) \end{aligned}$$

$$F_g(x, q_T, \mu_F) = f_g(x, \mu_F) \times \exp(-q_T^2 / \langle q_T^2 \rangle) / (\pi \langle q_T^2 \rangle)$$

Factorization approaches: PRA

Parton Reggeization Approach

Parton Reggeization Approach (PRA) is based on High-Energy Factorization (HEF)

$$d\sigma(p + p \rightarrow H + X) = \sum_{i,j=Q,\bar{Q},R} \int \frac{dx_1}{x_1} \int \frac{d^2\vec{q}_{1T}}{\pi} \Phi_i^p(x_1, t_1, \mu^2) \times \\ \times \int \frac{dx_2}{x_2} \int \frac{d^2\vec{q}_{2T}}{\pi} \Phi_j^p(x_2, t_2, \mu^2) d\hat{\sigma}(i + j \rightarrow H + X)$$

$$q_{1,2}^\mu = x_{1,2} P_{1,2}^\mu + q_{1,2T}^\mu, q_{1,2T} \neq 0, q_{1,2}^2 = -\vec{q}_{1,2T}^2 = q_{1,2T}^2 = t_{1,2}$$

- There is transverse momentum dependence, but initial partons are off-mass-shell instead of TMD factorization.

For details:

- M. A. Nefedov, V. A. Saleev and A. V. Shipilova, Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach, // Phys. Rev. D **87** (2013) no.9, 094030
- A.V. Karpishkov, M.A. Nefedov and V.A. Saleev, $B\bar{B}$ angular correlations at the LHC in the parton Reggeization approach merged with higher-order matrix elements // Phys. Rev. D **96**, 096019 (2017)
- M.A. Nefedov and V.A. Saleev, High-energy factorization for the Drell-Yan process in pp and $p\bar{p}$ collisions with new unintegrated PDFs // Phys. Rev. D **102**, 114018 (2020)

Amplitudes in PRA

Lipatov's field theory

For PRA amplitudes we can use effective gauge-invariance field theory with specific Feynman rules instead recover asymptotics of QCD amplitude on $s \rightarrow \infty$. This EFT is known as Lipatov's field theory.

Initial state factors:

$$\begin{aligned} \text{---} \rightarrow q &= \frac{q^\pm}{2\sqrt{-q^2}}, \\ \text{---} \rightarrow \bar{q} &= u(q^\mp). \end{aligned}$$

Propagators ($\hat{p}_\pm = \frac{1}{2}\hat{h}^\mp \hat{h}^\pm$):

$$\begin{aligned} \text{---} \rightarrow q &= \frac{\hat{p}_\pm i\hat{q}}{q^2}, \\ \text{---} \rightarrow \bar{q} &= \frac{i\hat{q} \hat{p}_\pm}{q^2}. \end{aligned}$$



$$q_{2i}^\pm \rightarrow q = -ig_s T^a \left(\hat{h}^\pm + 2\frac{\hat{q}_i}{Q_2^2} \right),$$



$$q_{2i}^\pm \leftarrow q = -2ieg_s T^a \frac{\hat{q}_i n_{2i}^\pm}{p^2 Q_2^2},$$



$$q_{2i}^\pm \rightarrow g = -ie \left(\gamma_\mu + \hat{q}_i \frac{n_{2i}^\pm}{p^2} + \hat{q}_2 \frac{n_{2i}^\pm}{p^2} \right),$$



$$q_{2i}^\pm \leftarrow g = -ie \left(\gamma_\mu + \hat{q}_i \frac{n_{2i}^\pm}{p^2} \right),$$



$$q_{2i}^\pm \rightarrow g = -ie^2 g_s T^a \frac{n_{2i}^\pm n_{2i}^\pm}{p_1^2 p_2^2},$$

$$\begin{aligned} q_{2i}^\pm \rightarrow g &= ie^2 \left(\hat{q}_2 \frac{n_{2i}^\pm n_{2i}^\pm}{p_1^2 p_2^2} - \hat{q}_1 \frac{n_{2i}^\pm n_{2i}^\pm}{p_1^2 p_2^2} \right), & q_{2i}^\pm \rightarrow g &= ie^2 \left(\hat{q}_2 \frac{n_{2i}^\pm n_{2i}^\pm}{p_1^2 p_2^2} + \hat{q}_1 \frac{n_{2i}^\pm n_{2i}^\pm}{p_1^2 p_2^2} \right), & q_{2i}^\pm \rightarrow g &= -2ie^2 g_s T^a \frac{\hat{q}_1 n_{2i}^\pm n_{2i}^\pm}{p_1^2 p_2^2}. \end{aligned}$$

ReggeQCD

Model-file ReggeQCD flv.mod was developed by Nefedov M.A. and Saleev V.A. for PRA calculations in FeynCalc with using of FeynArts in Wolfram Mathematica.

Automated amplitude generation with Reggeized quarks using
ReggeQuarks.

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For details:

- E.N. Antonov, L.N. Lipatov, E.A. Kuraev and I. O. Cherednikov, Feynman rules for effective Regge action // Nucl. Phys. B 721, 111 (2005)
- M.A. Nefedov, V.A. Saleev, Diphoton production at the Tevatron and the LHC in the NLO approximation of the parton Reggeization approach // Phys. Rev. D 92, 094033 (2015)

PRA: Unintegrated PDFs with exact normalization

Tree-level UPDF contains a collinear divergence

Standard definition in BFKL formalism is used to resolve this divergence:

$$\Phi_i(x, t, \mu^2) = \frac{d}{dt} \left[T_i(t, \mu^2, x) \tilde{f}_i(x, t) \right]$$

We ask **exact equivalence** between last ones and following (KMRW) prescription

$$\Phi_i(x, t, \mu_Y^2) = \frac{\alpha_s(t)}{2\pi} \frac{T_i(t, \mu^2, x)}{t} \sum_{j=q, \bar{q}, g} \int_x^1 dz P_{ij}(z) \tilde{f}_j\left(\frac{x}{z}, t\right) \theta(\Delta(t, \mu_Y^2) - z)$$

and Sudakov form-factor

$$T_i(q_T, \mu) = \text{Exp} \left(- \int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} (\tau_i(t', \mu^2)) + \Delta \tau_i(t', \mu^2, x) \right)$$

with $\tau_i(t, \mu^2) = \sum_j \int_0^1 dz z P_{ji}(z) \theta(\Delta(t, \mu^2) - z)$ and

$$\Delta \tau_i(t, \mu^2, x) = \sum_j \int_0^1 dz \theta(z - \Delta(t, \mu^2)) \left[z P_{ji}(z) - \frac{F_j(\frac{x}{z}, t)}{F_i(x, t)} P_{ij}(z) \theta(z - x) \right]$$

For details:

- M.A. Kimber, A.D. Martin and M.G. Ryskin, Unintegrated parton distributions and prompt photon hadroproduction // Eur.Phys.J.C12:655-661 (2000)
- M.A. Nefedov and V.A. Saleev, High-Energy Factorization for Drell-Yan process in pp and $p\bar{p}$ collisions with new Unintegrated PDFs // Phys. Rev. D 102 (2020)

Hadronization mechanisms: NRQCD and CSM

Basic idea - small relative momentum of $q\bar{q}$ -pair, which can be neglected

Fock states can be decomposed into a series according to the small relative velocity parameter v . For instance, J/ψ -meson series is:

$$|J/\psi\rangle = \mathcal{O}(v^0)|c\bar{c}[{}^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)|c\bar{c}[{}^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^1S_0^{(1,8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^3S_1^{(1,8)}]gg\rangle + \dots$$

For η_c it gives

$$|\eta_c\rangle = \mathcal{O}(v^0)|c\bar{c}[{}^1S_0^{(1)}]\rangle + \mathcal{O}(v^1)|c\bar{c}[{}^1P_1^{(8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^3S_1^{(1,8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^1S_0^{(1,8)}]gg\rangle + \dots$$

In **Color Singlet Model** (CSM) it's using only leading term of NRQCD series (singlet) without taking others terms (octets).

LDMEs

Long-Distance Matrix Elements defines transition of $q\bar{q}$ pair into final quarkonium. Singlet LDMEs connected with squared wave function as

$$\langle \mathcal{O}^H [c\bar{c}^{(1)}] \rangle = 2N_c(2J+1)|\Psi(0)|^2$$

There is symmetry of LDMEs between J/ψ and η_c final states:

- $\langle \mathcal{O}^{\eta_c} [{}^1S_0^{(1)} | {}^1S_0^{(8)}] \rangle = \frac{1}{3} \langle \mathcal{O}^{J/\psi} [{}^3S_1^{(1)} | {}^3S_1^{(8)}] \rangle$
- $\langle \mathcal{O}^{\eta_c} [{}^3S_1^{(8)}] \rangle = \langle \mathcal{O}^{J/\psi} [{}^1S_0^{(8)}] \rangle$
- $\langle \mathcal{O}^{\eta_c} [{}^1P_1^{(8)}] \rangle = 3 \langle \mathcal{O}^{J/\psi} [{}^3P_0^{(8)}] \rangle$

For details:

- G.T. Bodwin, E. Braaten, G. Peter Lepage, Rigorous QCD Analysis of Inclusive Annihilation and Production of Heavy Quarkonium // Phys. Rev. D 51, 1125 (1995)
- P. Cho, A.K. Lebovich, Color-octet quarkonia production // Phys. Rev. D 53, 150 (1996)

NRQCD: $q\bar{q}$ into quarkonium

Factorisation of η_c production in CSM from $c\bar{c}$ pair is presented in the form:

$$\hat{\sigma}(a + b \rightarrow c\bar{c}[13S_0^{1,8}] \rightarrow \eta_c) = \hat{\sigma}(a + b \rightarrow c\bar{c}[1S_0^{1,8}]) \frac{\langle \mathcal{O}^H[1S_0^{(1,8)}] \rangle}{N_{col} N_{pol}}$$

For singlet $N_{col} = 2N_c = 6$, $N_{pol} = 2J + 1 = 1$. For octet $N_{col} = N_c^2 - 1$

Amplitude of $c\bar{c}$ production can be obtained using projector on corresponds spin states

$$\Pi_1^\alpha = \frac{1}{\sqrt{8m_c^3}} \left(\frac{\hat{p}}{2} - \hat{q} - m_c \right) \gamma^\alpha \left(\frac{\hat{p}}{2} + \hat{q} + m_c \right)$$

where $\hat{p} = \gamma^\alpha p_\alpha$, p^α — 4-momenta of $c\bar{c}$ -pair, $\hat{q} = \gamma^\alpha q_\alpha$, q_α — relative 4-momenta of quarks, M — quarkonium mass, $m_c = M/2$ — mass of c -quark.

After convolution of amplitude with projector q is assumed **to be zero**.

For details:

- B.A. Kniehl, D.V. Vasin and V.A. Saleev Charmonium Production at High Energy in the k_T -Factorization Approach // Phys. Rev. D73 074022 (2006)
- B.A. Kniehl, V. Saleev, D. V. Vasin, Bottomonium production in the Regge limit of QCD // Phys. Rev. D 74, 014024 (2006)

Hadronization mechanisms: CEM and ICEM

In Color Evaporation Model, heavy quark pair is produced perturbatively with definite spin and color quantum numbers and color of pair "evaporates" to transform into quarkonium

In the **Improved Color Evaporation Model** (ICEM) cross section of quarkonium state can be presented in form:

$$\hat{\sigma}(J/\psi) = \hat{\sigma}(c\bar{c} : M_{\eta_c} < s < 4m_D^2) f_c^{\eta_c}$$

when charmonium momentum is distinguished from the momentum of a quark pair

through relation $p_T^{\eta_c} = \frac{M_{\eta_c}}{M_{c\bar{c}}} p_T^{c\bar{c}}$

f_c can be found from fit of experimental data. V. A. Saleev and A. A. Chernyshev showed that f_c depends on energy:

- $F_c = 0.02$ at big energies as 7,8 or 13 TeV
- $F_c = 0.07$ at PHENIX energy (200 GeV)
- $F_c = 0.2$ at energy 27 GeV of SPD NICA

For details:

- Y. Ma, R. Vogt, Quarkonium production in an improved color evaporation model // Phys. Rev. D 94,114029 (2016)
- A. A. Chernyshev and V. A. Saleev, Single and pair J/ψ production in the improved color evaporation model using the parton Reggeization approach, Phys. Rev. D 106 (2022) no.11, 114006

η_c production in pp-collisions

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)



CERN-EP-2019-214
LHCb-PAPER-2019-024
5 March 2020

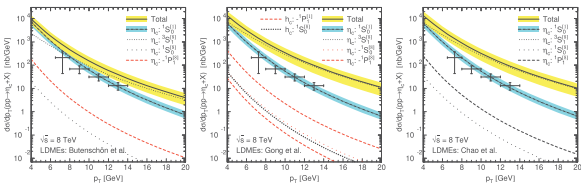
Measurement of the $\eta_c(1S)$ production cross-section in pp collisions at $\sqrt{s} = 13$ TeV

LHCb collaboration[†]

- LHCb 7 TeV, 8 TeV, 13 TeV - **There are experimental data only for 7 TeV, 8 TeV**
- ATLAS 7 TeV, 8 TeV, 13 TeV
- PHENIX 200 GeV
- SPD NICA 27 GeV

η_c production in CPM and GPM within framework of CSM η_c production at the LHC

[Butenschön, Kniehl, He, 2014] Experimental data from [LHCb, 2014]:
 $pp \rightarrow \eta_c (\rightarrow p\bar{p}) + X$ with $\sqrt{S} = 7$ and 8 TeV.



Conclusions:

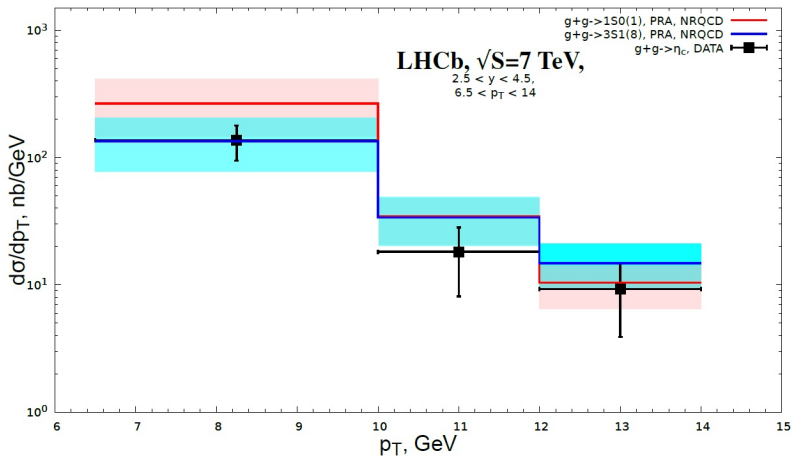
- ▶ CS-model ($^1S_0^{(1)}$) describes LHCb data! CO-contrs. lead to significant overshoot. \Rightarrow HQSS-relations fail!
- ▶ Feeddown from h_c is negligible

η_c production in CPM and GPM within framework of CSM

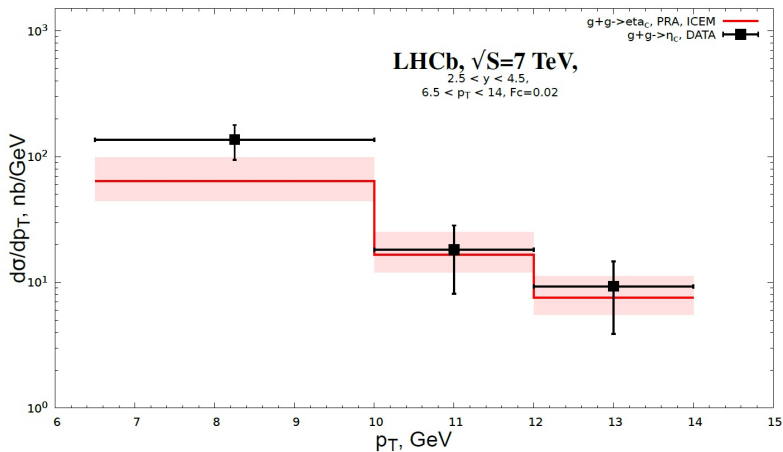
Analysis of η_c production in the NLO at the LHC leads to following conclusions:

- Color singlet LDME $\langle \mathcal{O}[^1S_0^{(1)}] \rangle$ can be calculated in a non-relativistic potential model ($|\Psi(0)|^2$), or extracted from the decay width $\Gamma(\eta_c \rightarrow \gamma\gamma)$. Color octet LDMEs are not needed.
- Final state is colorless and we can neglect final-state interactions with soft (Glauber) gluons, which destroy hard-soft factorization. The η_c production in two-gluon fusion may be considered as "Drell-Yan" process, only for initial gluons.
- There is only direct production of η_c , contributions from high-mass states can be neglected.

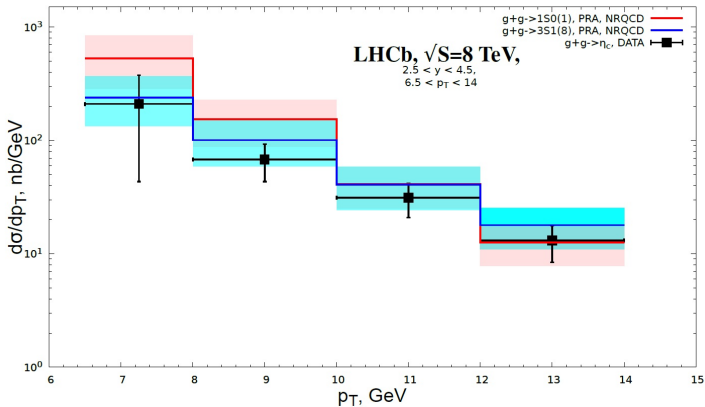
Comparison with LHCb data at the 7 TeV within NRQCD



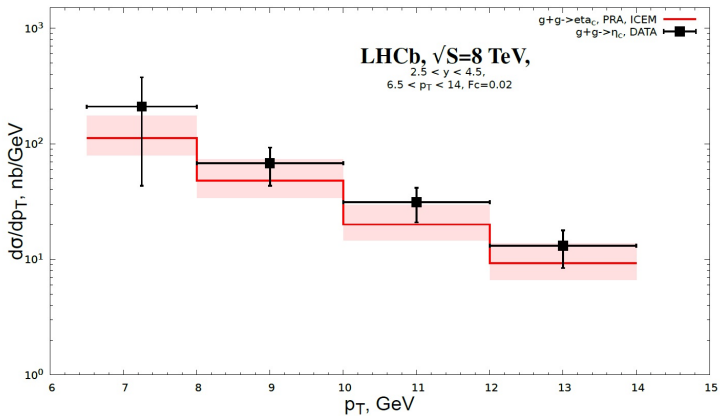
Comparison with LHCb data at the 7 TeV within ICEM



Comparison with LHCb data at the 8 TeV within NRQCD



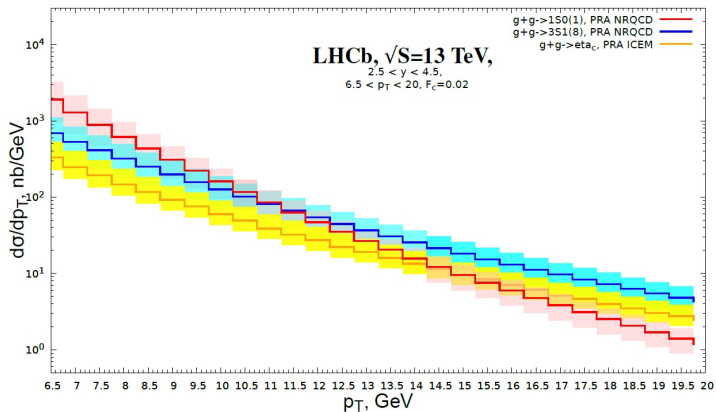
Comparison with LHCb data at the 8 TeV within ICEM



Predictions for LHCb at the 13 TeV

Total cross-section

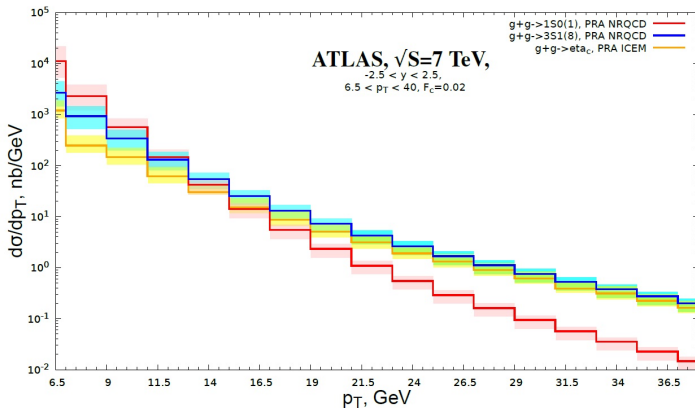
- PRA ICEM: $\sigma_{tot} = 0.95 \mu b$
- PRA CSM: $\sigma_{tot} = 15.1 \mu b$
- PRA NRQCD: $\sigma_{tot} = 19.5 \mu b$



Predictions for ATLAS at the 7 TeV

Total cross-section

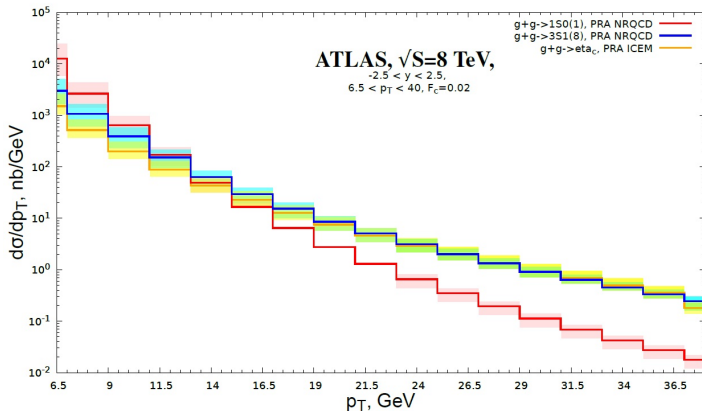
- PRA ICEM: $\sigma_{tot} = 0.5 \mu b$
- PRA CSM: $\sigma_{tot} = 8.1 \mu b$
- PRA NRQCD: $\sigma_{tot} = 11.2 \mu b$



Predictions for ATLAS at the 8 TeV

Total cross-section

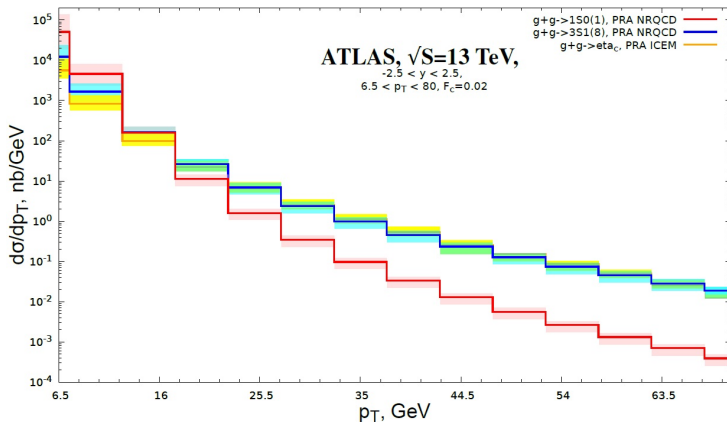
- PRA ICEM: $\sigma_{tot} = 0.6 \mu b$
- PRA CSM: $\sigma_{tot} = 9.3 \mu b$
- PRA NRQCD: $\sigma_{tot} = 12.3 \mu b$



Predictions for ATLAS at the 13 TeV

Total cross-section

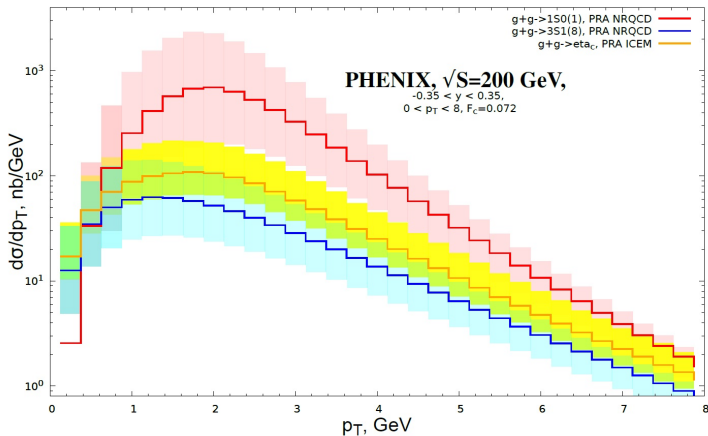
- PRA ICEM: $\sigma_{tot} = 1.1 \mu b$
- PRA CSM: $\sigma_{tot} = 15.02 \mu b$
- PRA NRQCD: $\sigma_{tot} = 18.9 \mu b$



Predictions for PHENIX

Total cross-section

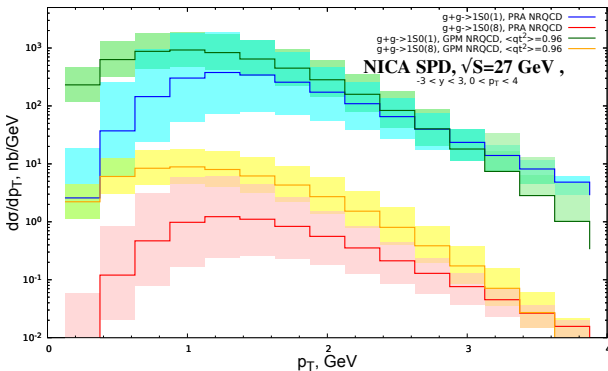
- PRA ICEM: $\sigma_{tot} = 0.2 \mu b$
- PRA CSM: $\sigma_{tot} = 1.4 \mu b$
- PRA NRQCD: $\sigma_{tot} = 1.6 \mu b$



Predictions for SPD NICA within NRQCD

Total cross-section

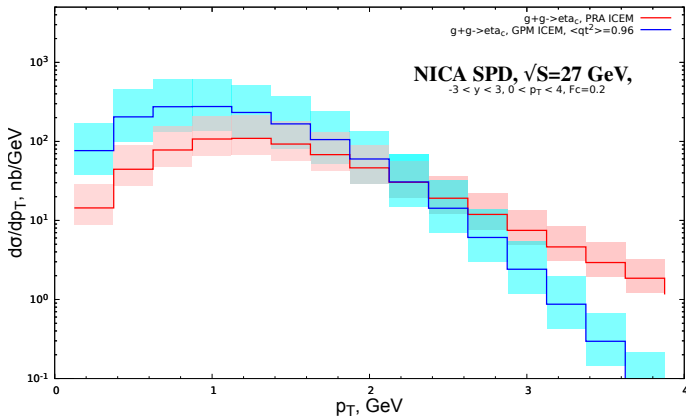
- PRA CSM: $\sigma_{tot} = 0.48 \mu b$
- PRA NRQCD: $\sigma_{to} = 0.49 \mu b$
- GPM CSM: $\sigma_{tot} = 1.3 \mu b$
- GPM NRQCD: $\sigma_{tot} = 1.31 \mu b$



Predictions for SPD NICA within ICEM

Total cross-section

- PRA ICEM: $\sigma_{tot} = 0.16 \mu b$
- GPM ICEM: $\sigma_{tot} = 0,37 \mu b$



Conclusions

- PRA calculations confirmed conclusions obtained in the NLO CPM. CSM is enough to describe η_c production at the LHC.
- CSM is approximately 80 % of NRQCD with CO NMEs taking accordingly HQS rules.
- PRA ICEM approximately 2 times less than data , and lies below CSM when $f_{\eta_c} = f_{J/\psi}$.
- PRA predictions for NICA are smaller by factor 2 than results within GPM used parameters obtained by fitting J/ψ data for the relevant energies.
- The predicted total cross section of η_c production is approximately from 0.5 μb to 1,3 μb

Thank you for your attention!