Precision calculations of Bhabha scattering for e^+e^- luminosity measurement

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Motivation

The standard problems of every collider experiment is luminosity monitoring.
At e^+e^- colliders the most common methods are:

1. small and large angle Bhabha scattering,
2. lepton-pair production in e^+e^- collisions,
3. large angle e^+e^- annihilation into photon pair.

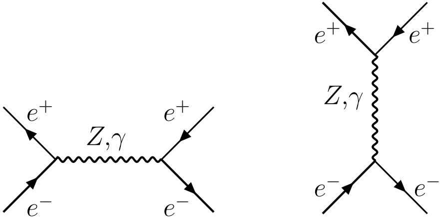
On the theory side, the most used and most advanced codes for Bhabha process are `BHLUMI`, `BHWIDE`, `BabaYaga`, `MCGPJ`, `ReneSANCe`.
Of the above codes, only `BHLUMI`, `BabaYaga` are used extensively to estimate the luminosity by Bhabha scattering at small angles.

Our group plans to develop a code for Bhabha process calculations which will include all the theoretical achievements up to date. Here we present our first step (complete one-loop results) in the study of the small angle Bhabha scattering (`SABS`) cross section by the means of events generator `ReneSANCe`.

We consider the process

$$e^+(p_1) + e^-(p_2) \to e^-(p_3) + e^+(p_4)(+\gamma(p_5))$$

at the complete one-loop electroweak level and evaluated the effects due to the working in the full phase space.



s and t channels for Bhabha scattering.

Features of SABS

- SABS is an almost purely electromagnetic process, which allows in principle to compute theoretical predictions for SABS within the perturbative theory approach with high accuracy.
- The differential distribution is strongly peaked at small angles where it behaves like $\sim 1/\theta^4$.
- Usually an MC generator has a cut-off for a minimum scattering angle to avoid divergence.

Experimental issues of SABS measurements

- The SABS cross section is usually measured with the help of calorimeters which are typically not equipped with tracking systems.
- Thus, it is impossible to distinguish a scattered electron from a radiated photon with the same energy.
- Potential background to Bhabha process are the events when one of the electrons is scattered by nearly zero angle, while an energetic photon is detected in the luminosity calorimeter.
- Another background is provided by the process of electron-positron annihilation into two (or more) photons which hit the luminosity calorimeters.

Cross-check with the 1996 LEP Workshop

Firstly, we compare the technical precision of our codes with results presented in the proceedings of the CERN Workshop for SABS at LEP era. For the tuned comparison we used non-calorimetric event selection (ES) called BARE1 and calorimetric ES called CALO1.

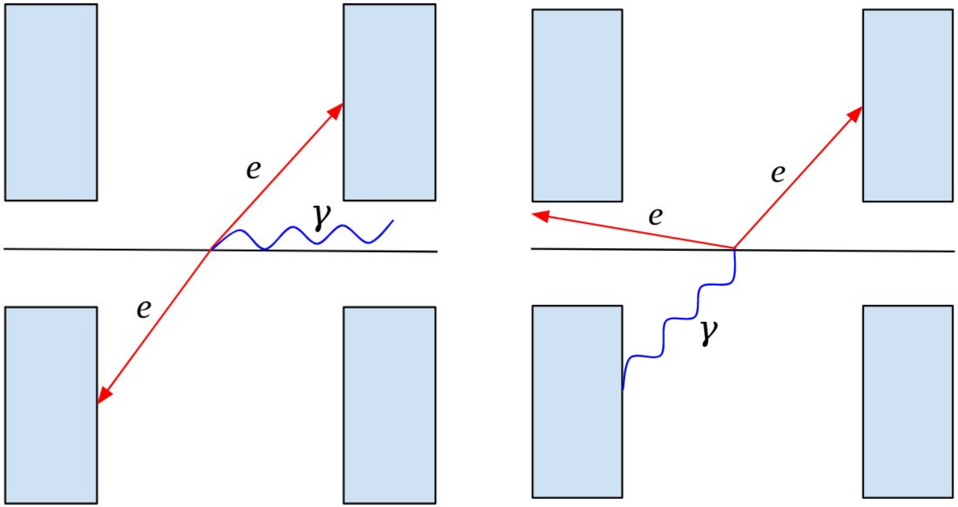
All numbers produced in setup the Workshop [1] for the $\mathcal{O}(\alpha)$ matrix element without contribution of the Z exchange, up-down interference and vacuum polarization. The results are shown with various values of the energy-cut $z_{min} = s'/s$, where s' is the collision energy after ISR.

Geometry and acceptance event selection:

- **BARE1** – each arm of the luminometer is hit by an electron (positron) with the energy $E_{e^\pm} > 0.5E_{\text{beam}}$ without taking into account photons.
- **CALO1** – each arm of the luminometer is hit by electron and/or photon(s) with total energy $E_{\text{CALO}} > 0.5E_{\text{beam}}$.

[1] Jadach, S. and others, *Event generators for Bhabha scattering*, CERN Workshop on LEP2 Physics (followed by 2nd meeting, 15-16 Jun 1995 and 3rd meeting 2-3 Nov 1995), hep-ph/9602393

Scheme of the two cases under consideration: BARE1 (left) and CALO1 (right)



Technical agreement: ReneSANCe vs BHLUMI

Comparison of BARE1 and CALO1 for the $\mathcal{O}(\alpha)$ matrix element. Z exchange, up-down interference and vacuum polarization are switched off. The center of mass energy is $\sqrt{s} = 92.3$ GeV. The results are shown with various values of the energy-cut $z_{min} = s^{'}/s$.

z_{min}	ReneSANCe	BHLUMI	ReneSANCe	BHLUMI
	BARE1: σ [nb]		CALO1: σ [nb]	
.100	166.01(1)	166.05(2)	166.33(1)	166.33(2)
.300	164.71(1)	164.74(2)	166.05(1)	166.05(2)
.500	162.19(1)	162.24(2)	165.26(1)	165.29(2)
.700	155.41(1)	155.43(2)	161.77(1)	161.79(2)
.900	134.36(2)	134.39(2)	149.91(1)	149.93(2)

Generator cuts and input parameter sets

We generated Bhabha events, where each arm of the luminometer registered an energy shower from an electron or photon. **No restriction on minimum scattering angle was set.**

- Electrons were allowed to scatter by any angle, down to zero.
- Luminosity acceptance was assumed 30 mrad $< \vartheta <$ 174.5 mrad.

Numerical results were obtained in the full phase space, in the $\alpha(0)$ electroweak scheme for the following set of input parameters:

$$
\begin{array}{llllll}
\alpha^{-1}(0) = & 137.035999084 & \Gamma_W = & 2.0836\ \text{GeV} & \Gamma_Z = & 2.4952\ \text{GeV} \\
M_H = & 125.0\ \text{GeV} & M_W = & 80.379\ \text{GeV} & M_Z = & 91.1876\ \text{GeV} \\
m_u = & 0.062\ \text{GeV} & m_d = & 0.083\ \text{GeV} & m_s = & 0.215\ \text{GeV} \\
m_c = & 1.5\ \text{GeV} & m_b = & 4.7\ \text{GeV} & m_t = & 172.76\ \text{GeV} \\
m_e = & 0.51099895\ \text{MeV} & m_\mu = & 0.1056583745\ \text{GeV} & & \\
m_\tau = & 1.77686\ \text{GeV}. & & & &
\end{array}
$$

Results: BARE1 vs CALO1

- The CALO1 cross-section at $\sqrt{s} = 240$ GeV is 3% larger than BARE1 Bhabha cross-section, when both beam particles must hit the luminometer.
- Majority of the effect is due to the events with collinear photon or due to the events when electron is scattered by an angle LARGER than the luminosity acceptance, while hard ISR photon hits the luminometer.
- Such events can be (and have been!) taken into account by any NLO Bhabha generator.
- LEP luminosity measurement (hopefully) was not affected by those events.

Results for the BARE1

To represent numerical result of each contributions we evaluate corresponding relative corrections defined as

$$\delta = \frac{\sigma^{\text{contr.}}}{\sigma^{\text{Born}}} - 1, \%$$

$\sqrt{s}$, GeV	91.18	240
σ^{Born}, pb	135008.457(1)	19473.628(2)
$\delta^{\text{one-loop}}$, %	−1.509(2)	−0.828(3)
δ^{QED}, %	−6.170(2)	−7.002(3)
δ^{weak}, %	0.00918(3)	−0.00675(2)
δ^{VP}, %	4.65262	6.18667
$\delta^{\text{weak+VP}}$, %	4.66180(3)	6.17992(3)

New effect

- About $\sim \mathbf{1.4}$ permille of the total cross-section (both at $\sqrt{s} = 91.18$ and 240 GeV) is represented by of events with electron scattering angle **below** the luminometer acceptance.
- Those events have been missed by earlier generators.
- In particular, this effect was a bias of the LEP luminosity measurement.

Integrated cross section and relative corrections

- BARE1: each arm of the luminometer is hit by an electron,
- CALO1: each arm of the luminometer is hit by an electron or photon,
- $\vartheta < 0.030$: same as CALO1, but electron is scattered into the luminometer acceptance **or below**.

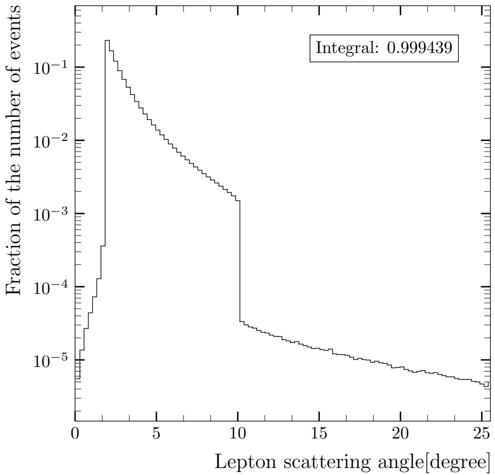
$\sqrt{s}$, GeV	91.18	240
$\sigma^{\rm Born}$, pb	135008.457(1)	19473.6283(2)
$\delta^{\rm QED}$(BARE1), %	−6.170(2)	−7.002(3)
$\delta^{\rm QED}$(CALO1), %	−3.486(4)	−3.992(4)
$\delta^{\rm QED}(\vartheta < 0.030)$, %	−3.349(2)	−3.854(4)

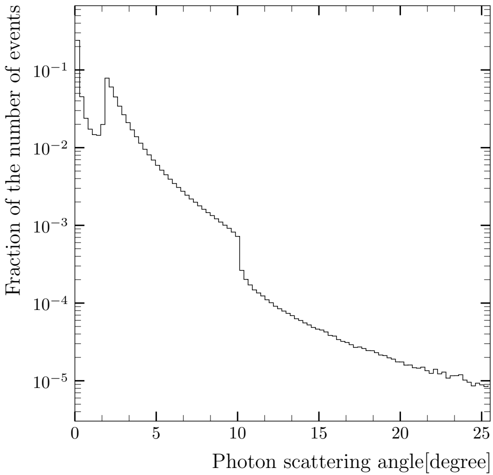
Angular distributions

We illustrate the results with several angular distributions at two energies of 91.18 GeV and 240 GeV:

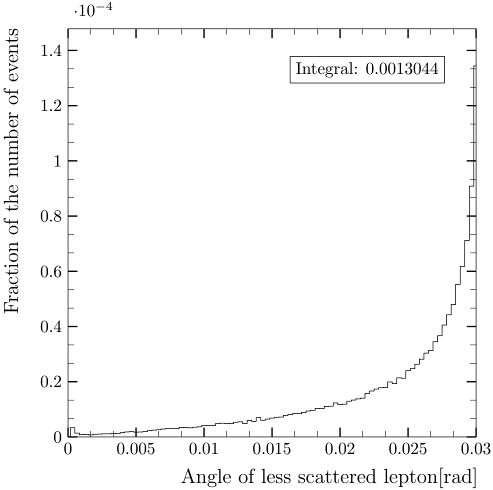
- Distribution of fraction of the number of Bremsstrahlung photon events over the photon scattering angle $\vartheta_{15} = \vartheta_{\gamma}$, i.e. the angle between particle p_1 (positron e^+) and particle p_5, photon.
- Distribution of fraction of the number of lepton events over the lepton scattering angle ϑ_{14}, i.e. the angle between particle p_1 (positron e^+) and particle p_4, positron.

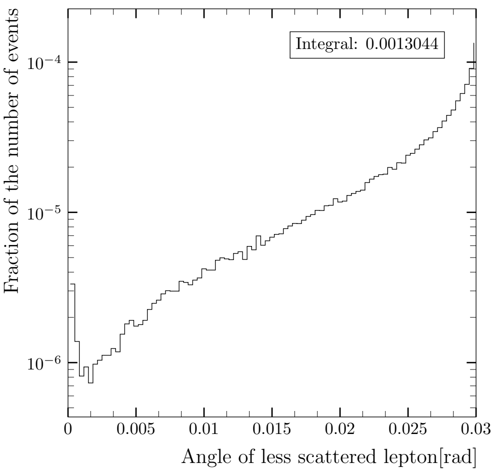
Distributions of lepton (left) and photon (right) scattering angles at $\sqrt{s} = 91.18$ GeV



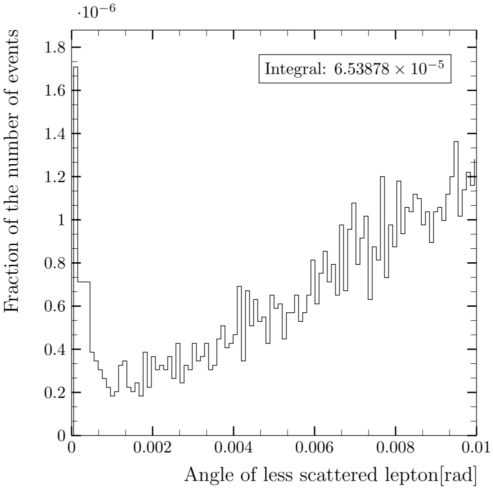


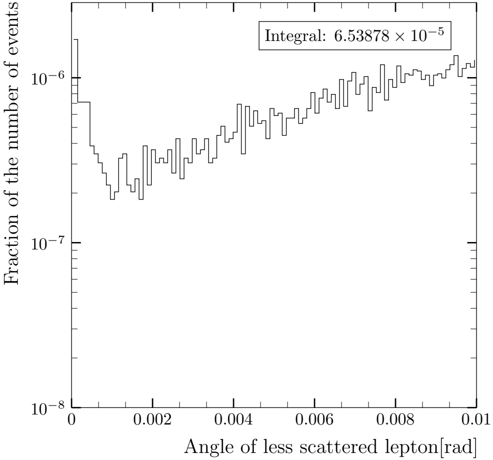
Distribution of lepton scattering angle at $\sqrt{s} = 91.18$ GeV



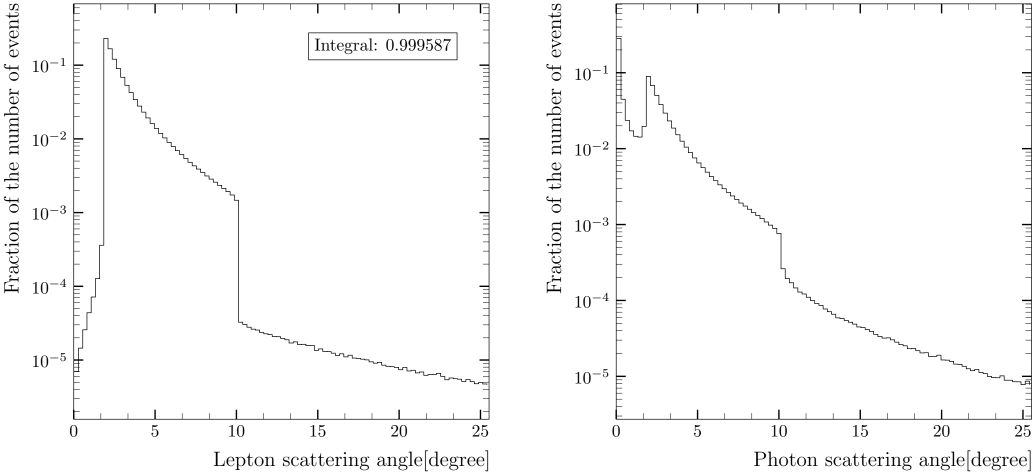


Distribution of lepton scattering angle at $\sqrt{s} = 91.18$ GeV

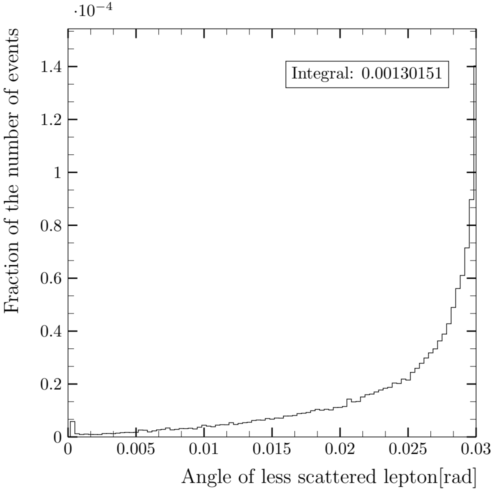


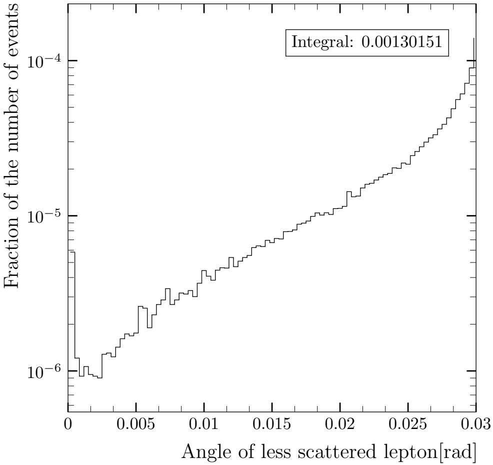


Distributions of lepton (left) and photon (right) scattering angles at $\sqrt{s} = 240$ GeV

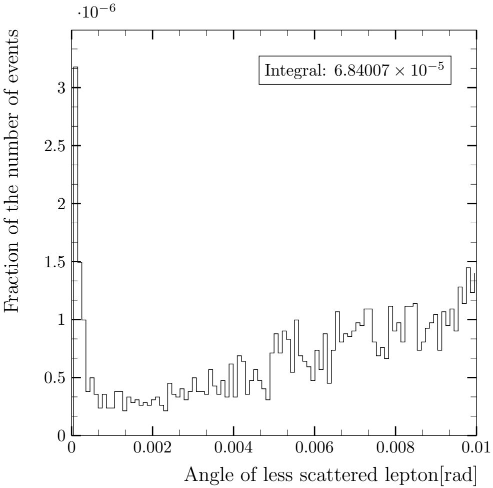


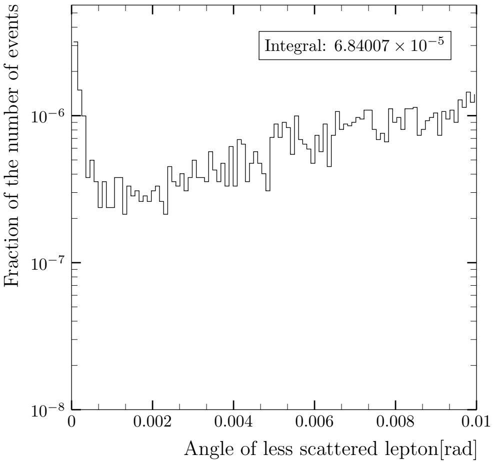
Distribution of lepton scattering angle at $\sqrt{s} = 240$ GeV





Distribution of lepton scattering angle at $\sqrt{s} = 240$ GeV





Conclusions

- Bhabha generator at complete 1-loop EW level is ready
- Technical agreement with LEP-era generators is achieved
- A unique feature of the ReneSANCe generator is the possibility to simulate electron scattering by infinitely small angles. This allows to take into account events in which one arm of the luminosity calorimeter is fired by an energetic ISR photon, while an electron is scattered by very small angle and escapes detection. If not taken into account, this effect leads to a luminosity (and N_ν) bias of 1.3-1.4 ‰, both at Z pole and at 240 GeV.
- The bias can be avoided by generating events down to 10 mrad scattering angles. In this case the bias value becomes less than 10^{-4}.
- Further development: higher order radiative corrections and exponentiation.

Foundation

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Thank you for your attention!