Probing a core of the interaction potential of two Λ-hyperons with femtoscopic correlations at LEP

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The analysis of ALEPH data on femtoscopic correlations of two Λ-hyperons in Z-boson decays yields a very small source radius of 0.11 ± 0.03 fm if taking into account only the repulsion due to the Fermi-Dirac quantum statistics. Such a small source radius is counter-intuitive in the string picture of particle production due to a moderate string tension of $\sim$1 GeV/fm. It is shown that the ALEPH data can be described with an acceptable source radius of $>\sim$ 0.3 fm if taking into account the repulsive final state interaction between hyperons at distances smaller than a femtometer. Information on the potential core of two-hyperon interaction is difficult to obtain otherwise.

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Introduction

In case of a poor knowledge of the two–particle strong interaction, the correlation measurements can be used to study the latter [1]. Such studies can essentially improve the information on the interaction involving strange particles, highly required to understand the properties of neutron stars [2]. Particularly important is the measurement of $\Lambda\Lambda$ interaction, to clarify the H-dibaryon problem and describe the properties of double Λ hypernuclei [3]. In the context of two-hyperon interaction, we elaborate further the analysis [1] of the data on two-Λ momentum and spin correlations in Z^0 decays obtained by ALEPH collaboration at LEP [4]. In present analysis, we take into account that the momentum correlations of parent hyperons are practically completely transferred to their daughter Λ's provided that these correlations are much wider than the decay momenta of ~ 0.1 GeV/c.

Momentum correlations of identical particles

Consider first the correlation function $\mathcal{R}$ of two identical particles on the absence of their final state interaction (FSI). The $\mathcal{R}$-behaviour at small relative momenta $Q = 2k$ in pair rest frame is then determined by the symmetry requirement of quantum statistics (QS): at $Q \to 0$, the contribution of the even total spin S is enhanced by a factor of 2, while that of the odd S vanishes. Assuming sufficiently smooth momentum dependence of the production amplitude within the region of correlation effect (i.e. a small space-time extent of the particle emitters as compared with their space-time

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separation) - so called "smoothness" assumption, the $\mathcal{R}$-contribution at a given total spin S reduces to a square of the symmetrised wave function of the free motion, $\psi^S_{-\mathbf{k}}(\mathbf{r}) = [\exp(-i\mathbf{kr}) + (-1)^S \exp(i\mathbf{kr})]/\sqrt{2}$, averaged over the vector of the relative distances $\mathbf{r}$ between the emission points in pair rest frame (PRF) [5,6]:

$$\begin{aligned} \mathcal{R}_S &= \widetilde{\rho}_S\langle\left|\psi^S_{-\mathbf{k}}(\mathbf{r})\right|^2\rangle = \widetilde{\rho}_S[1+(-1)^S\left\langle\cos(2\mathbf{kr})\right\rangle], \\ \mathcal{R} &= \textstyle\sum_S \mathcal{R}_S = 1 + (\sum_{S-\mathrm{even}}\widetilde{\rho}_S - \sum_{S-\mathrm{odd}}\widetilde{\rho}_S)\left\langle\cos(2\mathbf{kr})\right\rangle, \end{aligned} \tag{1}$$

where $\widetilde{\rho}_S$ is the intrinsic probability of the total spin S in the absence of QS and FSI. As for the "smoothness" assumption, it is certainly justified for heavy-ion collisions, being however questionable for electro- or hadro-production processes. In any case, for processes with high numbers of participating particles, the neglect of space-time coherence in Eq. (1) can be justified based on the statistical concept [7,8].

For $\Lambda\Lambda$ system and in the case of independent emission, the intrinsic singlet ($S = 0$) and triplet ($S = 1$) probabilities are determined by the square of the intrinsic Λ polarisation vector $\widetilde{\mathbf{P}}$ [6]:

$$\widetilde{\rho}_s = (1-\widetilde{\mathbf{P}}^2)/4, \qquad \widetilde{\rho}_t = (3+\widetilde{\mathbf{P}}^2)/4. \tag{2}$$

Taking into account that Λ-hyperons are momentum-correlated only in a fraction λ of the pairs, adding further a free normalisation parameter $\mathcal{N}$ and assuming that the $\mathbf{r}$-distribution is described by a spherically symmetrical Gaussian with dispersion $\langle\mathbf{r}^2\rangle = 2r_0^2$, i.e. $\langle\cos(2\mathbf{kr})\rangle = \exp(-r_0^2Q^2)$, one arrives at the experimental correlation function:

$$\mathcal{R}^{\mathrm{exp}} = \mathcal{N}[(1-\lambda)+\lambda\mathcal{R}] = \mathcal{N}\left[1-\frac{1}{2}(1+\mathbf{P}^2)\lambda\exp(-r_0^2Q^2)\right], \tag{3}$$

characterised at small Q by a dip of a width $1/r_0$.

To estimate the pair fraction of Λ's that are momentum-correlated due to QS, one can use the data on the production rates in e^+e^- collisions at Z^0 mass [9] to get the fraction 0.445 ± 0.026 of pairs of directly produced Λ's; the remaining pairs contain at least one Λ from $\Sigma^0 \to \Lambda\gamma$ or $\Xi \to \Lambda\pi$ decays. Since the correlation Q-width of ~ 2 GeV/c (Fig. 1) is much larger than the Σ^0- and Ξ-decay momenta of ~ 0.1 GeV/c, the parent momentum correlations are practically completely transferred to the daughter Λ's. Therefore, the Λ's from pairs of identical parents also contribute to the QS correlation function, increasing the correlated fraction to $\lambda = 0.50 \pm 0.03$. Here we assume a universal $\mathbf{r}$-distribution for all hyperon pairs and take into account that the effect of the Coulomb FSI for $\Xi^-\Xi^-$ pairs is negligible at $Q > 0.1$ GeV/c. Taking further into account the experimental pair purity of 86% [4], the momentum-correlated fraction due to QS is decreased to $\lambda = 0.43 \pm 0.03$. As for the Λ polarisation, it appears to be less than 10%, except for a small fraction of very energetic Λ's [10], so that the contribution of the polarisation squared in the intrinsic singlet and triplet probabilities in Eqs. (2) is

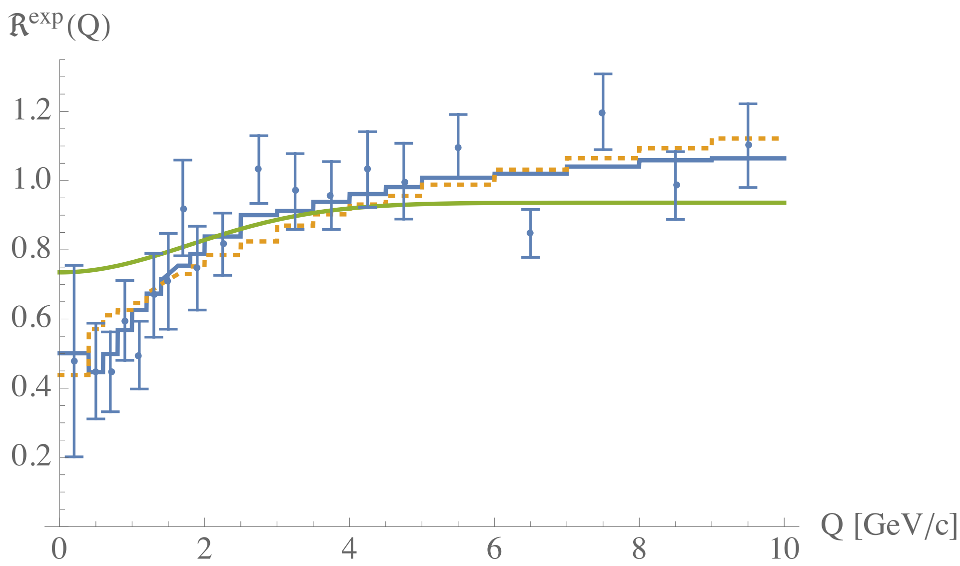

Fig. 1. The ALEPH two–lambda correlation function [4] and example fits: the curve - pure QS fit according to Eq. (3) at fixed zero polarisation and $\lambda = 0.43$ ($r_0 = 0.08 \pm 0.03$ fm); the histograms - fits with λ = purity = 0.86 and the FSI modelled by a three-range Gaussian s-wave potential [3] with the singlet potential $V^s(\gamma = 0.5463)$ reproducing the NSC97e one [18]; the triplet potential $V^t = V^s$ (dashed histogram, $r_0 = 0.26 \pm 0.02$ fm) and $V^t = V^s(\gamma = -2)$ with the widened repulsive part, (full histogram, $r_0 = 0.39 \pm 0.04$ fm).

less than 1% and can be neglected; these probabilities then become equal to 1/4 and 3/4 as for a statistical spin mixture.

The ALEPH fit results [4], $r_0 = 0.11 \pm 0.02_{\mathrm{stat}} \pm 0.01_{\mathrm{sys}}$ and $\lambda = 1.18 \pm 0.18_{\mathrm{stat}} \pm 0.08_{\mathrm{sys}}$, point to a problem of a simple fit with the neglected FSI. First, the fitted fraction λ of correlated pairs is $\sim$ 4 standard deviations higher than the estimated value of 0.43 ± 0.03; as a result, the fit with a fixed $\lambda = 0.43$ yields the unsatisfactory $\chi^2/\mathrm{ndf} = 41.7/18$ (see Fig. 1). Second, the fitted source size parameter r_0 of ~ 0.1 fm appears to be too small in the string model due to a separation of the identical hyperons by a state with anti-hyperon quantum numbers, corresponding to a separation by a string length of $\gtrsim 1$ fm in case of the on-mass-shell state. It indicates a need for the FSI repulsion between any hyperons, including the Λ-parents, with a width of the corresponding momentum anti-correlation much larger than $1/r_0$.

Final State Interaction

In first approximation, the two-particle FSI can be taken into account like in β-decay, substituting the wave function of the free motion in Eq. (1) by the solution of the corresponding scattering problem viewed in the opposite time direction - so the negative sign of the vector **k**; also, when taking into account possible inelastic transitions, the detected channel should be considered as the entrance one [11]. A similar generalisation of Eq. (1) was done in [12] to account for mutual FSI of the protons emitted with non-relativistic momenta.

However, contrary to $t = 0$ in β-decay, the time separation t in PRF may be quite substantial in multiparticle production. It can be taken into account in corresponding Bethe-Salpeter amplitude, which vanishes at large $|t|$ and reduces to the wave function on condition [6] $|t| \ll mr^2$. This condition is usually fulfilled for particles with sufficiently high mass m, like kaons or baryons; even for pions, the substitution of the Bethe-Salpeter amplitude by the wave function usually leads to the error in the strong FSI contribution to the correlation function less than 10% [13].

In heavy ion collisions, r_0 can be considered larger than the range d of the strong interaction potential. The strong FSI contribution $\Delta\mathcal{R}_S^{\mathrm{FSI}}$ to the correlation function is then independent of the actual potential form [14] and, at small Q, it is determined by the s-wave scattering amplitudes $f^S(k)$ [6]. In case of $|f^S| > r_0$, the FSI contribution is of the order of $|f^S/r_0|^2$ and dominates over the effect of QS. In the opposite case, the sensitivity of the correlation function to the scattering amplitude is determined by the linear term f^S/r_0.

The two-Λ correlations have been measured in Au+Au and p+p(Pb) collisions by STAR [15] and ALICE [16] collaborations, respectively. Both the STAR and ALICE fits prefer the s-wave singlet scattering lengths $f^s(0)$ less than $\sim$ 1 fm in magnitude but differ in sign. The STAR fit yields a negative sign, allowing for a possible bound state, while the ALICE fit prefers the positive sign, in agreement with potential models describing the double-Λ hypernuclei, though still leaving a small door for a negative sign.

A possible reason for the discrepancy may be the oversimplified treatment of residual correlations of daughter Λ's from Σ^0- and Ξ-decays. In ALICE analysis, they are considered flat, while in STAR analysis, they are somewhat arbitrarily parametrised by a Gaussian. Also, in proton collisions, the extent of the correlation up to Q of several hundreds MeV/c and the small source size r_0 of $1.2 - 1.4$ fm may invalidate the assumptions of the s-wave FSI dominance and independence of $\Delta\mathcal{R}_S^{\mathrm{FSI}}$ on the potential form; though the latter assumption was found to be fulfilled on a percent level [16], one may expect a significant violation of the former one at $Q > 100 - 150$ MeV/c (see below).

In e^+e^- collisions at LEP, $r_0 < d$, so the two–particle FSI is sensitive to the form of the interaction potential, particularly - to the presence of the potential core. Since now the correlation effect extends up to $k = Q/2$ of several GeV/c, the waves with orbital angular momentum up to $L \sim 15$ contribute to $\Delta\mathcal{R}_S^{\mathrm{FSI}}$ and the calculation is more complicated, requiring the knowledge of the corresponding potentials to solve the "relativistic" Schrodinger equation for the non-symmetrised radial wave functions $\psi_{k,L}^S(r)$; in this equation, the masses are substituted by the energies so that it coincides with the Klein-Gordon equation for the potentials much smaller than the total pair energy.

For the assumed spherically symmetric Gaussian source and taking into account only elastic transitions with the L, S conservation, the respective strong FSI contributions to the correlation functions of non-interacting identical ($\mathcal{R}$ in Eq. (1)) and non-identical ($\mathcal{R}=1$) particles are

$$\begin{aligned} \Delta\mathcal{R}_S^{\rm FSI} &= 2\widetilde{\rho}_S \textstyle\sum_{(L+S)-{\rm even}}(2L+1)\langle[[\psi_{k,L}^S(r)]^2-[j_L(kr)]^2)\rangle, \\ \Delta\mathcal{R}_S^{\rm FSI} &= \widetilde{\rho}_S \textstyle\sum_{L}(2L+1)\langle[[\psi_{k,L}^S(r)]^2-[j_L(kr)]^2)\rangle, \end{aligned} \tag{4}$$

where $j_L(x)$ are the spherical Bessel functions.

To model two-hyperon potentials, we have used a three-range Gaussian s-wave potential with the negative height of the intermediate-range Gaussian multiplied by a scale parameter γ [3], well reproducing various Nijmegen potentials [17, 18] generated by one-boson exchanges with the couplings fixed by SU(3)-symmetry. All these potentials contain repulsive core or repulsive regions, generated by vector-boson exchanges. From such example $\mathcal{R}$-calculations and corresponding fits of the two-Λ correlation function, one may conclude: (i) at $r_0 < 1$ fm, the Q-dip generated by a repulsive potential region is much wider than $1/r_0$; (ii) the anti-correlation is widened due to the contribution of higher L-waves and - the relativistic effect at $Q > 2$ GeV/c; (iii) the anti-correlation of identical and non-identical hyperons is practically the same at $Q > 1/r_0$ for potentials with similar repulsion, thus allowing one, as a first approximation, to fit the ALEPH data with a universal two-hyperon correlation function and λ equal to pair purity; (iv) the fitted r_0 increases with increasing the effective heights and the upper boundaries $r_B^{s,t}$ of the repulsive regions of the singlet and triplet potentials, with the χ^2/ndf vs $r_B^t(\gamma)$ approaching a wide plateau ~ 1 at $r_B^t > 1$ fm ($\gamma < -0.2$); e.g. for the singlet NSC97e potential [18], $r_B^s = 0.75$ fm ($\gamma = 0.5463$), the fitted $r_0 = 0.26 \pm 0.02$ fm (χ^2/ndf $= 25.1/18$) and 0.39 ± 0.04 fm (χ^2/ndf $= 16.1/18$) for $r_B^t = r_B^s$ and $r_B^t = 1.75$ fm ($\gamma = -2$), respectively (Fig. 1).

Spin correlations

For spin–1/2 particles, the information on the system size and the two–particle interaction can be achieved also with the help of spin correlation measurements using as a spin analyser either the asymmetric (weak) particle decay [19,20] or the particle scattering [21,22]. Since these techniques require no construction of the uncorrelated reference sample, they can serve as an important consistency check of the standard correlation measurements.

Particularly, for $\Lambda \to p\pi^-$ decays, characterised by the decay asymmetry parameter $\alpha = 0.642$, the distribution of the cosine of the relative angle θ between the directions of the decay protons in the respective Λ rest frames,

$$\frac{dN}{d\cos\theta} = \frac{N}{2}\left[1+\alpha^2\left(\frac{4}{3}\rho_t - 1\right)\cos\theta\right], \tag{5}$$

allows one to determine the triplet fraction

$$\rho_t = \mathcal{R}_t/\mathcal{R}. \tag{6}$$

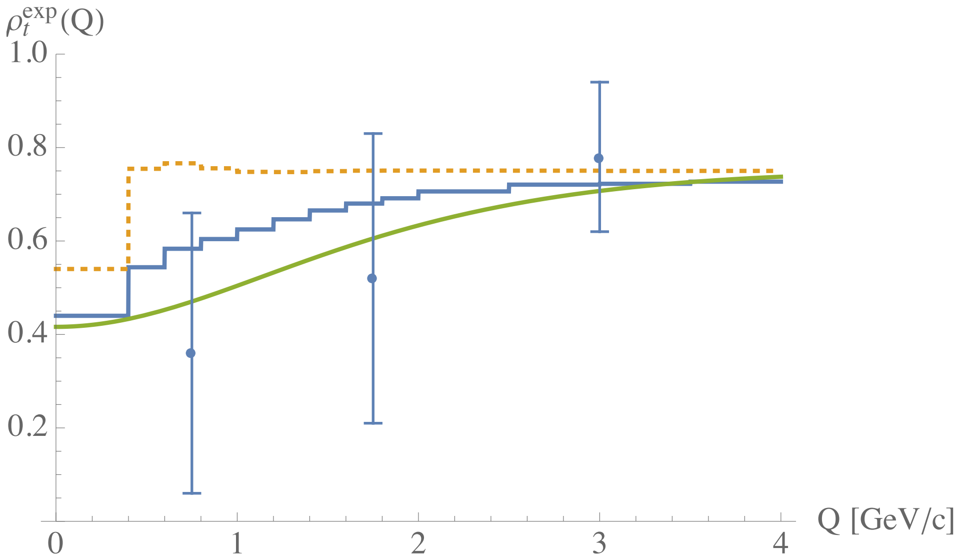

Fig. 2. The ALEPH two–Λ triplet fraction [4] diluted by daughter Λ's from hyperon decays. The curve and histograms correspond to those in Fig. 1, they are calculated according to Eqs. (6), (7) with $\lambda' = 0.445$.

This fraction vanishes at small Q and approaches $\widetilde{\rho}_t \doteq 3/4$ with the increasing Q, thus demonstrating a similar dip at small Q as the correlation function $\mathcal{R} = \mathcal{R}_s + \mathcal{R}_t$. Taking into account that Λ-hyperons are spin-correlated only in a fraction λ' of the pairs, the experimental triplet fraction

$$\rho_t^{\rm exp} = \frac{3}{4}(1-\lambda') + \lambda' \rho_t, \tag{7}$$

where ρ_t is given in Eq. (6). Since the possible parent spin correlations are practically not transferred to the daughter Λ's, λ' coincides with the fraction of directly produced Λ's, $\lambda' = 0.445 \pm 0.026$. Note that this fraction is not multiplied by pair purity since the ALEPH angular distributions are corrected for the acceptance and impurity effects using Monte Carlo simulations.

The ALEPH triplet fraction [4] is compared with the example calculations in Fig. 2. It is consistent with the value of $3/4$, except for a suppression at $Q < 2$ GeV/c, in agreement with the theoretical expectations according to Eqs. (6), (7) and example calculations presented in Fig. 1. Despite rather large errors, the triplet fraction data somewhat prefer a larger extent of the triplet repulsion potential (the solid histogram). Even better agreement of the pure QS calculation (the curve) is invalidated by a failure to describe the correlation function (Fig. 1).

Conclusions

The ALEPH data on $\Lambda\Lambda$ momentum and spin correlation provide a strong evidence on the essential repulsive regions in the two-hyperon strong interaction potentials. The repulsion is solving the problems due to: (i) a substantial fraction of Λ's from hyperon decays; (ii) an unacceptably small radius of the Λ source if neglecting FSI. Further quantification of two-hyperon strong interaction potentials requires to account for their differences, a more complicated spin structure (including spin-orbit and tensor interaction) and presence of coupled channels. A complementary analysis of two-hyperon correlation functions in electro (hadro)-production processes and heavy-ion collisions can reduce the uncertainties.

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