Model investigation of transverse momentum cumulants of different orders in nuclear-nuclear collisions

Модельное исследование кумулянтов поперечного импульса различного порядка в ядро-ядерных столкновениях

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В данной работе мы анализируем кумулянты поперечного импульса разных порядков (K_n) для инклюзивных заряженных адронов, образующихся в столкновениях $Bi + Bi$, с использованием событий, полученных с помощью модели UrQMD-3.4. Анализ выполняется двумя способами: с помощью стандартного метода, в котором исследование проводится по всему промежутку быстроты, а также с помощью метода подсобытий, подавляющего влияние ближних корреляций. Были получены зависимости кумулянтов второго, третьего и четвёртого порядков от энергии. Также в работе приводится сравнение $p+p$ и $Bi+Bi$ столкновений, показавшее, что для обеих систем предсказывается насыщение при $\sqrt{s_{NN}} \approx 5$ ГэВ.

In this research, we analyze transverse momentum cumulants of different orders (K_n) for charged inclusive hadrons produced in $Bi + Bi$ collisions using the UrQMD-3.4 model events. The analysis is carried out in two ways: with the standard method, in which the study is carried out over the entire rapidity interval, and also with the sub-event method, which suppresses the influence of short-range correlations. Dependencies of second-, third-, and fourth-order cumulants on collision energy were obtained. The paper also provides a comparison of $p + p$ and $Bi + Bi$ collisions that indicated the simultaneous saturation for both systems at $\sqrt{s_{NN}} \approx 5$ GeV.

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Introduction

Quark-gluon plasma [1], which according to modern concepts is an almost ideal liquid of free quarks and gluons, is expected to be in a phase different from ordinary hadronic gas, when quarks and gluons are in a state of confinement. In addition, it was predicted [2], that there will be a critical point on the phase diagram of strongly interacting matter. A first order transition to the quark-gluon plasma is expected to occur in certain regions of the phase diagram, which can be accessed by scanning the beam energies to lower values, as is being done at the CERN-Super Proton Synchrotron (SPS) [3] and

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the RHIC beam energy scan [4, 5] program, or at FAIR and NICA in the future. The main characteristic of a phase transition is the divergence of the correlation radius - the system in the region of the critical point is characterized by an infinite length of correlations, it becomes scale-invariant. This leads to large fluctuations in such quantities as particle multiplicity, mean transverse momentum, and net charge. In the area of the critical point, fluctuations have specific characteristics [6, 7]. Thus, the properties of the system should be sensitive to the location of the system in the vicinity of the critical point [8]. That is, when scanning a phase diagram, an increase in fluctuations can become a signal that the system is near a critical point or critical line.

In this work, fluctuations of the event mean transverse momentum, $\overline{p_T} = \frac{1}{N}\sum_{i=1}^{N} p_{T,i}$, are studied. There are trivial statistical fluctuations of $\overline{p_T}$ due to the fact that the average is estimated over a finite sample of particles, and there are also excess fluctuations - these are called dynamic fluctuations. Inter-event fluctuations at the early stage of the collision produce dynamic $\overline{p_T}$ fluctuations in hydrodynamic models [9, 10]. Hydrodynamic model studies of $\overline{p_T}$ fluctuations offer a direct method of monitoring initial state oscillations. Fluctuations of $\overline{p_T}$ arise due to fluctuations of the total energy of the system in the initial state, E_0. Momentum fluctuations can be estimated numerically using cumulants. Experimentally, the first and second cumulants are usually measured [11], however, recently it has been proposed to study higher order cumulants [12]. Also, in order to understand what contribution short-range correlations make, a comparison is made of two methods: the standard method and the subevent method. In the subevent method, events are selected from a certain interval based on their rapidity.

Model

In this work, the cascade version of UrQMD model (version 3.4) was used for comparative analysis.

UrQMD-3.4 is a microscopic transport model which includes covariant hadron propagation on classical pathways, stochastic binary scatterings, color string production, and resonance decay. It is the Monte Carlo solution of a significant number of linked partial integro-differential equations for the temporal evolution of multiple phase space densities $f(x, p)$ [14].

In terms of interactions between recognized hadrons and their resonances, this microscopic transport model characterizes the phenomenology of hadronic interactions at low and intermediate energies ($\sqrt{s} < 5$ GeV). The UrQMD-3.4 model's multiple generation of particles is dominated at higher energies, $\sqrt{s} > 5$ GeV, by the excitation of color strings and their subsequent fragmentation into hadrons.

Definitions

Cumulants of p_T fluctuations are intensive quantities, i.e. they are independent of the volume of created matter. In order to examine the impact of short-range correlations, two methods are proposed in this work: the first is based on the standard definition of cumulants for the whole rapidity acceptance, and the second is based on the sub-event method. In this study, collisions of two types: $p+p$ and $Bi+Bi$ have been investigated using the UrQMD model events.

We consider the cumulants of the second, third and fourth order. Their most general definition is introduced via n-particle correlator [12]:

$$C_n = \frac{\sum_{i_1 \neq \ldots \neq i_n} \omega_{i_1} \ldots \omega_{i_n} (p_{T,i_1} - \langle\langle p_T \rangle\rangle) \ldots (p_{T,i_n} - \langle\langle p_T \rangle\rangle)}{\sum_{i_1 \neq \ldots \neq i_n} \omega_{i_1} \ldots \omega_{i_n}} \tag{1}$$

where ω_i is the weight for particle i. In this work, all weights are taken equal to one: $\omega_{i_1} = \ldots = \omega_{i_n} = 1$, i.e. we consider all produced charged hadrons.

To simplify the calculation, auxiliary expressions are introduced:

$$p_{mk} = \sum_i \omega_i^k p_i^m / \sum_i \omega_i^k, \tau_k = \frac{\omega_i^{k+1}}{\left(\sum_i \omega_i\right)^{k+1}} \tag{2}$$

denoting $p \equiv p_T$:

$$\overline{p}_{1k} \equiv p_{1k} - \langle\langle p_T \rangle\rangle \tag{3}$$

$$\overline{p}_{2k} \equiv 2p_{1k} \langle\langle p_T \rangle\rangle + \langle\langle p_T \rangle\rangle^2 \tag{4}$$

$$\overline{p}_{3k} \equiv p_{3k} - 3p_{2k} \langle\langle p_T \rangle\rangle + 3p_{1k} \langle\langle p_T \rangle\rangle^2 - \langle\langle p_T \rangle\rangle^3 \tag{5}$$

$$\overline{p}_{4k} \equiv p_{4k} - 4p_{3k} \langle\langle p_T \rangle\rangle + 6p_{2k} \langle\langle p_T \rangle\rangle^2 - 4p_{1k} \langle\langle p_T \rangle\rangle^3 + \langle\langle p_T \rangle\rangle^4 \tag{6}$$

Note that $\langle\langle p_T \rangle\rangle = \langle p_{11} \rangle$ is the mean value of $\overline{p_T}$ averaged over ensemble of events.

$$C_2 = \frac{\overline{p}_{11}^2 - \overline{p}_{22}}{1 - \tau_1}, \; C_3 = \frac{\overline{p}_{11}^3 - 3\overline{p}_{22}\overline{p}_{11} + 2\overline{p}_{33}}{1 - 3\tau_1 + 2\tau_2} \tag{7}$$

$$C_4 = \frac{\overline{p}_{11}^4 - 6\overline{p}_{22}\overline{p}_{11}^2 + 3\overline{p}_{22}^2 + 8\overline{p}_{33}\overline{p}_{11} - 6\overline{p}_{44}}{1 - 6\tau_1 + 3\tau_1^2 + 8\tau_2 - 6\tau_3}, \tag{8}$$

where particles are taken from the whole available kinematic acceptance and only unique combinations of particles in the event are taken into account.

Cumulants are calculated by averaging C_n over a given ensemble of events using additional normalization.

$$K_2 = \frac{\langle C_2 \rangle}{\langle\langle p_T \rangle\rangle^2}, \; K_3 = \frac{\langle C_3 \rangle}{\langle\langle p_T \rangle\rangle^3}, \; K_4 = \frac{\langle C_4 \rangle - 3\langle C_2 \rangle^2}{\langle\langle p_T \rangle\rangle^4} \tag{9}$$

Thus, the cumulants are determined in terms of moments and are normalized to the average momenta not only in one event, but also over all events. Cumulants are interesting because they isolate n-particle correlations by removing the influence of lower rank correlations. The moment of order n scales as N^n and the associated cumulant is proportional to N if a nucleus-nucleus collision is a superposition of a fixed number N of separate nucleon-nucleon collisions [15].

For the subevent method, it is necessary to take two rapidity intervals, separated by a certain interval. The letters f and b respectively mean the intervals of rapidity from which events are taken. The cumulants for the subevent method are respectively given by the expressions:

$$K_{2,sub} = \frac{\langle c_{2,2sub}\rangle}{\langle\langle p_T\rangle\rangle_f \langle\langle p_T\rangle\rangle_b}, \quad K_{3,2sub1} = \frac{\langle c_{3,2sub1}\rangle}{\langle\langle p_T\rangle\rangle_f^2 \langle\langle p_T\rangle\rangle_b}, \tag{10}$$

$$K_{3,2sub2} = \frac{\langle c_{3,2sub2}\rangle}{\langle\langle p_T\rangle\rangle_f \langle\langle p_T\rangle\rangle_b^2}, \quad K_{4,2sub} = \frac{\langle c_{4,2sub}\rangle - 2\langle c_{2,2sub}\rangle^2 - \langle c_2\rangle_a \langle c_2\rangle_c}{\langle\langle p_T\rangle\rangle_f^2 \langle\langle p_T\rangle\rangle_b^2} \tag{11}$$

Rapidity gap between two subevents allows to suppress a contribution of short-range correlations, e.g. resonance decays, Bose-Einstein correlations etc.

Criteria of selection

To compare in the future the obtained model results with the NICA experiments, we used selection criteria for produced particles in event that would correspond the rapidity (y)-transverse momentum acceptance of the MPD experiment. These criteria for the direct and subevent methods are given in the table1.

We selected $\sqrt{s}$ = 3, 3.25, 3.5, 3.75, 4, 4.25, 4.5, 4.75, 5, 6, 7, 10, 17.3, 19.6 GeV for UrQMD-3.4 simulations of $p + p$ and $Bi + Bi$ collisions. In order to obtain the results that would be most opposite to the $p + p$ case we chose the impact parameter $b = 0$ fm for the $Bi + Bi$ case. The direct comparison with the future experimental results would require to repeat the experimental procedure of centrality determination.

Direct method		Subevent method
$0.15 < p_T < 2$ GeV/c		$0.15 < p_T < 2$ GeV/c
$-1 < y < 1$		$-0.8 < y_b < -0.3$, $0.3 < y_f < 0.8$

Table 1. Criteria of selection for UrQMD-3.4.

Results

As a result, graphs of the dependence of second-, third- and fourth-order cumulants on energy were obtained. Comparisons between the standard approach and the subevent method for proton-proton collisions were also attained.

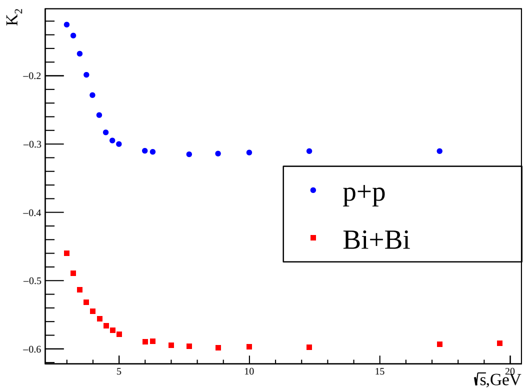

Fig. 1. Dependence of K_2 on collision energy for Bi-Bi and proton-proton collisions.

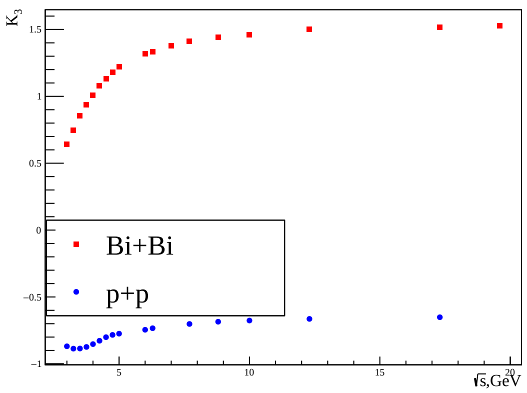

Fig. 2. Dependence of K_3 on collision energy for Bi-Bi and proton-proton collisions.

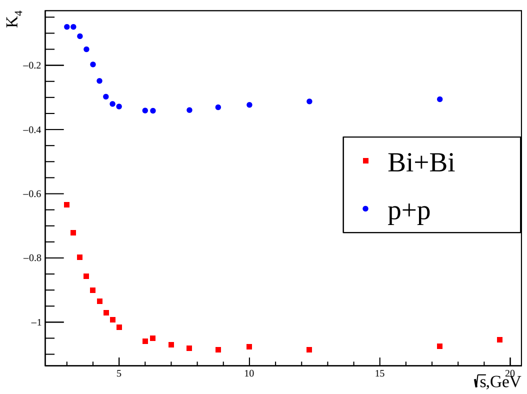

Fig. 3. Dependence of K_4 on collision energy for Bi-Bi and proton-proton collisions.

The graphs of the dependence of the second, third and fourth order cumulants clearly show that the dependence after 5 GeV becomes flat. The qualitative trends in the dependences for proton-proton and Bi-Bi collisions coincide. In the HIJING model it was shown that $K_2 > 0$, $K_3 > 0$ [12]. From Fig. 1, 2 it can be seen that the prediction of the UrQMD-3.4 model for $K_2 > 0$ does not coincide with the prediction of the HIJING model, but for $K_3 > 0$ it coincides [12].

On Fig. 4, 5 one can see comparison two methods for proton-proton collisions: standard method and sub-event method. These two methods show a quantitative difference - K_2 changes sign and K_3 is almost suppressed to 0. It is worth noting that at lower collision energies the significant fluctations of K_2 and K_3 are present.

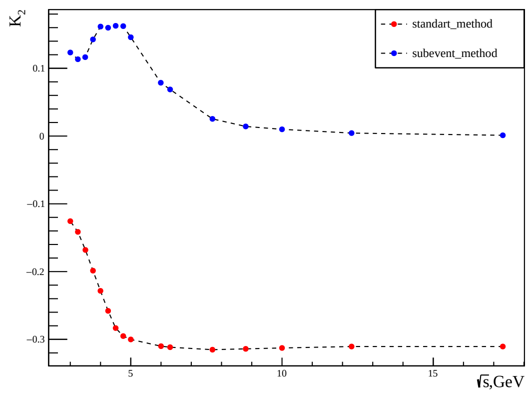

Fig. 4. Dependence of K_2 on collision energy for proton-proton collisions for standard and sub-event method.

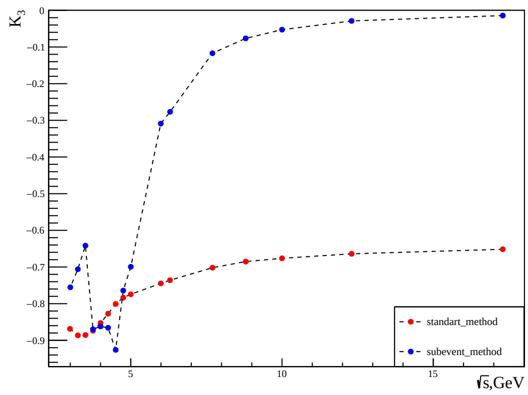

Fig. 5. Dependence of K_3 on collision energy for proton-proton collisions for standard and sub-event method.

Summary

In summary, we have analyzed transverse momentum cumulants of different orders (K_n) for charged inclusive hadrons produced in $Bi + Bi$ and $p+p$ collisions using the UrQMD-3.4 model events. We compared two methods: the standard method and the subevent method. There was a clear quantitative discrepancy between the two methods.

For the dependencies of cumulants, a qualitative coincidence of trends was observed for $p + p$ and $Bi + Bi$ collisions. For collision energies larger than 5 GeV, the values of cumulants of different orders reach saturation. It could indicate that in this energy domain the mechanism of particle production from strings fragmentation is dominating.

One of the reasons for the discrepancy between the traditional method and the subevent method may be the choice of interval of the subevent method, as well as the physical mechanisms inherent in the UrQMD model. Future results from the NICA experiment may clarify the reason for this discrepancy. In the future it is also planned to compare different versions of EoS in the UrQMD model to test sensitivity of cumulants to simulation of the hydrodynamical phase.

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