Gravitational chiral anomaly in a vortical quantum fluid

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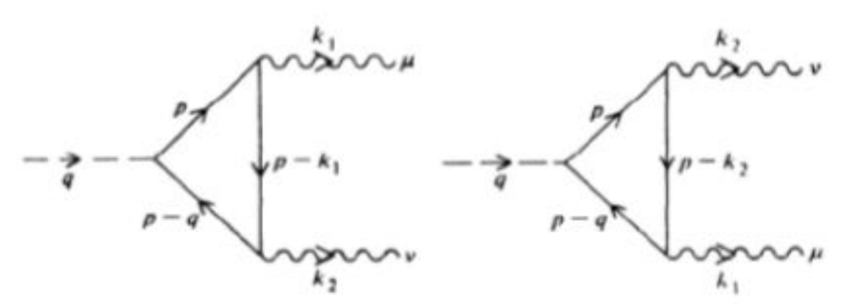
Abstract

We construct a hydrodynamic gradient expansion for the axial current and the stress-energy tensor of massless fermions in a fluid with rotation and acceleration in a curved space-time. We establish a duality between the currents induced by the cosmological constant and the finite acceleration in flat space-time. We also verify the duality between the current in a rotating and accelerated medium, the so-called kinematical vortical effect (KVE), and the gravitational chiral anomaly. Finally, we construct the hydrodynamic expansion for the stress-energy and show brand new derivation of Unruh-Deser temperature, which depends on the acceleration in five-dimensional space-time.

Quantum anomalies

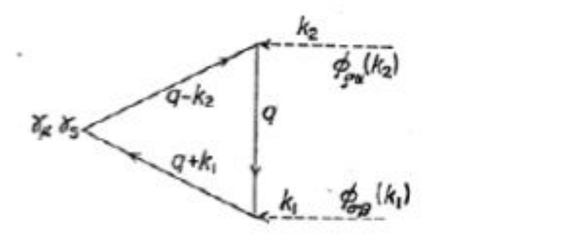
The phenomenon of non-conservation of the number of left and right particles at the quantum level in the presence of external fields. Quantum anomalies are at the basis of new-type transport phenomena that are being actively sought at accelerators

Electromagnetic anomaly



$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\widetilde{F}^{\mu\nu} = \frac{e^2}{32\pi^2}\varepsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}.$$

Gravitational anomaly



$$\nabla_\mu j^\mu_A = \mathcal{N}\epsilon^{\alpha\beta\mu\nu}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho} = \frac{1}{768\pi^2}\epsilon^{\alpha\beta\mu\nu}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}.$$

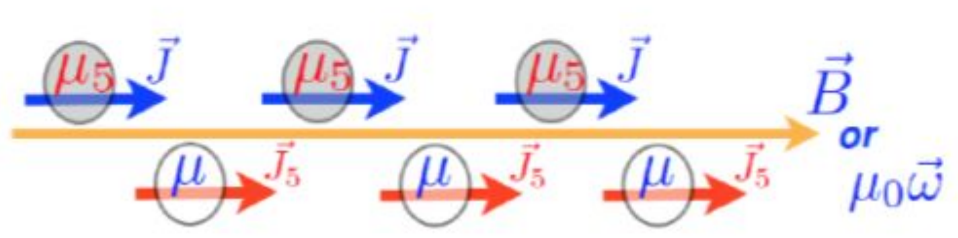
$$\nabla_\mu j^\nu = \partial_\mu j^\nu + \Gamma^\nu_{\alpha\mu}j^\alpha, \quad \epsilon^{\alpha\beta\mu\nu} = \frac{1}{\sqrt{g}}\varepsilon^{\alpha\beta\mu\nu}$$

Chiral transport phenomena

Chiral magnetic effect (CME): The phenomenon of vector current flow along the magnetic field in a system with broken axial symmetry. Can be treated as a fifth Maxwell equation in extreme conditions.

$$\boldsymbol{j} = \frac{e^2\mu_5}{2\pi^2}\boldsymbol{B}$$

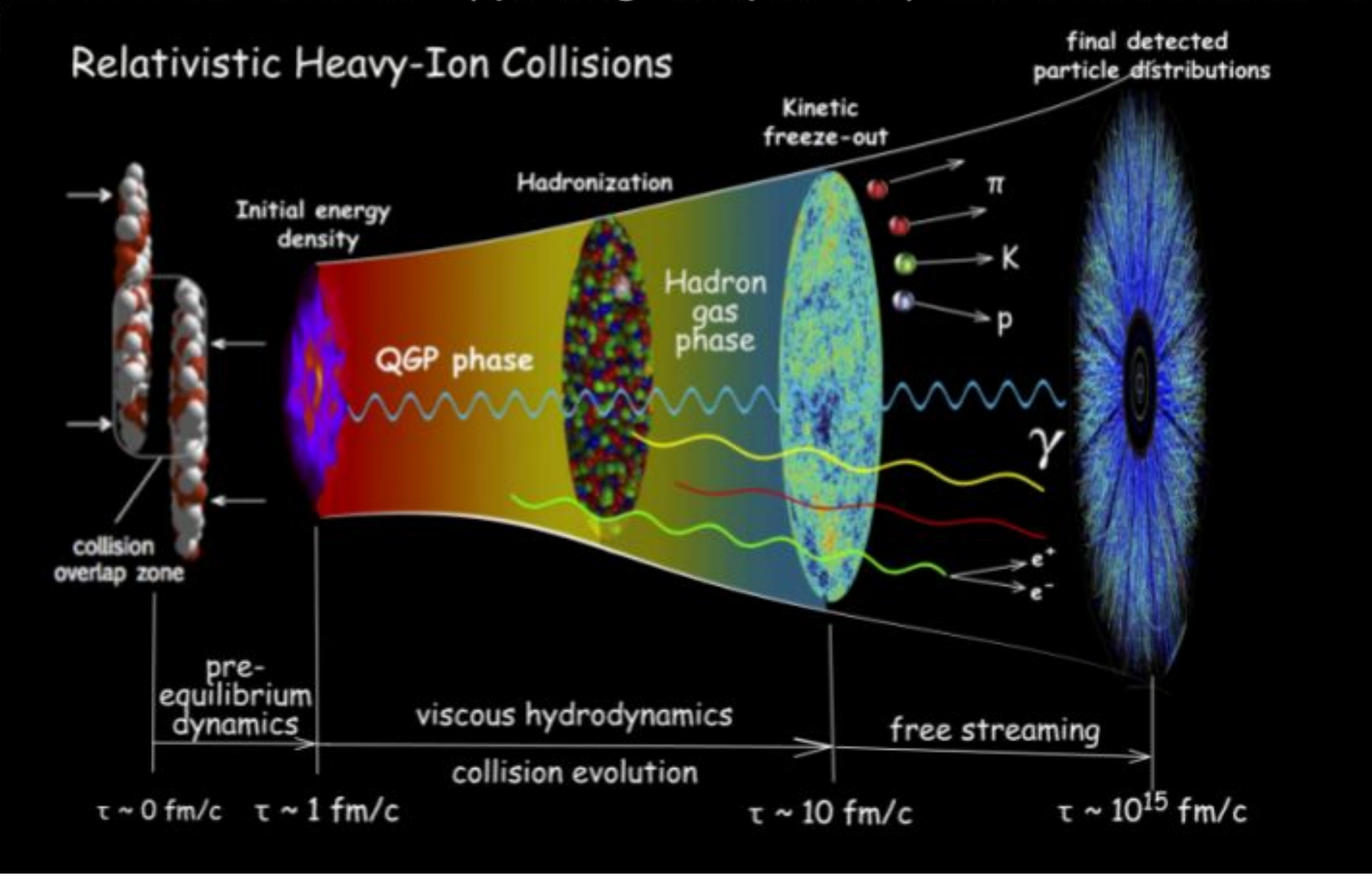
Chiral vortical effect (CVE): Appearance of axial current in rotating medium, directed along the vorticity



$$\vec{\mathbf{J}} = \frac{1}{\pi^2}\mu_5\mu\vec{\omega}$$

Hydrodynamics at RHIC

In RHIC, the intermediate state of matter (quark gluon plasma) can be described as a near-perfect liquid



New gravitational anomaly phenomena: Kinematical vortical effect

The brand new anomalous effect was recently founded by JINR scientists G. Prokhorov, O. Teryaev and V. Zakharov.

It consists in the fact that the linear combination of kinematic coefficients of the axial current without gravity is proportional to the coefficient of the gravitational anomaly. This important result was obtained in a case of Ricci-flat spacetime

$$j^A_\mu = \left(\lambda_1\omega^2 + \lambda_2 a^2\right)\omega_\mu \qquad \nabla_\mu j^\mu_A = \mathcal{N}\widetilde{R}_{\alpha\beta\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

KVE with non-zero cosmological constant

Generalization of derivation of KVE on a case non-trivial Ricci tensor, proportional to the cosmological constant (Einstein manifolds) has shown that KVE takes place not only in case

$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$

But also we found out that there will be new term in axial current, connected with curvature o space and new connection between curvature and acceleration coefficients, that can be thought as Einstein equivalence principle in a second order

$$j^{A(3)}_\mu = \xi_1(T)w^2 w_\mu + \xi_2(T)\alpha^2 w_\mu + \xi_3(T)(\alpha w)\alpha_\mu + \xi_4(T)\widetilde{A}_{\mu\nu}w^\nu + \xi_5(T)B_{\mu\nu}\alpha^\nu + \xi_\Lambda(T)\Lambda w_\mu$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

KVE Still exist

$$j^{\mu(3)}_A = \left(-\frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2} + \frac{R}{96\pi^2}\right)\omega^\mu$$

New gravitational term in axial current

$$\lambda_\Lambda = -\frac{\lambda_2}{3}$$

Curvature and acceleratio are connected

Energy-momentum tensor: Derivation of Unruh-Deser temperature

Hydrodynamic consideration of Energy-momentum tensor and its conservation in Einstein manifolds gave us an alternative way of derivation well known Unruh temperature in maximally symmetric curved spacetimes

$$T_{\mu\nu} = C_T T^4 + \left[\left(A_1 a^2 + A_2 R\right)T^2 + \left(B_1 a^4 + B_2 a^2 R + B_3 R^2\right)\right]\left(4u_\mu u_\nu - g_{\mu\nu}\right) + \frac{k}{4}R^2 g_{\mu\nu}$$

conservation law will give us

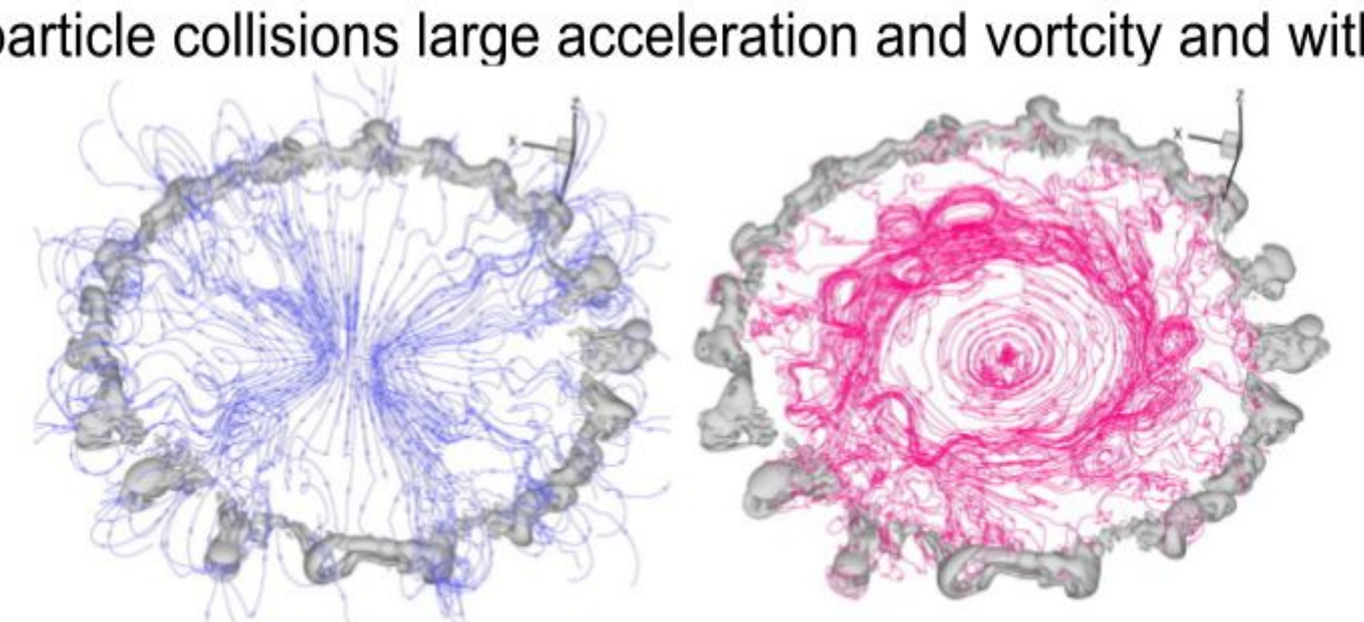
$$\nabla_\mu T_{\mu\nu} = 0 \Rightarrow T_{\mu\nu} = C_T T^4 + \left[\left(A_1 a^2 + A_2 R\right)T^2 + \left(B_1 a^4 + B_2 a^2 R + B_3 R^2\right)\right]\left(4u_\mu u_\nu - g_{\mu\nu}\right) + \frac{k}{4}R^2 g_{\mu\nu}$$

To nullify EMT in vacuum we have to choose the temperature

$$T_U = \frac{a_5}{2\pi} = \frac{\sqrt{a^2 + \frac{R^2}{12}}}{2\pi} = \frac{\sqrt{a^2 + \frac{1}{r^2}}}{2\pi}$$

Conclusion

As far as kinematical vortical effect doesn't depend on external gravitationa field, it can be useful to study gravitational effects in quark gluon plasma, obtained in particle collisions large acceleration and vortcity and without gravity.



Also, further theoretical and experimental study of this effect will help to understand the kinematic nature of the mechanism of chiral symmetry breaking and provide new methods for analyzing chiral transport effects