Synchrotrons: Concept and Applications in Experiment with Hadron Beam

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Accelerator Physics School of JINR
June 19-23, 2023
Lipnya, Moscow reg.

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- Ways to accelerate charged particles
- Basics of transverse focusing and beam dynamics
- Basics of longitudinal focusing and beam dynamics
- Types of synchrotrons
- Magnets and RF
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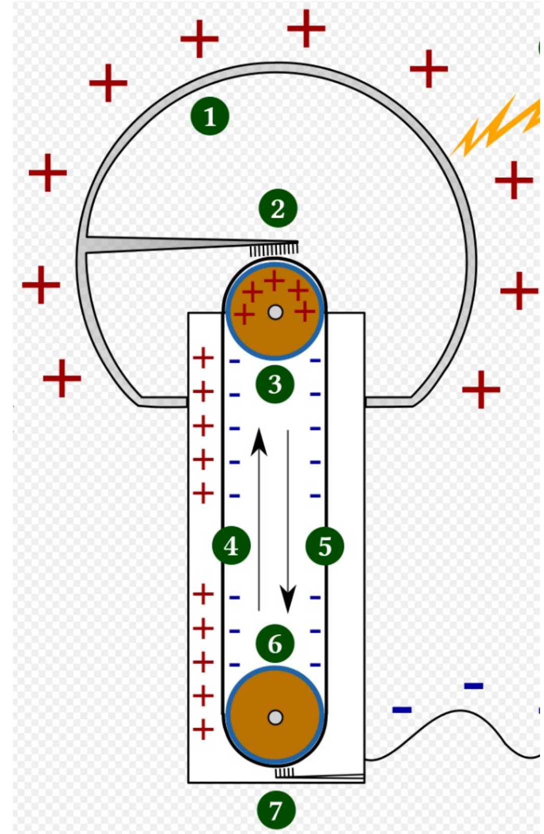
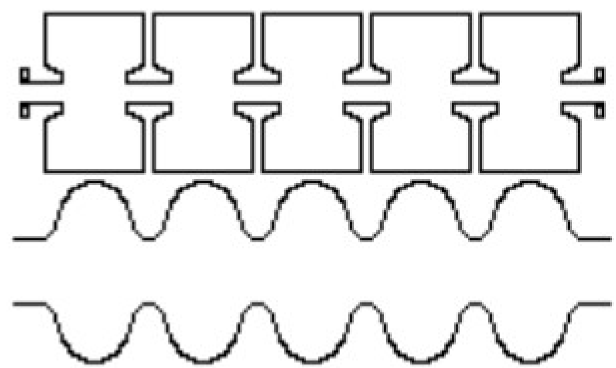
How One Can Accelerate Charged Particles

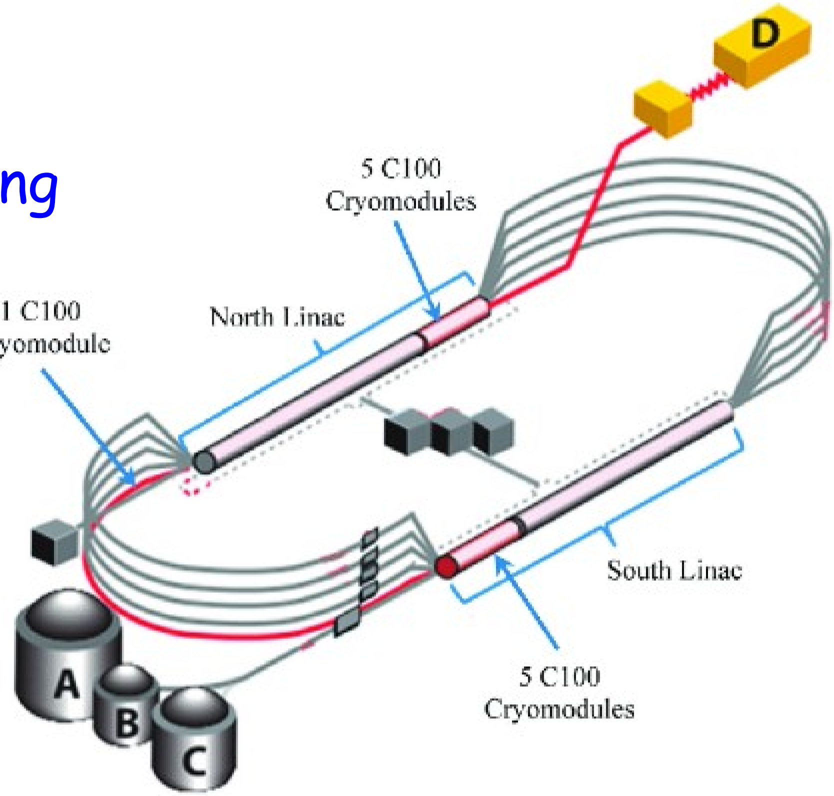
- Electrostatic acceleration
    - Van de Graaff generator, up to 5 MV (1929)
    - Pelletron, up to 25 MeV (mid 1960s, developed by Ray Herb)
  - Energy can be doubled with ion stripping: H^- -> p
- Linear accelerators
    - First proposal: 1924 by Gustav Ising
    - First implementation: 1927 by Rolf Wideroe
  - Used as injectors to synchrotrons
    - Protons: RFQ, drift tubes, side coupled cav.
    - Electrons: different types, but v=c
  - Achieved energies:
    - e: SLC, 2*45 GeV, e^-e^+ (end of 1980s)
    - p: ~800 MeV: LANSCE (Los Alamos), SNS (Oakridge)
- Cyclotrons

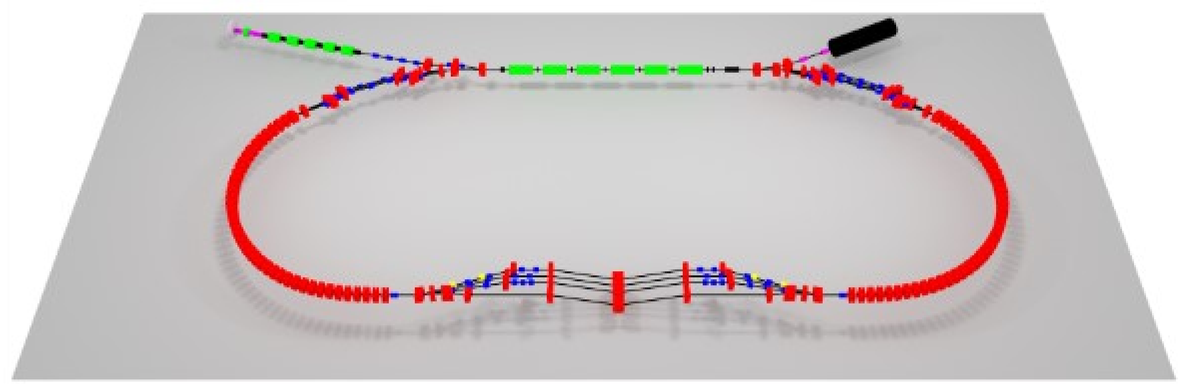




Recirculators

- To support sufficiently high accelerating gradient normal conducting linacs require very large powers (100 kW – 100 MW scale)
  - ⇨ Pulsed operation
- Superconductivity enables obtaining large accelerating gradients in CW
  - In this case a usage of recirculators looks very attractive
    - CEBAF - first recirculator: polarized electrons, 12 GeV, 1 MW
- Energy recovery linacs
  - The beam is sent to the same cavities with 180 deg. phase shift
  - CEBAF FEL (Newport News, VA), CBETA (Cornel university, NY)

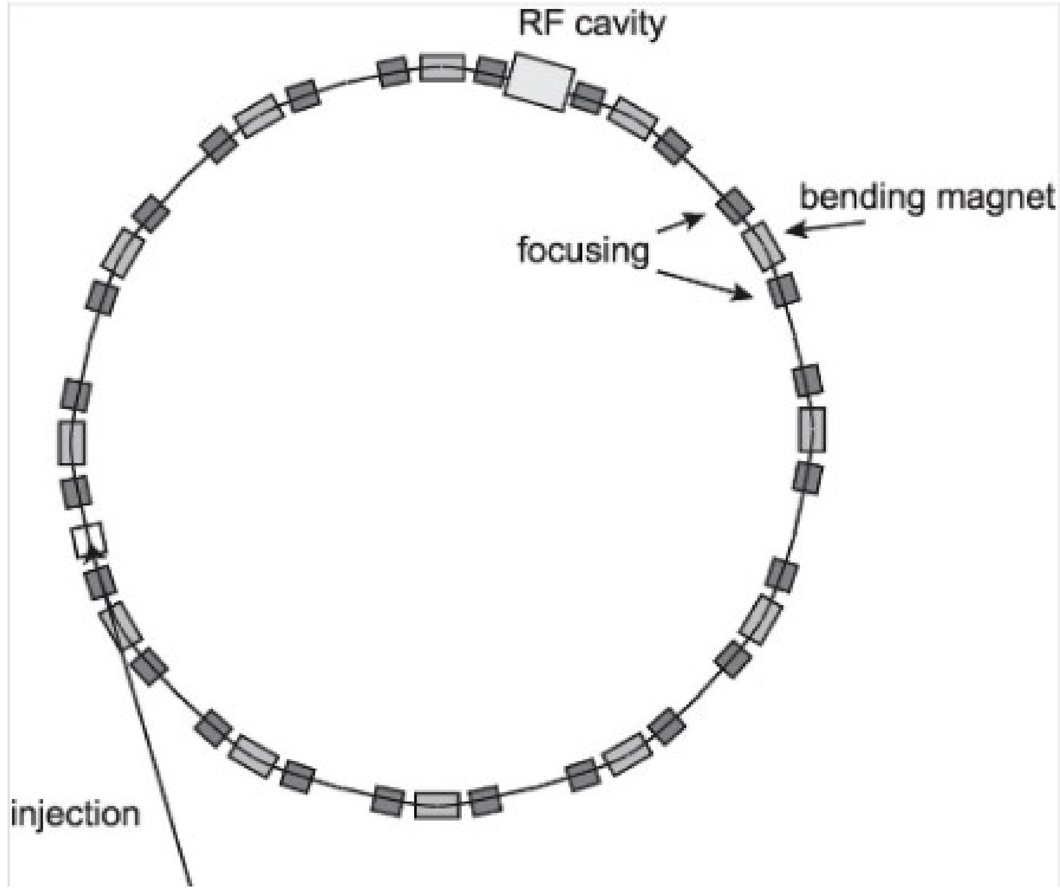




The Cornell-BNL FFAG-ERL Test Accelerator: White Paper, arXiv:1504.00588

Synchrotrons

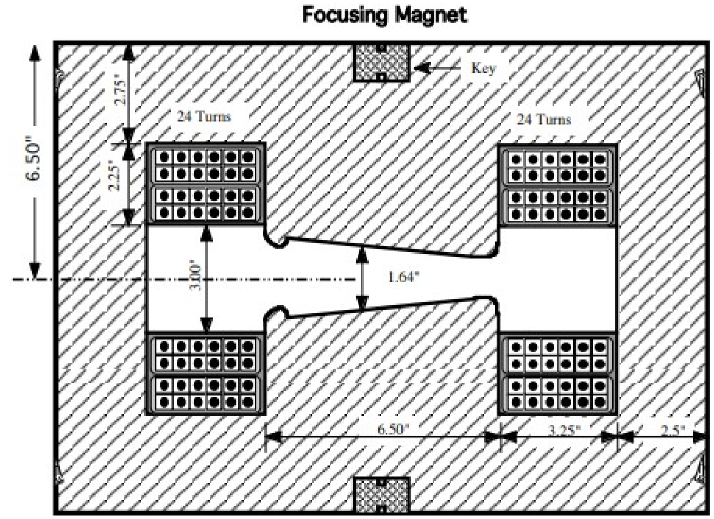
- Transition from linacs to synchrotrons enables energy increase by orders of magnitude,
  - LHC – 7 TeV
  - Still quite high beam power
    - FNAL MI – 900 kW
- Principle of longitudinal focusing
  - Acceleration is impossible without it
  - Suggested by V. Veksler (1944)
  - Edwin McMillan constructed the first electron synchrotron in 1945,
    arriving at the idea independently

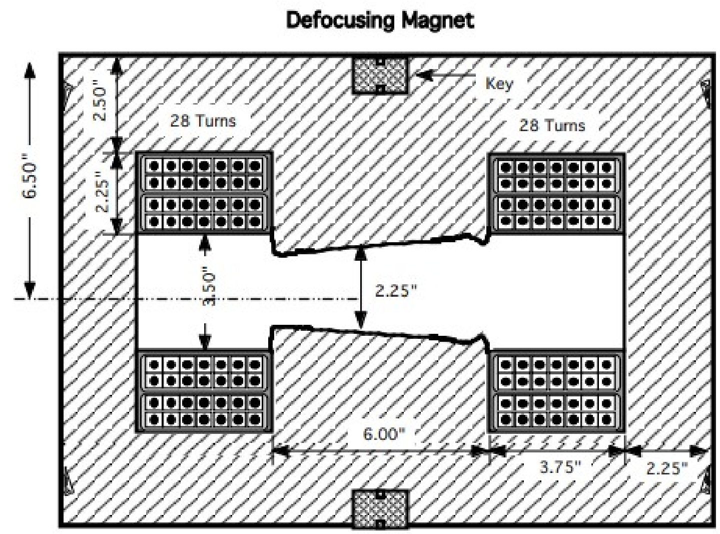


Equations of motion in beam frame

- Magnetic potential & magnetic field for sufficiently large bending radius

$$\Delta\varphi = 0 \quad \Rightarrow \quad \varphi = -\left(B_0 y + Gxy\right) \quad \Rightarrow \quad \begin{cases} B_x = Gy \\ B_y = B_0 + Gx \end{cases}$$





- Force acting on particle

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \mathrm{v}_0 / c \\ Gy & Gx & 0 \end{bmatrix} \Rightarrow \begin{cases} F_x = -e\dfrac{\mathrm{v}_0}{c}\left(B_0 + Gx\right) \\ F_y = e\dfrac{\mathrm{v}_0}{c} Gy \end{cases}$$

- Centripetal acceleration

$$F_{cp} = -F_0 \frac{x}{R_0} = -\left(m\gamma \frac{\mathrm{v}_0^{\,2}}{R_0}\right)\frac{x}{R_0}$$

- Equations of motion in the frame of reference particle

$$m\gamma \frac{d\mathrm{v}_x}{dt} = -m\gamma \frac{\mathrm{v}_0^{\,2}}{R_0}\frac{x}{R_0} - e\frac{\mathrm{v}_0}{c} Gx \xrightarrow[m\gamma \frac{\mathrm{v}_0^2}{R_0} = \frac{e\mathrm{v}_0 B_0}{c}]{\mathrm{v}_x = \mathrm{v}_0 dx/ds,\; d/dt = \mathrm{v}_0 d/ds} \begin{cases} \dfrac{d^2x}{ds^2} = -\dfrac{x}{R_0^{\,2}} - kx \\ \dfrac{d^2x}{ds^2} = ky \end{cases}$$

$$k = \frac{eG}{m\gamma c \mathrm{v}_0} = \frac{1}{R_0}\frac{G}{B_0} = \frac{n}{R_0^{\,2}}$$

Soft Focusing Synchrotron

- Soft Focusing Synchrotron

$$\begin{cases} \dfrac{d^2x}{ds^2} = -(1+n)\dfrac{x}{R_0{}^2} \\ \dfrac{d^2x}{ds^2} = \dfrac{y}{R_0{}^2} \end{cases} \xrightarrow[n<0]{1+n>0} \begin{cases} x = x_0 \cos\left(\sqrt{1+n}\,\dfrac{s}{R_0} + \psi_x\right) \\ y = y_0 \cos\left(\sqrt{-n}\,\dfrac{s}{R_0} + \psi_y\right) \end{cases} \xrightarrow[\nu_y=\sqrt{-n}]{\nu_x=\sqrt{1+n}} \begin{cases} x = x_0 \cos\left(\nu_x \dfrac{s}{R_0} + \psi_x\right) \\ y = y_0 \cos\left(\nu_y \dfrac{s}{R_0} + \psi_y\right) \end{cases}$$

  - Particle motion is stable for $-1 < n < 0$
  - Beam emittance

$$\theta_{x,y}^{\max} = x_0 \frac{\nu_{x,y}}{R_0} \quad \Rightarrow \quad \varepsilon_x \equiv \theta_x^{\max} x_0 = x_0{}^2 \frac{\nu_x}{R_0}$$

- Synchrophasotron versus Booster (Dubna): C=208 m

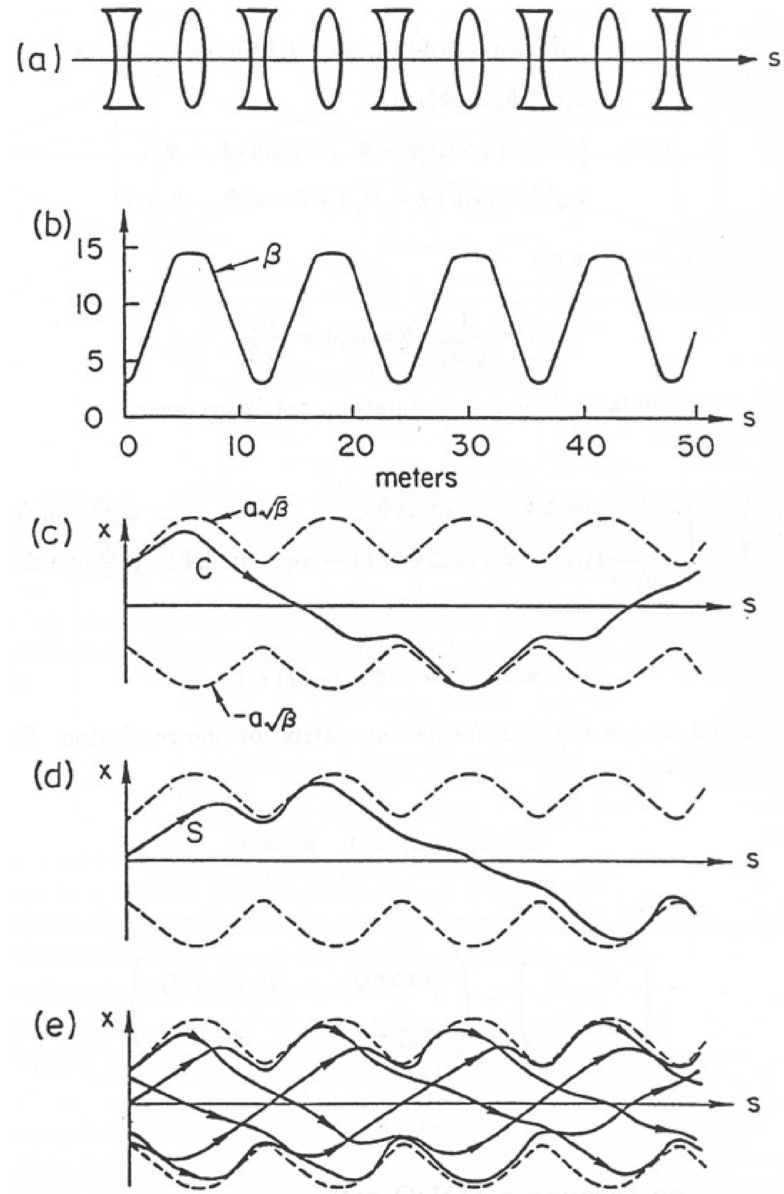
	2w [cm]	ε_{x_max} [mm mrad]
Synchrophasotron (1957)	120	7690
Booster (2020)	12	260

* In real life the aperture is decreased due to closed orbit distortions

- Soft focusing machine requires very large apertures
- The cost and weight grow at least as A^2. (10^2=100)

Strong Focusing or Alternating-gradient Focusing

- Strong focusing was conceived by N. Christofilos in 1949, not published but patented.
- In 1952, the strong focusing principle was independently developed by Courant, Livingston, Snyder and Blewett at BNL
- AGS (BNL, 1960, 33 GeV) – the first strong focusing machine
- Laplace's equation prohibits simultaneous quad focusing in both H&V planes
- The principle of focusing in both planes is based on a sequence of positive and negative quadrupoles
  - ♦ Due to larger particle displacements in focusing quads the focusing overtakes



$$\begin{pmatrix} 1 & 0 \\ -\frac{\Phi}{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \Phi & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{\Phi}{2} & 1 \end{pmatrix} = \begin{bmatrix} 1 - \frac{L^2 \cdot \Phi^2}{2} & L \cdot (L \cdot \Phi + 2) \\ -\frac{L \cdot \Phi^2 \cdot (2 - L \cdot \Phi)}{4} & 1 - \frac{L^2 \cdot \Phi^2}{2} \end{bmatrix}$$

Beta-functions and Betatron Tunes

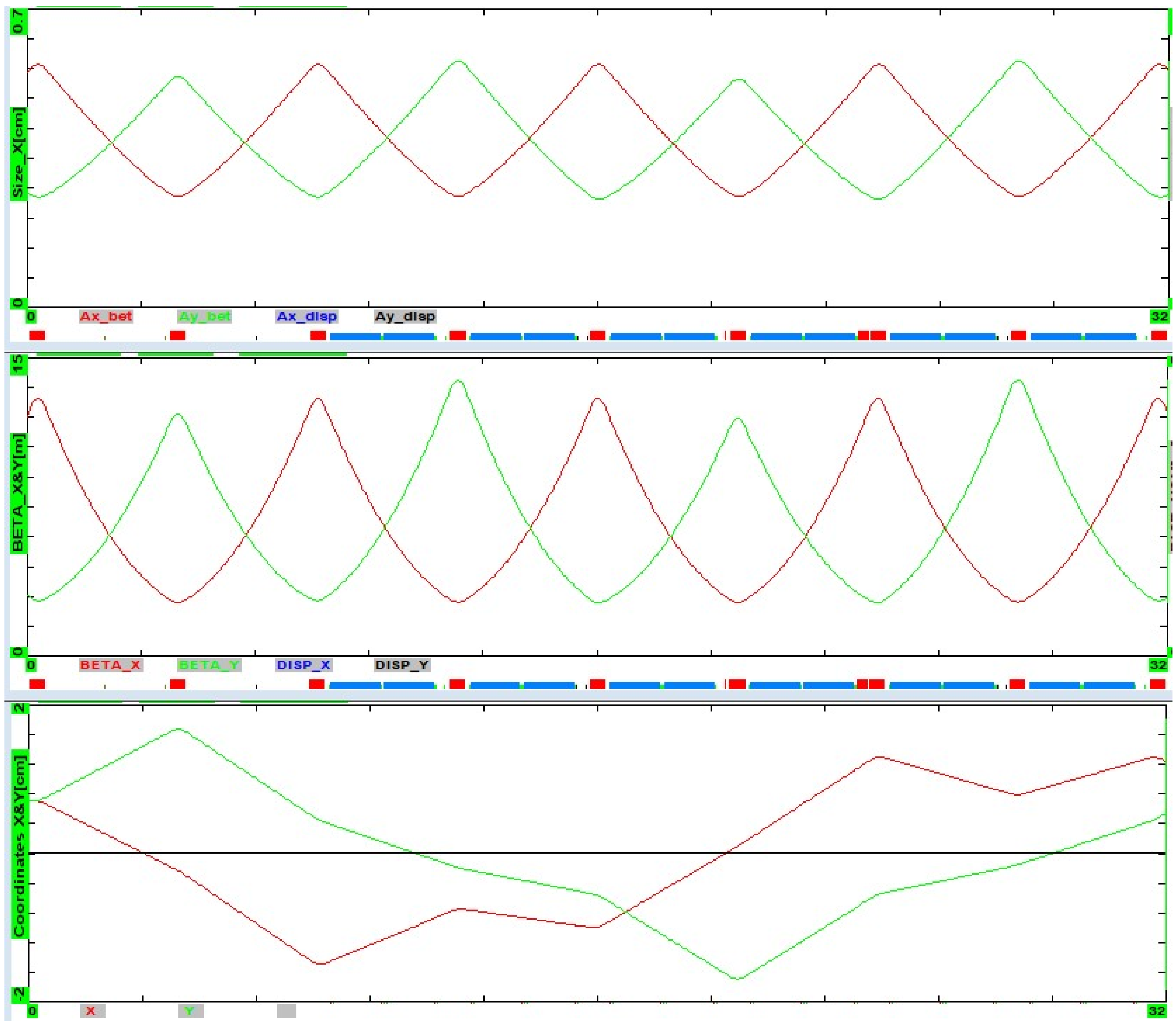
- Since particle motion is determined by 2nd order linear ordinary differential equation, the change of particle coordinates from point 1 to point 2 for each plane (H & V) can be described by a 2nd order matrix called the transfer matrix: $\begin{bmatrix} x \\ \theta \end{bmatrix}_2 = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix}_1$ or in matrix form $\mathbf{x}_2 = \mathbf{M}\mathbf{x}_1$
  - Liouville theorem requires $|\mathbf{M}| = 1$, *i.e.* only 3 out of 4 elements are independent
  - For a ring the multiturn particle motion is: $\mathbf{x}_n = \mathbf{M}^n \mathbf{x}_0$
- Following a general recipe one can present an arbitrary initial vector as: $\mathbf{x} = A_1\mathbf{v}_1 + A_2\mathbf{v}_2$

  where $\mathbf{v}_{1,2}$ are the eigen-vectors for a given point: $\mathbf{M}\mathbf{v} = \lambda\mathbf{v}$
  - For stable motion: $|\lambda_{1,2}| = 1$ and since M is the real matrix $\lambda_2 = \lambda_1^*$
  - We introduce the betatron tune as: $\lambda_1 = e^{-2\pi i \nu}$
- Then a multiturn motion is $\mathbf{x}_n = \mathbf{M}^n \left(A_1\mathbf{v}_1 + A_2\mathbf{v}_2\right) = \lambda_1{}^n A_1\mathbf{v}_1 + \lambda_2{}^n A_2\mathbf{v}_2 = \mathrm{Re}\left(A\lambda_1{}^n \mathbf{v}_1\right)$
- Similar for a motion along the ring: $\mathbf{x}(s) = \mathrm{Re}\left(Ae^{-i\mu}\mathbf{v}(s)\right)$

  where the normalized eigen-vector is determined as: $\mathbf{v}(s) \equiv \mathbf{v} = \begin{bmatrix} \sqrt{\beta} \\ -\dfrac{i+\alpha}{\sqrt{\beta}} \end{bmatrix}$

  ⇨ $x(s) = \mathrm{Re}\left(\sqrt{2I\beta(s)}\,e^{-i(\mu(s)+\psi)}\right)$
- $\beta(s)$ is called the beta-function. In smooth approximation: $\beta = C/(2\pi\nu)$

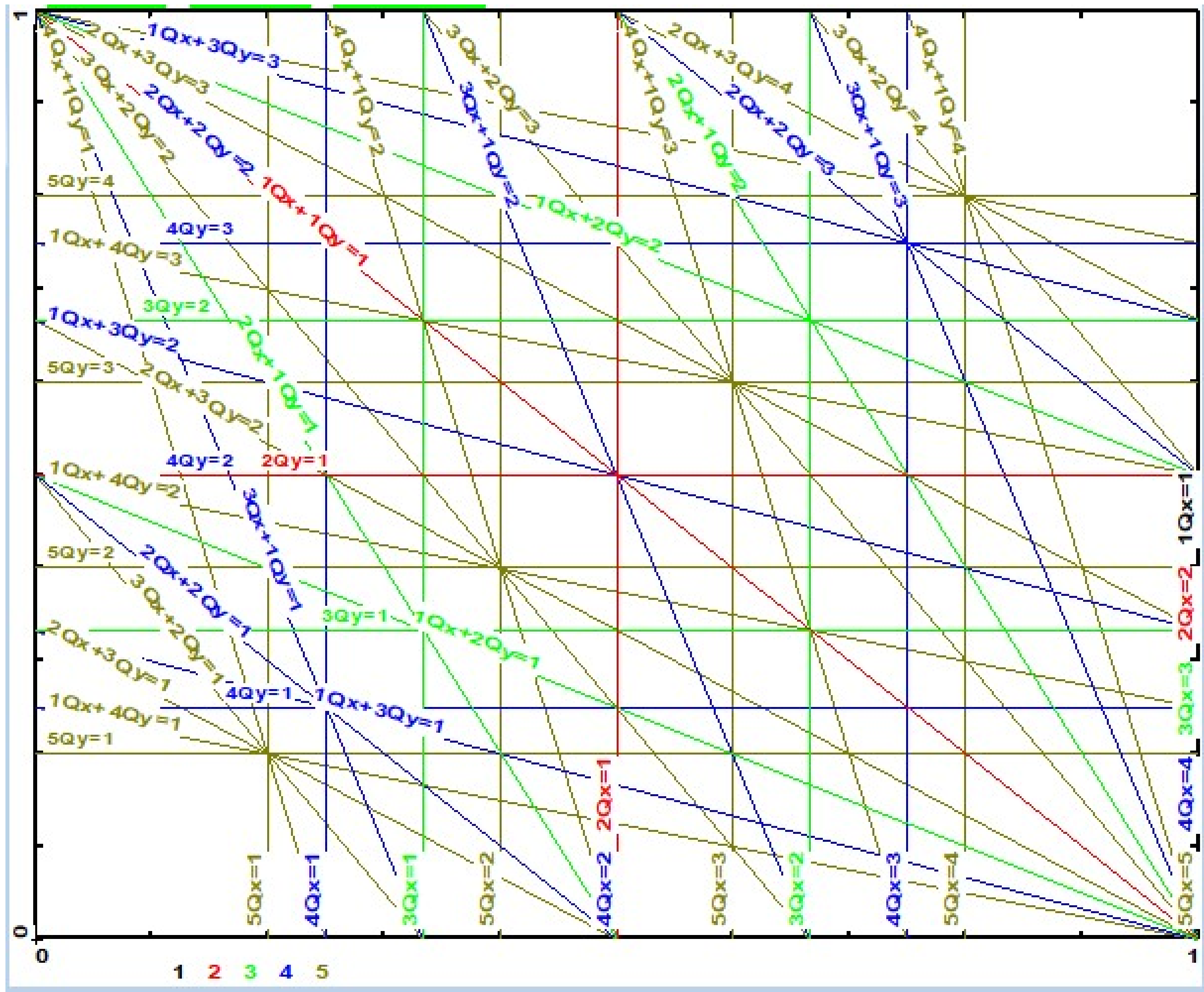
Beta-functions and Betatron Tunes (2)



Beam sizes, beta-functions and a particle trajectory for a Nuclotron half-superperiod

Non-linear Resonances

- Presence of small non-linearities in bending and focusing fields results in growth of particle amplitudes
- The growth is accelerated with amplitude
- To avoid problems, one needs to choose the betatron tunes far from resonances



Nonlinear Resonances Driven by Space Charge

- Fields of beam space charge results in betatron tune shifts
- Dependence of betatron tunes on the betatron amplitude results in that the tunes of some particles are at non-linear resonances
  - Consequently, particle amplitudes grow resulting in beam loss
  - SC effect is diminishing fast with beam energy

$$\begin{bmatrix} \delta\nu_{SC_X} \\ \delta\nu_{SCY} \end{bmatrix} = \frac{r_p Z^2 N_i}{2\pi A \beta^2 \gamma^3} \frac{C}{\sqrt{2\pi}\sigma_s} \left\langle \frac{1}{\left(\sigma_x + \sigma_y\right)} \begin{bmatrix} \beta_x / \sigma_x \\ \beta_y / \sigma_y \end{bmatrix} \right\rangle_s , \quad \sigma_{x,y} = \sqrt{\beta_{x,y}\varepsilon_{x,y} + \left(D_{x,y}\sigma_p\right)^2}$$

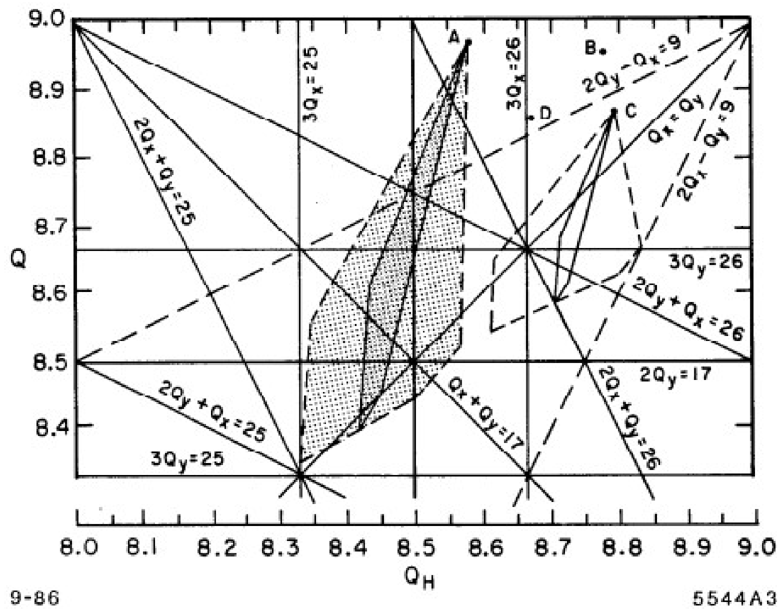


Fig. 3. Space charge tune shift of the AGS.

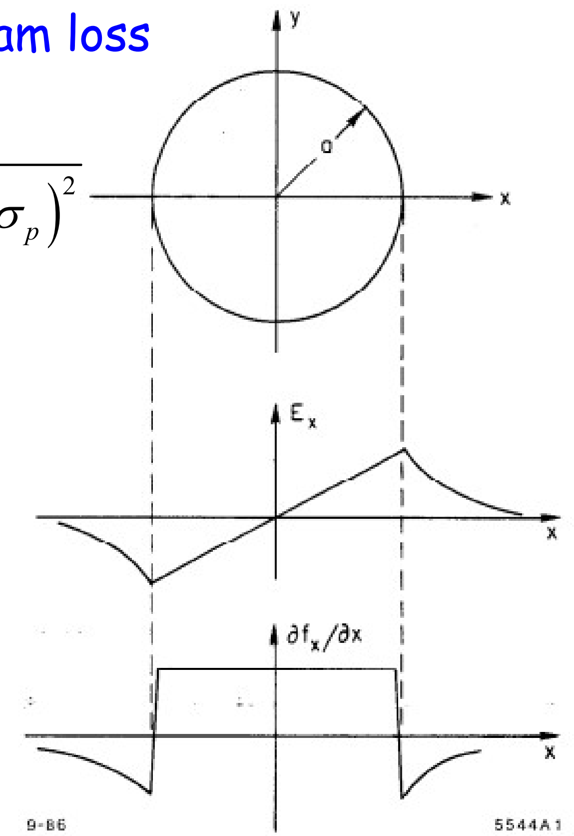


Fig. 1. Space Charge force of a uniform cylindrical beam.

- If there is no time dependense of focusing on time (smooth focusing approximation) there are no emittance growth and beam loss
  - time (actually on long. coordinate) dependence of the force leads to problems

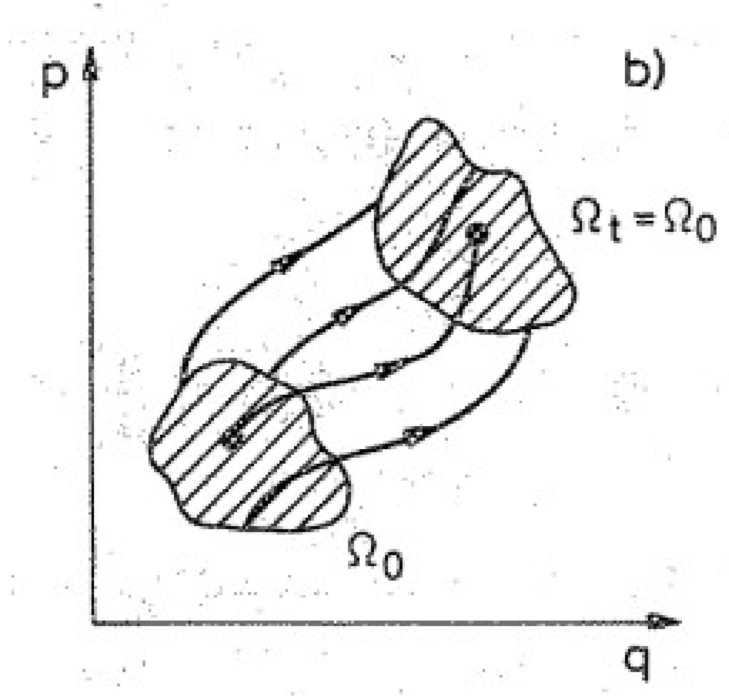
Possible Values of Tune Shifts

- Achieved values of tune shifts
  - Space charge
    - NAPM ~0.15 (strong el. cooling)
    - Fermilab Booster ~0.3 (only ~2000 turns at low energy)
    - J-PARC, PS Booster ~ 0.5-0.6
      (High accuracy of super-periodicity, 3 periods for J-PARC)

Conservation of the Phase Space

- Liouville theorem results in conservation of the phase space
  In other words, the Jacobian does not depend on time

$$\frac{\partial(p,q)}{\partial(p_0,q_0)}=1 \quad \Rightarrow \quad \int pdq = const$$



- In the absence of acceleration, the beam emittance is conserved $\varepsilon = \int \theta dx = const$
- Accounting acceleration requires an introduction of normalized emittance
  $\varepsilon_n = \beta\gamma\int \theta dx = const$, which is "always" conserved
- For linear motion in equilibrium the beam phase portrait is represented by an ellipse which does not change with time at a given place. The area of this ellipse, i.e. the beam emittance, is invariant of motion along the ring.
- For a single particle the ellipse "described" in the course of its turn-by-turn motion determines emittance which is equal to its Courant-Snyder invariant

$$\varepsilon = \beta\theta^2 + 2\alpha x\theta + \left(1+\alpha^2\right)\frac{x^2}{\beta}, \quad \alpha = -\frac{1}{2}\frac{d\beta}{ds}$$

Longitudinal Motion

- Beam acceleration, in addition to transverse beam focusing requires the longitudinal one
  - Principle of autophasing was suggested by V. Veksler in 1944

- Momentum compaction: $\dfrac{\Delta C}{C} = \alpha \dfrac{\Delta p}{p}, \quad \alpha = \dfrac{M_{51}D + M_{52}D' + M_{56}}{C}$

- Slip-factor: $\dfrac{\Delta T}{T} = \dfrac{\Delta C}{C} - \dfrac{\Delta v}{v} = \eta \dfrac{\Delta p}{p} \xrightarrow[\frac{\Delta C}{C} = \alpha \frac{\Delta p}{p}]{\frac{\Delta v}{v} = \frac{1}{\gamma^2}\frac{\Delta p}{p}} \left(\alpha - \dfrac{1}{\gamma^2}\right)\dfrac{\Delta p}{p} \quad \Rightarrow \quad \eta = \alpha - \dfrac{1}{\gamma^2}$

- Equations for the longitudinal motion (no acceleration)

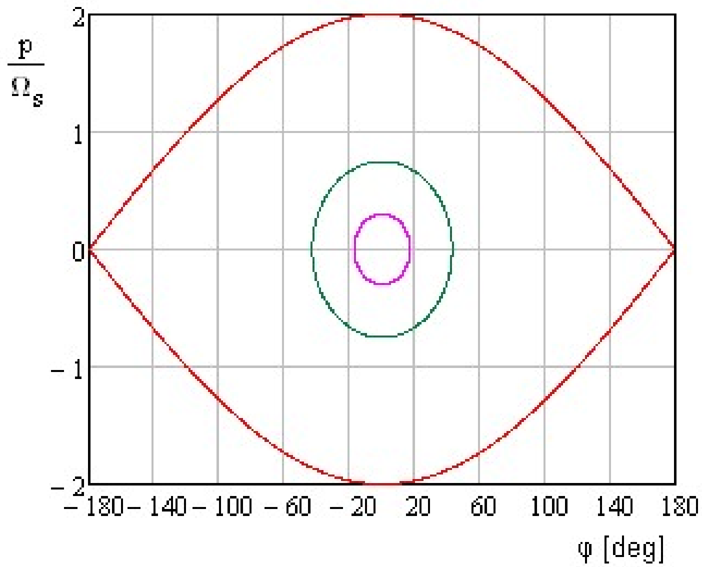
$$\begin{cases} \dfrac{d\varphi}{dt} = q\omega_0 \eta \dfrac{\Delta p}{p} \\ \dfrac{d}{dt}\dfrac{\Delta p}{p} = \dfrac{1}{\beta^2}\dfrac{d}{dt}\dfrac{\Delta E}{E} = -\dfrac{1}{\beta^2 E} eV_0 \dfrac{\omega_0}{2\pi}\sin\varphi \end{cases} \Rightarrow \quad \frac{d^2\varphi}{dt^2} = -\Omega_s^{\,2}\sin\varphi, \quad \Omega_s = \omega_0\sqrt{\frac{eV_0 q\eta}{2\pi mc^2\beta^2\gamma}}$$

Adding acceleration, one finally obtains

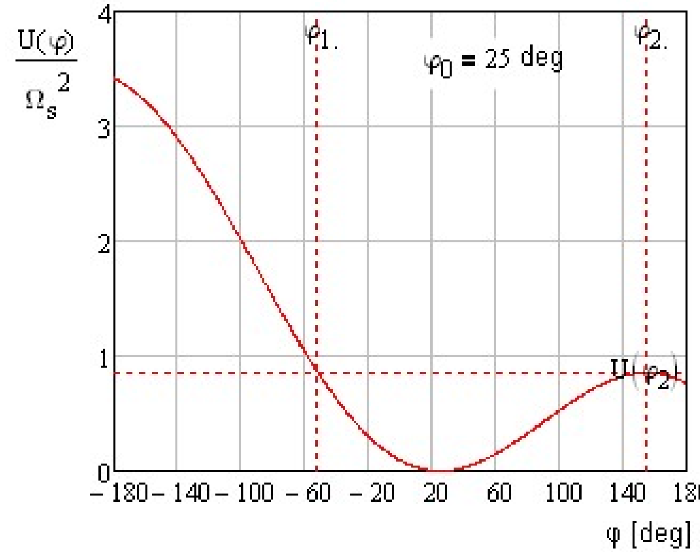
$$\frac{d^2\varphi}{dt^2} = -\Omega_s^{\,2}\left(\sin\varphi - \sin\varphi_0\right), \quad \sin\varphi_0 = \frac{V_{SR}}{V_0}$$

Acceptance of RF Bucket

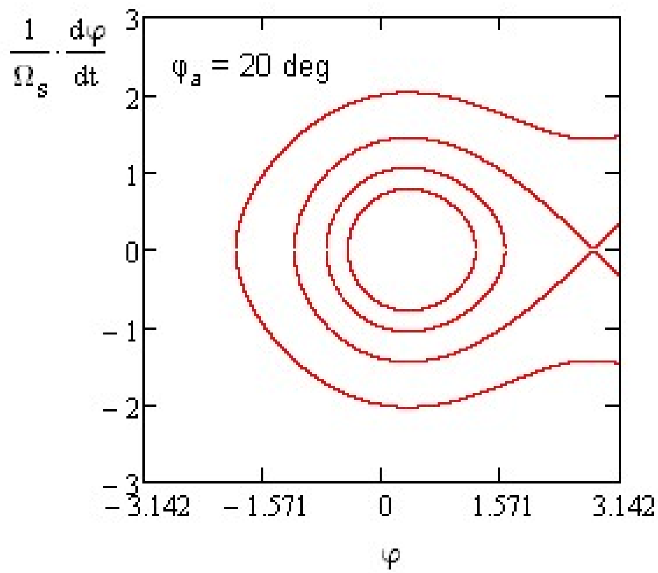
- In difference to transverse motion there is no resonances in the longitudinal plane
- An acceleration results in a reduction of RF bucket size
  - *A reason of particle loss in recent Run of NICA injection complex*

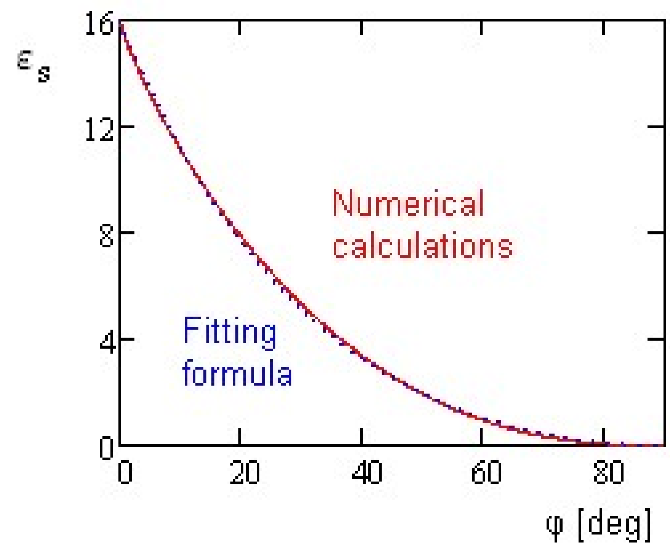


Phase trajectories in the absence of acceleration



Potential well for accelerating phase of 25 deg.





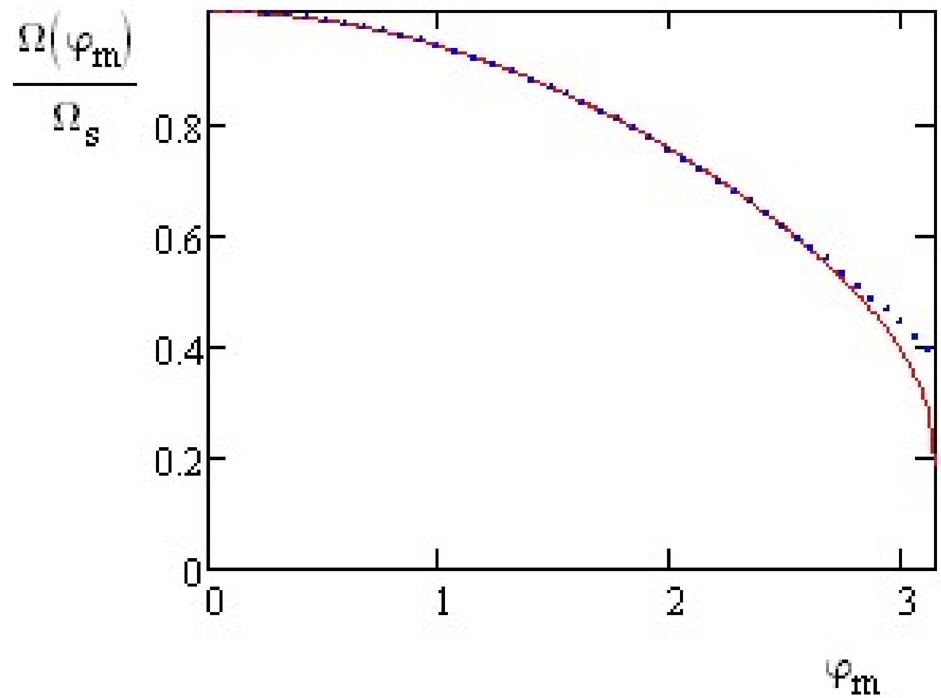
Dependence of RF bucket area on accelerating phase

Dependence of Synchrotron Frequency on Amplitude

- For small amplitude particle

$$\frac{\Omega(\varphi_m)}{\Omega_s} \approx 1 - \frac{\varphi_m^{\ 2}}{16}$$

  - Desynchronization is accelerated with amplitude



- Dependence of synchrotron tune on particle amplitudes
  - Helps to fight longitudinal instabilities
  - Enables to shape the longitudinal particle distribution (LHC). Off course such shaping results in an increase of longitudinal emittance

Beam Interaction with Accelerator Elements

- A particle beam interacts with surrounding structures through its electric and magnetic fields
- For non-relativistic beams their direct electric field makes the major contribution
  - With energy increase the effect of the direct beam electric field is compensated by the beam magnetic field that results in total force decrease as $1-\beta^2 = 1/\gamma^2$
- Therefore with energy increase other mechanisms become important
  - For ultrarelativistic particles an interaction is transferred downstream
- We introduce the longitudinal and transverse impedances:

$$U_\omega \equiv \oint E_{\parallel\omega} ds = Z_\parallel(\omega) I_\omega$$

$$\oint E_{\perp\omega} ds = Z_\perp(\omega) I_\omega x$$

- In modern synchrotrons a careful design of vacuum chamber and its elements (BPMs, vacuum ports, bellows, etc.) is done to reduce the beam impedances
  - Feedbacks suppressing the beam instabilities are other important parts of accelerators

Types of Synchrotrons

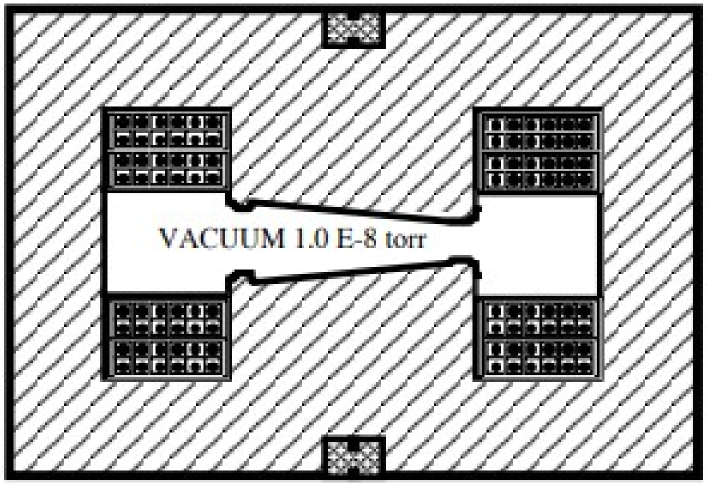
- Electron vs hadron
  - Electrons run at v=c

  ⇨ Constant revolution frequency, much easier RF system
- Fast cycling versus normal cycling
  - Fast cycling synchrotrons (15-50 Hz) have ceramic vacuum chamber or do not have it at all (FNAL Booster). All existing fast cycling synchrotrons have normal conducting dipoles
- Combined functions versus separate functions
  - Combined function synchrotrons do not require good relative stabilization between dipole and quad circuits, since only trim (correcting) quads are used
- Colliders, SR sources and storage rings stays at the top energy for long time

  ⇨ Good cooling of magnets and large-power power supplies or SC magnets
- Extraction type
  - Typically, slow extraction continues for few seconds and operates at the 3^{rd} order resonance
  - In high power synchrotrons, fast (1 turn) extraction needs abort gap

Magnets and their Powering

- FNAL Booster
  - Combined function, operates at 15 Hz
  - Dipoles put inside vacuum box
    => no vacuum chamber
    => beam sees laminations
    => huge impedances
    However the only real problem with stability is at the transition crossing where the beam generates 150 kV/turn
    (1 MV total RF voltage)
  - Powering DC + excitation of resonance LC circuit (Q=40) saves a lot of actual power



Magnets and their Powering

- CERN PS Booster
  - Four rings stacked on the top of each other
  - separated function magnets
  - Filling PS in one cycle
  - Superperiodicity enabled doubling of space charge tune shift

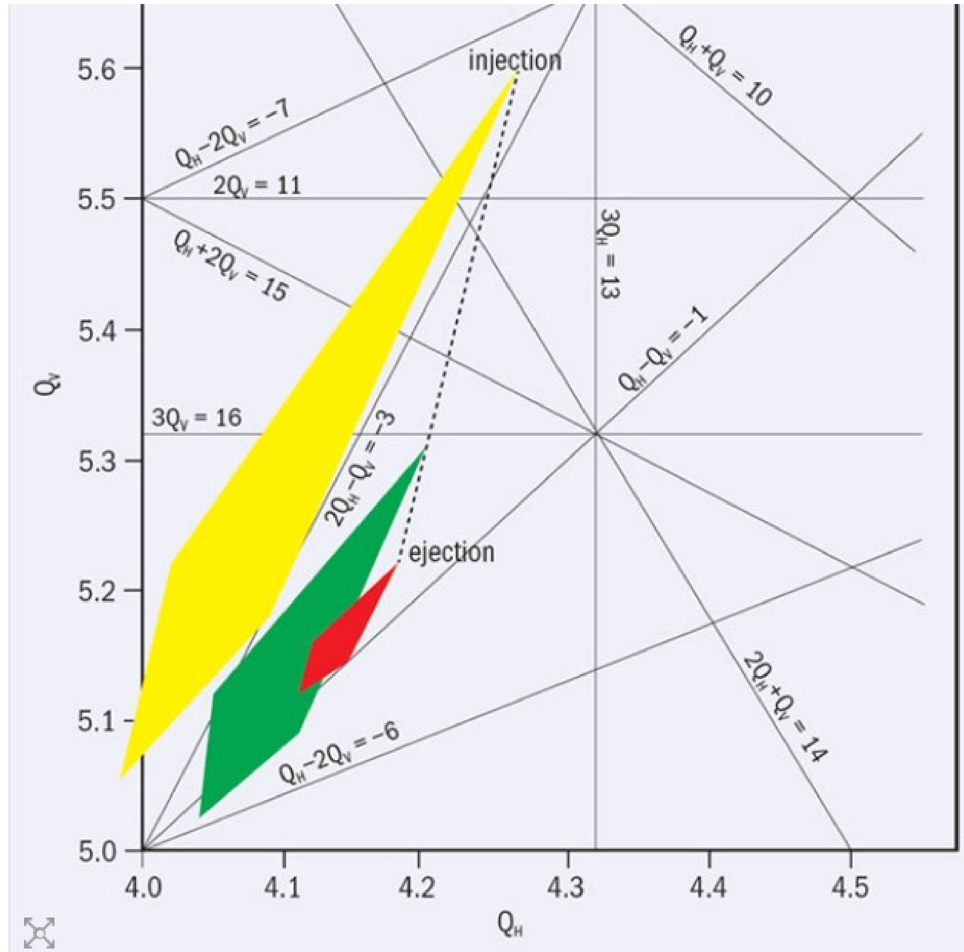
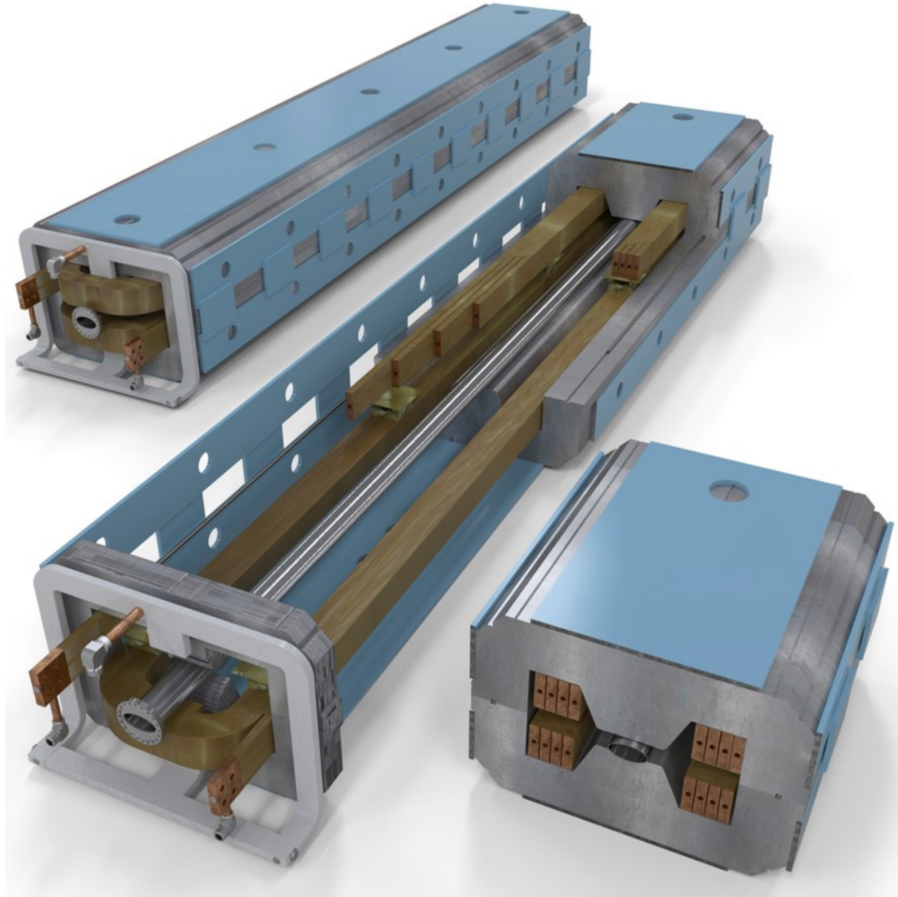


Fig. 3. Horizontal and vertical tunes Q_H, Q_V and the space-charge tune-spread ΔQ during acceleration in the PSB. The coloured areas depict ΔQ at injection (yellow), after 120 ms acceleration (green) and after 400 ms at 1.4 GeV (red) for 10^{13} protons in one ring. While ΔQ is around 0.55 at injection, covering many stop-bands, it shrinks significantly during acceleration, enabling the working point to be moved rapidly to an area clear of harmful stop-bands.

Magnets and their Powering

- FNAL Main injector
  - Represents typical synchrotron magnets
  - Separate functions
  - Normal powering
  - 2 s accelerating cycle
  - Insufficient water cooling to stay at the top energy for long time



Highest Energy = Highest Field SC Magnets

4.5T

Tevatron,
6 m, 76 mm
774 dipoles

4.5 K He, NbTi
+ warm iron
small He-plant

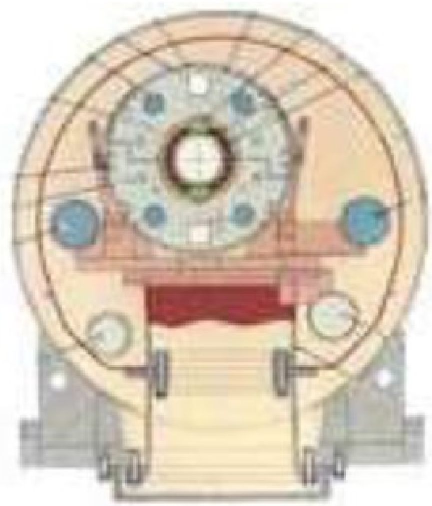
5.3T

HERA,
9 m, 75 mm
416 dipoles

NbTi cable
cold iron
Al collar

3.5T

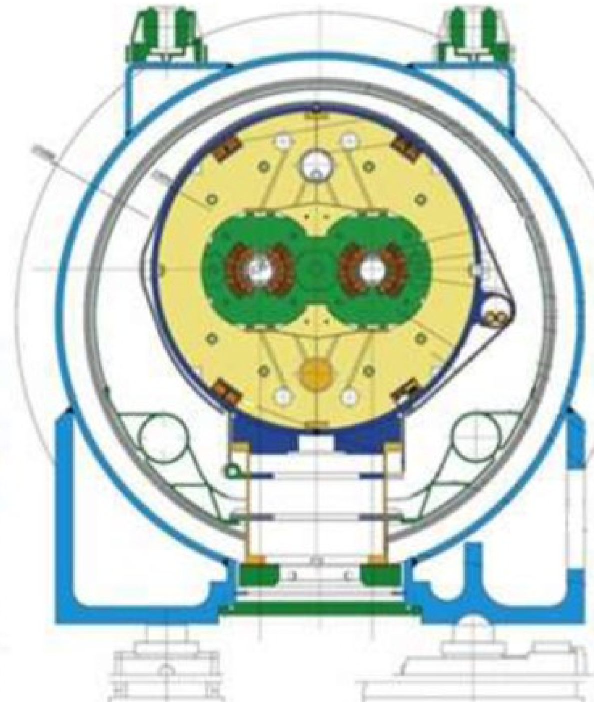
RHIC,
9 m, 80 mm
264 dipoles



NbTi cable
simple &
cheap

8.3T

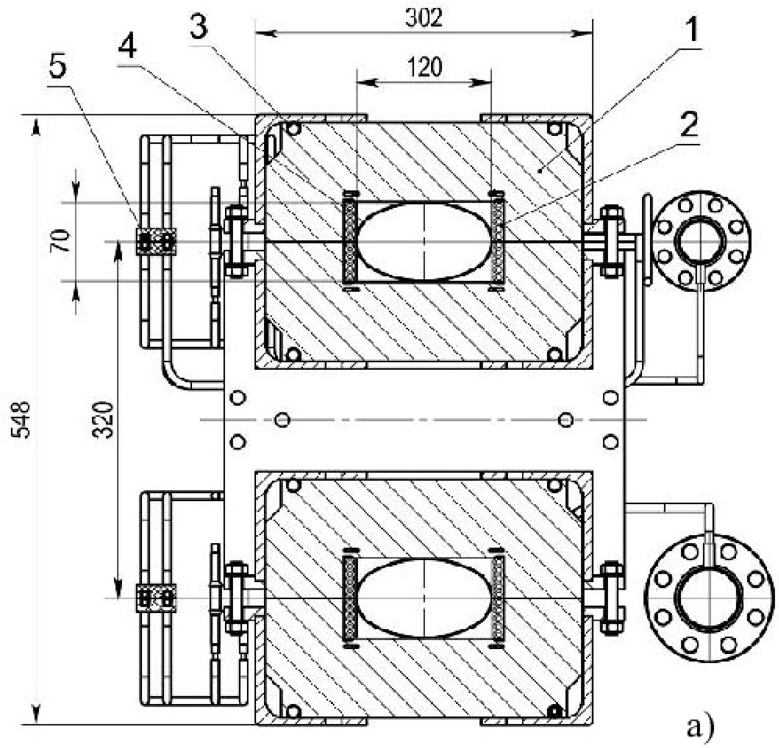
LHC,
15 m, 56 mm
1276 dipoles

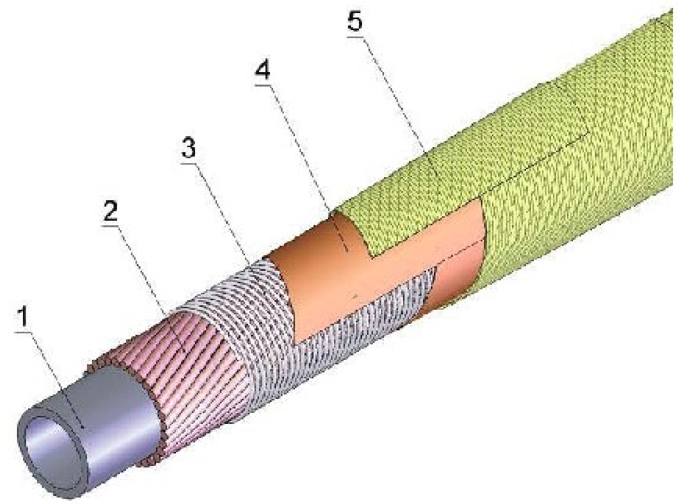


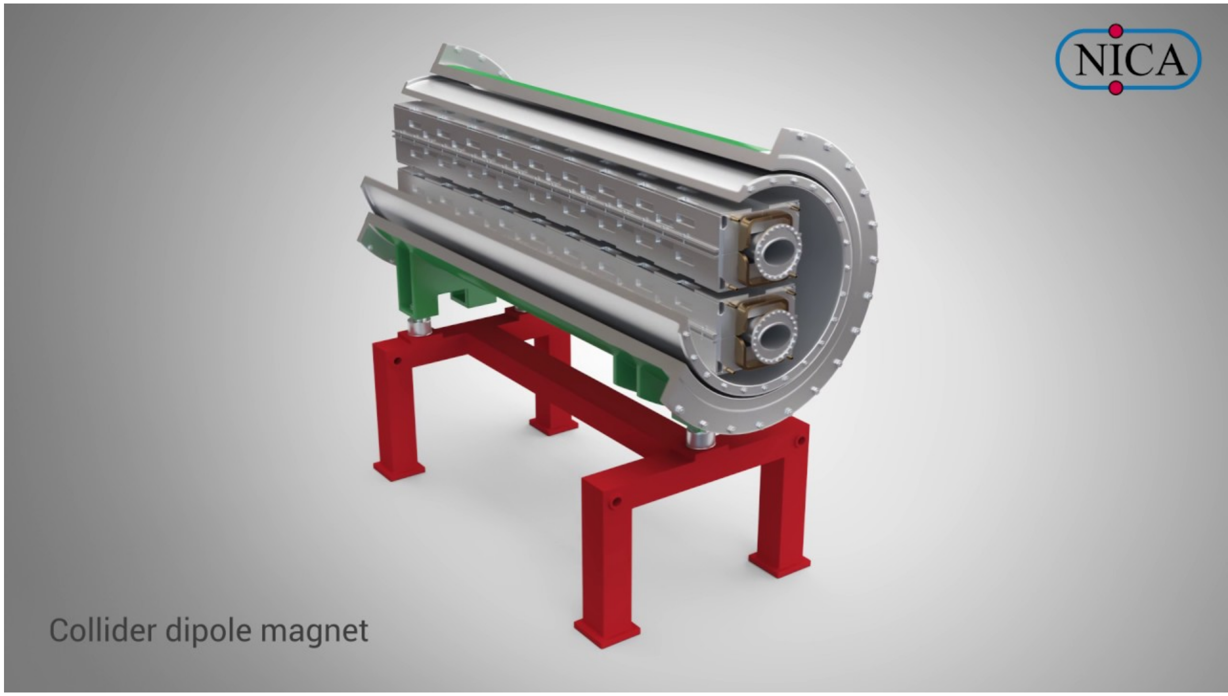
NbTi cable
2K He
two bores

Fermilab

NICA Superferric Magnets

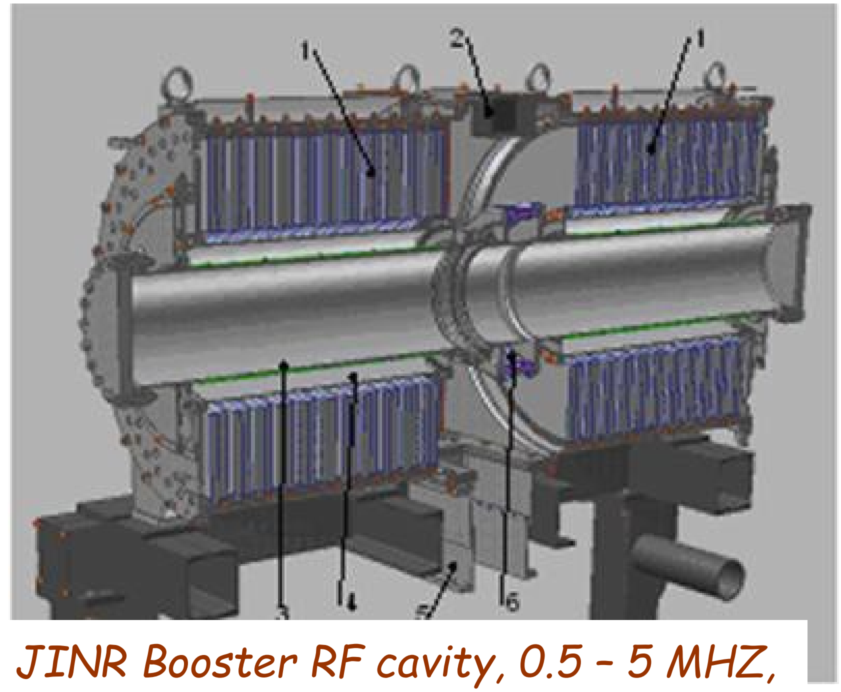




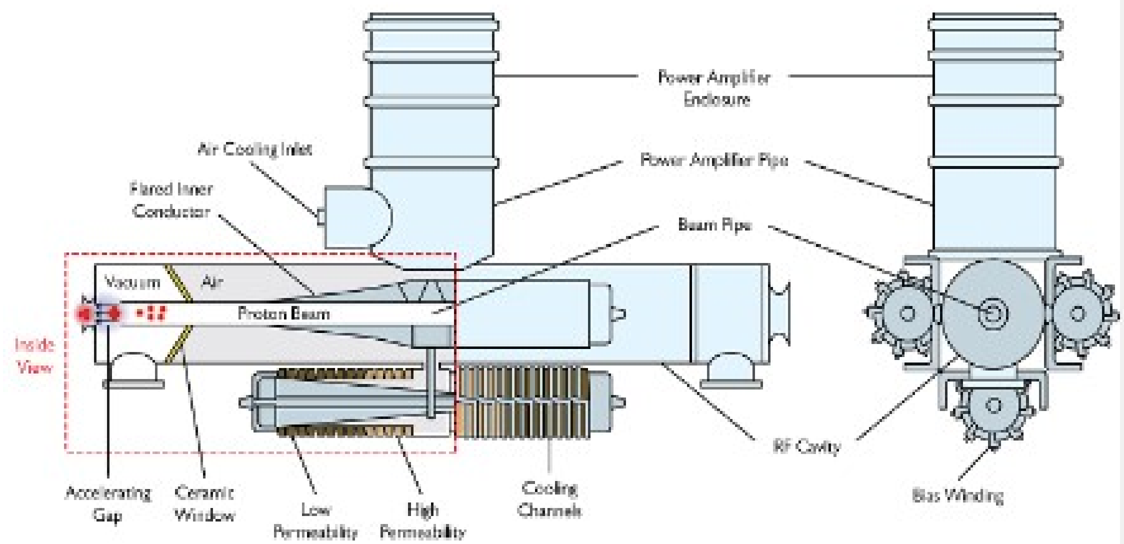


Types of RF Cavities

- Choice of RF cavities is determined by their voltage, rate of frequency change, and frequency range
- Single turn transformer
  - Compact, enables large tuning range but high voltage requires very high power
  - Fits well for moderate voltage or high beam current so that the beam takes large fraction of large power
- Drift tube (Nuclotron, FNAL Booster)
  - Resonance frequency supported by bias current affecting permeability of ferrites
  - Fast rate of frequency change



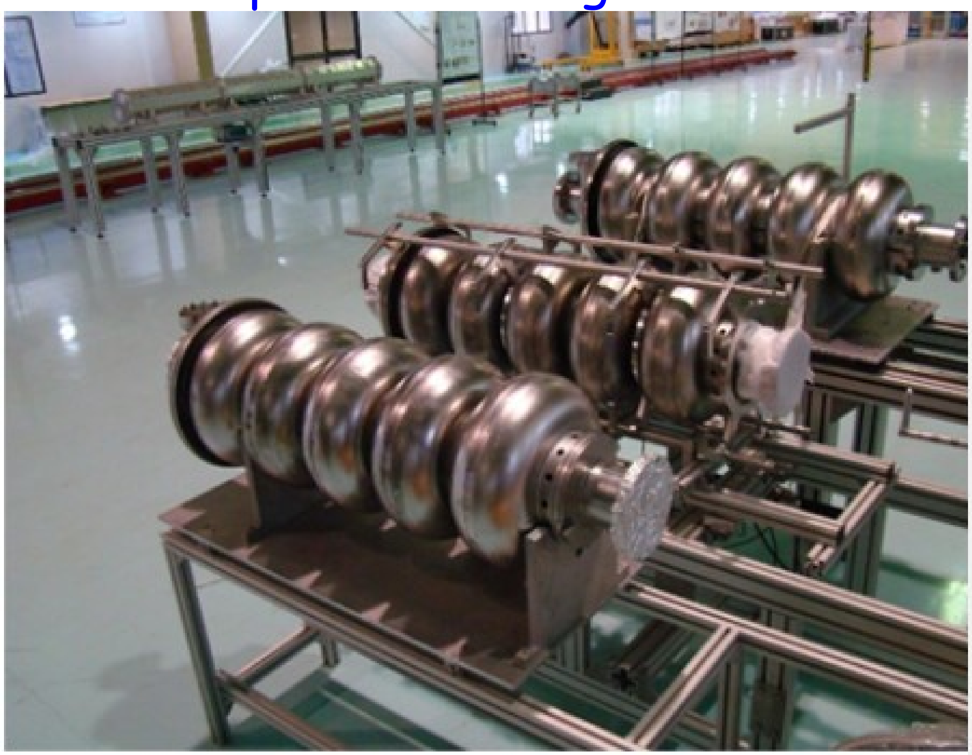
JINR Booster RF cavity, 0.5 – 5 MHZ, 5 kV, ~10 kW; Amorphous iron, 5B-M



FNAL Booster RF cavity

Types of RF Cavities (2)

- For ultra relativistic synchrotrons one can use cavities having little or zero tuning range
- PEP-II cavity is built to minimize its high order modes
- LHC cavity is superconducting to achieve high voltage with small number of cavities
  - Another way to reduce the total impedance of high order modes



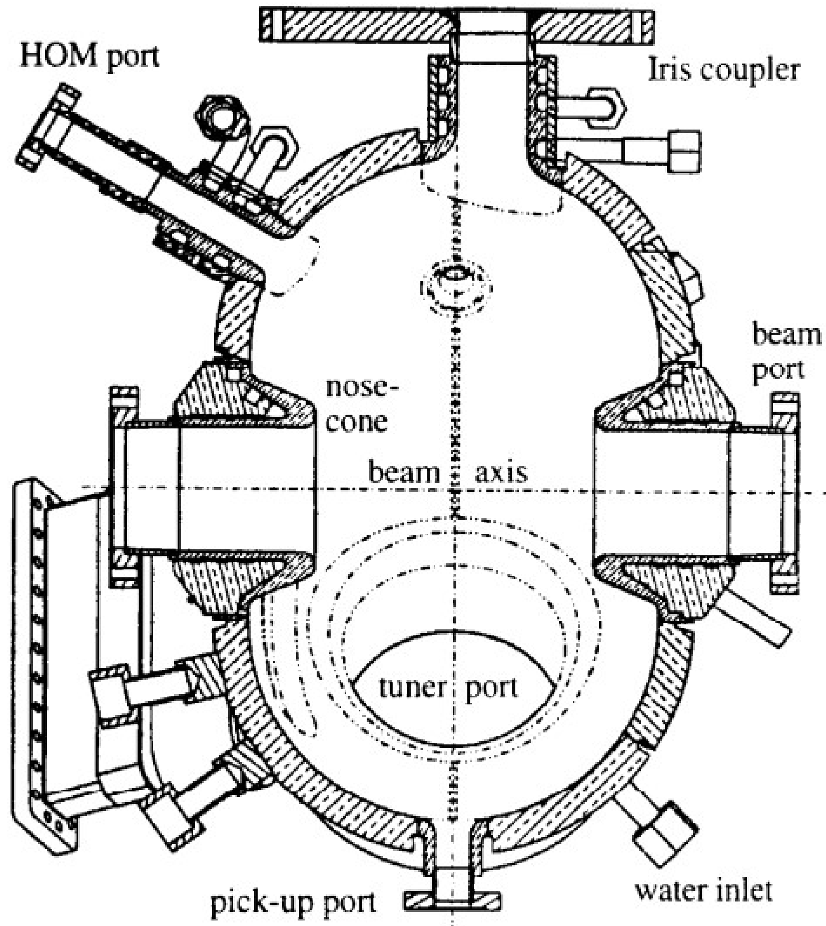
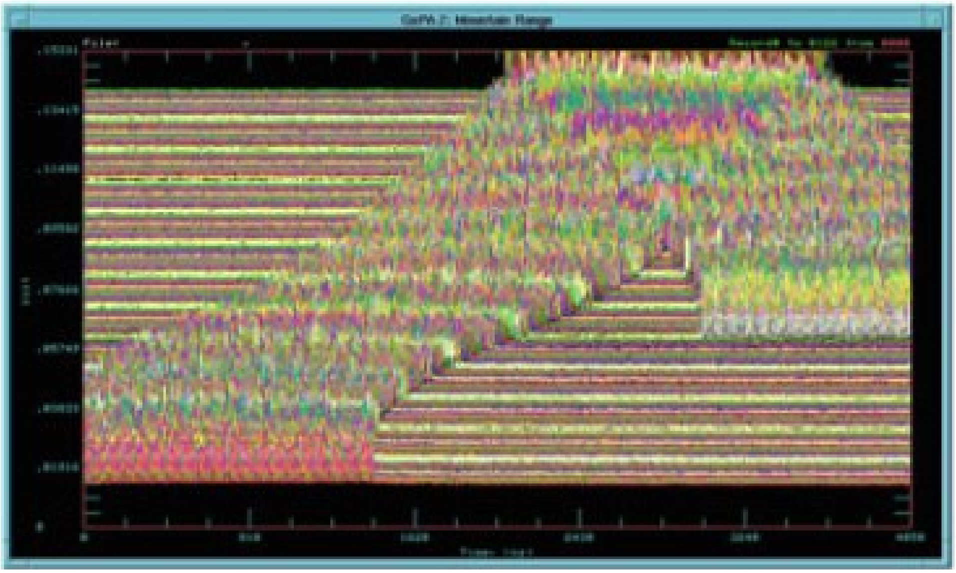
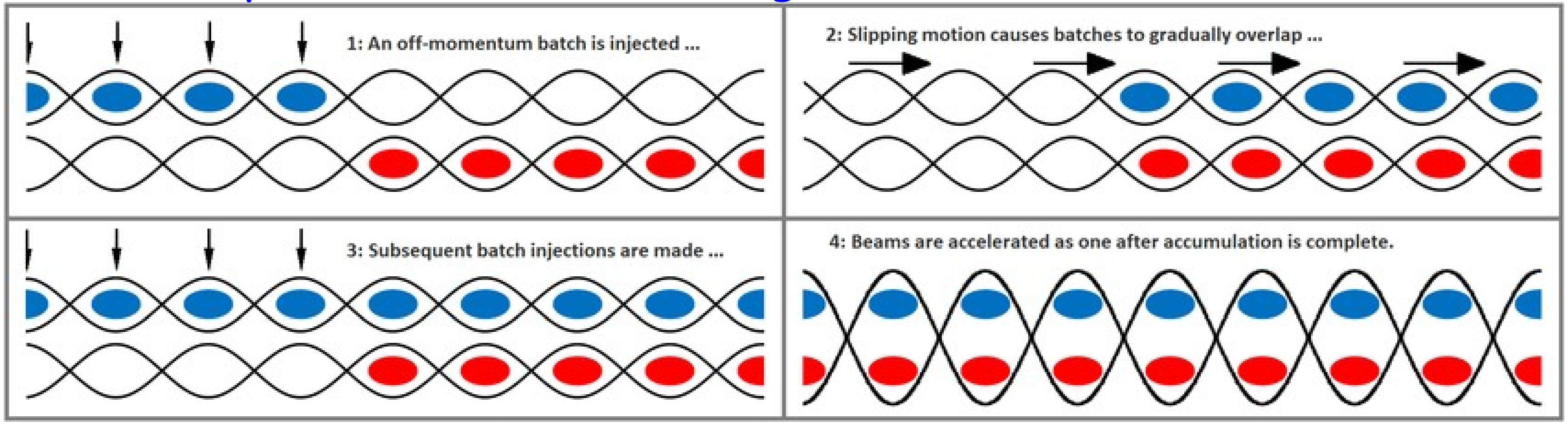


Figure 1: Horizontal cross section through the PEP-II high-power cavity showing the coupler, HOM port, beam ports and small pick-up port.

Slip-stacking



- Typically, a next stage synchrotron gets few injections from the previous one
- Slip-staking enables additional doubling of beam intensity in a high-energy synchrotron
  - Two RF frequencies from two RF systems are present simultaneously
  - Then, recapture into one much larger RF bucket

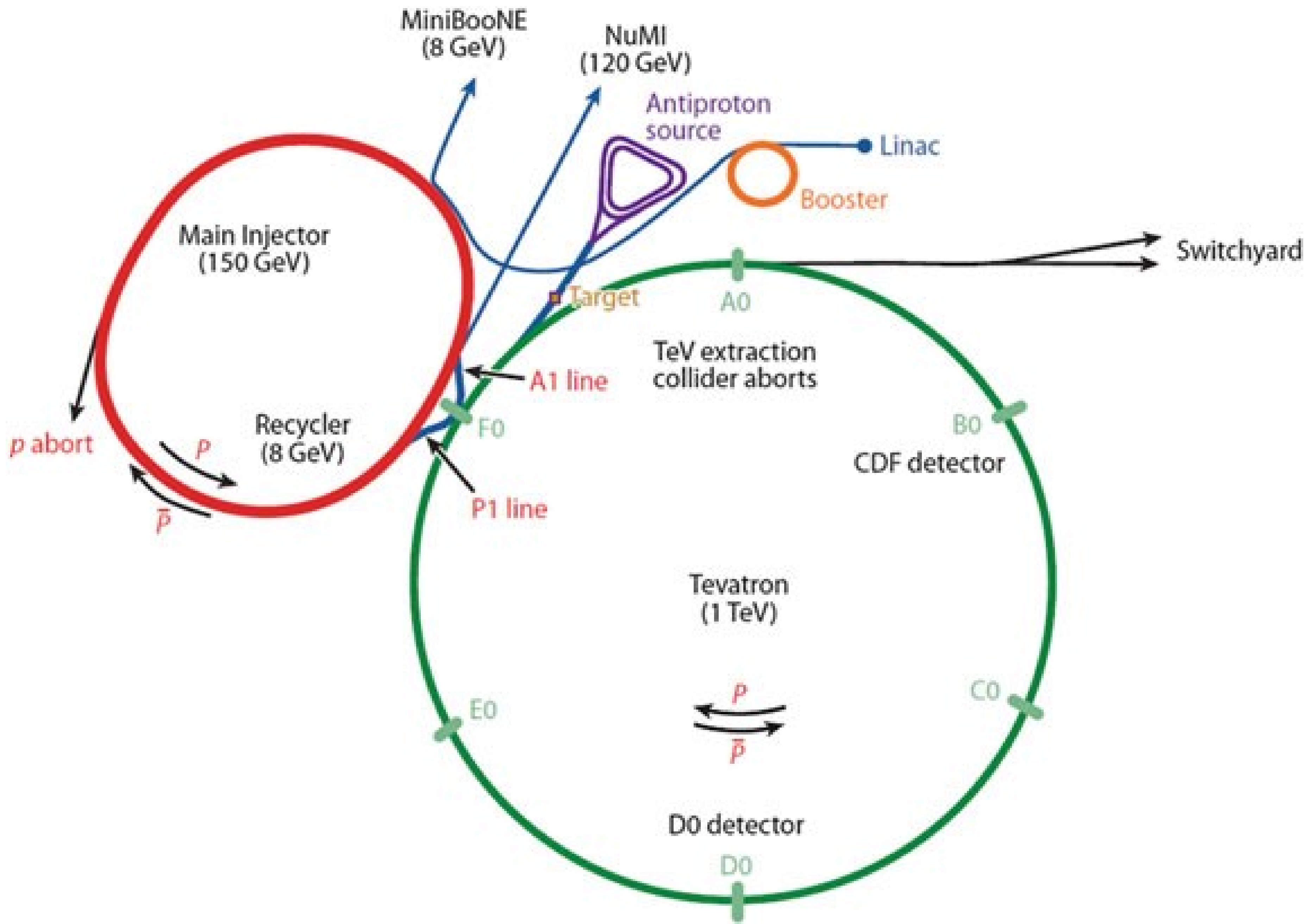


Application of Synchrotrons

- High energy physics
  - Fast extraction: Neutrino (dark sector)
  - Slow resonant extraction: rear decays, “normal” detectors(BM@N)
    - high luminosity, low energy
  - Colliders
- Nuclear physics
  - Colliders
    - Spin physics, quark-gluon plasma
  - Spallation neutron sources
- Atomic physics
- Sources of synchrotron radiation
- Medical accelerators
- R&D in accelerators
  (el. cooling, OSC, integrable optics, instabilities, …)

- Transmutation of nuclear waste

Fermilab Accelerator Complex in the Course of Run II



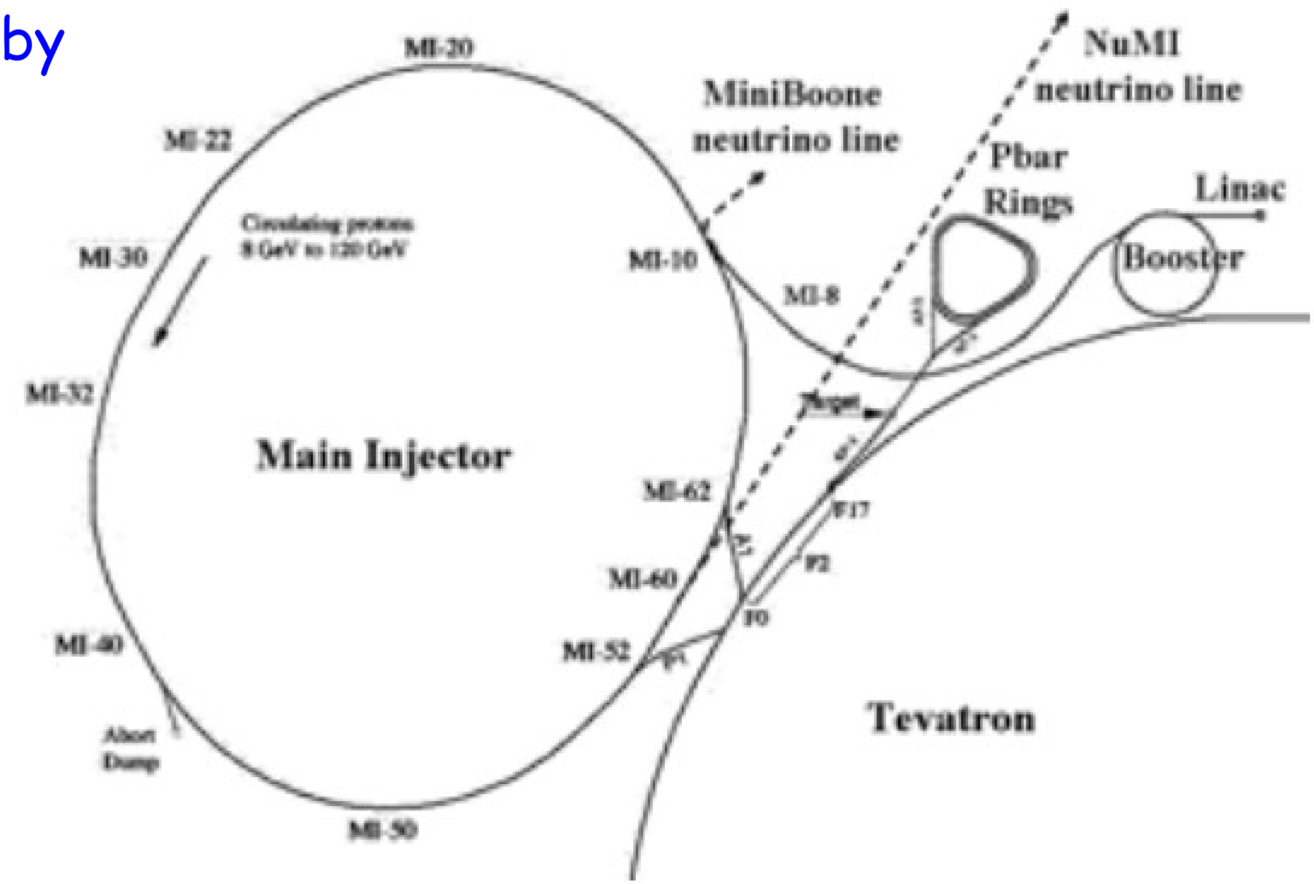
Highlights of Tevatron Run II Accelerator Technologies

- Suggested and developed in CERN
  - Stochastic cooling
- Suggested and developed in USSR
  - Strip H^- injection to Booster
  - Lithium lens for focusing of antiprotons outgoing from target
  - Electron cooling
  - RFQ linac (actually came after the Run II)
- Fermilab
  - Slip-stacking (2001)
  - Barrier bucket (1983)

- Each technology was polished to perfection in the course of Run II

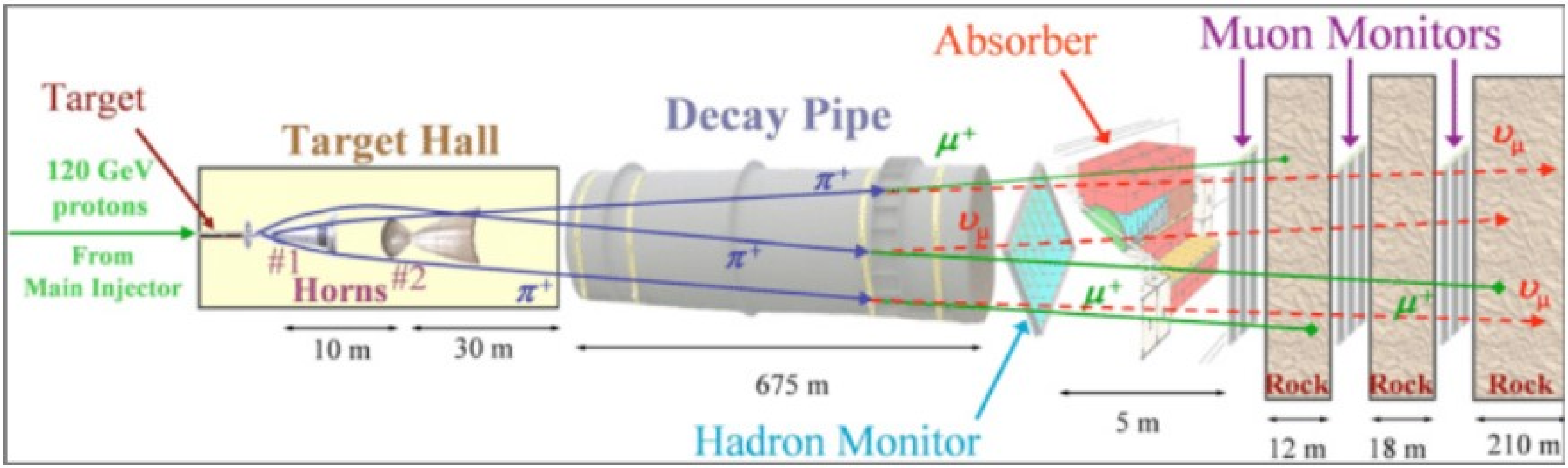
Fermilab Accelerator Complex Now

- Tevatron is decommissioned
- Rings of the antiproton source are transformed to support mu2e and g-2 experiments
- Beam power for the 120 GeV neutrino line grew up by about 3 times up to ~800 kW
- 8 GeV neutrino line continues to operate
- Slow extraction of 120 GeV protons to the MI continues to operate



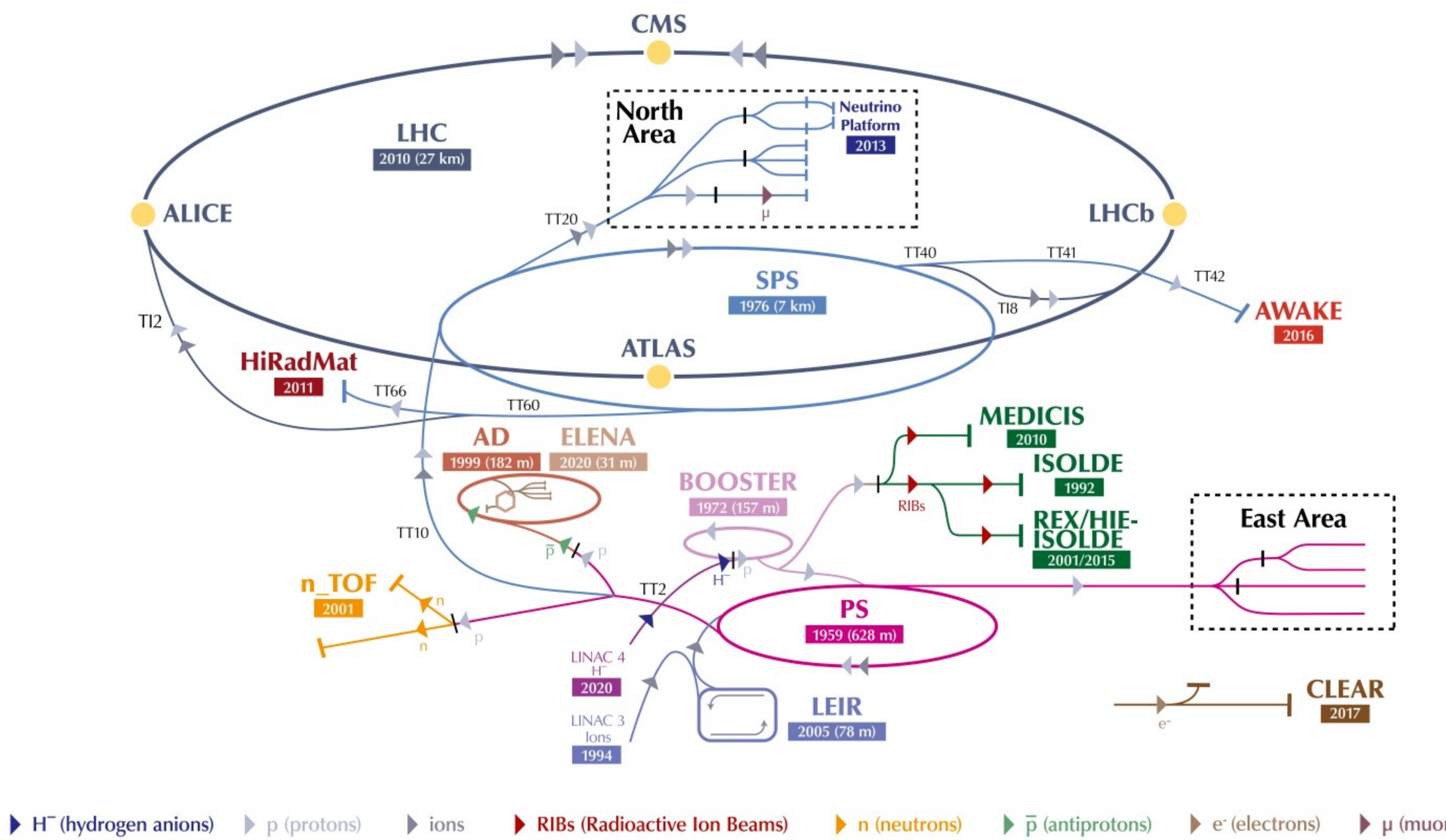
Neutrino Production

- Neutrino questions
  - Mass ierarhy
  - CP-violation
  - Are neutrino and antineutrino the same particle? Dirak or Majorana particle?



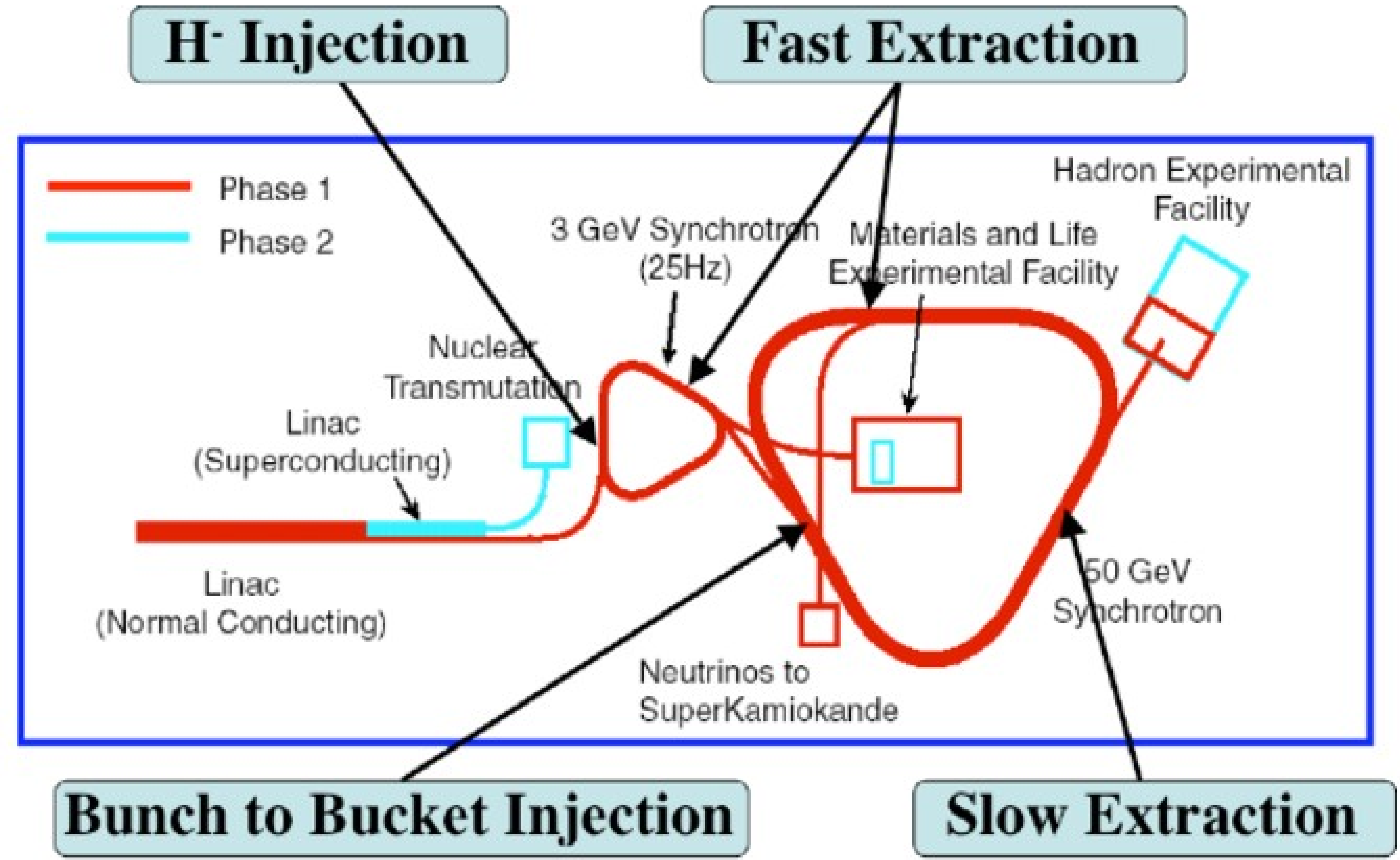
- The most effective nutrino production is based on the decay of pions
- Single turn extraction
  - Reduces background
  - Enables usage of horns for focusing

The CERN accelerator complex
Complexe des accélérateurs du CERN



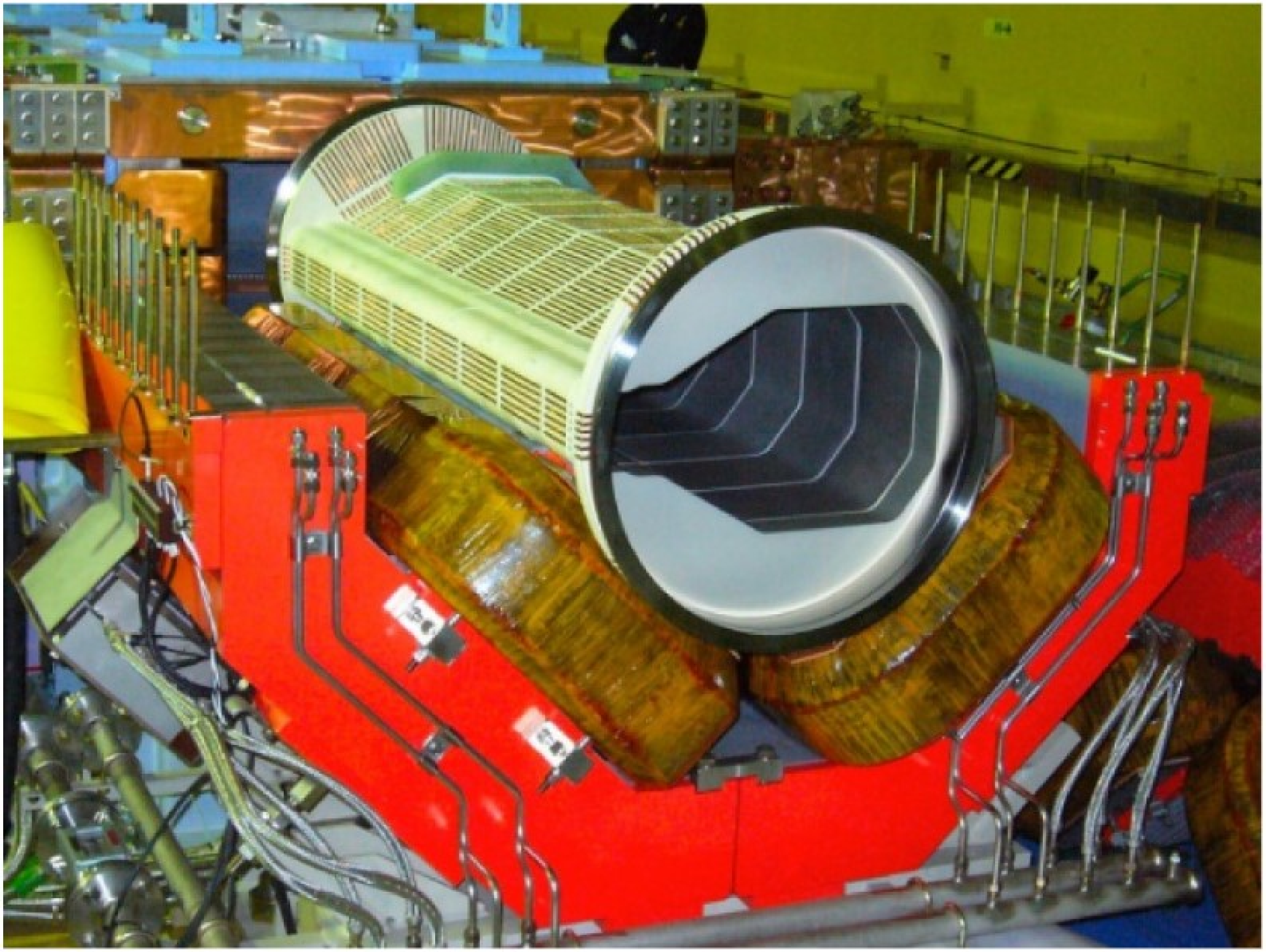
LHC - Large Hadron Collider // SPS - Super Proton Synchrotron // PS - Proton Synchrotron // AD - Antiproton Decelerator // CLEAR - CERN Linear Electron Accelerator for Research // AWAKE - Advanced WAKefield Experiment // ISOLDE - Isotope Separator OnLine // REX/HIE-ISOLDE - Radioactive EXperiment/High Intensity and Energy ISOLDE // MEDICIS // LEIR - Low Energy Ion Ring // LINAC - LINear ACcelerator // n_TOF - Neutrons Time Of Flight // HiRadMat - High-Radiation to Materials // Neutrino Platform

J-PARC



- Modern high power accelerator complex, > 1 MW beam power is the goal
- Exceptional neutrino program

J-PARC



Ceramic chamber of the injection region