

Study of multiquark fluctons in dd collisions at SPD

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Cumulative Particle Production

Production of particles from nuclei in a region, kinematically forbidden for reactions with free nucleons.

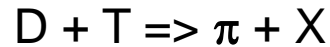
Cumulative Pion Production

1970 - Dubna – beams of relativistic deuterons ($p_0=5 \text{ GeV}/c/\text{nucleon}$)

Stavinskiy V.S. =>

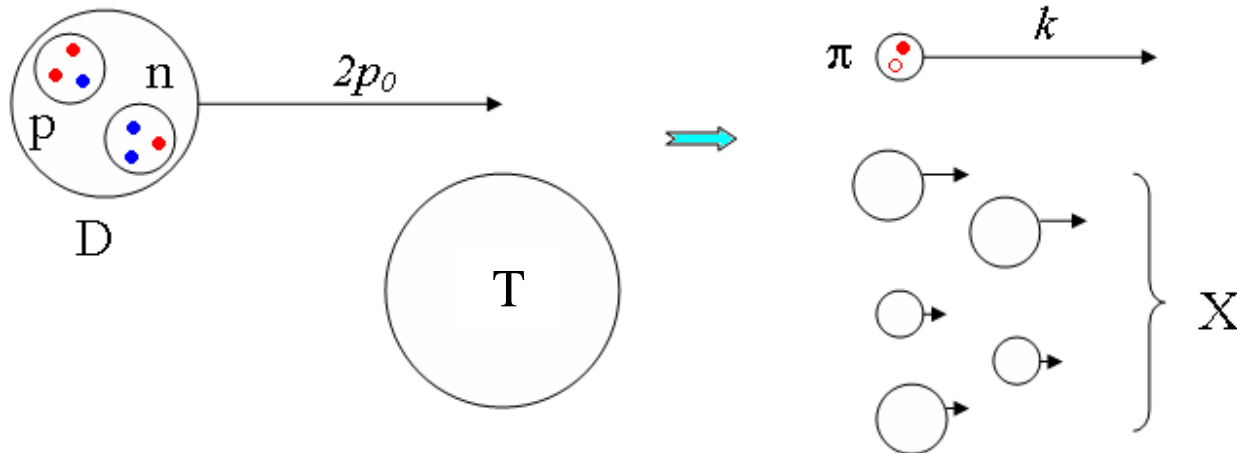
Fragmentation of projectile deuterons, D, on target, T.

Baldin A.M. et al., Yad.Fiz. 18 (1973) 79



$p_0 \gg m_N$: $p_0 < k < 2p_0$ - cumulative pions

Later – superconducting Nuclotron. Now – NICA.



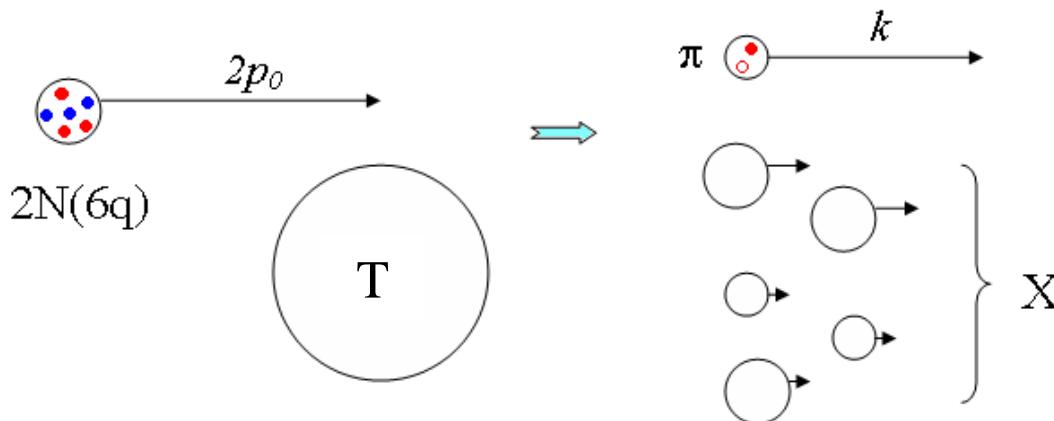
Flucton – intrinsic droplet of dense cold nuclear matter in a nucleus

Blokhintsev D.I., JETP 33 (1957) 1295

($2N$ flucton – 6 quark state)

~5% $2N(6q)$ – flucton
admixture in D

Common $6q$ -bag



Fragmentation of **projectile** nucleus \Leftrightarrow Fragmentation of **target** nucleus
(the same phenomenon in different frames of reference)

Cumulative fragmentation of **target** nucleus:

The 1st experimental observations of the **backward** particle production
in $p+A$ collisions on a **fixed target** nucleus:

G.A. Leksin et al., ZhETF 32, 445 (1957)

L.S. Azhgirej et al., ZhETF 33, 1185 (1957)

Yu.D. Bayukov et al., Izv. AN SSSR 30, 521 (1966)

The Reserford-like experiments indicating the presence of **droplets of dense nuclear matter in a target nucleus** (fluctons).

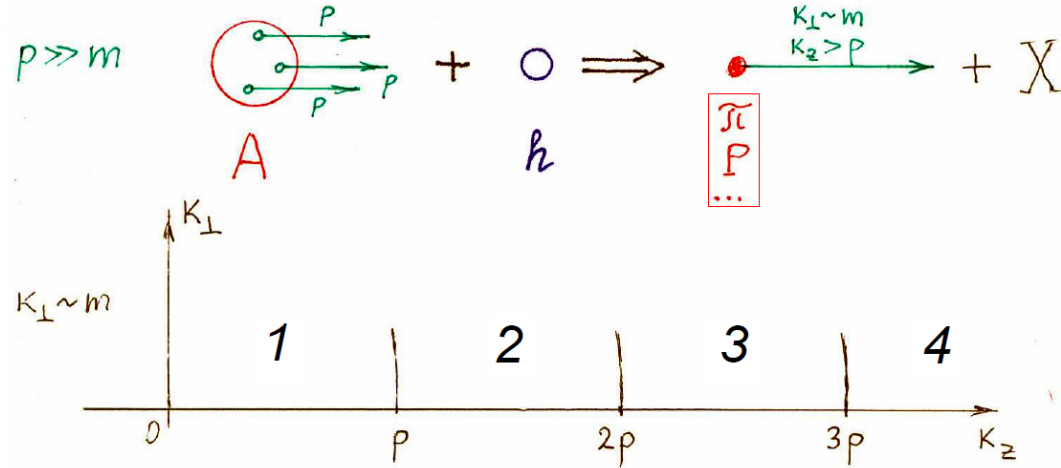
Kinematics of cumulative production in nucleus fragmentation region

Fragmentation of *projectile* nucleus

$$x \equiv \frac{k_+}{p_+} = \frac{k_0 + k_z}{p_0 + p_z} \approx \frac{k_z}{p}$$

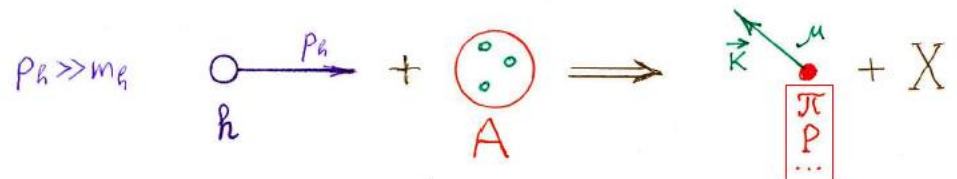
$k_z, p \gg m$, m – nucleon mass

$$x = 1, 2, 3, \dots, A$$



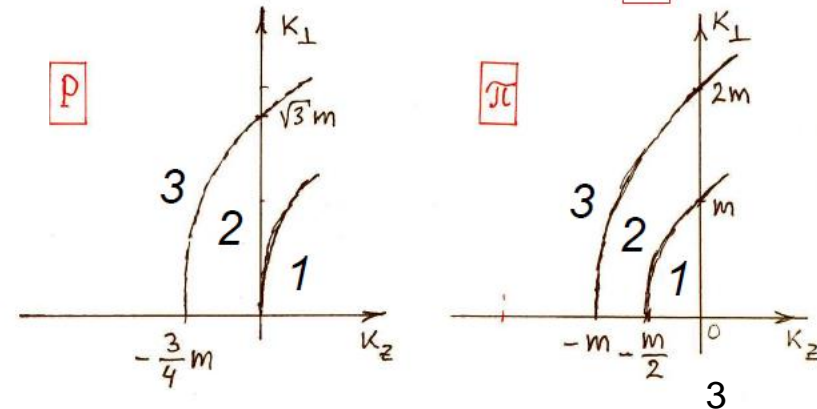
The borders increase with p

Fragmentation of *target* nucleus



$$x \equiv \frac{k_-}{p_-} = \frac{\tilde{k}_0 - \tilde{k}_z}{m} = \frac{\sqrt{\tilde{k}_z^2 + k_{\perp}^2 + \mu^2} - \tilde{k}_z}{m}$$

$$\tilde{k}_z = -\frac{xm}{2} + \frac{k_{\perp}^2 + \mu^2}{2xm}$$

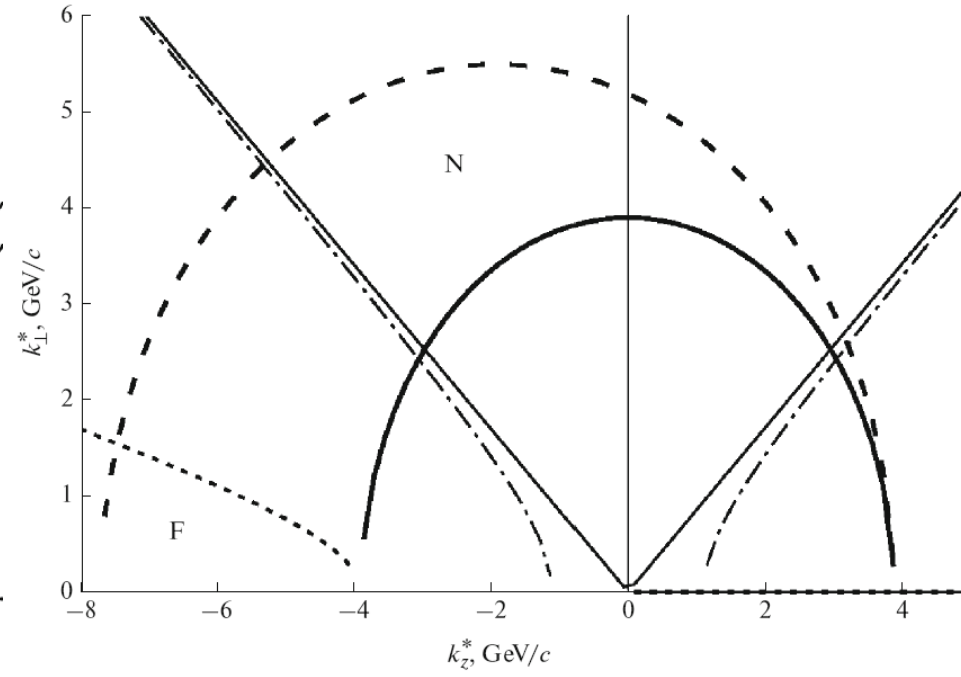
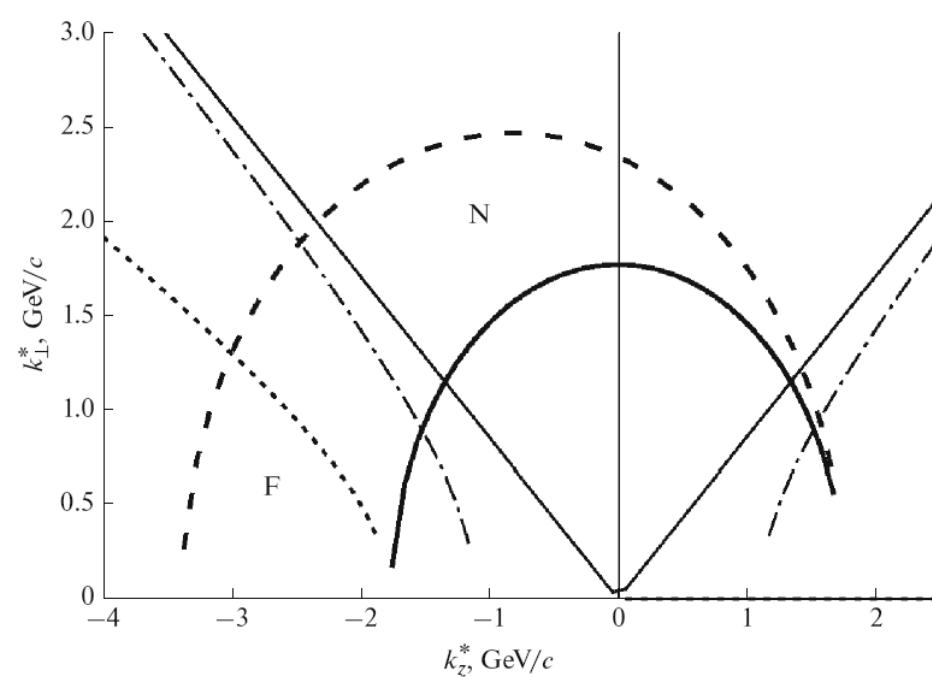


The borders are fixed at $p \gg m$

**New cumulative region -
 – central rapidities and large transverse momenta
 at NICA energies (in the c.m. system)**

$\sqrt{s_{NN}} = 4 \text{ GeV}$

$\sqrt{s_{NN}} = 8 \text{ GeV}$



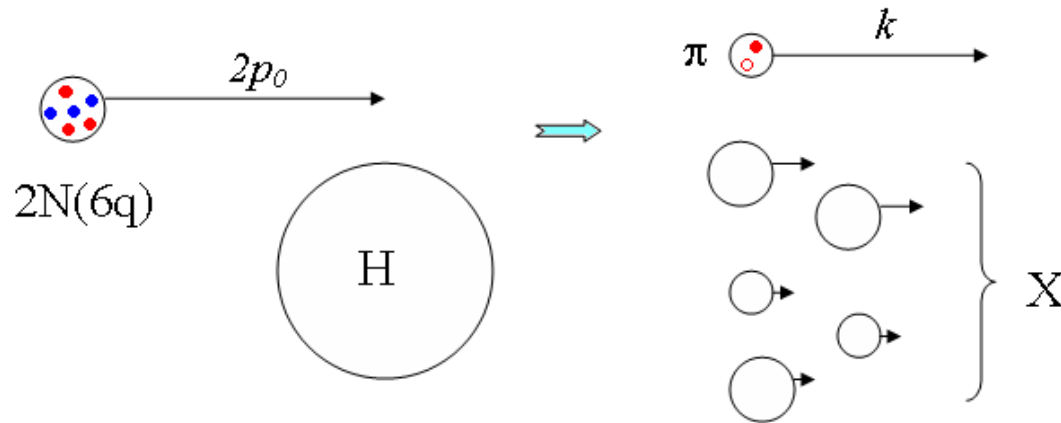
V.V. Vechernin, Phys. Part. Nuclei 53, 433-440 (2022).

$N+2N \Rightarrow p+X$

**Cumulative production cannot be studied
 in collider experiments at RHIC and LHC energies,
 but can be studied at NICA energies.**

**Braun M.A., Vechernin V.V.,
 Investigation of Cold Quark-Gluon Plasma in Cumulative Particle Production Processes,
 CERN-ALICE-INT-1993-04.**

Limiting fragmentation of light nuclei. Quark counting rules.



$1 < x < 2$ - the cumulative region ($1 < x < f$ - for the fN flucton)

Theoretical description near upper threshold: for $2N(6q)$ flucton $k \rightarrow 2p_0$, $x = k/p_0 \rightarrow 2$
 (Limiting fragmentation of a nucleus)

Quark counting rules: $I \sim \Delta^{2p-1}$

Δ – the deviation of x from its maximal value f , $\Delta = f - x$

p – the number of “donors”, stopped quarks, $p = n - 1$

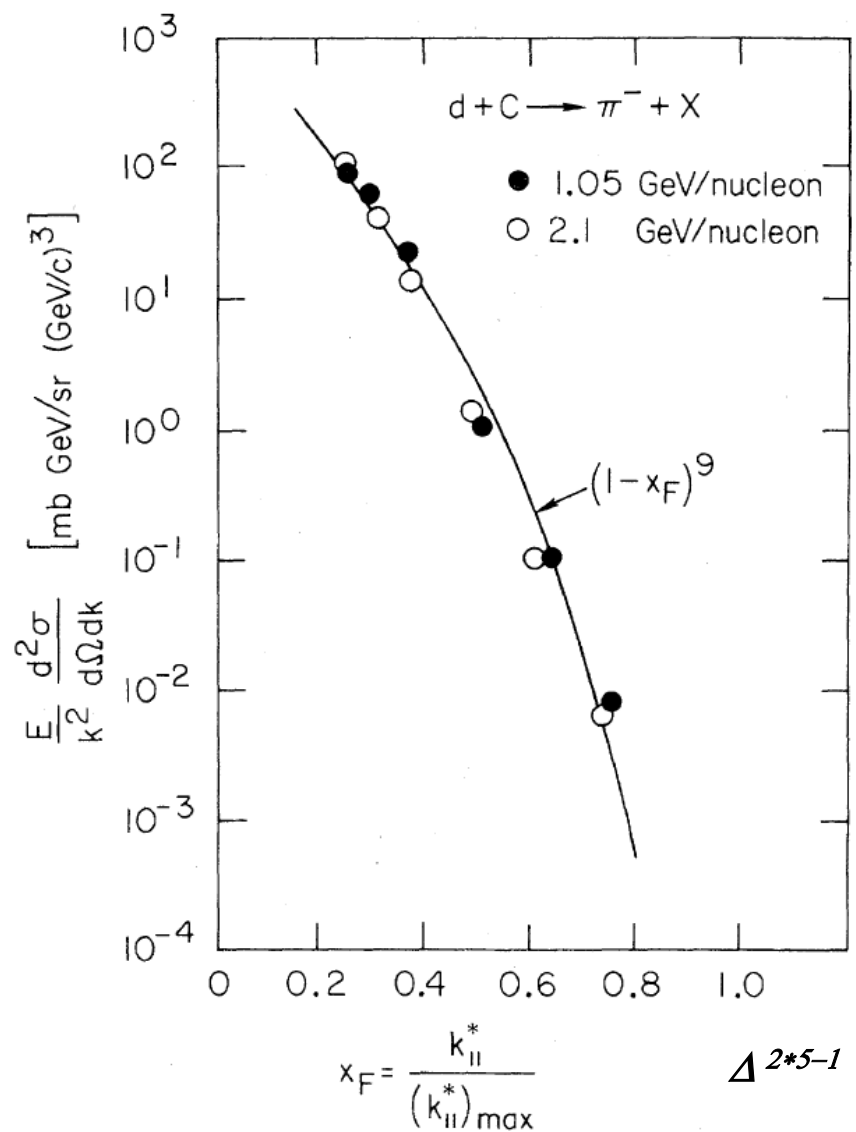
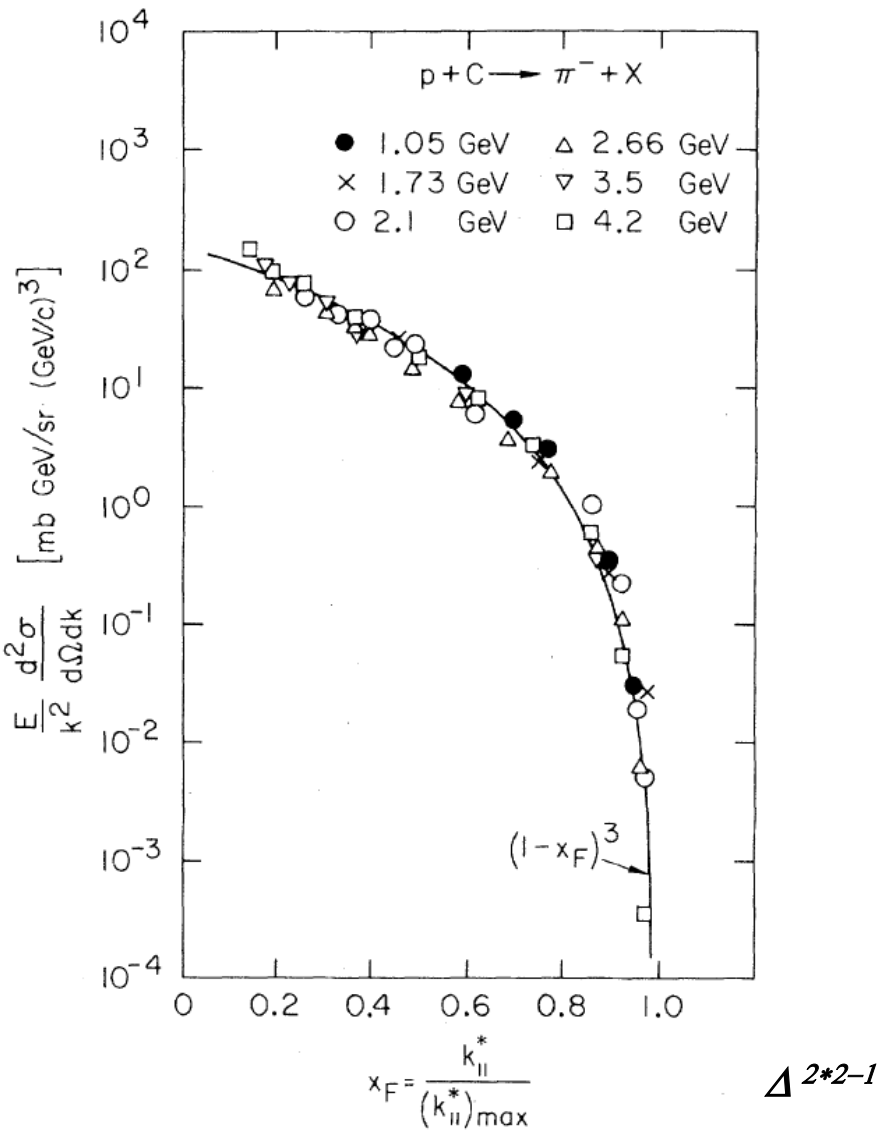
n – the number of constituents.

For $2N(6q)$ flucton $f = 2$, $n = 6$, $p = 5$, then $I \sim (2-x)^{2 \cdot 5 - 1} = (2-x)^9 = \Delta^9$

Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719

Brodsky S.J., Chertok B.T. Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269



The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).

Description of the hadron asymptotics at $x \rightarrow 1$

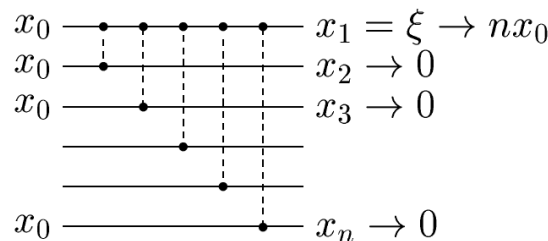
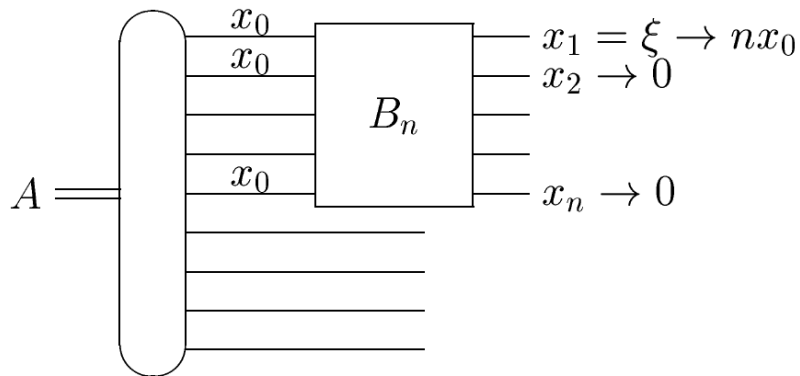
by the intrinsic diagrams of QCD in light-cone gauge
with low- x spectator quarks interact with the target

Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl.Phys. B369 (1992) 519

Description of the flucton asymptotic at $x \rightarrow f$,

f - the number of nucleons in flucton, n - the number of quarks in flucton, $x_0 = f/n (=1/3)$.

M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614. (DIS in cumulative region)

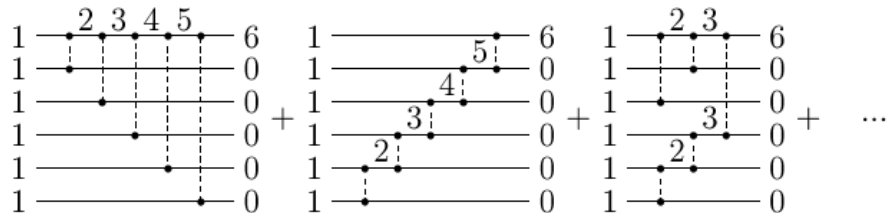


$$\sim \Delta^{2p-1}$$

$$p=n-1$$

$$\Delta = f - \mathbf{x} = nx_0 - \xi$$

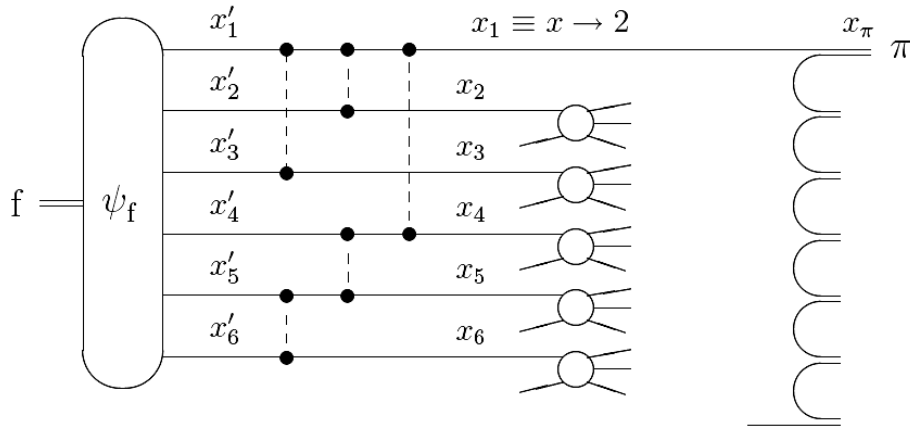
At $x_1 \rightarrow f \Rightarrow$ all $x_2, \dots, x_n \rightarrow 0 \Rightarrow$ all $|q_i| \gg m \Rightarrow$
pQCD works \Rightarrow min.number of hard exchanges.
Simple instantaneous Coulomb part dominates
in light-cone gauge.



$$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} n \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} X_n \end{array} = \sum_{k=1}^{n-1} C_{n-2}^{k-1} \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} k \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} X_k \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} n-k \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} X_{n-k} \end{array}$$

- the recurrence relation

Coherent Quark Coalescence and Production of Cumulative Protons

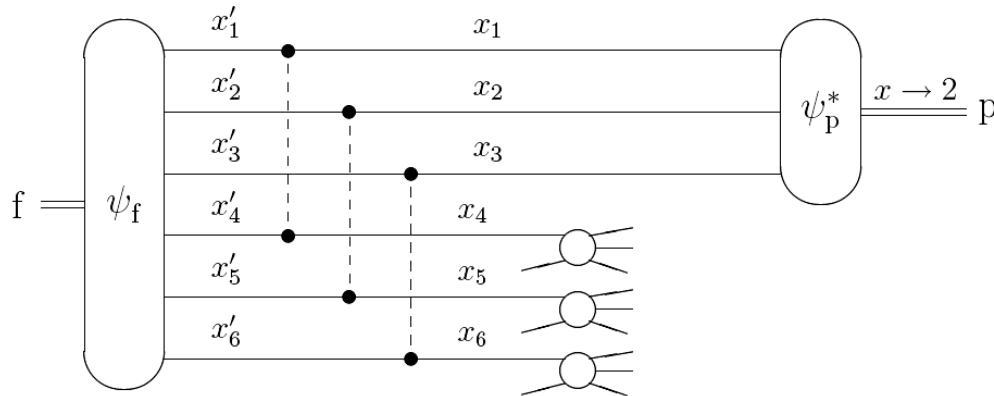


- the cumulative pion production by hadronization of one fast quark

M.A. Braun, V.V. Vechernin, Nucl.Phys.B 427, 614 (1994); Phys.Atom.Nucl. 60, 432 (1997); 63, 1831 (2000)

- the cumulative proton production by **coherent** quark coalescence mechanism:

M.A. Braun, V.V. Vechernin, Nucl.Phys.B 92, 156 (2001); Theor.Math.Phys 139, 766 (2004); V.Vechernin, AIP Conf.Proc.1701 (2016) 060020.



The last **recalls** the few nucleon **short-range correlations** in a nucleus

L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).

But instead of using the relativistic generalization of non-relativistic NN wave function

the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge

Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. B369 (1992) 519.

was developed and applied.



$$W_j: j=1,2,3.$$

$$n=n_1+n_2+n_3$$

$$p_1=n_1-1$$

$$p_2=n_2-1$$

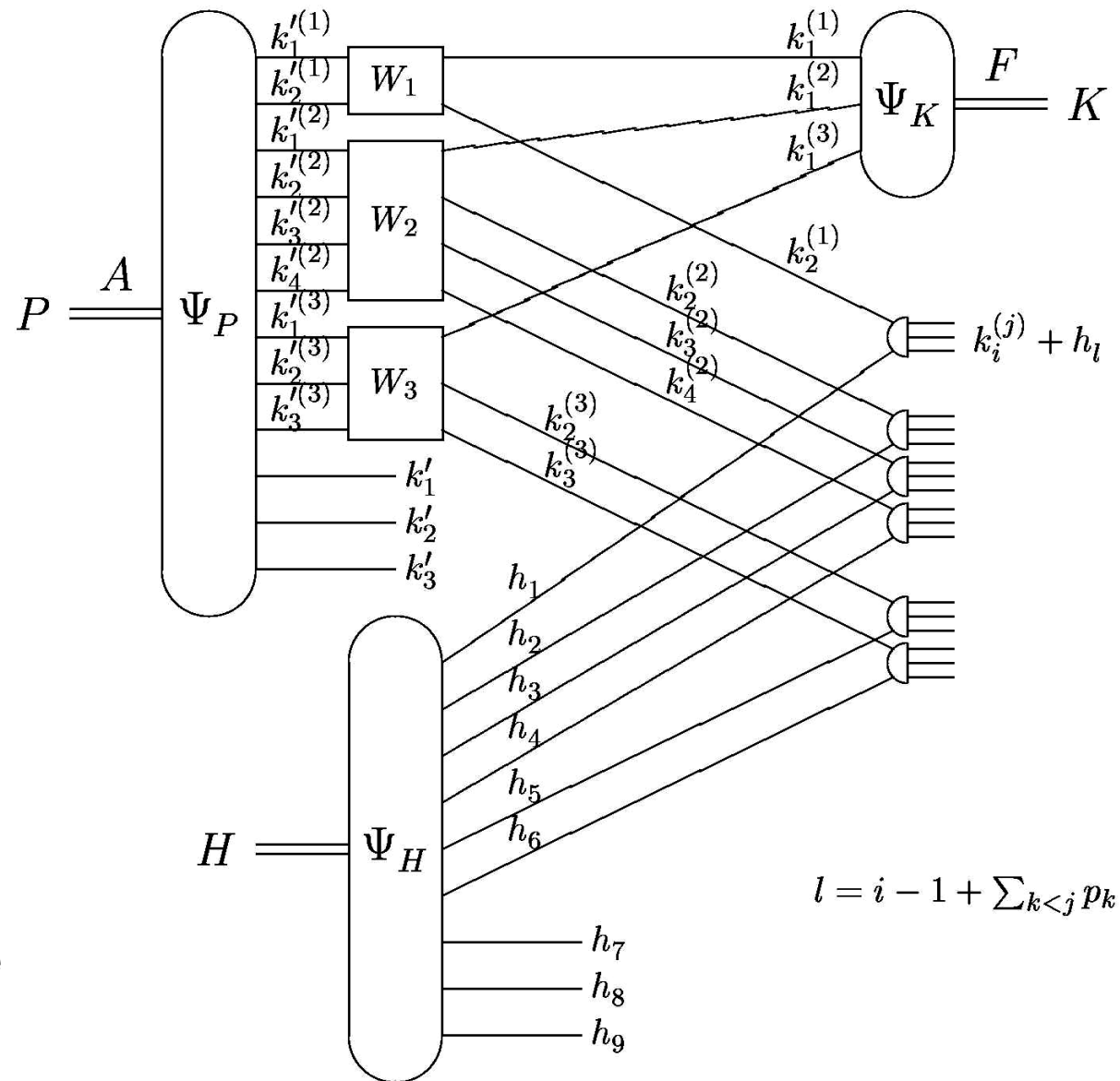
$$p_3=n_3-1$$

$$p=p_1+p_2+p_3=1+3+2=6$$

$$n=p+3=9$$

The analysis of the diagrams shows that all donor quarks:

p_1, p_2, p_3 have large virtuality and must to interact with the projectile H .



$$\sigma_{pion}(x, k_{\perp}; p) = C(p) (x_{frag} - x)^{2p-1} f_p \left(\frac{k_{\perp}}{m} \right)$$

$x < x_{frag}(p) = 1/3 + p/3$ Quark counting rules near the cumulative thresholds $p=n-1$

M.A. Braun, V.V. Vechernin, Phys.Atom.Nucl. 63, 1831 (2000)

$$\sigma_{prot}(x, k_{\perp}; p_1, p_2, p_3) = C(p_1, p_2, p_3) (x_{coal} - x)^{2p-1} f_{p_1} \left(\frac{k_{\perp}}{3m} \right) f_{p_2} \left(\frac{k_{\perp}}{3m} \right) f_{p_3} \left(\frac{k_{\perp}}{3m} \right)$$

$x < x_{coal}(p) = 1 + p/3$, $p = p_1 + p_2 + p_3$

M.A. Braun, V.V. Vechernin, Theor.Math.Phys. 139, 766 (2004)

$p=n-3$

$$f_p(t) = 2\pi \int_0^{\infty} dz z J_0(tz) [z K_1(z)]^p$$

$J_0(z)$ - the Bessel function, $K_1(z)$ - the modified Bessel function.

$$(2\pi)^{-2} \int f_p(|\mathbf{b}|) d^2\mathbf{b} = (2\pi)^{-1} \int_0^{\infty} f_p(t) t dt = 1$$

$$e^{-b_s x} = 10^2,$$

$$b_s \approx 7, \quad x = -2/3$$

Note that for $p=1$ it can be simplified to $f_1(t) = 4\pi/(t^2 + 1)^2$

$$\varphi_{pion}(k_{\perp}, p) \equiv \sigma_{pion}(x, k_{\perp}; p) / \sigma_{pion}(x, 0; p) = f_p \left(\frac{k_{\perp}}{m} \right) / f_p(0)$$

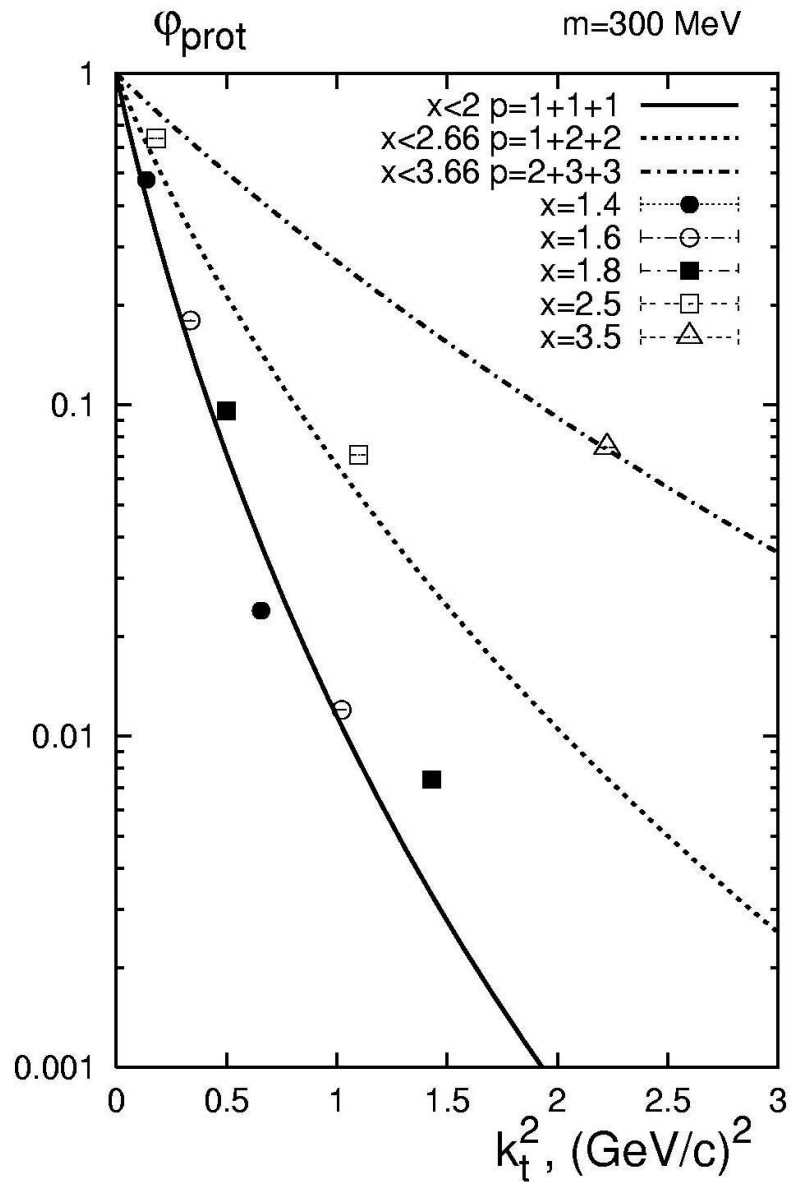
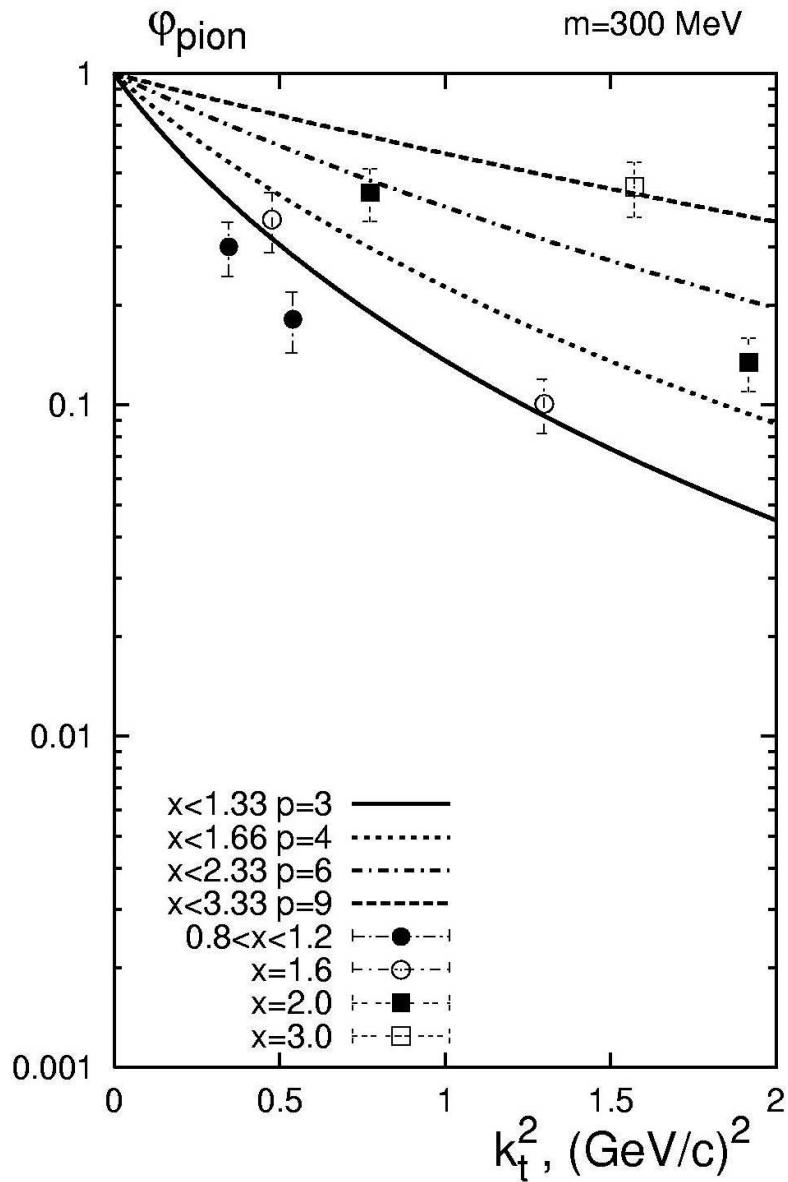
$$\varphi = \sigma(x, \theta) / \sigma(x, \theta=180^{\circ})$$

$$\varphi_{prot}(k_{\perp}, p) \equiv \sigma_{prot}(x, k_{\perp}; p) / \sigma_{prot}(x, 0; p)$$

$$\varphi_{prot}(k_{\perp}, p) = \frac{\sum_{p_1, p_2, p_3} \delta_{p, p_1+p_2+p_3} C(p_1, p_2, p_3) f_{p_1} \left(\frac{k_{\perp}}{3m} \right) f_{p_2} \left(\frac{k_{\perp}}{3m} \right) f_{p_3} \left(\frac{k_{\perp}}{3m} \right)}{\sum_{p_1, p_2, p_3} \delta_{p, p_1+p_2+p_3} C(p_1, p_2, p_3) f_{p_1}(0) f_{p_2}(0) f_{p_3}(0) \dots}$$

$$\varphi_{prot}(k_{\perp}, p_1, p_2, p_3) \equiv \frac{\sigma_{prot}(x, k_{\perp}; p_1, p_2, p_3)}{\sigma_{prot}(x, 0; p_1, p_2, p_3)} = \frac{f_{p_1} \left(\frac{k_{\perp}}{3m} \right) f_{p_2} \left(\frac{k_{\perp}}{3m} \right) f_{p_3} \left(\frac{k_{\perp}}{3m} \right)}{f_{p_1}(0) f_{p_2}(0) f_{p_3}(0)}$$

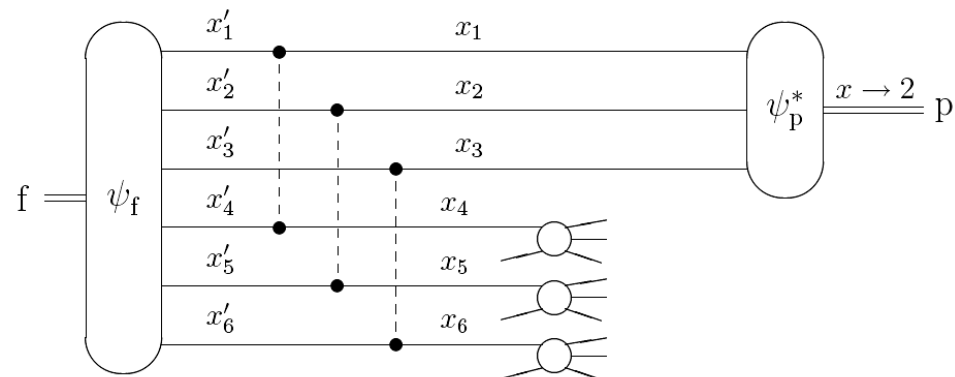
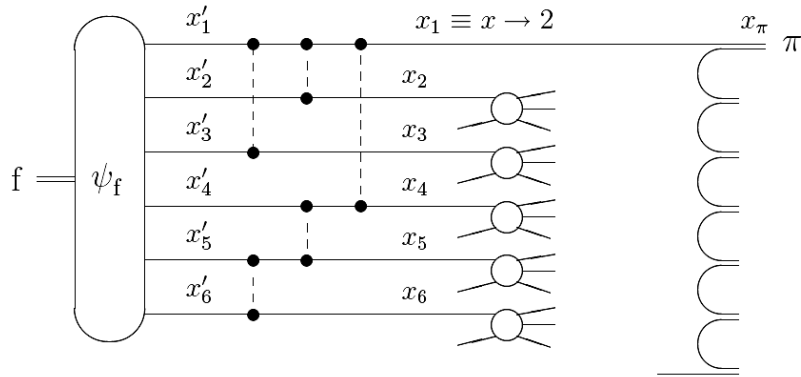
No free parameters (!) only m – the constituent quark mass: $m = 300 \text{ MeV}$.



V.Vechernin,
AIP Conference Proceedings
1701 (2016) 060020.

S.V. Boyarinov et al., *Sov.J.Nucl.Phys.* **46**, 871 (1987)
S.V. Boyarinov et al., *Physics of Atomic Nuclei* **57**, 1379 (1994)
S.V. Boyarinov et al., *Sov.J.Nucl.Phys.* **55**, 917 (1992)

dd collisions



$$f_\pi(x, k_\perp) \equiv \frac{k_0 d^3 \sigma_\pi}{d^3 \mathbf{k}} = C_\pi (2-x)^9 \Phi_5 \left(\frac{k_\perp}{m_q} \right) / \Phi_5(0)$$

$$f_p(x, k_\perp) \equiv \frac{k_0 d^3 \sigma_p}{d^3 \mathbf{k}} = C_p (2-x)^5 \Phi_1^3 \left(\frac{k_\perp}{3m_q} \right) / \Phi_1^3(0)$$

(1)

$$\Phi_p(t) = 2\pi \int_0^\infty dz z J_0(tz) [z K_1(z)]^p$$

(2)

$$\Phi_1(t) = \frac{4\pi}{(t^2 + 1)^2}$$

$$x \equiv 2x_+$$

$$x_+ = 1$$

- exact kinematic
boundary for dd reaction

$$x_+ \equiv \frac{k_+}{k_+^{max}},$$

$$k_+ \equiv \frac{k_0 + k_z}{\sqrt{2}}.$$

$x = \frac{k_+}{p_+}$ - light cone variable

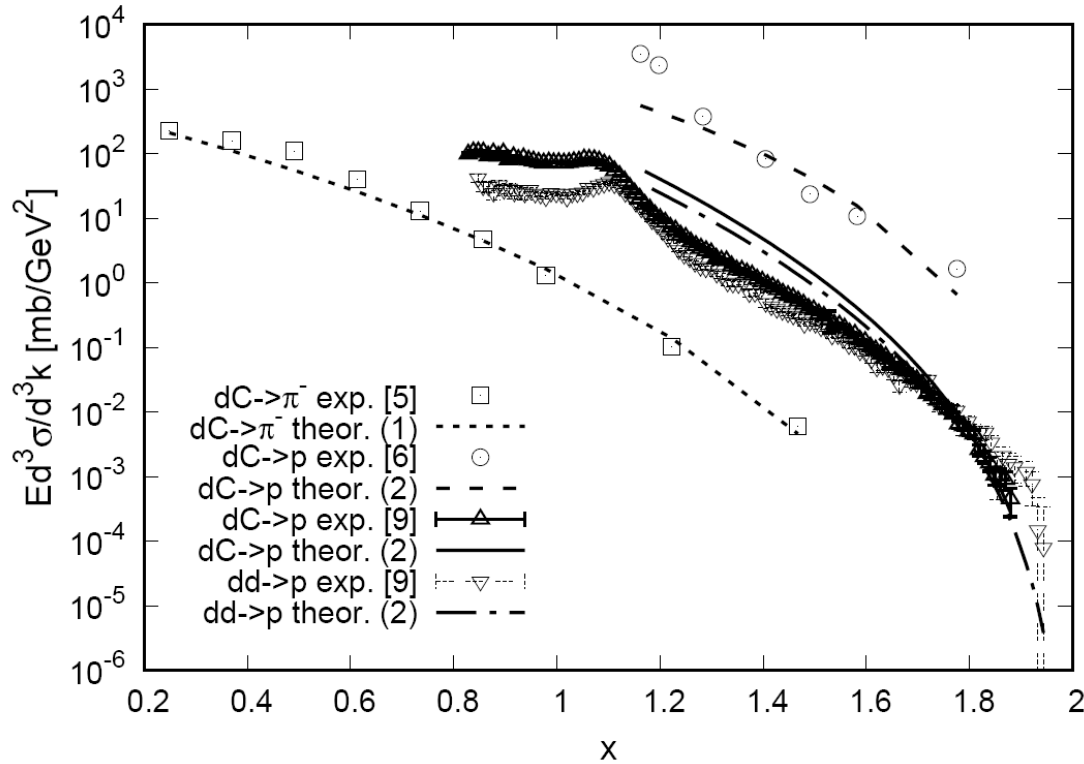
$x_F = \frac{k_z}{k_z^{max}}$ - Feynman variable

$M_f^{min} = X m_N$ - cumulative number

$x \approx x_F \approx X$ at $s \rightarrow \infty$

$$\frac{m_N^2}{E^{*2}} = \frac{4m_N^2}{s}$$

Fixation of normalization constants



$$C_{\pi}^{dC} = 1.4 \text{ mb/GeV}^2 ,$$

$$C_p^{dC} = 1500 \text{ mb/GeV}^2$$

[9] L.S. Azhgirei et al., *Sov. J. Nucl. Phys.* **46**, 661 (1987)

$$p_{lab}^d = 9.0 \text{ GeV} \quad (\sqrt{s_{NN}} = 3.2 \text{ GeV}) \quad 0.139 \text{ rad} = 8^\circ$$

$$d+C \Rightarrow p+X, \quad d+d \Rightarrow p+X$$

[5] J. Papp et al., *Phys. Rev. Lett.* **34**, 601 (1975)

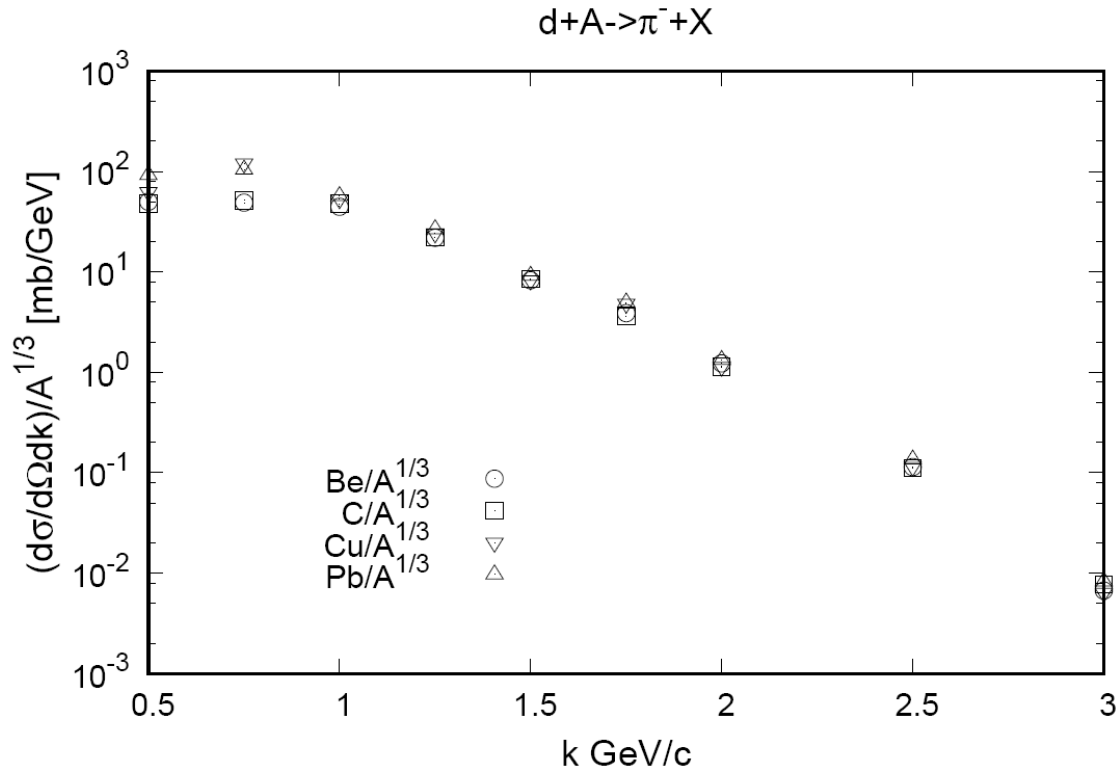
[6] J. Papp, Ph. D. thesis, Univ. of California, Berkeley, Report No. LBL-3633, 1975

[I.A. Schmidt and R. Blankenbecler, *Phys. Rev. D* **15**, 3321-1326 (1977)]

$$E_{lab}^{kin} = 2.1 \text{ GeV} \quad \sqrt{s_{NN}} = 2.7 \text{ GeV} \quad 2.5^\circ$$

$$d+C \Rightarrow \pi^- + X, \quad d+C \Rightarrow p+X$$

A-dependence of the deuteron fragmentation



$$[5] \sim A^{1/3} \Rightarrow$$

$$C_{\pi} = C_{\pi}^{dd} = 0.77 \text{ mb/GeV}^2$$

$$C_p = C_p^{dd} = 825 \text{ mb/GeV}^2$$

Estimation of pion and proton multiplicities in the new cumulative region of central rapidities and large transverse momenta in dd collisions at SPD

$$\langle n \rangle_{\text{dd}}^{\Omega} \cdot \sigma_{\text{dd}}^{\text{tot}} = \int_{\Omega} \frac{d^3 \mathbf{k}}{k_0} f(x, k_{\perp}) = \int_{\Omega} \frac{dk_z^*}{k_0^*} d^2 \mathbf{k}_{\perp} f(x, k_{\perp})$$

$$\langle n \rangle_{\text{dd}}^{\Omega} \cdot \sigma_{\text{dd}}^{\text{tot}} = 2\pi \int_{\Omega} dy dk_{\perp} k_{\perp} f(x(y, k_{\perp}), k_{\perp}) \quad y \equiv \frac{1}{2} \ln \frac{k_0^* + k_z^*}{k_0^* - k_z^*},$$

$$k_z^* = \mu_{\perp} \sinh y, \quad k_0^* = \mu_{\perp} \cosh y, \quad \mu_{\perp} \equiv \sqrt{k_{\perp}^2 + \mu^2}.$$

$$0.5 < |y| < 1$$

$$1 < x < 2$$

$$(1.2 < x < 2; 1.5 < x < 2)$$

$$\langle n \rangle_{\text{dd}} \cdot \sigma_{\text{dd}}^{\text{tot}} = 4\pi \int_{0.5}^1 dy \int_{k_{\perp}^{\text{min}}(y)}^{k_{\perp}^{\text{max}}(y)} dk_{\perp} k_{\perp} f(x(y, k_{\perp}), k_{\perp}).$$

$$k_{\perp}^{\text{min}}(y) = k_{\perp}(y, x = 1), \quad k_{\perp}^{\text{max}}(y) = k_{\perp}(y, x = 2).$$

$\sqrt{s_{NN}}$	4 GeV			8 GeV		
	y	k_{\perp}^{min}	k_{\perp}^{max}	y	k_{\perp}^{min}	k_{\perp}^{max}
dd \rightarrow π	0.5	1.728	2.752	0.5	4.197	6.672
dd \rightarrow π	1.0	1.102	2.002	1.0	2.687	4.86
dd \rightarrow p	0.5	1.741	2.999	0.5	4.218	6.803
dd \rightarrow p	1.0	0.852	2.089	1.0	2.605	4.915

$$\sigma_{dd}^{tot} = 120 \text{ mb.}$$

	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$\langle n_{\pi^-} \rangle_{dd}$	$x > 1.0$	$9 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$
	$x > 1.2$	$6.6 \cdot 10^{-5}$	$1.2 \cdot 10^{-5}$
	$x > 1.5$	$3.6 \cdot 10^{-7}$	$5.8 \cdot 10^{-8}$
$\langle n_p \rangle_{dd}$	$x > 1.0$	$2.3 \cdot 10^{-2}$	$9 \cdot 10^{-6}$
	$x > 1.2$	$1.2 \cdot 10^{-3}$	$4.6 \cdot 10^{-7}$
	$x > 1.5$	$1.04 \cdot 10^{-5}$	$4.2 \cdot 10^{-9}$

**Estimation of pion and proton yields in the new cumulative region
of central rapidities and large transverse momenta in dd collisions at SPD
for $t = 1$ hour**

$$Y_{dd} = 0.1 \cdot L_{dd} \cdot \sigma_{dd}^{tot} \cdot \langle n \rangle_{dd} \cdot t$$

$$L_{dd} = 10^{30} \text{ cm}^{-2} \text{ c}^{-1} \quad \text{at 8 GeV and 100 times lower at 4 GeV}$$

V.M. Abazov, et al. [The SPD collaboration], "Conceptual design of the Spin Physics Detector ArXiv:2102.00442v3 [hep-ex], 2022.

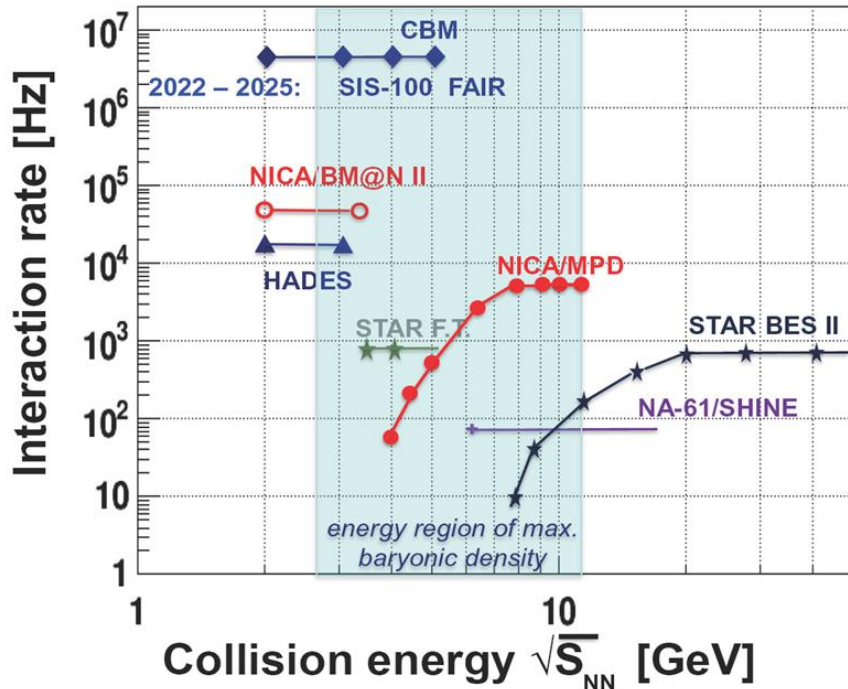
	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$Y_{dd} \rightarrow \pi^-$	$x > 1$	400	8 000
	$x > 1.2$	30	500
	$x > 1.5$	0.16	2.5
$Y_{dd} \rightarrow p$	$x > 1$	10 000	400
	$x > 1.2$	500	20
	$x > 1.5$	4.5	0.18

Comparison of Interaction Rates in AuAu (BiBi) collisions at MPD and in dd collisions at SPD

$$\text{MPD: } L_{AuAu} = 10^{27} \text{ cm}^{-2} \text{ c}^{-1}$$

$$\sigma_{AuAu}^{tot} \cong 7000 \text{ mb}$$

Present and future HI experiments



$$I_{AuAu} = L_{AuAu} \sigma_{AuAu}^{tot} = 7 \text{ KHz}$$

V. Kekelidze, A. Kovalenko, R. Lednicky, V. Matveev, I. Meshkov, A. Sorin, G. Trubnikov, Feasibility study of heavy-ion collision physics at NICA, Nuclear Physics A 967 (2017) 884–887.

$$\sigma_{dd}^{tot} \cong 120 \text{ mb}$$

$$\text{SPD: } L_{dd} = 10^{30} \text{ cm}^{-2} \text{ c}^{-1}$$

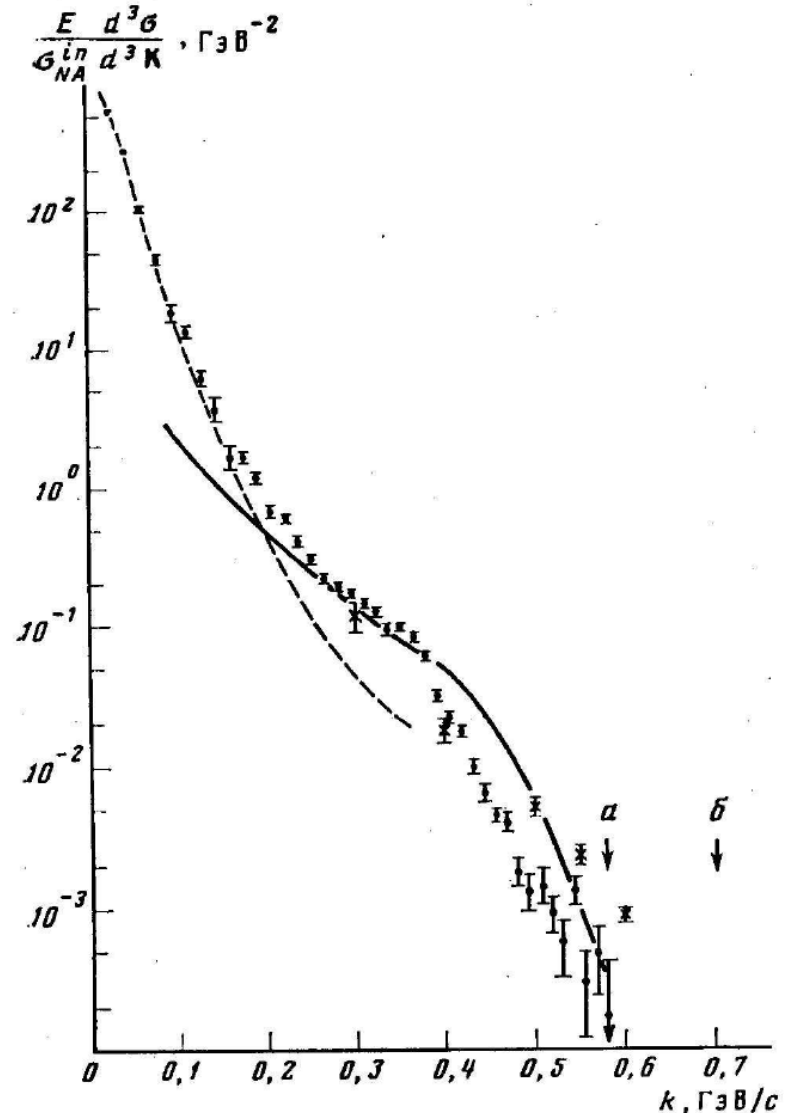
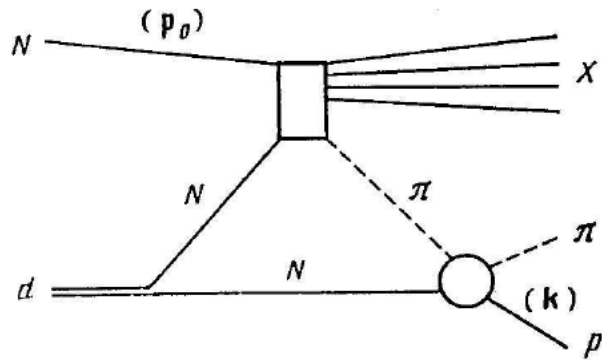
$$I_{dd} = L_{dd} \sigma_{dd}^{tot} = 120 \text{ KHz}$$

V.M. Abazov, et al. [The SPD collaboration], "Conceptual design of the Spin Physics Detector ArXiv:2102.00442v3 [hep-ex], 2022.

Conclusions (part I)

- We have made **estimates of pion and proton production in a new cumulative region of central rapidities and large transverse momenta in dd collisions** using the theoretical results for the transverse momentum dependence of cumulative particle with given x , obtained in the microscopic (at the quark level) **model of the nucleon – flucton interaction**.
- It is shown that **the observation** of particle yields in this new cumulative region is **accessible for study in dd collisions at the SPD**, due to high luminosity available at NICA SPD, which is important for registering rare cumulative processes.
- The multiplicities of cumulative particles **drop with increase of initial energy** due to general increase of transverse momenta. That in region $\sqrt{s_{NN}}$ **from 4 and 8 GeV** can be **partially compensated by the increase of luminosity**. In this new cumulative region studies are not possible for colliders with large initial energy.
- It is shown also that in this new cumulative region the **yields of pions in comparison with the yields of protons are not suppressed so strongly** as in the nuclear fragmentation region, what can be explained by the different mechanisms of the formation of these cumulative particles. (However, it should be noted that the possible contribution of rescattering processes at large distances to cumulative protons has not yet been taken into account.)

Contribution of pion rescattering to cumulative proton production from deuteron (long distance contribution !)



Prediction:

Braun M.A., Vechernin V.V., Yad.Fiz. 28 (1978) 1466.

Experiment:

Ableev V.G. et al., Nucl.Phys.A 393 (1983) 491.

Preprint JINR EI-82-377, Dubna, 1982.

Confirmation:

Braun M.A., Vechernin V.V., Yad.Fiz. 40 (1984) 1588.

Braun M.A., Vechernin V.V., Yad.Fiz. 43 (1986) 1579.

The shoulder in the spectrum is due to the contribution of the Δ -resonance to elastic πN scattering amplitude

Remarks and todo

- The **advantage of studying fluctons in dd collisions at SPD** is that when the deuteron is in the flucton state (6-quark bag), there is **no admixture of other single nucleons left**, in contrast to the situation with collisions of heavier nuclei. This reduces the background and makes it possible to register, in addition to the cumulative particle itself, **also particles formed from fragmentation of the flucton residue**.

V.V. Vechernin, *Phys. Part. Nuclei* 54, 528-535 (2023).

- In this new cumulative region it is possible to study a new process of **flucton-flucton interaction**, that is impossible in nucleus fragmentation region.

I. Alekseev et al., *Phys. At. Nucl.* 71, 1848-1859 (2008); O. Denisovskaya et al., "Dense cold nuclear matter study with cumulative trigger," arXiv:0911.1658, 2009; [25] A. Stavinskiy, "Dense cold matter with cumulative trigger," *Phys. Part. Nucl. Lett.* 8, 912 (2011).

- In case of **dd collisions** the flucton-flucton scattering corresponds to a very clean process, when **both** colliding deuteron at the moment of collision are **simultaneously in a compressed 6-quark configuration**.

- Generalization of this approach to the case of zero rapidities of cumulative particles.

- Development of a microscopic approach based on these principles for the case of flucton-flucton interaction.

Modeling of the dd scattering within the framework of the Glauber approach

Both analytical and MC modeling (Belokurova S.N.)

$$T_A(a_1, \dots, a_A) = \prod_{j=1}^A T_A(a_j). \quad \Rightarrow \quad T_{d_1}(a_1, a_2) = T_{d_1}(a_1)T_{d_1}(a_2)\delta(a_1 - a_2).$$

$$T(a) = \int |\Psi(a, z)|^2 dz \quad \Psi(r) = C(e^{-\gamma r} - e^{-\mu r})/r, \quad C^2 = \frac{\gamma(\gamma + \mu)\mu}{2\pi(\mu - \gamma)^2},$$

$\gamma = 45,8 \text{ M}\text{\AA}\text{B}, \quad \mu = 140 \text{ M}\text{\AA}\text{B}.$

$$\sigma(a) = \exp\left(-\frac{a^2}{r_N^2}\right) \quad \sigma_{NN} \equiv \int db \sigma(b), \quad \sigma_{NN} = \pi r_N^2.$$

$$\langle N_{coll}(\beta) \rangle = 4\chi(\beta) \quad \chi(\beta) \equiv c^{-1} \int \sigma(a - b + \beta) (T_{d_1}(a))^2 da (T_{d_2}(b))^2 db,$$

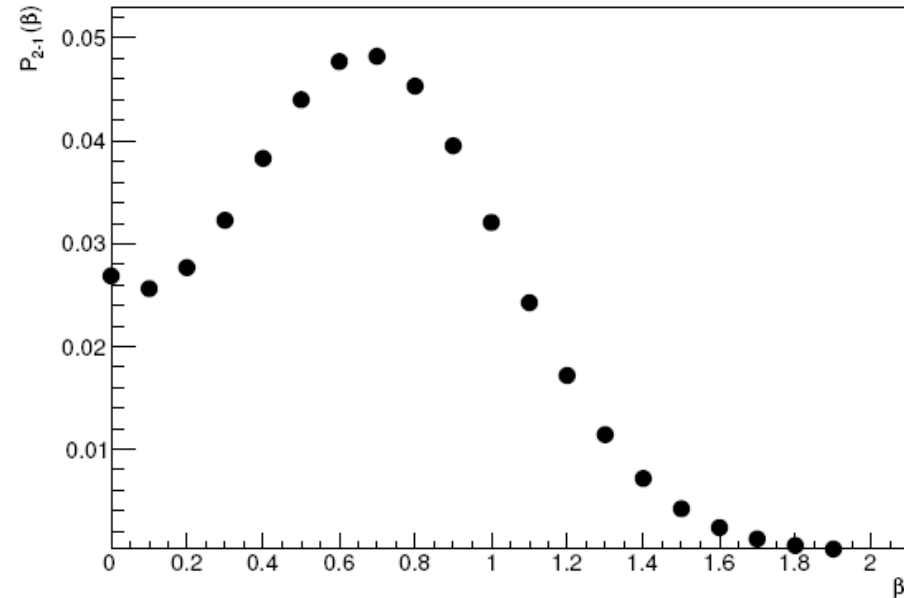
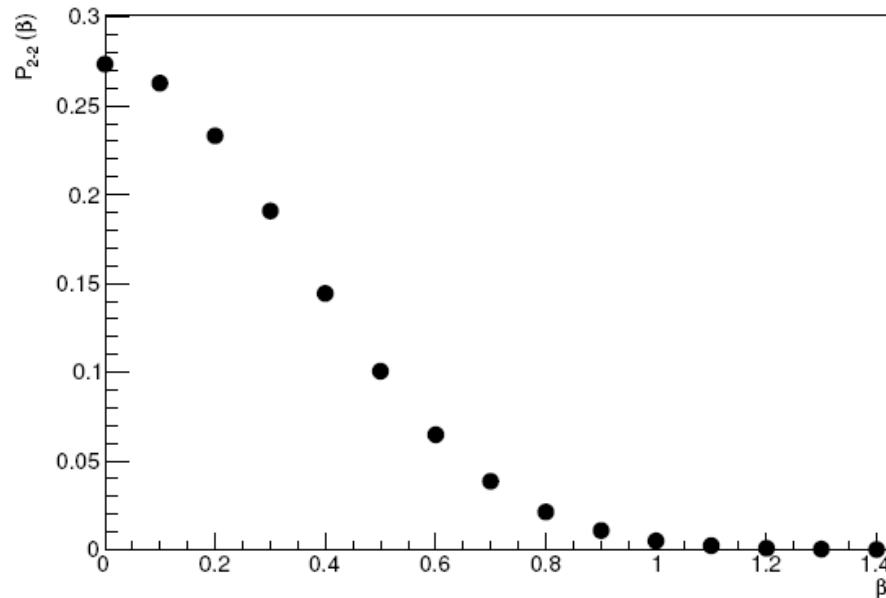
$$V[N_{coll}(\beta)] = \dots$$

$$\langle N_w^{d_1}(\beta) \rangle + \langle N_w^{d_2}(\beta) \rangle = \dots$$

$$V[N_w^{d_1}(\beta) + N_w^{d_2}(\beta)] = \dots$$

$$P_{2-2}(\beta) = c^{-1} \int \sigma(a - b + \beta) \sigma(a + b + \beta) \sigma(-a + b + \beta) \sigma(-a - b + \beta) (T_{d_1}(a))^2 da (T_{d_2}(b))^2 db$$

$$P_{2-1}(\beta) = 2c^{-1} \int \sigma(a - b + \beta) [1 - \sigma(a + b + \beta)] [1 - \sigma(-a + b + \beta)] \sigma(-a - b + \beta) (T_{d_1}(a))^2 da (T_{d_2}(b))^2 db$$



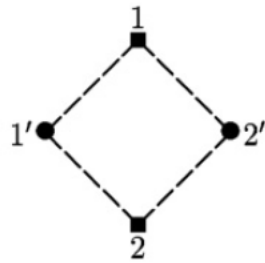
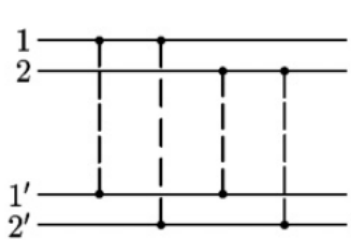
min.bias:

$$P_{2-1}^{min.bias} = 0.067$$

$$P_{2-2}^{min.bias} = 0.0046$$

Variation of the number of participant and NN collisions in AA (dd) interactions

Vechernin, V.V. and Nguyen, H.S. Phys. Rev. C 84 (2011) 054909



$$\chi(\beta) \equiv \int da db T_A(a) T_B(b) \sigma(a - b + \beta)$$

$$\approx \sigma_{NN} \int da T_A(a) T_B(a + \beta)$$

$$\mathcal{P}_{\text{opt}}(N_{\text{coll}}) = C_{AB}^{N_{\text{coll}}} \chi(\beta)^{N_{\text{coll}}} [1 - \chi(\beta)]^{AB - N_{\text{coll}}}$$

$$\sigma_{NN} \equiv \int db \sigma(b),$$

\Rightarrow

$$I(a) \equiv \int db \sigma(b) \sigma(b + a).$$

!!!

So for variation of the number of participant and NN collisions in AA (dd) interactions the general analytical formulas from textbooks (“the optical approximation”):

C.-Y. Wong, Introduction to High-Energy Heavy-Ion Collisions (World Scientific, Singapore, 1994).

R. Vogt, Ultrarelativistic Heavy-Ion Collisions (Elsevier, Amsterdam, 2007).

are not correct and are not supported by MC simulations:

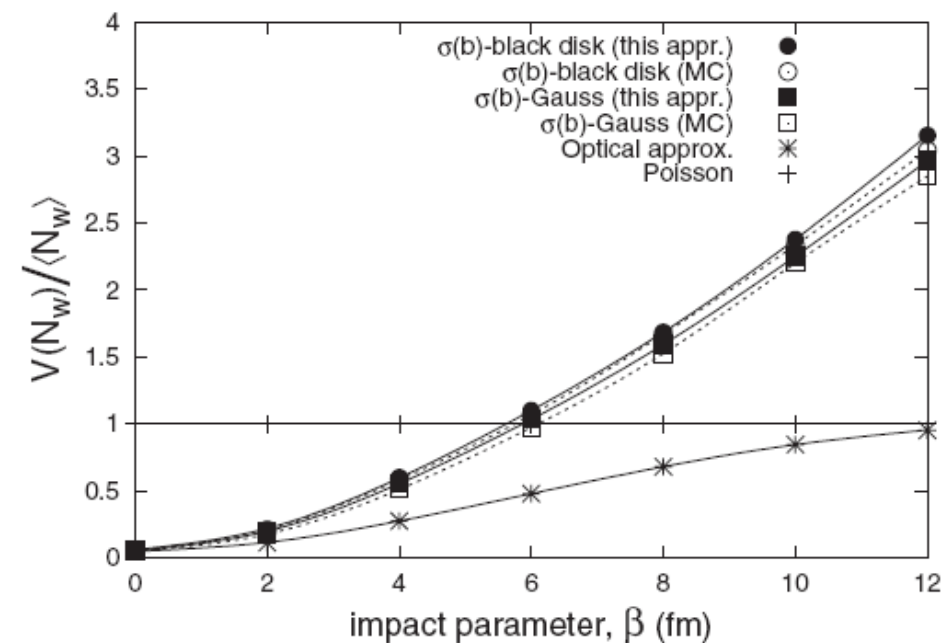
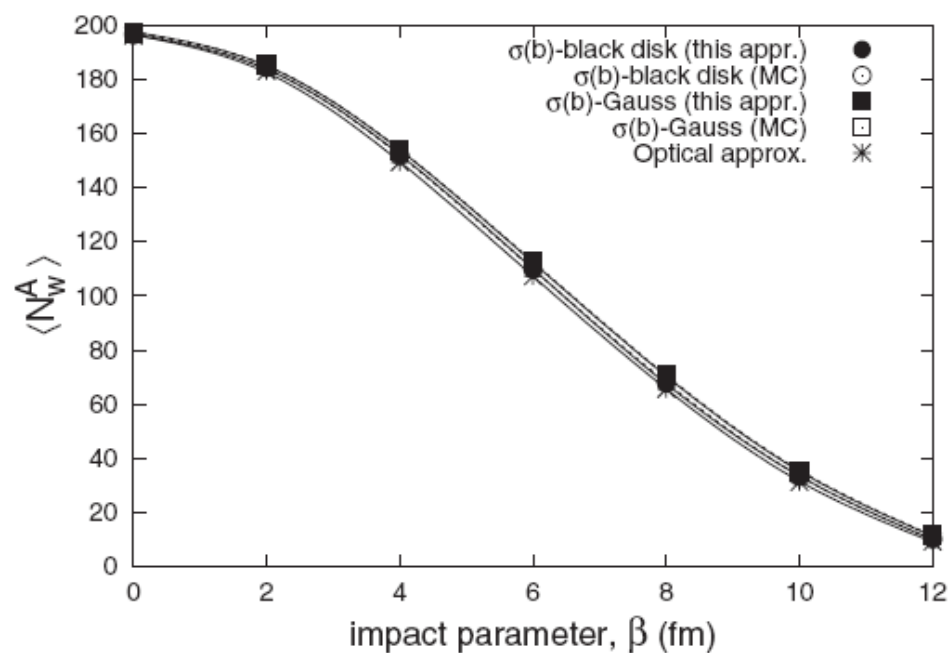
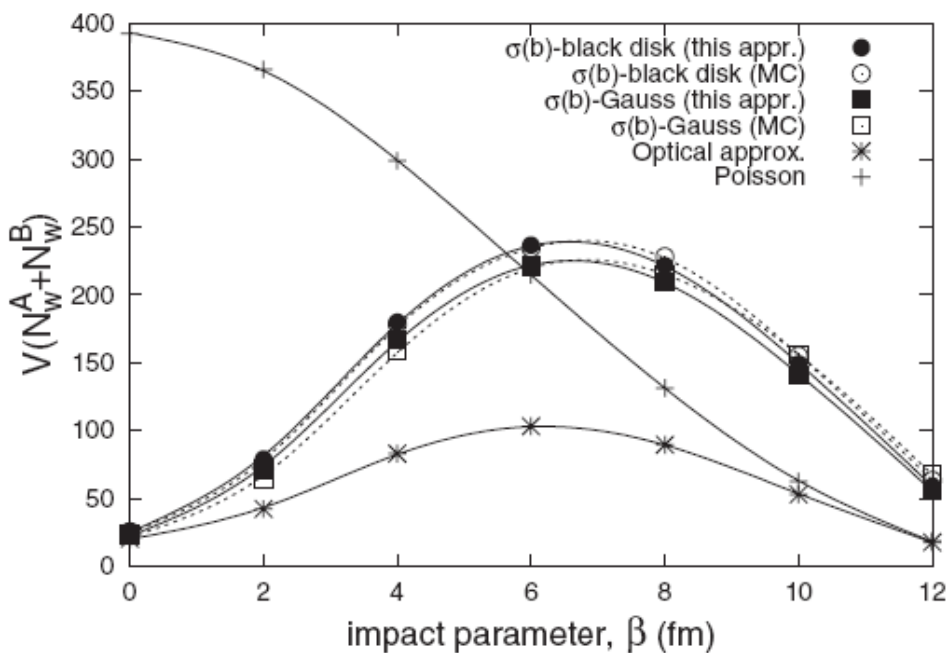


FIG. 1. Variance of the number of wounded nucleons in one nucleus for PbPb collisions at SPS energies ($\sigma_{NN} = 31$ mb) as a function of the impact parameter β (fm). The points \bullet and \blacksquare are results of numerical calculations from the analytical formulas (2)–(4), (11), and (12) using, respectively, the black-disk (14) and Gaussian (15) approximations for NN interactions; \circ and \square are results of independent Monte Carlo (MC) simulations using the black-disk (14) or Gaussian (15) approximation for NN interactions; $*$ is the optical approximation result (8) [the first term in formula (2)]; and $+$ is the Poisson variance $V[N_w^A(\beta)] = \langle N_w^A(\beta) \rangle$. The curves are shown to guide the eyes.

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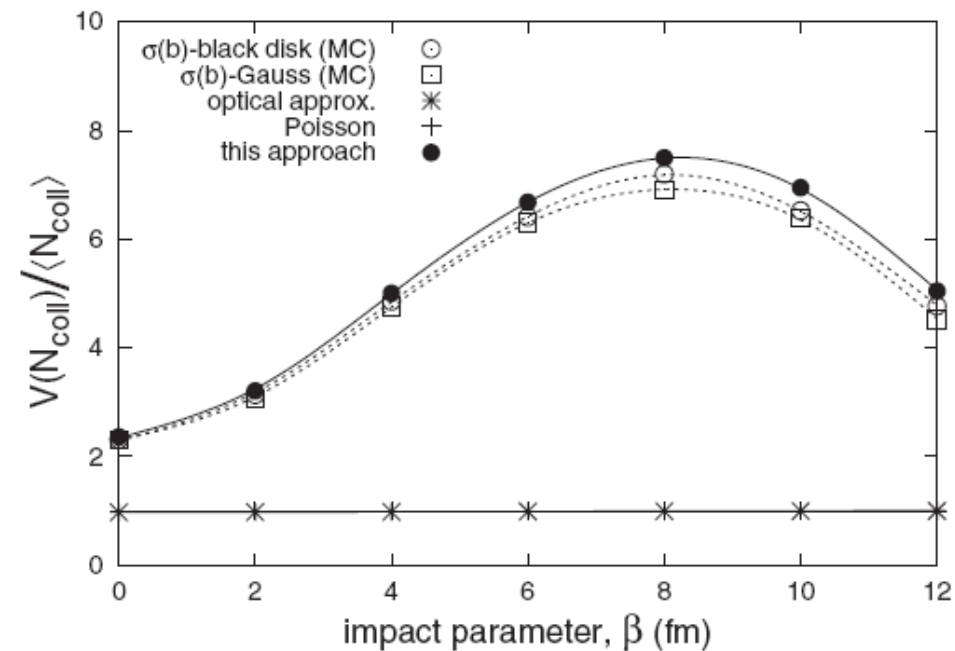
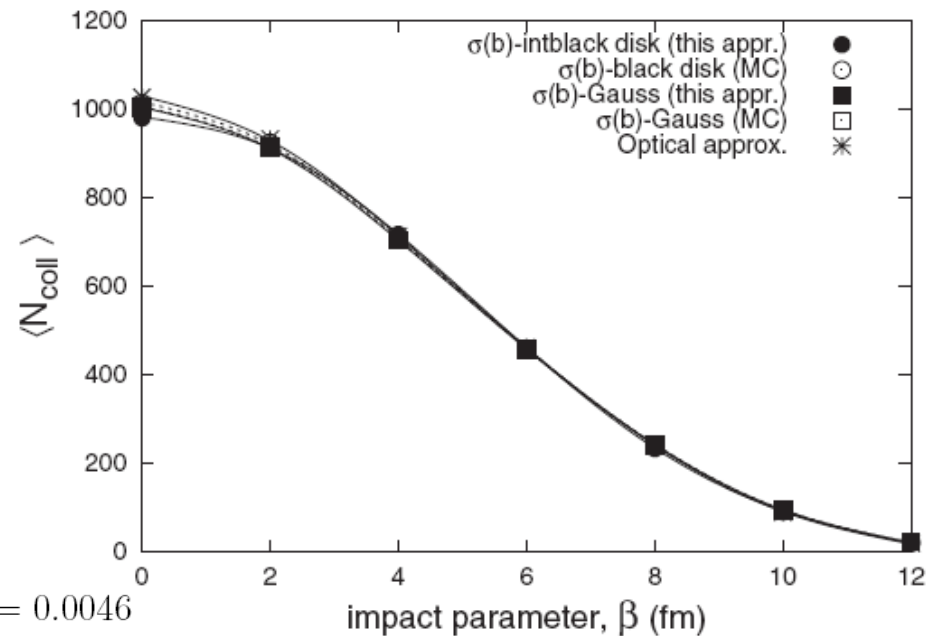
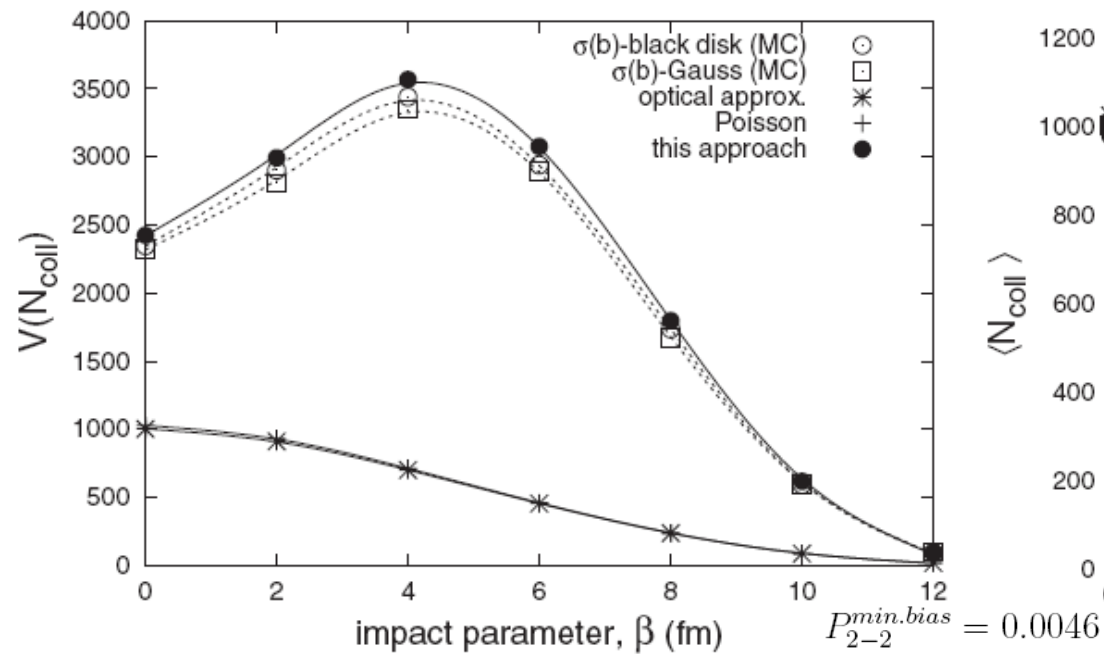


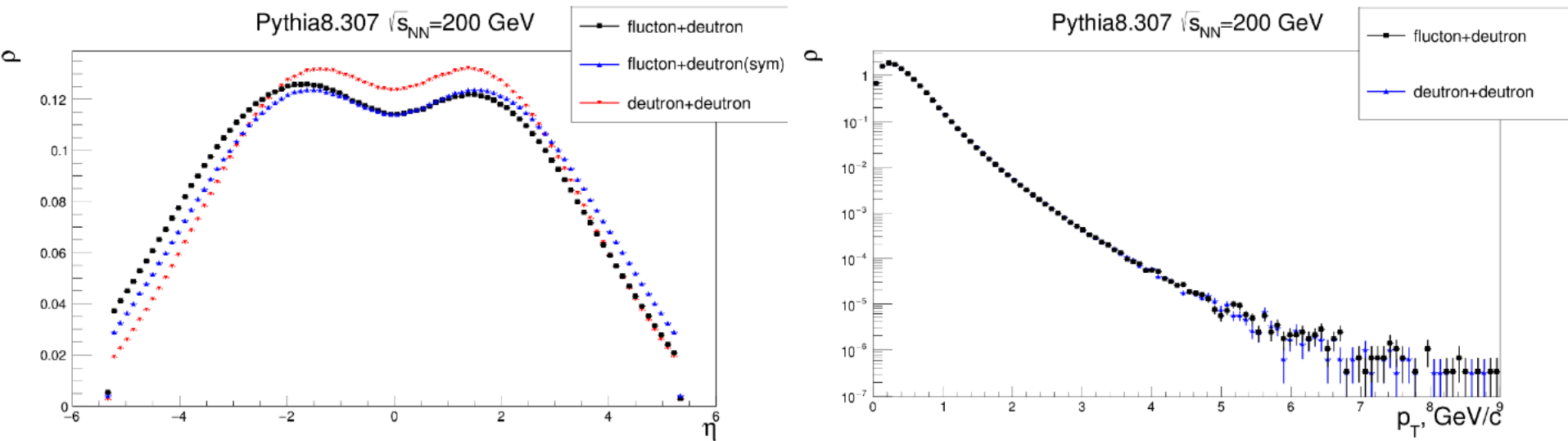
FIG. 8. The variance of the number of NN collisions in PbPb interactions at SPS energies as a function of the impact parameter β (fm). \bullet is the result of calculations from analytical formulas (24)–(27); $*$ is the optical approximation result, calculated from formulas (24) and (30); $+$ is the Poisson variance $V[N_{\text{coll}}(\beta)] = \langle N_{\text{coll}}(\beta) \rangle$. The symbols are the same as in Fig. 1.

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Phys. Rev. C 84 (2011) 054909

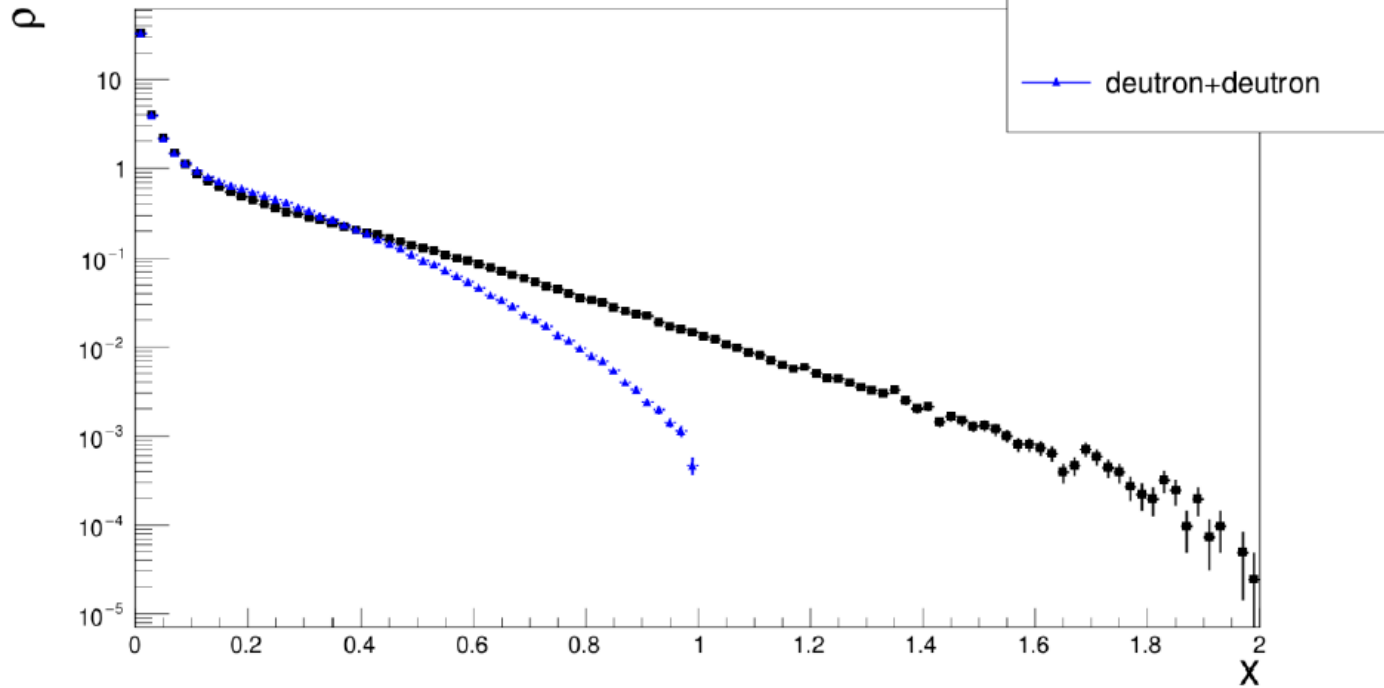
Simulations of particle production in the cumulative region using the PYTHIA8 event generator (Yurchenko S.V.)

The code adding two-nucleon states, imitating the presence of 6-quark fluctons:

```
pythia.particleData.addParticle (1000010010, "2_flucton", 1, 3, 0, 1.876);
```



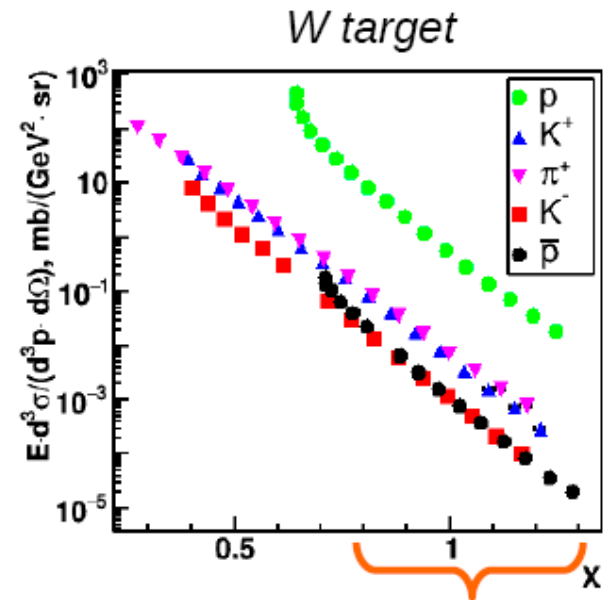
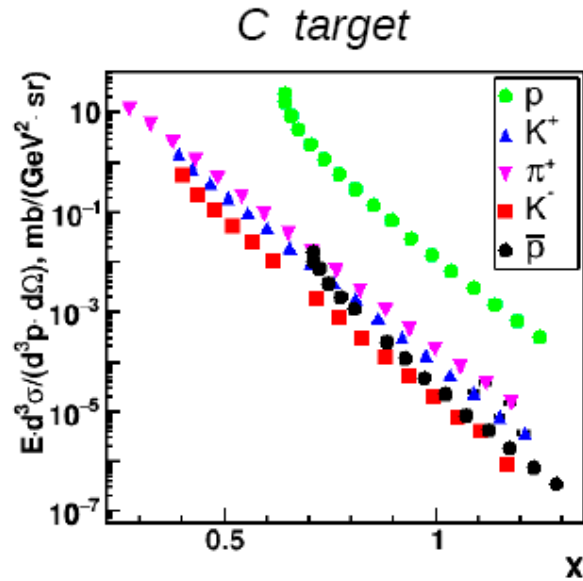
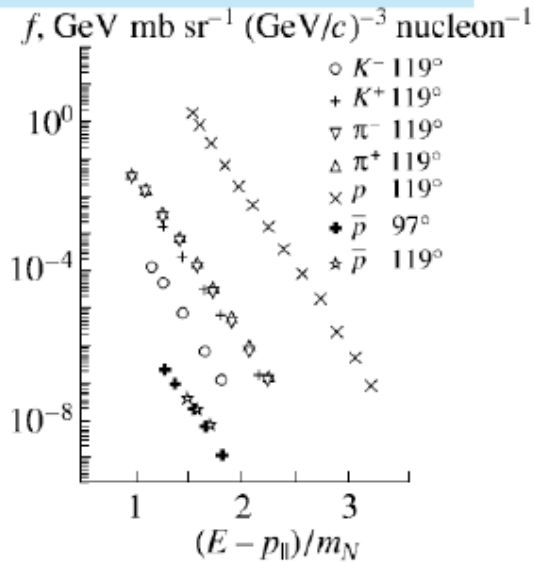
Pythia8.307 $\sqrt{s_{NN}}=200$ GeV



$$x = \frac{m_p^2 - E \cdot E_0 + p_z \cdot P_0}{m_p^2 + E \cdot E_0 + p_z \cdot P_0 - s/2}$$

Backup slides

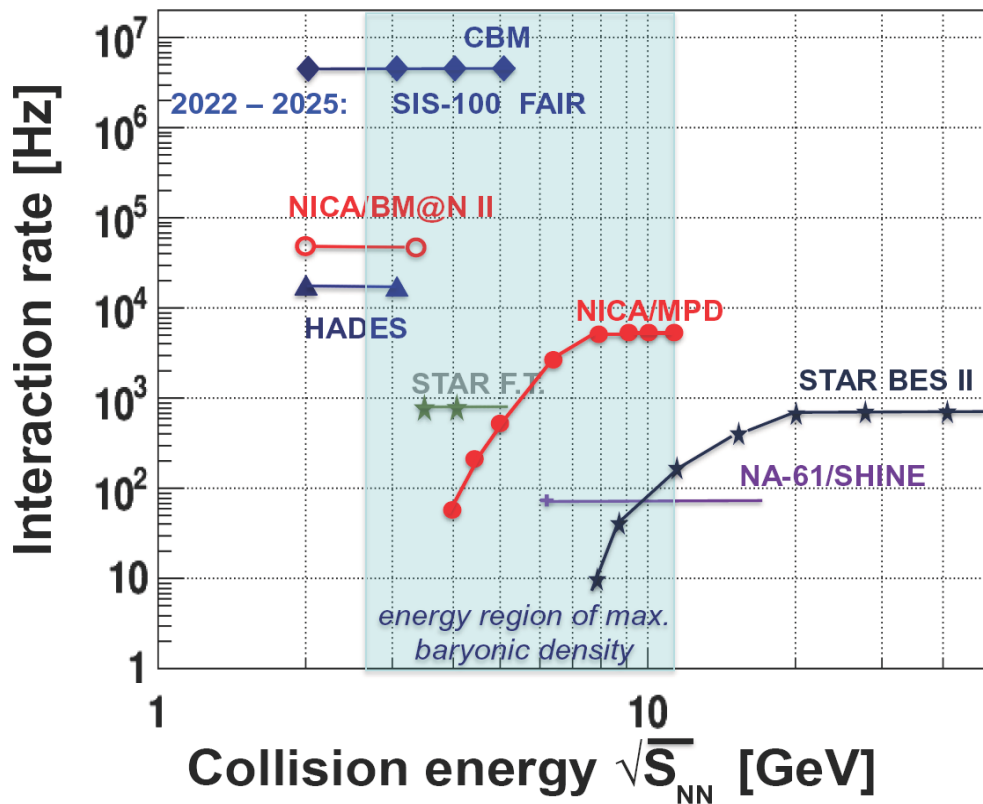
Г.А. Лексин, ЯФ, т. 65, № 11,
2002, стр. 2042 — 2051



N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhanov, V. Viktorov “High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c” (National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino)
LXX International Conference “NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies”, St Petersburg, October 11-17, 2020.

- The lower multiplicity of cumulative particle production at 8 GeV compared to 4 GeV at mid-rapidities can possibly be compensated by a 100 times higher luminosity of the NICA collider at an energy of 8 GeV than at an energy of 4 GeV.

Present and future HI experiments



V. Kekelidze, A. Kovalenko, R. Lednicky, V. Matveev, I. Meshkov, A. Sorin, G. Trubnikov, Feasibility study of heavy-ion collision physics at NICA, Nuclear Physics A 967 (2017) 884–887.

Different cumulative variables

$x = \frac{k_+}{p_+}$ - light cone variable

$x_F = \frac{k_z}{k_z^{max}}$ - Feynman variable

$M_f^{min} = X m_N$ - cumulative number

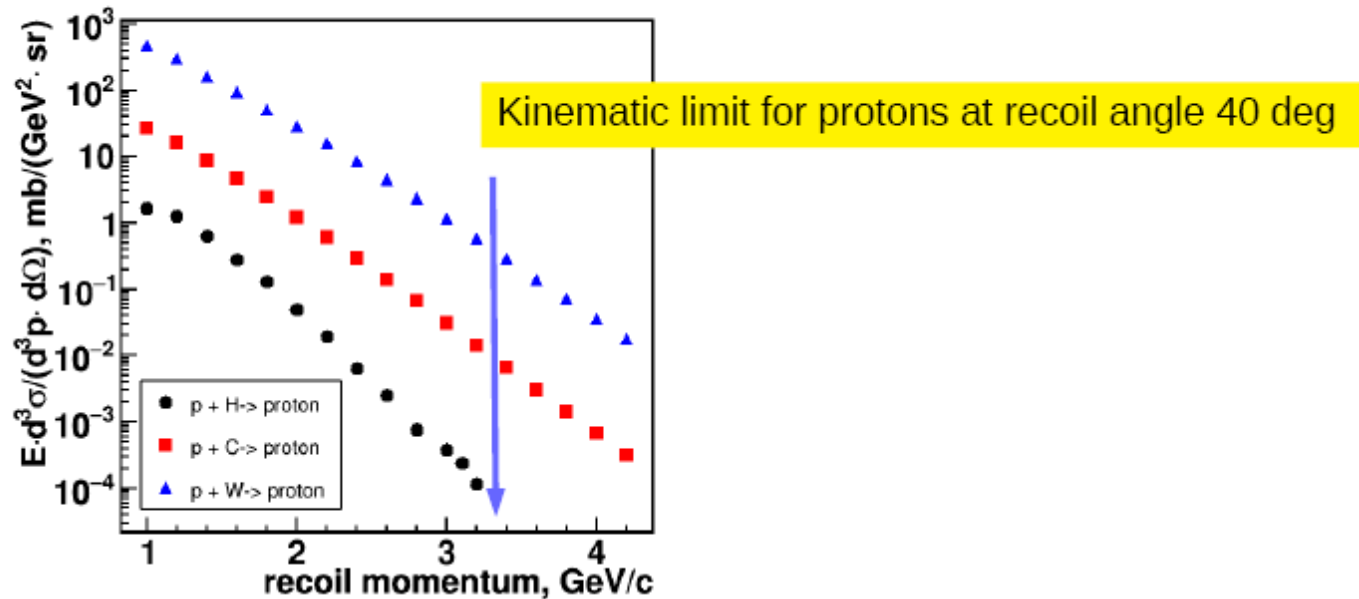
$x \approx x_F \approx X$ at $s \rightarrow \infty$

$$\frac{m_N^2}{E^{*2}} = \frac{4m_N^2}{s}$$

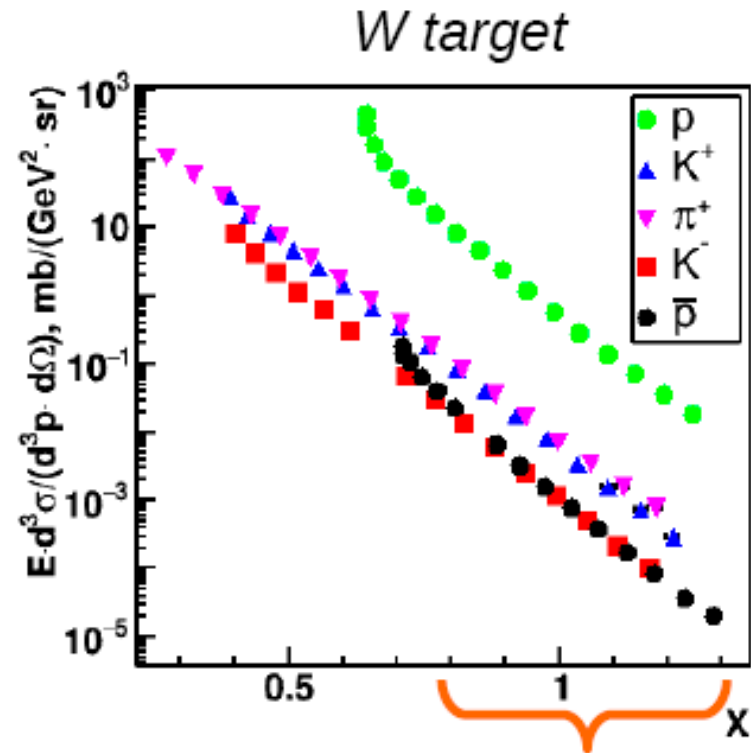
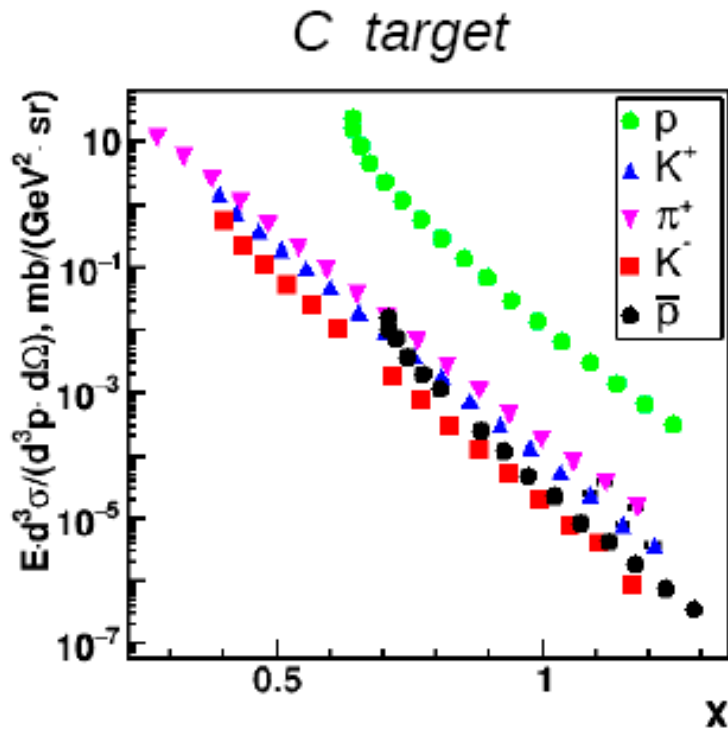
p+Au => Au+Au

Based on the estimates made for the multiplicity of cumulative particles with large transverse momenta at mid rapidities in p+Au collisions, we can obtain estimates for their multiplicity in this region in the Au+Au reaction. When replacing an incident proton with a nucleus, the number of projectile nucleons interacting with the flucton in another nucleus increases, that can be taken into account by introducing the corresponding factor γ . The value of this factor was estimated through the ratio of the number of nucleon-nucleon collisions in p+Au and Au+Au reactions: $\gamma_{coll} = \langle N_{coll} \rangle_{AuAu} / \langle N_{coll} \rangle_{pAu}$, which was chosen equal to 20. The obtained estimates for the multiplicity of cumulative particles with $X > 1.6$ at mid rapidities in Au+Au collisions at the NICA collider energies are given in Table 6. In performing these estimates, we, of course, took into account the symmetric contribution when the flucton in the first nucleus interacts with the nucleon in the second nucleus. Note also that for now, we leave without consideration a new physically interesting contribution arising from flucton-flucton scattering as an object for future research.

Invariant cross-sections of protons knocked out from H, C and W targets.



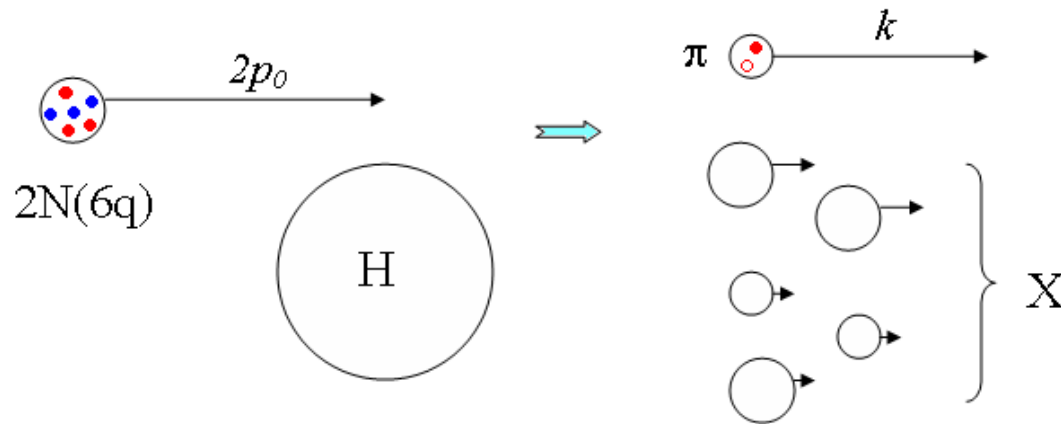
N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhanov, V. Viktorov “High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c” (National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino) LXX International Conference “NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies”, St Petersburg, October 11-17, 2020.



Large fraction of cumulative processes

N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhonov, V. Viktorov “High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c” (National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino)
 LXX International Conference “NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies”, St Petersburg, October 11-17, 2020.

Limiting fragmentation of light nuclei. Quark counting rules.



$1 < x < 2$ - the cumulative region ($1 < x < f$ - for the fN flucton)

Theoretical description near upper threshold: for $2N(6q)$ flucton $k \rightarrow 2p_0$, $x = k/p_0 \rightarrow 2$
 (Limiting fragmentation of a nucleus)

Quark counting rules: $I \sim \Delta^{2p-1}$

Δ – the deviation of x from its maximal value f , $\Delta = f - x$

p – the number of “donors”, stopped quarks, $p = n - 1$

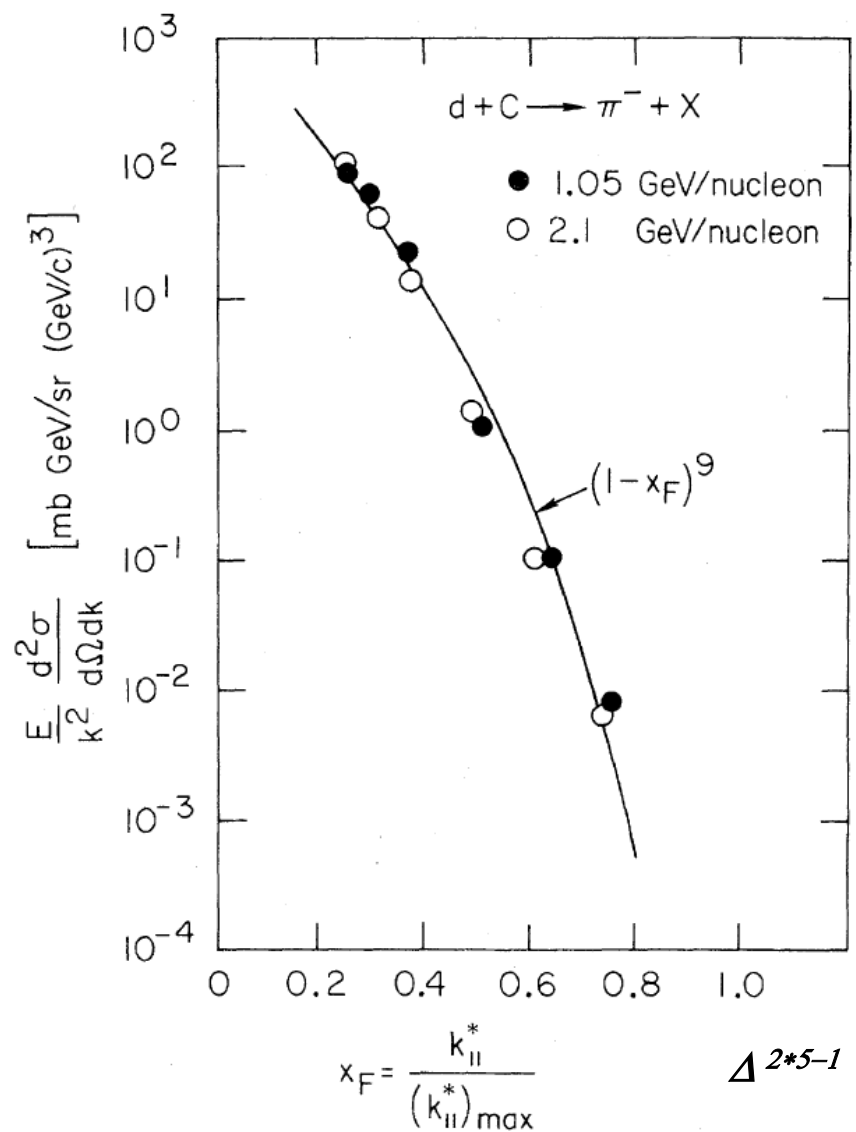
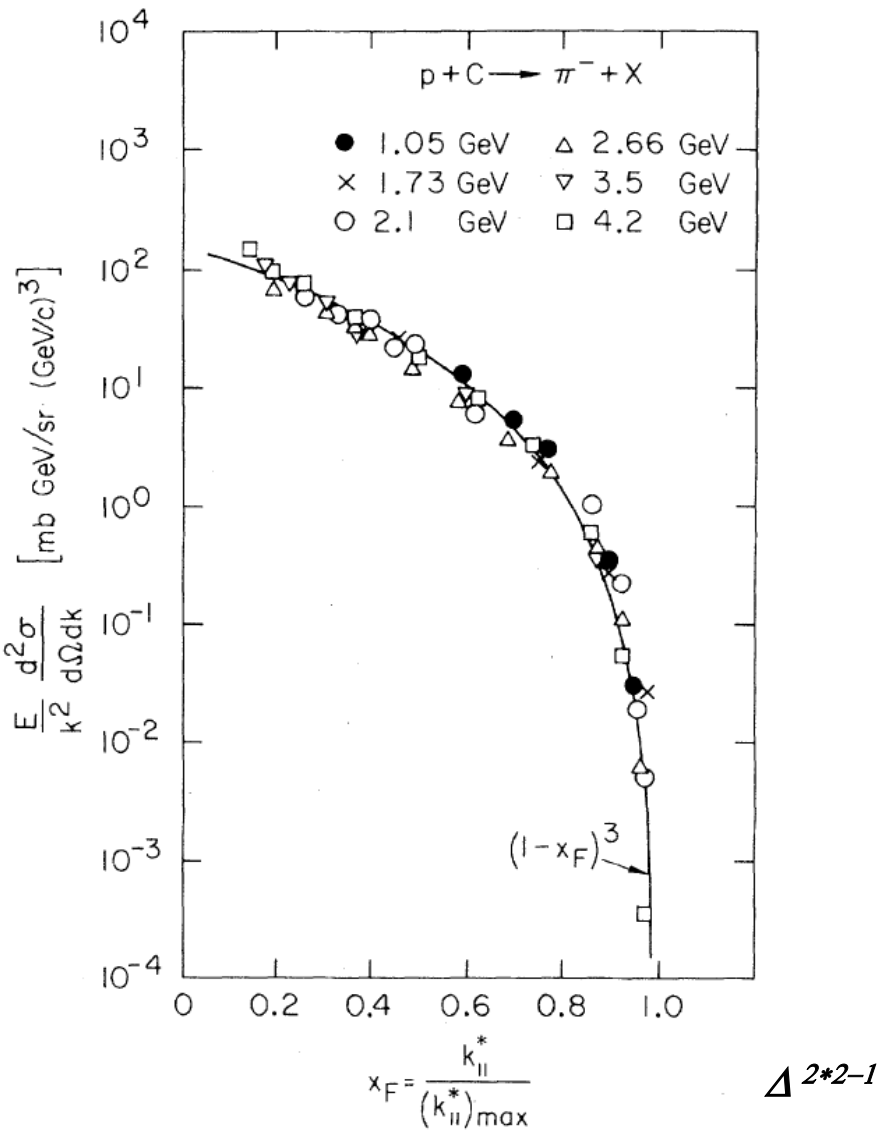
n – the number of constituents.

For $2N(6q)$ flucton $f = 2$, $n = 6$, $p = 5$, then $I \sim (2-x)^{2 \cdot 5 - 1} = (2-x)^9 = \Delta^9$

Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719

Brodsky S.J., Chertok B.T. Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269



The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).

Description of the hadron asymptotics at $x \rightarrow 1$

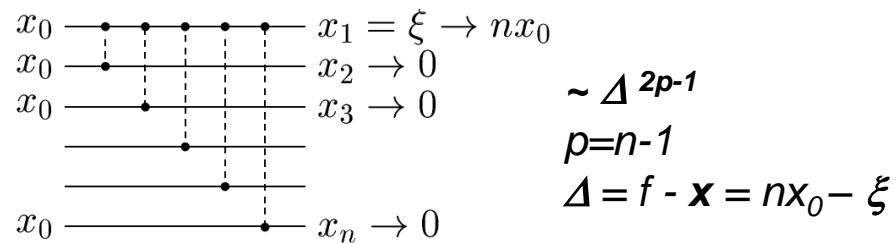
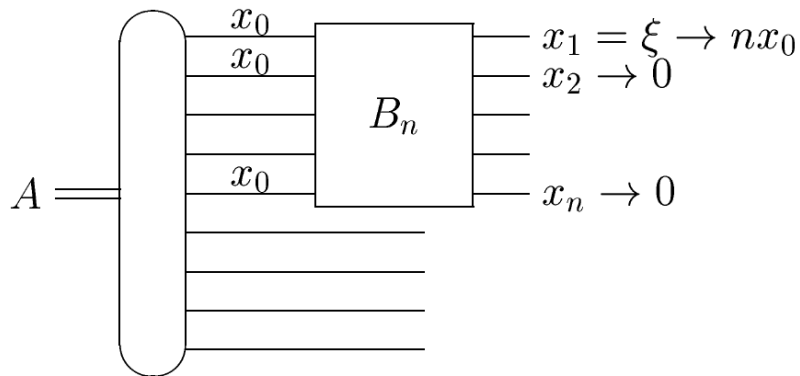
by the intrinsic diagrams of QCD in light-cone gauge
with low- x spectator quarks interact with the target

Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl.Phys. B369 (1992) 519

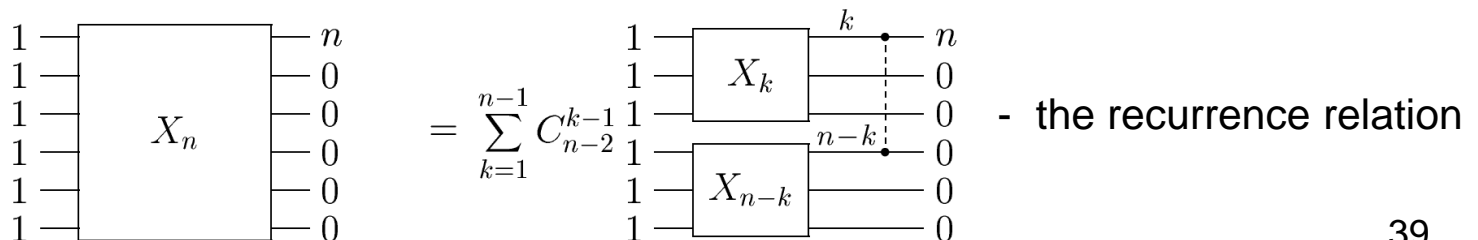
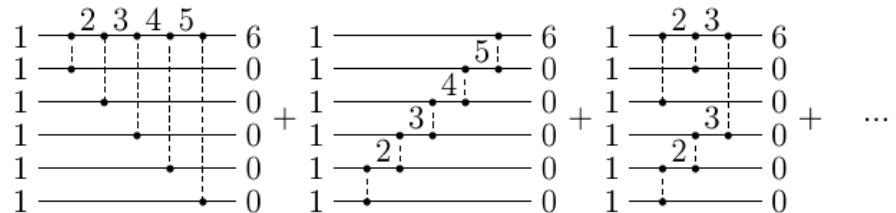
Description of the flucton asymptotic at $x \rightarrow f$,

f - the number of nucleons in flucton, n - the number of quarks in flucton, $x_0 = f/n (=1/3)$.

M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614. (DIS in cumulative region)

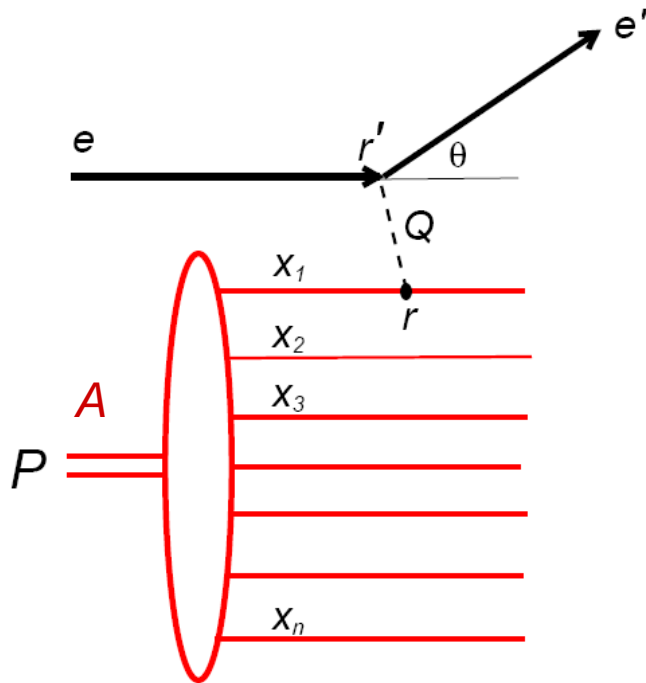


At $x_1 \rightarrow f \Rightarrow$ all $x_2, \dots, x_n \rightarrow 0 \Rightarrow$ all $|q_i| \gg m \Rightarrow$
pQCD works \Rightarrow min.number of hard exchanges.
Simple instantaneous Coulomb part dominates
in light-cone gauge.



Deep Inelastic Scattering (DIS) in cumulative region

Lehman E., *Phys.Lett.62B (1976) 296* – connection of the limiting fragmentation of deuteron into pions with deuteron DIS structure function F_2 (5% $2N(6q)$ -flucton admixture in D)



$$|\mathbf{r} - \mathbf{r}'| \sim 1/|Q|, \quad |Q| \gg m$$

$$\xi \equiv \frac{-Q^2}{2(pQ)} = \frac{-Q^2}{2mQ_0}, \quad p = P/A$$

$$0 < \xi < A, \quad 1 < \xi < A - \text{cumulative region}$$

$$Q^2 = -4EE' \sin^2 \frac{\theta}{2}, \quad Q_0 = E - E'$$

$$x_1 \equiv \frac{k_{1+}}{p_+} \approx \frac{k_{1z}}{p} \geq \xi - \text{Bjorken scaling variable}$$

($x_1 = \xi$ for elastic γq)

Experimental observations of DIS in cumulative region:

Shuetz W.P. et al., *Phys.Rev.Lett.*, 38 (1977) 259 [D]

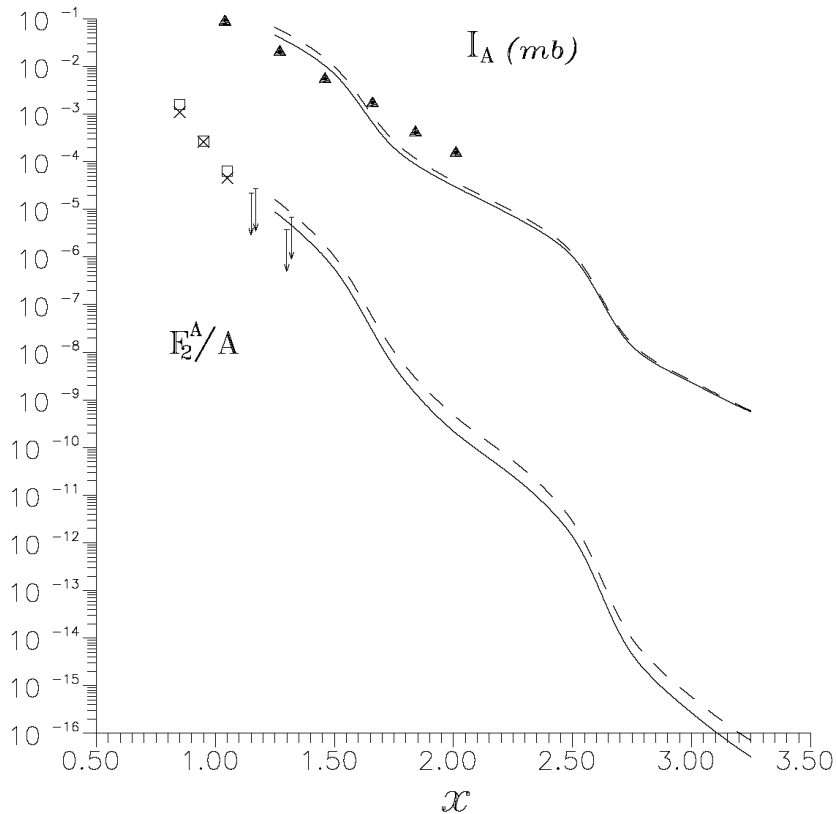
Filippone B.W. et al., *Phys.Rev.C*, 45 (1992) 1582 [Fe]

Benvenuti A.C. et al. (BCDMS collaboration) *Z. Phys.* C63 (1994) 29 [C]

Egiyan K.S., et al., *Phys.Rev.Lett.* 96 (2006) 082501 [$^3\text{He}, ^4\text{He}, \text{C}, \text{Fe}$]

The different slopes of spectra for DIS and for particle production in cumulative region

M.A. Braun, V.V. Vechernin, Phys.Atom.Nucl. 63 (1997) 432



$$F_2^A(x) \sim \exp(-b_0 x) \quad b_0 \sim 16$$

$$I_A(x) \sim \exp(-b_s x) \quad b_s \sim 6 \div 8$$

The experimental points from

Benvenuti A.C. et al. (BCDMS collaboration)

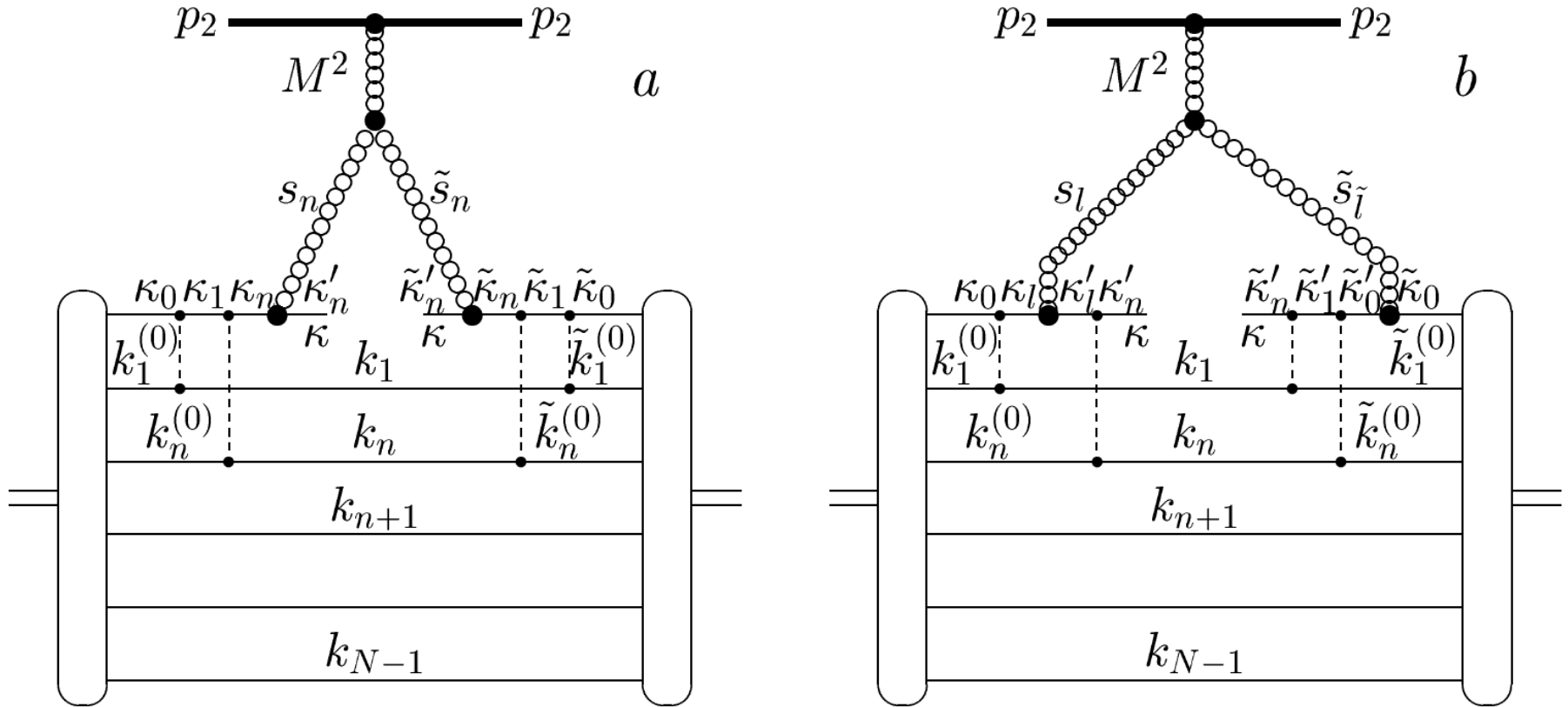
Z. Phys. C63 (1994) 29

[^{12}C , $q^2 = 61 \text{ GeV}^2$, 150 GeV^2].

Nikiforov N.A. et al. Phys. Rev. C22 (1980) 700

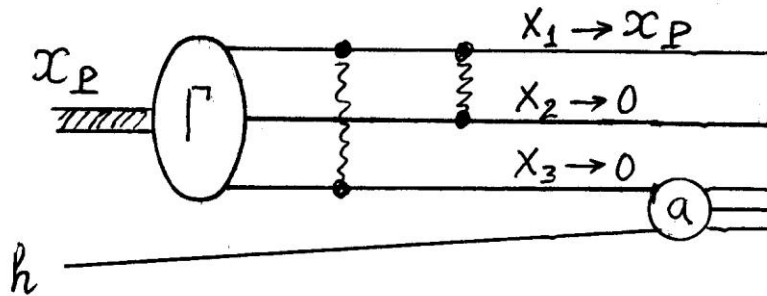
[$p+^{181}\text{Ta} \rightarrow p + X$, $400 \text{ GeV}/c$]

Cancellation of the direct contributions to a cumulative quark formation



M.A. Braun, V.V. Vechernin, Phys.Atom.Nucl. 63 (1997) 432

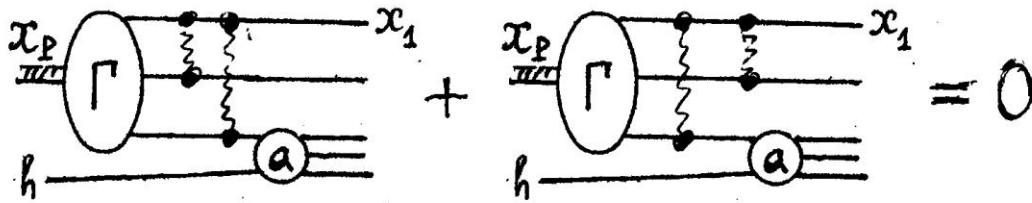
Cancellations in spectator contributions to a cumulative quark formation
 => all donor quarks must to interact with the projectile!



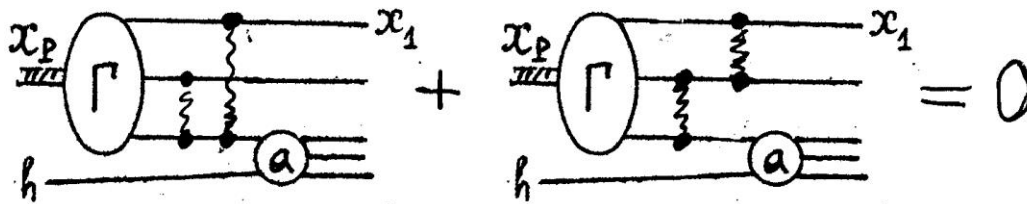
$$\sim (x_P - x_1)^{2p-1}$$

$$x_P = 1, 2, \dots$$

$$p=2 \Rightarrow \sim (1-x_1)^3$$

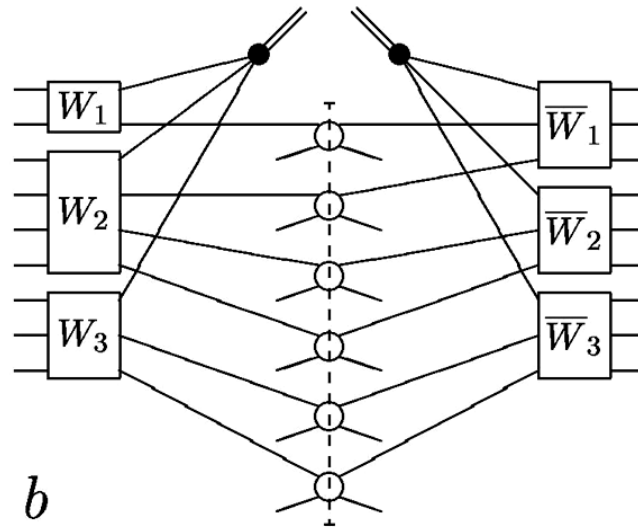
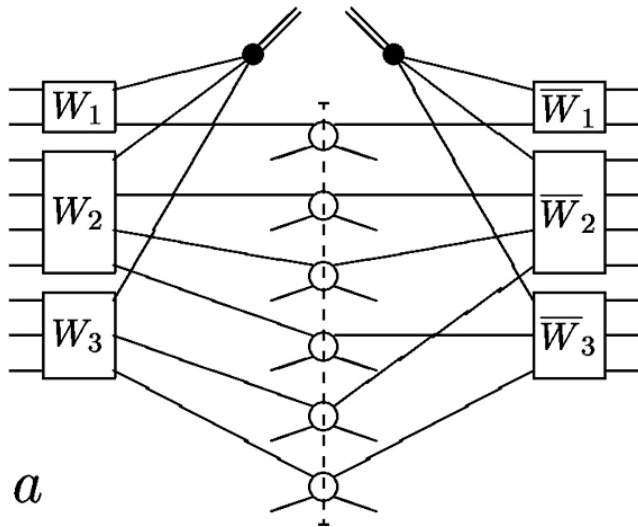
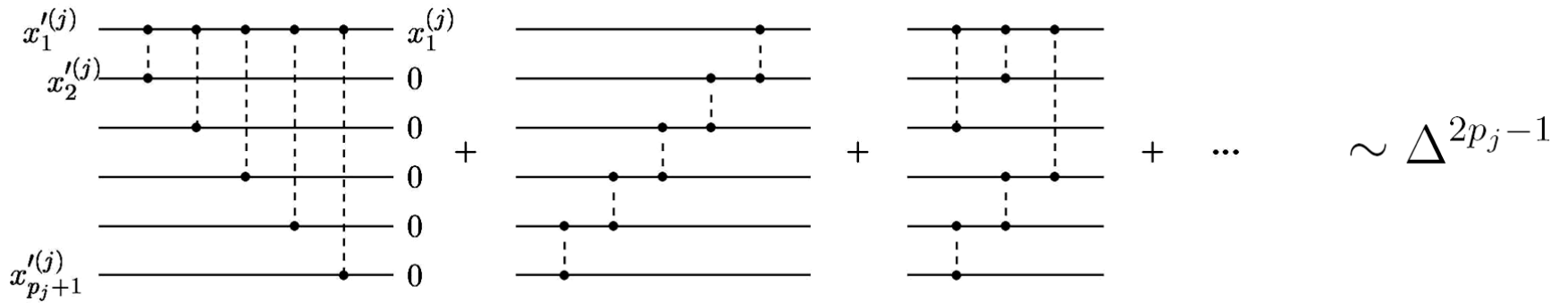


$$\sim (1-x_1)^q$$



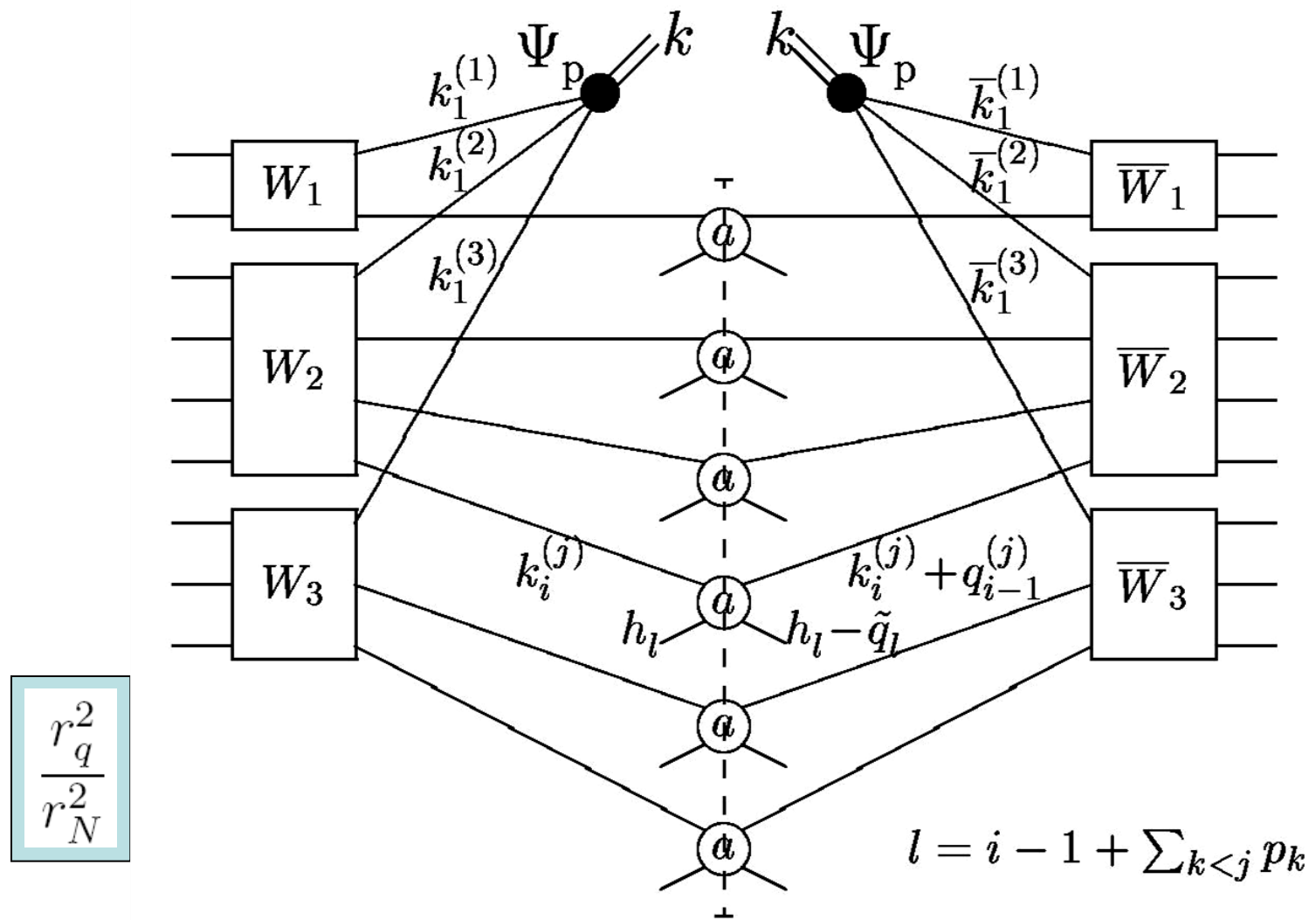
$$q > 3$$

Contributions to the blobs W_j :



The examples of two types of non-diagonal contributions to the cross section of cumulative proton production:

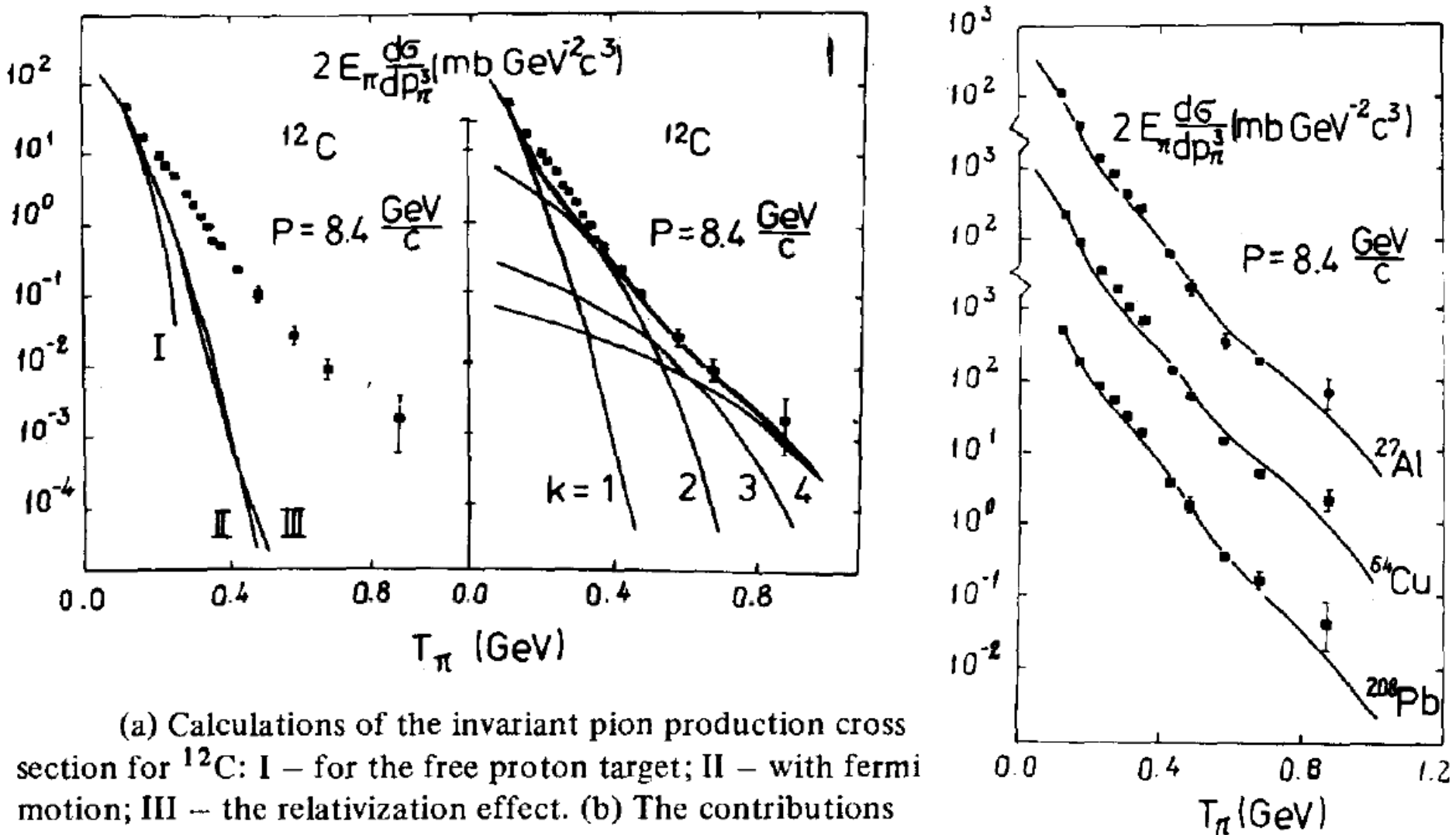
a – all $p_j = \bar{p}_j$, *b* – some $p_j \neq \bar{p}_j$



$$\frac{r_q^2}{r_N^2}$$

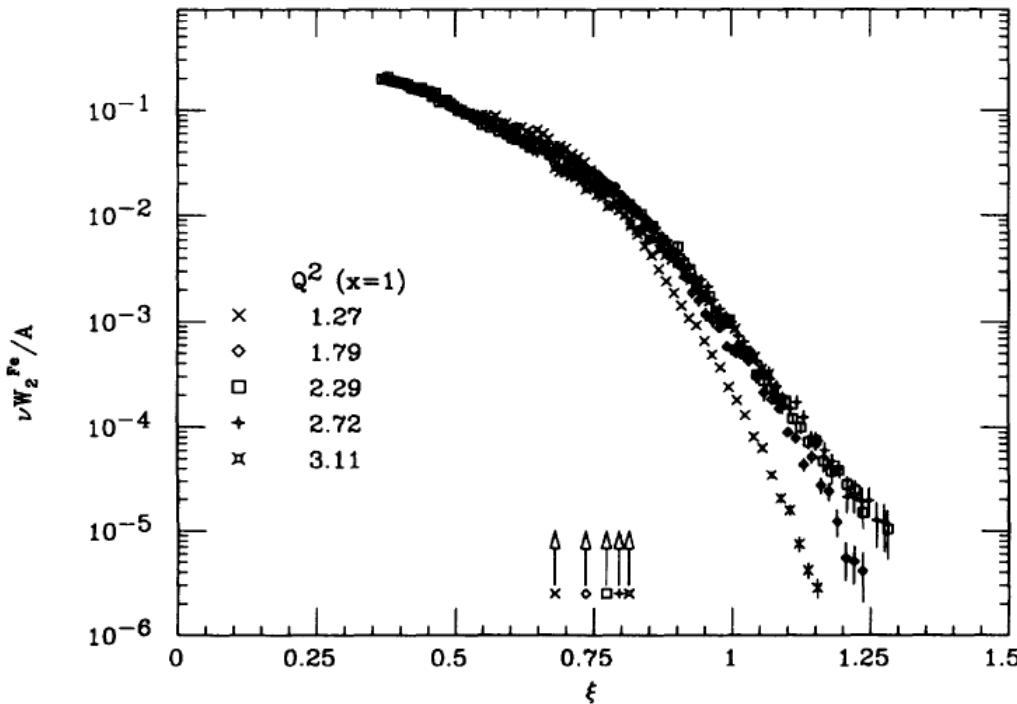
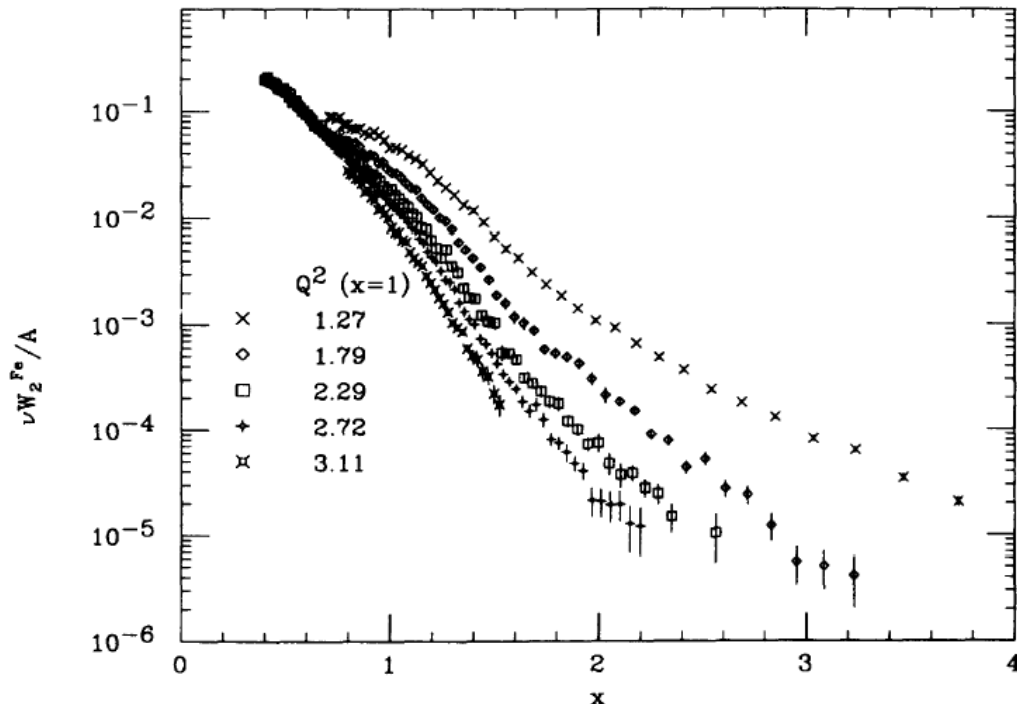
The diagonal contribution to the cross section of cumulative proton production.

Note the presence of the interference effects also in this case!



(a) Calculations of the invariant pion production cross section for ^{12}C : I – for the free proton target; II – with fermi motion; III – the relativization effect. (b) The contributions of separate fluctuons with mass $M_k = km_p$ where k is the order of cumulativity.

Filippone B.W. et al.,
Phys.Rev.C, 45 (1992) 1582



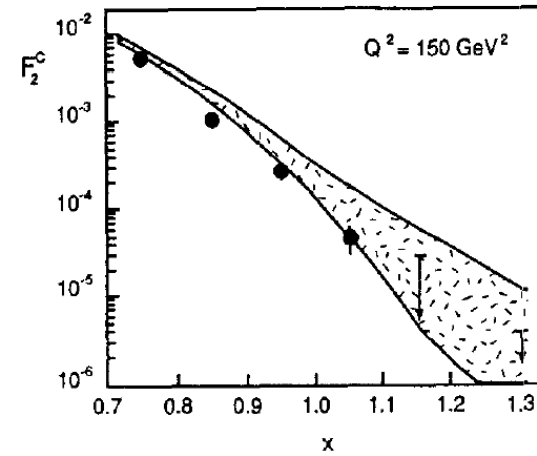
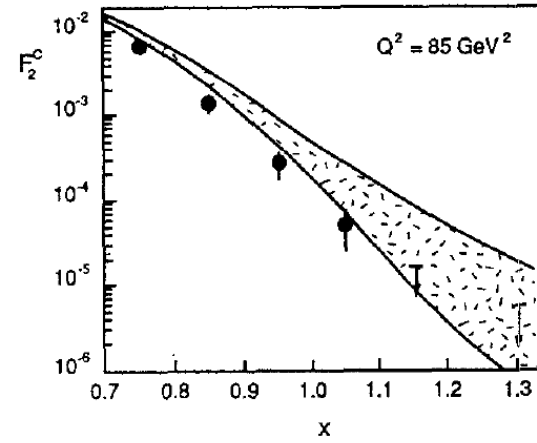
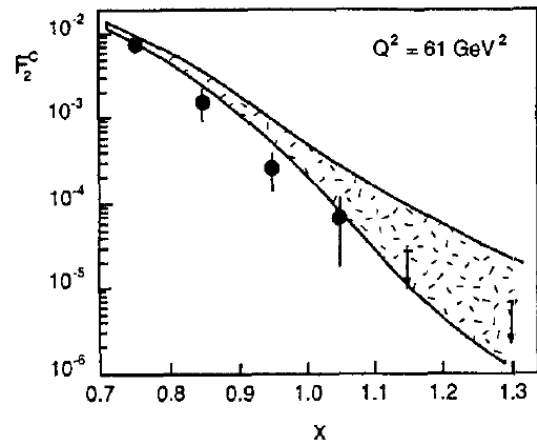
*Freedom at moderate energies:
 Masses in color dynamics*

Georgi H., Politzer H.D.

Phys. Rev. D 14, 1829 (1976)

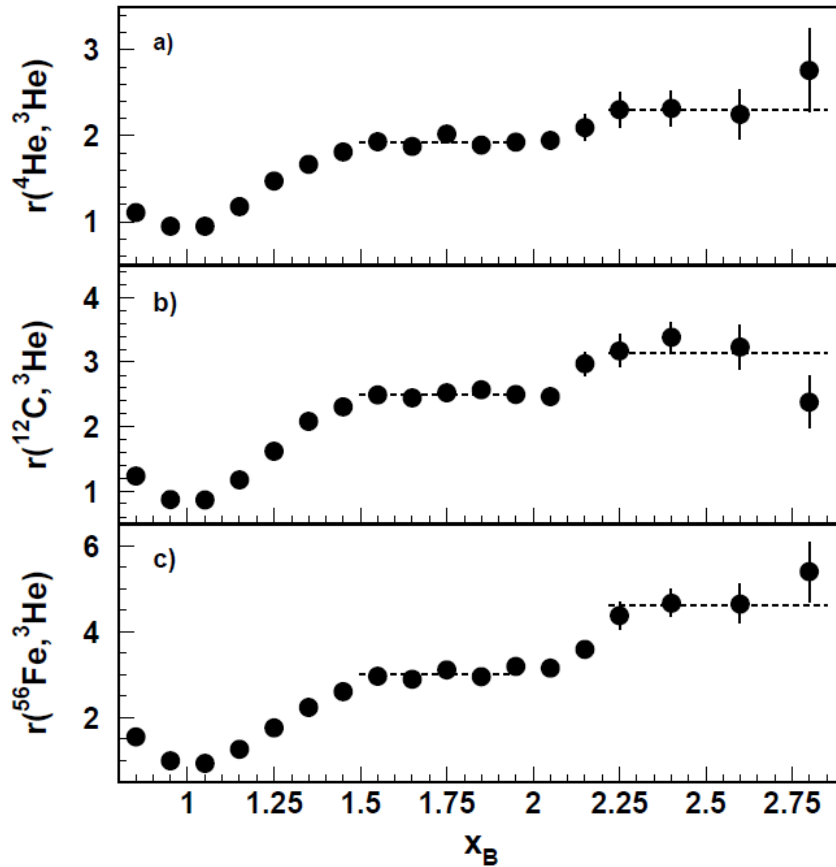
$$\xi = 2x / [1 + (1 + 4M^2x^2/Q^2)^{1/2}]$$

Benvenuti A.C. et al. (BCDMS collaboration) Z. Phys. C63 (1994) 29



L. Frankfurt, M. Strikman, Phys. Rep. 160 (1988) 325

*K.S. Egiyan, et al.,
Phys.Rev.Lett. 96 (2006) 082501*



$$r(A, {}^3\text{He}) = \frac{A(2\sigma_{ep} + \sigma_{en})}{3(Z\sigma_{ep} + N\sigma_{en})} \frac{3\mathcal{Y}(A)}{A\mathcal{Y}({}^3\text{He})} C_{\text{rad}}^A$$

	$a_2(A/{}^3\text{He})$	$a_{2N}(A)(\%)$	$a_3(A/{}^3\text{He})$	$a_{3N}(A)(\%)$
${}^3\text{He}$	1	8.0 ± 1.6	1	0.18 ± 0.06
${}^4\text{He}$	$1.93 \pm 0.01 \pm 0.03$	15.4 ± 3.2	$2.33 \pm 0.12 \pm 0.04$	0.42 ± 0.14
${}^{12}\text{C}$	$2.49 \pm 0.01 \pm 0.15$	19.8 ± 4.4	$3.18 \pm 0.14 \pm 0.19$	0.56 ± 0.21
${}^{56}\text{Fe}$	$2.98 \pm 0.01 \pm 0.18$	23.9 ± 5.3	$4.63 \pm 0.19 \pm 0.27$	0.83 ± 0.27

Quark counting rules for elastic and quasi elastic reactions with nuclei

Brodsky S., Farrar G. *Phys.Rev.Lett.* 31 (1973) 1153

Brodsky S., Chertok B.T., *Phys.Rev.* **D14** (1976) 3003

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. *Lett. Nuovo Cimento* 7 (1973) 719

$s \rightarrow \infty$, t/s fixed

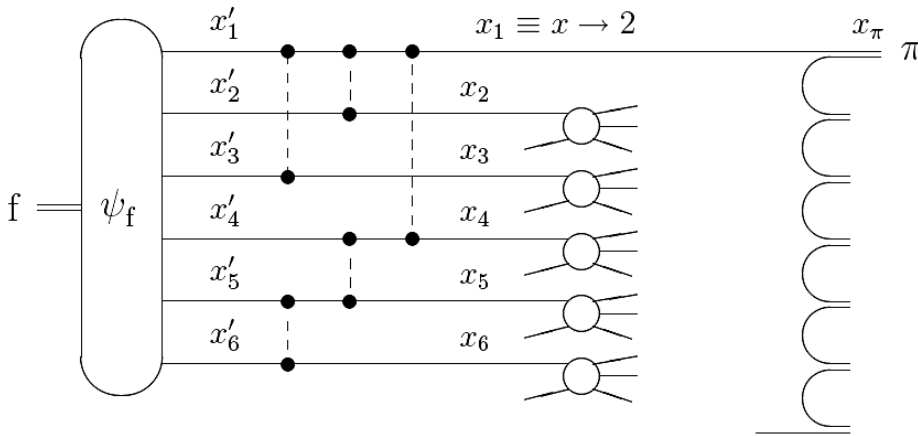
$$(d\sigma/dt)_{\pi p \rightarrow \pi p} \sim s^{-8}, (d\sigma/dt)_{pp \rightarrow pp} \sim s^{-10}, (d\sigma/dt)_{\gamma p \rightarrow \pi p} \sim s^{-7}, (d\sigma/dt)_{\gamma p \rightarrow \gamma p} \sim s^{-6}$$

$$\sim s^{-n} \quad A+B \rightarrow C+D \quad n=n_A+n_B+n_C+n_D-2 \quad n_p=3 \quad n_\pi=2 \quad n_\gamma=1$$

$$\frac{d\sigma}{dt}(A+B \rightarrow C+D) \rightarrow \frac{1}{t^{N-2}} f(t/s)$$

$$N=n_A+n_B+n_C+n_D$$

Transverse momentum spectra of cumulative pions



- the cumulative pion production

k_{\perp} – dependence:

*M.A. Braun, V.V. Vechernin,
Phys.Atom.Nucl. **63**, 1831 (2000)*

$$\sigma_{pion}(x, k_{\perp}; p) = C(p) (x_{frag} - x)^{2p-1} f_p \left(\frac{k_{\perp}}{m} \right)$$

$$x < x_{frag}(p) = 1/3 + p/3$$

p – the number of “donors”, stopped quarks

m – the constituent quark mass

$$f_p(t) = \frac{1}{\pi^p} \int \prod_{i=1}^p \frac{d^2 t_i}{(t_i^2 + 1)^2} (2\pi)^2 \delta^{(2)} \left(\sum_{i=1}^p t_i + t \right)$$

$$t = k_{\perp}/m, \quad t_i = k_{i\perp}/m$$

$$f_p(t) = 2\pi \int_0^{\infty} dz z J_0(tz) [z K_1(z)]^p$$

$$\langle |K_{\perp}| \rangle = pm \int_0^{\infty} dz K_0(z) (z K_1(z))^{p-1}$$

