

# Production of $\eta_c$ mesons at energy of the SPD NICA within generalized parton model and with various models of hadronization

A. Anufriev<sup>1</sup>, V. Saleev<sup>1,2</sup>

<sup>1</sup> Samara National Research University

<sup>2</sup> Joint Institute for Nuclear Research

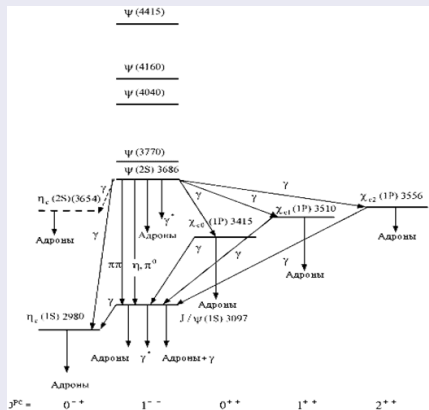
25 October 2023,  
Samara

**Talk at SPD NICA meeting**

## Outline

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## Introduction

$$\eta_c = c\bar{c}[^1S_0], M(\eta_c) = 2.981 \text{ GeV}, \Gamma = 29.7 \text{ MeV}$$


$$Br(\eta_c \rightarrow p\bar{p}) = 1.4 \times 10^{-3}$$

$$Br(\eta_c \rightarrow \Lambda\bar{\Lambda}) = 9.4 \times 10^{-4}$$

$$Br(\eta_c \rightarrow K\bar{K}\pi) = 7.2 \times 10^{-2}$$

$$Br(\eta_c \rightarrow \gamma\gamma) = 1.78 \times 10^{-4}$$

## Factorization approaches: CPM, TMD and GPM

Hard (factorization) scale  $\mu_F \sim M$

Intrinsic parton transverse momentum  $\langle q_T^2 \rangle \sim 1 \text{ GeV}^2$

- **Collinear parton model:**  $q_{1,2T} \ll p_T$  and  $\mu_F = M_T \geq M$

$$\sigma(pp \rightarrow \eta_c X) = \int dx_1 \int dx_2 f_g(x_1, \mu_F) f_g(x_2, \mu_F) \hat{\sigma}(g + g \rightarrow \eta_c + g)$$

- **TMD PM** by Collins, Soper, Stermann:  $q_{1,2T} \sim p_T$  and  $p_T \ll \mu_F$

$$\begin{aligned} \sigma(pp \rightarrow \eta_c X) = & \int dx_1 d^2 q_{1T} \int dx_2 d^2 q_{2T} F_g(x_1, q_{1T}, \mu_F, \mu_Y) \times \\ & \times F_g(x_2, q_{2T}, \mu_F, \mu_Y) \hat{\sigma}(g + g \rightarrow \eta_c) \end{aligned}$$

- **Generalized parton model:**  $q_{1,2T} \sim p_T$  and  $p_T \sim \mu_F$

$$\begin{aligned} \sigma(pp \rightarrow \eta_c X) = & \int dx_1 d^2 q_{1T} \int dx_2 d^2 q_{2T} F_g(x_1, q_{1T}, \mu_F) \times \\ & \times F_g(x_2, q_{2T}, \mu_F) \hat{\sigma}(g + g \rightarrow \eta_c) \end{aligned}$$

$$F_g(x, q_T, \mu_F) = f_g(x, \mu_F) \times \exp(-q_T^2 / \langle q_T^2 \rangle) / (\pi \langle q_T^2 \rangle)$$

# Factorization approaches: PRA

## Parton Reggeization Approach

Parton Reggeization Approach (PRA) is based on High-Energy Factorization (HEF)

$$d\sigma(p + p \rightarrow H + X) = \sum_{i,j=Q,\bar{Q},R} \int \frac{dx_1}{x_1} \int \frac{d^2\vec{q}_{1T}}{\pi} \Phi_i^p(x_1, t_1, \mu^2) \times \\ \times \int \frac{dx_2}{x_2} \int \frac{d^2\vec{q}_{2T}}{\pi} \Phi_j^p(x_2, t_2, \mu^2) d\hat{\sigma}(i + j \rightarrow H + X)$$

$$q_{1,2}^\mu = x_{1,2} P_{1,2}^\mu + q_{1,2T}^\mu, q_{1,2T} \neq 0, q_{1,2}^2 = -\vec{q}_{1,2T}^2 = q_{1,2T}^2 = t_{1,2}$$

- There is transverse momentum dependence, but initial partons are off-mass-shell instead of TMD factorization.
- For PRA amplitudes we can use effective gauge-invariance field theory with specific Feynman rules instead recover asymptotics of QCD amplitude on  $s \rightarrow \infty$ . This EFT is known as Lipatov's field theory.

Initial state factors:		
$\text{---} \xrightarrow{q} \pm = \frac{q^\pm}{2\sqrt{-q^2}}$	$\downarrow \text{---} = -ig_s T^a \hat{n}^\pm$	$q_{1\downarrow}^\pm = -ie \left( \gamma_\mu + \hat{q}_1 \frac{n_\mu^\mp}{p^\mp} + \hat{q}_2 \frac{n_\mu^\pm}{p^\pm} \right)$
$\text{---} \xrightarrow{q} \pm = u(q^\parallel)$	$q_{1\uparrow}^\pm = -ig_s T^a \left( \hat{n}^\pm + 2 \frac{\hat{q}_1}{q_2^\pm} \right)$	$q_{2\uparrow}^\pm = -ie \left( \gamma_\mu + \hat{q}_1 \frac{n_\mu^\mp}{p^\mp} \right)$
Propagators ( $\hat{P}_\pm = \frac{1}{2} \hat{n}^\mp \hat{n}^\pm$ ):		
$\text{---} \xrightarrow{q} \pm = \hat{P}_\pm \frac{i\hat{q}}{q^2}$	$q_{2\downarrow}^\pm = -2ie g_s T^a \frac{\hat{q}_1 n_\mu^\mp}{p^\mp q_2^\mp}$	$q_{1\downarrow}^\pm = -ie^2 \hat{q}_1 \frac{n_{\mu_1}^\mp n_{\mu_2}^\mp}{p_1^\mp p_2^\mp}$
$\text{---} \xrightarrow{q} \pm = \frac{i\hat{q}}{q^2} \hat{P}_\pm$		

$q_{2\downarrow}^\pm = ie^2 \left( \hat{q}_2 \frac{n_{\mu_1}^\pm n_{\mu_2}^\pm}{p_1^\pm p_2^\pm} - \hat{q}_1 \frac{n_{\mu_1}^\mp n_{\mu_2}^\mp}{p_1^\mp p_2^\mp} \right)$	$q_{2\downarrow}^\pm = -ie^3 \left( \hat{q}_2 \frac{n_{\mu_1}^\pm n_{\mu_2}^\pm n_{\mu_3}^\pm}{p_1^\pm p_2^\pm p_3^\pm} + \hat{q}_1 \frac{n_{\mu_1}^\mp n_{\mu_2}^\mp n_{\mu_3}^\mp}{p_1^\mp p_2^\mp p_3^\mp} \right)$	$q_{2\downarrow}^\pm = -2ie^2 g_s T^a \frac{\hat{q}_1 n_{\mu_1}^\mp n_{\mu_2}^\mp}{p_1^\mp p_2^\mp}$
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## Hadronization mechanisms: NRQCD and CSM

Basic idea - small relative momentum of  $q\bar{q}$ -pair, which can be neglected

Fock states can be decomposed into a series according to the small relative velocity parameter  $v$ . For instance,  $J/\psi$ -meson series is:

$$|J/\psi\rangle = \mathcal{O}(v^0)|c\bar{c}[{}^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)|c\bar{c}[{}^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^1S_0^{(1,8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^3S_1^{(1,8)}]gg\rangle + \dots$$

For  $\eta_c$  it gives

$$|\eta_c\rangle = \mathcal{O}(v^0)|c\bar{c}[{}^1S_0^{(1)}]\rangle + \mathcal{O}(v^1)|c\bar{c}[{}^1P_1^{(8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^3S_1^{(1,8)}]g\rangle + \mathcal{O}(v^2)|c\bar{c}[{}^1S_0^{(1,8)}]gg\rangle + \dots$$

In **Color Singlet Model** (CSM) it's using only leading term of NRQCD series (singlet) without taking others terms (octets).

## LDMEs

Long-Distance Matrix Elements defines transition of  $q\bar{q}$  pair into final quarkonium. Singlet LDMEs connected with squared wave function as

$$\langle \mathcal{O}^H[c\bar{c}^{(1)}] \rangle = 2N_c(2J+1)|\Psi(0)|^2$$

There is symmetry of LDMEs between  $J/\psi$  and  $\eta_c$  final states:

- $\langle \mathcal{O}^{\eta_c}[{}^1S_0^{(1)}|{}^1S_0^{(8)}] \rangle = \frac{1}{3} \langle \mathcal{O}^{J/\psi}[{}^3S_1^{(1)}|{}^3S_1^{(8)}] \rangle$
- $\langle \mathcal{O}^{\eta_c}[{}^3S_1^{(8)}] \rangle = \langle \mathcal{O}^{J/\psi}[{}^1S_0^{(8)}] \rangle$
- $\langle \mathcal{O}^{\eta_c}[{}^1P_1^{(8)}] \rangle = 3 \langle \mathcal{O}^{J/\psi}[{}^3P_0^{(8)}] \rangle$

Factorisation of  $\eta_c$  production in CSM from  $c\bar{c}$  pair is presented in the form:

$$\hat{\sigma}(a+b \rightarrow c\bar{c}[13S_0^{1,8}] \rightarrow \eta_c) = \hat{\sigma}(a+b \rightarrow c\bar{c}[{}^1S_0^{1,8}]) \frac{\langle \mathcal{O}^H[{}^1S_0^{(1,8)}] \rangle}{N_{col} N_{pol}}$$

For singlet  $N_{col} = 2N_c = 6$ ,  $N_{pol} = 2J+1 = 1$ . For octet  $N_{col} = N_c^2 - 1$

## Transformation into colorless charmonium

Amplitude of  $c\bar{c}$  production can be obtained using projector on corresponds spin states

$$\Pi_1^\alpha = \frac{1}{\sqrt{8m_c^3}} \left( \frac{\hat{p}}{2} - \hat{q} - m_c \right) \gamma^\alpha \left( \frac{\hat{p}}{2} + \hat{q} + m_c \right)$$

where  $\hat{p} = \gamma^\alpha p_\alpha$ ,  $p^\alpha$  — 4-momenta of  $c\bar{c}$ -pair,  $\hat{q} = \gamma^\alpha q_\alpha$ ,  $q_\alpha$  — relative 4-momenta of quarks,  $M$  — quarkonium mass,  $m_c = M/2$  — mass of  $c$ -quark.

After convolution of amplitude with projector  $q$  is assumed **to be zero**.

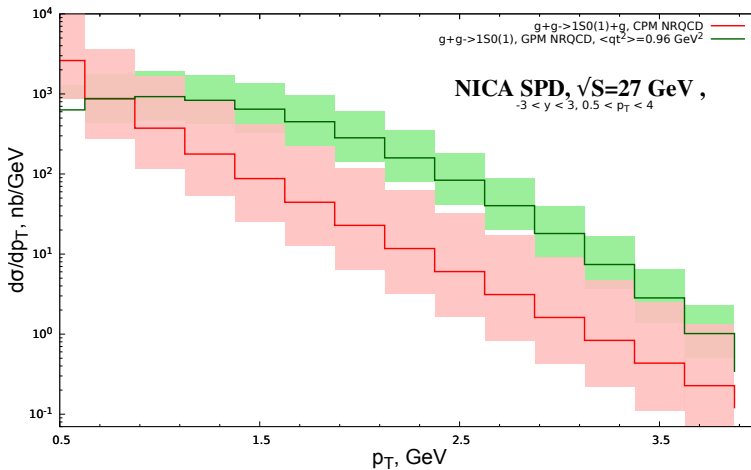
In Color Evaporation Model, heavy quark pair is produced perturbatively with definite spin and color quantum numbers and color of pair "evaporates" to transform into quarkonium

In the **Improved Color Evaporation Model** (ICEM) cross section of quarkonium state can be presented in form:

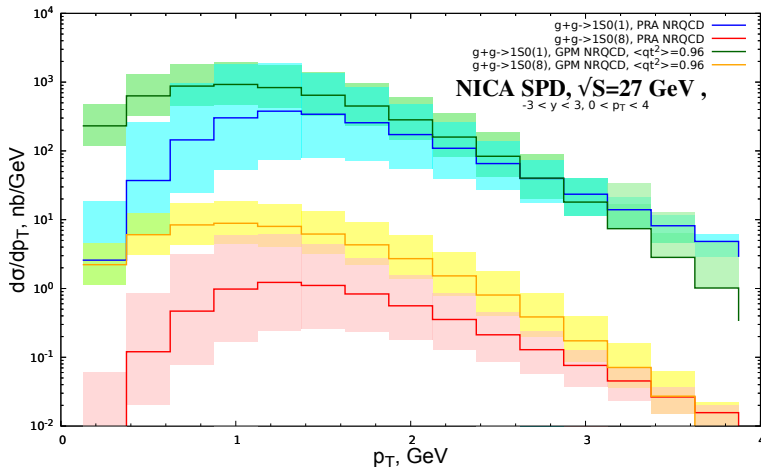
$$\hat{\sigma}(J/\psi) = \hat{\sigma}(c\bar{c} : M_{\eta_c} < s < 4m_D^2) f_c^{\eta_c}$$

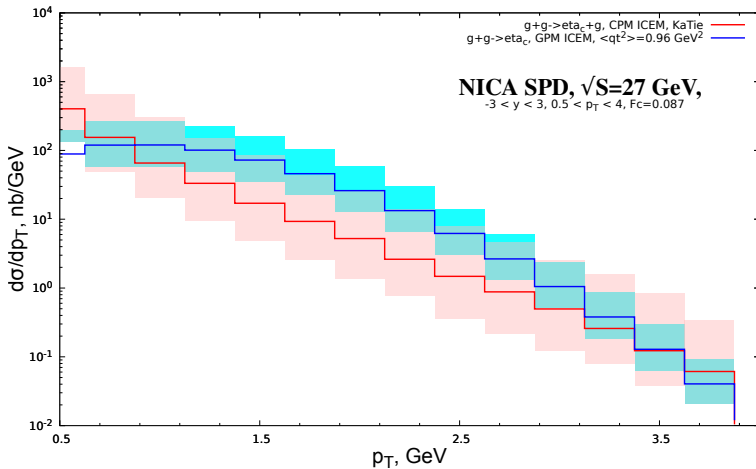
when charmonium momentum is distinguished from the momentum of a quark pair through relation  $p_T^{\eta_c} = \frac{M_{\eta_c}}{M_{c\bar{c}}} p_T^{c\bar{c}}$

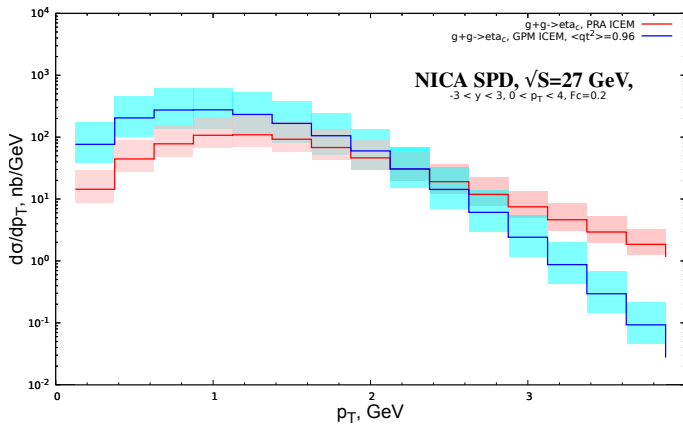
$f_c$  can be found from fit of experimental data. V. A. Saleev and A. A. Chernyshev showed that  $f_c$  depends on energy:  $\mathbf{F}_c = \mathbf{0.2}$  at energy 27 GeV of SPD NICA

Comparison of differential cross-section of  $\eta_c$ -production at NICA in GPM and CPM within NRQCD



Comparison of differential cross-section of  $\eta_c$ -production at NICA in GPM and PRA within NRQCD

Comparison of differential cross-section of  $\eta_c$ -production at NICA in GPM and CPM within ICEM

Comparison of differential cross-section of  $\eta_c$ -production at NICA in GPM and PRA within ICEM

## Conclusions

- CSM is approximately 80 % of NRQCD with CO NMEs taking accordingly HQS rules.
- PRA predictions for NICA are smaller by factor 2 than results within GPM used parameters obtained by fitting  $J/\psi$  data for the relevant energies.
- There is a good agreement between LO CPM, LO GPM and LO PRA calculations using both hadronization models NRQCD and ICEM.
- New experimental data is needed to answer some questions about NRQCD and ICEM phenomenology for  $\eta_c$  production case.

**Thank you for your attention!**