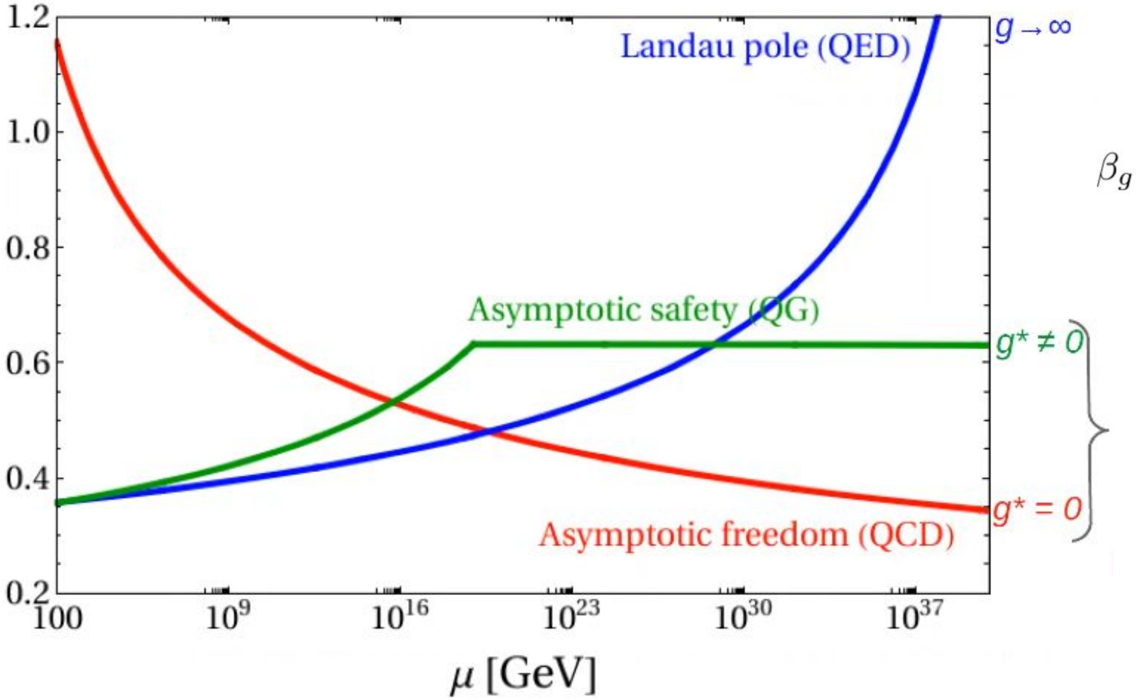
How robust are particle physics predictions in asymptotic safety?

arXiv: 2204.08959

Motivation

Asymptotic behaviours



$$\beta_g = \frac{dg}{d \ln k}$$

$$\beta_i(\{\alpha_j^*\}) = 0$$

- AS originally advocated by Weinberg to improve the UV behavior of G_N
- Advocated in QFT as solution to $U(1)_Y$ triviality problem

Fixed point and critical surface

$\beta_a = -a(a^*-a)$

irrelevant directions = predictions

$\beta_i(\{\alpha_j^*\}) = 0 \longrightarrow M_{ij} = \partial\beta_i/\partial\alpha_j\big|_{\{\alpha_i^*\}} \longrightarrow \{-\theta_i\}$

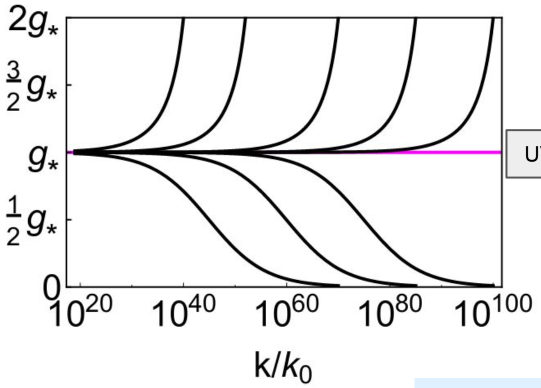
stability matrix

critical exponents

$\beta_a = a(a^*-a)$

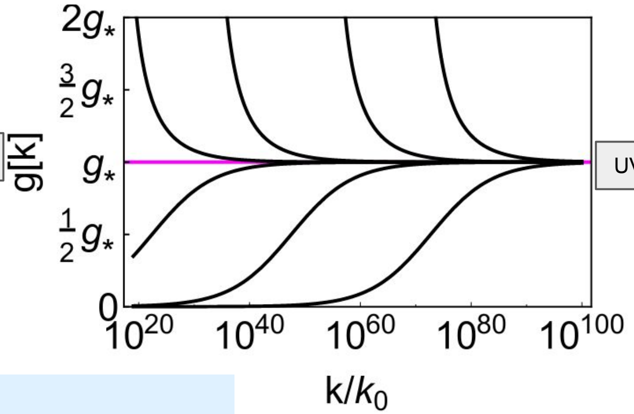
relevant directions = free parameters

IR attractive



UV repulsive

IR repulsive

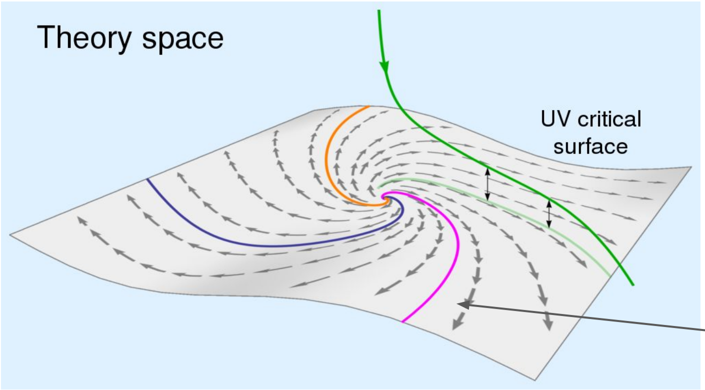


UV attractive

from Wikipedia

$\theta_i < 0$

$\theta_i > 0$



can only deviate from the FP along the critical surface

Span the UV critical surface

$g_{irrelevant} = g_{irrel}(g_{rel}, g_{rel})$

… can be uniquely fixed

once determined by the experiment

Asymptotic safety in quantum gravity

Quantum gravity and quantum gravity + matter might feature interactive UV fixed points

[Reuter '96, Reuter, Saueressig '01, Litim '04, Codello, Percacci, Rahmede '06, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Manrique, Rechenberger, Saueressig '11, Falls, Litim, Nikolakopoulos '13, Dona', Eichhorn, Percacci '13, Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, Pawlowski *et al.* '18 … many more]

Prototype example: Einstein-Hilbert gravity

$$S_{\rm EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(-R(g)+2\Lambda)$$

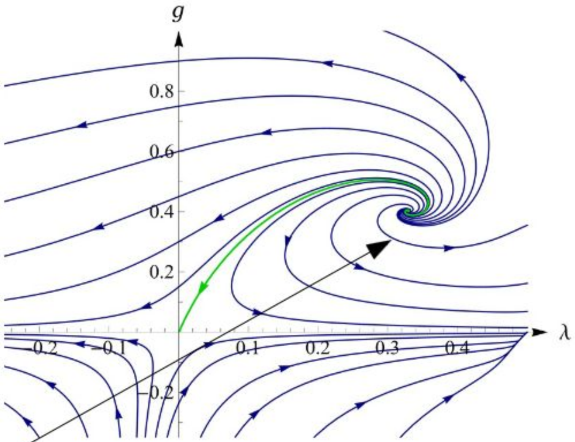
Functional renormalization group techniques (Wetterich Equation) lead to 2 fixed points

$$\beta_g \equiv \frac{dg}{d\ln k} = 0 \qquad \beta_\lambda \equiv \frac{d\lambda}{d\ln k} = 0$$

(gaussian) $g = 0 \;\; \lambda = 0$

(interactive) $\boxed{g = g^* \;\; \lambda = \lambda^*}$

Reuter, Saueressig, hep-th/0110054



Fixed point persists under the addition of new interactions

Asymptotic safety in QG with matter

Gravity affects matter:

Gauge-Yukawa system coupled to gravity

Modification to RGEs @ $k > M_{\rm Pl}$

$$\beta_g = \beta_g^{\rm SM+NP} - g\, f_g$$

$$\beta_y = \beta_y^{\rm SM+NP} - y\, f_y$$

$$\beta_\lambda = \beta_\lambda^{\rm SM+NP} - \lambda\, f_\lambda$$

Quantum-gravitational contribution
(in principle via FRG)

[Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, Wetterich, Yamada '16, Hamada, Yamada '17, Pawlowski et al. '18, Eichhorn, Versteegen '17, Eichhorn, Held '17-'18 ...]

EXAMPLE : U(1) + Φ + E-H:

$$f_g = G\frac{1-4\Lambda}{4\pi(1-2\Lambda)^2}$$

Asymptotic safety in QG with matter

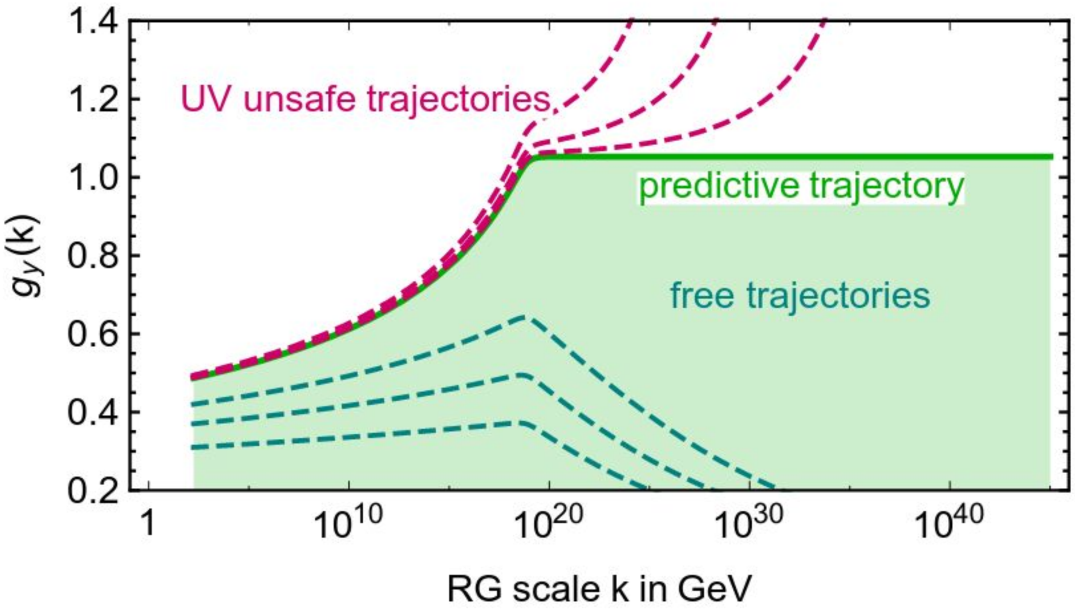
Gravity affects matter: $$\frac{dg_i}{dt} = \beta_i^{(\mathrm{matter})} - f_g\, g_i$$

matter & gravity fluctuations compete

strong gravity: asymptotically freedom

strong matter: UV unsafe

balance: UV safe & interacting



A.Eichhorn, F. Versteegen (PLB ‘18, 1709.07252)

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EXAMPLE : U(1) + Φ + E-H:

$$f_g = G\frac{1-4\Lambda}{4\pi(1-2\Lambda)^2}$$

In practice *fg*, *fy* are subject to large uncertainties
(truncation in number of operators, cut-off scheme dependence, etc.)

[Lauscher, Reuter '02, Codello, Percacci, Rahmede '07-'08, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Dona', Eichhorn, Percacci '13, Falls, Litim, Schroeder '18, ...]

↓

f_g, f_y free parameters determined by matching to the low-energy data

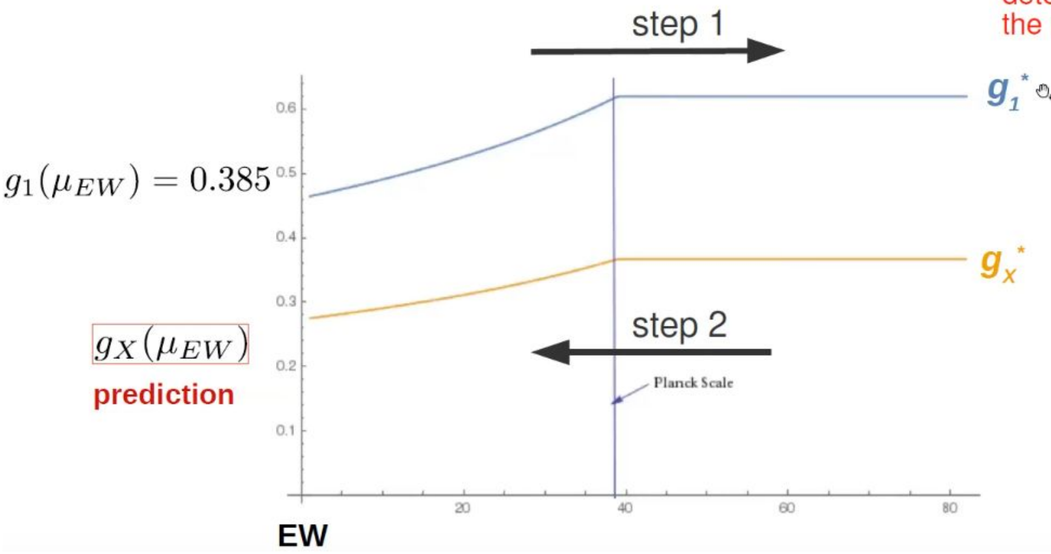
applied in SM and simple SM extensions

see *e.g.* Eichhorn, Held, 1707.01107, 1803.04027; Reichert, Smirnov, 1911.00012; Alkofer *et al.* 2003.08401

Asymptotic safety in QG with matter: example

Predictions from AS:

$$\text{SM} + \text{U}(1)_X \quad \begin{cases} \dfrac{dg_1}{dt} = \dfrac{41}{6}\dfrac{g_1^3}{16\pi^2} - f_g\, g_1 \\ \dfrac{dg_X}{dt} = 11\dfrac{g_X^3}{16\pi^2} - f_g\, g_X \end{cases}$$



determined from the matching

$$g_1^* \rightarrow f_g = \frac{1}{4\pi}\sqrt{\frac{41\, g_1^*}{6}}$$

universality

$$g_X^* = 4\pi\sqrt{\frac{f_g}{11}}$$

Robustness of the asymptotic safety predictions

Assumptions of the fixed point analysis:

- 1-loop RGEs;
- Planck scale set at M_{Pl} = 10^19 GeV;
- Gravity parameters *f* are constant;
- Gravity decouples instantaneously.

How robust the predictions derived in this way?
What extent dropping any of the approximations may affect to test these predictions at the low scale?

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What extent dropping any of the approximations may affect to test these predictions at the low scale?

2304.08959

1. The inclusion of higher-order corrections in the matter sector
2. Changing the position of the Planck scale by a few orders of magnitude
3. The non-trivial functional dependence of the running gravitational couplings, $f_{g,y}(t)$, resulting in the non-instantaneous decoupling of the trans-Planckian UV completion

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Models

Gauged B-L *[JHEP 02 (2016) 135]*

- SU(3)xSU(2)xU(1)xU(1)$_{B-L}$
- Right-handed neutrinos and complex scalar field S
- The Yukawa part is extended by

$$\mathcal{L} \supset -Y_\nu N \left(\epsilon H^*\right)^\dagger L - \frac{1}{2} Y_N S N N + \text{H.c.}\,,$$

- The abelian gauge part

$$\mathcal{L} \;\; \supset \;\; -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{\epsilon}{2} B_{\mu\nu} X^{\mu\nu} \\ + i \bar{f} \left(\partial^\mu - i g_Y Q_Y \tilde{B}^\mu - i g_{B-L} Q_{B-L} \tilde{X}^\mu\right) \gamma_\mu f\,,$$

- From $g_Y^{\text{SM},\overline{\text{MS}}}(M_t = 173.1\,\text{GeV}) = 0.36$ and $y_t^{\text{SM},\overline{\text{MS}}}(M_t) = 0.95$

$\longrightarrow$ $g_Y \to g_Y\,, \quad g_d = \dfrac{g_{B-L}}{\sqrt{1-\epsilon^2}}\,, \quad g_\epsilon = -\dfrac{\epsilon\, g_Y}{\sqrt{1-\epsilon^2}}\,.$

- $f_g\,(1 \text{ loop}) = 0.0097\,, \quad f_y\,(1 \text{ loop}) = 0.0020\,,$

$g_Y^*\,(1 \text{ loop}) = 0.4734\,, \quad g_d^*\,(1 \text{ loop}) = 0.4420\,, \quad g_\epsilon^*\,(1 \text{ loop}) = -0.3450\,,$

$y_t^*\,(1 \text{ loop}) = 0.2901\,, \quad y_\nu^*\,(1 \text{ loop}) = 0.5398\,, \quad y_N^*\,(1 \text{ loop}) = 0.3868\,.$

Leptoquark S$_3$ *[Phys. Rev. D **43**, 225]*

- SM + S(3,3,1/3)
- The Yukawa interaction of S$_3$ with SM fermions

$$\mathcal{L} \ni -Y_{\text{LQ}}\, Q^T \epsilon S_3\, L + \text{H.c.}\,,$$

- $f_g\,(1 \text{ loop}) = 0.0106\,, \qquad f_y\,(1 \text{ loop}) = -0.0004\,,$
- $g_Y^*\,(1 \text{ loop}) = 0.4823\,, \quad y_t^*\,(1 \text{ loop}) = 0.2340\,, \quad y_{\text{LQ}}^*\,(1 \text{ loop}) = 0.1132\,.$
- $y_{\text{LQ}}\,(M_t, 1 \text{ loop}) = 0.4270$

Estimation of uncertainties

1. Impact of higher-order corrections

Gauge couplings

- RGEs in the B-L model:

$$\begin{aligned}
\frac{dg_Y}{dt} &= \frac{1}{16\pi^2}\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right) g_Y^3 - f_g\, g_Y \\
\frac{dg_d}{dt} &= \frac{1}{16\pi^2}\left[\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right) g_d g_\epsilon^2 + \left(b_d + \Pi^{(d)}_{n\geq 2}\right) g_d^3 + \left(b_\epsilon + \Pi^{(\epsilon)}_{n\geq 2}\right) g_d^2 g_\epsilon\right] - f_g\, g_d \\
\frac{dg_\epsilon}{dt} &= \frac{1}{16\pi^2}\left[\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right)\left(g_\epsilon^3 + 2g_Y^2 g_\epsilon\right) + \left(b_d + \Pi^{(d)}_{n\geq 2}\right) g_d^2 g_\epsilon \right. \\
&\quad \left. + \left(b_\epsilon + \Pi^{(\epsilon)}_{n\geq 2}\right)\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.
\end{aligned}$$

- Generic n-loop contribution

$$\Pi^{(i)}_{n\geq 2} = \frac{1}{16\pi^2}\sum_{l,k}\alpha^{(i)}_{lk}\, c_l\, c_k + \sum_{n>2}\sum_{l_1\ldots l_{n-1}}\frac{1}{(16\pi^2)^{n-1}}\alpha^{(i)}_{l_1\ldots l_{n-1}} c_{l_1}^2\ldots c_{l_{n-1}}^2\,,$$

$\alpha_{l_1 \ldots l_{n-1}}$ loop coefficients; c_l - the gauge and Yukawa couplings

- $$f_g(n\ \text{loops}) \approx \frac{g_Y^{*2}(n\ \text{loops})}{16\pi^2}\left(b_Y + \Pi^{(Y)*}_{n\geq 2}\right)\,,$$

- $$\begin{aligned}
r^*_{g,d}(n\ \text{loops}) &\equiv \frac{g_d^*}{g_Y^*}(n\ \text{loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}\,, \\
r^*_{g,\epsilon}(n\ \text{loops}) &\equiv \frac{g_\epsilon^*}{g_Y^*}(n\ \text{loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}\,,
\end{aligned}$$

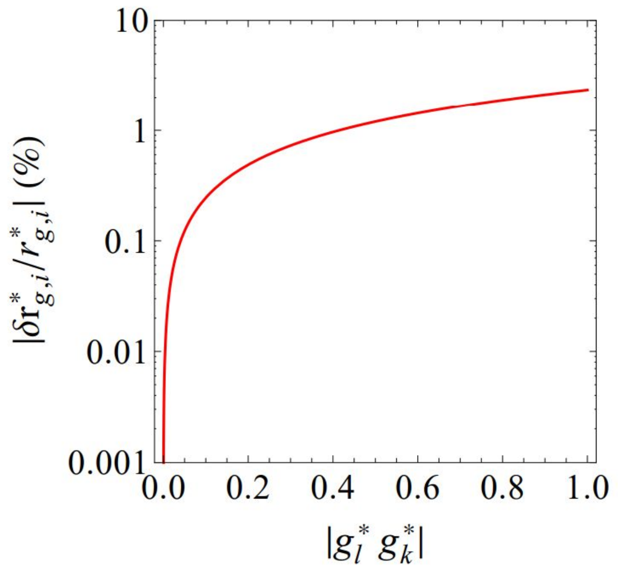
$$\tilde{b}_i \equiv b_i + \Pi^{(i)*}_{n\geq 2}\,.$$

The uncertainty (retain 2-loop corrections)

$$\frac{\delta r^*_{g,i}}{r^*_{g,i}} = \frac{r^*_{g,i}(2\ \text{loops}) - r^*_{g,i}(1\ \text{loop})}{r^*_{g,i}(1\ \text{loop})}\,.$$

In the B-L model: $b_Y = 41/6,\ b_d = 12,\ b_\epsilon = 32/3.$

$$\delta r^*_{g,d}/r^*_{g,d} = -0.41\% \text{ and } \delta r^*_{g,\epsilon}/r^*_{g,\epsilon} = -0.44\%.$$



Yukawa couplings

- Yukawa coupling RGEs of a generic SM+NP theory

$$\frac{dy_r}{dt} = \frac{y_r}{16\pi^2}\left(\sum_j a_j^{(r)} y_j^2 - \sum_{l,k} a'^{(r)}_{lk} g_l g_k + \sum_{n\geq 2}\widetilde{\Pi}_n^{(r)}\right) + \sum_{m,p,q\neq r}\frac{y_m y_p y_q}{16\pi^2}\left(a''_{mpq} + \sum_{n\geq 2}\tilde{\Delta}_n^{(mpq)}\right) - f_y y_r,$$

$y_{r(j)=1,2\ldots}$ the set of Yukawa couplings; $g_{l(k)}$ the gauge couplings

- The genetic multiplicative n-loop contribution

$$\widetilde{\Pi}_n^{(r)} \equiv \frac{1}{(16\pi^2)^{n-1}} \sum_{l_1 l_2 \ldots l_{2n}} \alpha'^{(r)}_{l_1 l_2 \ldots l_{2n}} c_{l_1} c_{l_2} \ldots c_{l_{2n}} ,$$

- The generic additive n-loop piece

$$\tilde{\Delta}_n^{(mpq)} \equiv \frac{1}{(16\pi^2)^{n-1}} \sum_{l_1 \ldots l_{2n-n}} \alpha''^{(mpq)}_{l_1 \ldots l_{2n-n}} c_{l_1} \ldots c_{l_{2n-2}} ,$$

n-loop coefficients — all gauge, Yukawa, quartic couplings

- The 2-loop correction

$$f_y(2\text{ loops}) \approx \frac{1}{16\pi^2}\left[y_1^{*2}(2\text{ loops}) \times F_0\left(a_j^{(r)}\right) + G_0\left(\sum_{l,k} a'^{(r)}_{lk} g_l^* g_k^*(2\text{ loops})\,;\, a^{(r)}_{j\neq 1}\right) + H_0\left(\widetilde{\Pi}_2^{(r)*}\,;\, a^{(r)}_{j\neq 1}\right)\right],$$

rational functions

shift $\quad y_1^{*2}(2\text{ loops}) = y_1^{*2}(1\text{ loop}) + \delta y_1^{*2},$

$$y_2^*(2\text{ loops}) \approx \left[F_1\left(a_j^{(r)}\right)\left(y_1^{*2}(1\text{ loop}) + \delta y_1^{*2}\right) + G_1\left(\sum_{l,k} a'^{(r)}_{lk} g_l^* g_k^*\,;\, a^{(r)}_{j\neq 1}\right) + H_1\left(\widetilde{\Pi}_2^{(r)*}\,;\, a^{(r)}_{j\neq 1}\right)\right]^{1/2},$$

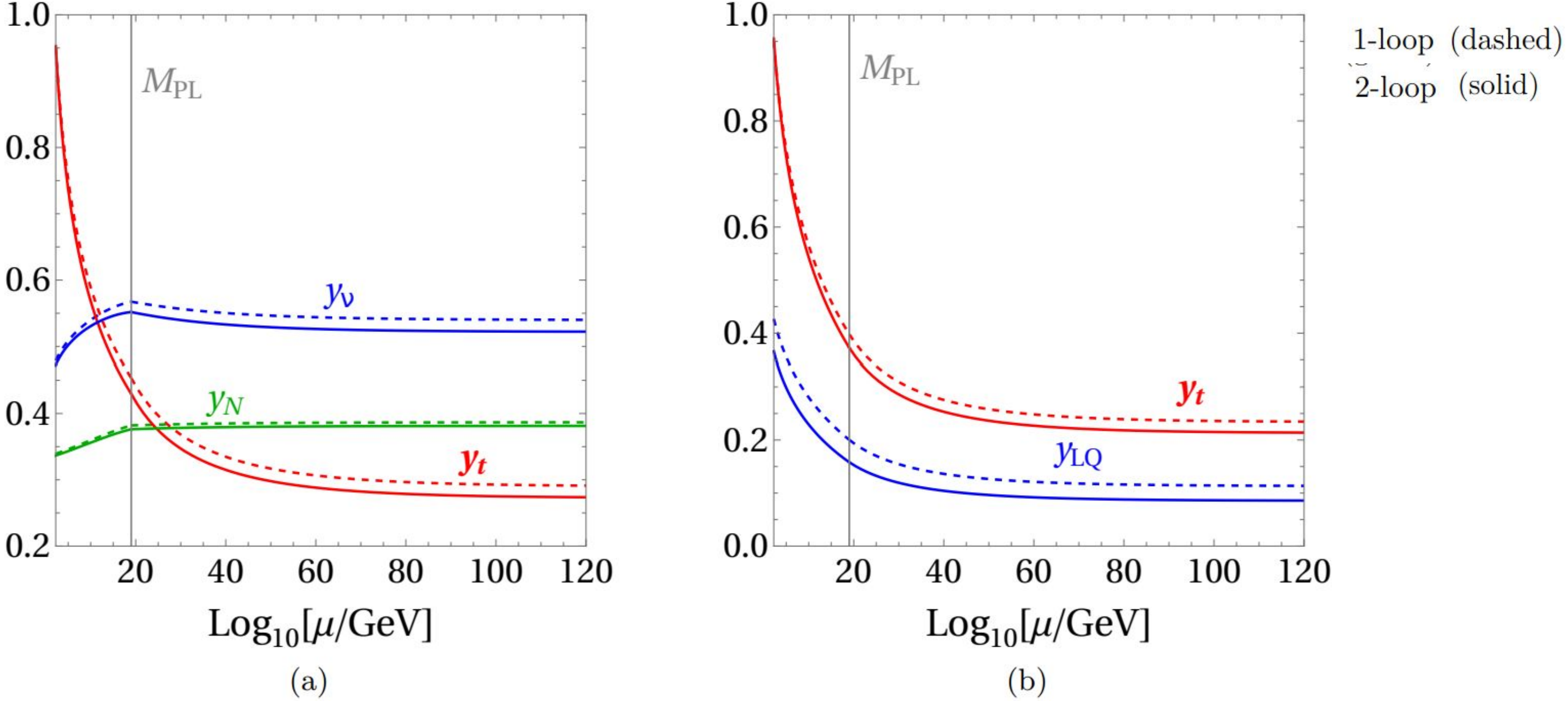
$$y_2^*(2\text{ loops}) \approx \left[\frac{a_1^{(2)} - a_1^{(1)}}{a_2^{(1)} - a_2^{(2)}}\left(y_1^{*2}(1\text{ loop}) + \delta y_1^{*2}\right) + \frac{\left(a'^{(1)}_{11} - a'^{(2)}_{11}\right) g_1^{*2} + \widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}}{a_2^{(1)} - a_2^{(2)}}\right]^{1/2}$$

$$a_1^{(1)} = 9/2,\quad a_2^{(1)} = 3/2,\; a'^{(1)}_{11} = 17/12,\; a_1^{(2)} = 1/2,\; a_2^{(2)} = 8,\; a'^{(2)}_{11} = 5/6$$

$$\delta y_1^{*2} \approx -9\times 10^{-3} \longrightarrow \frac{\delta y_2^*}{y_2^*} = \frac{y_2^*(2\text{ loops}) - y_2^*(1\text{ loop})}{y_2^*(1\text{ loop})} \approx -25\%.$$

$B-L$	f_g	g_Y^*	g_d^*	g_ϵ^*	$\delta g_Y^*/g_Y^*$	$\delta g_d^*/g_d^*$	$\delta g_\epsilon^*/g_\epsilon^*$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
	0.0098	0.4748	0.4415	−0.3445	0.3%	−0.1%	−0.1%	−0.4%	−0.5%
	f_y	y_t^*	y_ν^*	y_N^*	$\delta y_t^*/y_t^*$	$\delta y_\nu^*/y_\nu^*$	$\delta y_N^*/y_N^*$	$\delta y_\nu/y_\nu(M_t)$	$\delta y_N/y_N(M_t)$
	0.0016	0.2727	0.5220	0.3813	−6.0%	−3.3%	−1.4%	−1.4%	−0.8%
S_3 LQ	f_y	y_t^*	$y_{\rm LQ}^*$		$\delta y_t^*/y_t^*$	$\delta y_{\rm LQ}^*/y_{\rm LQ}^*$		$\delta y_{\rm LQ}/y_{\rm LQ}(M_t)$	
	−0.0007	0.2133	0.0855		−8.8%	−24.5%		−14.3%	

Table 1: 2-loop determination of the gravity parameters f_g and f_y, fixed-point values of the reference and the to-be-predicted couplings, percent uncertainty at the fixed point, and percent uncertainty at the low scale for the models introduced in Sec. 2. The uncertainties are defined w.r.t. the 1-loop results of Sec. 2, cf. Eq. (39).



1.0
0.8
0.6
0.4
0.2
M_{PL}
y_ν
y_N
y_t
20
40
60
80
100
120
$\text{Log}_{10}[\mu/\text{GeV}]$
(a)
1.0
0.8
0.6
0.4
0.2
0.0
M_{PL}
y_t
y_{LQ}
20
40
60
80
100
120
$\text{Log}_{10}[\mu/\text{GeV}]$
(b)
1-loop (dashed)
2-loop (solid)

2. Dependence on the position of the Planck scale

Gauge couplings

$$r^*_{g,d}(n \text{ loops}) \equiv \frac{g^*_d}{g^*_Y}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}^2_\epsilon}},$$

$$r^*_{g,\epsilon}(n \text{ loops}) \equiv \frac{g^*_\epsilon}{g^*_Y}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}^2_\epsilon}},$$

$\tilde{b}_i \equiv b_i + \Pi^{(i)*}_{n\geq 2}$. → moving the Planck scale back and forth does not affect the predicted ratios

Yukawa couplings

$$y^*_2(2 \text{ loops}) \approx \left[F_1\left(a^{(r)}_j\right)\left(y^{*2}_1(1 \text{ loop}) + \delta y^{*2}_1\right) + G_1\left(\sum_{l,k} a'^{(r)}_{lk} g^*_l g^*_k \,;\, a^{(r)}_{j\neq 1}\right) + H_1\left(\tilde{\Pi}^{(r)*}_2 \,;\, a^{(r)}_{j\neq 1}\right)\right]^{1/2},$$

→ does affect the predicted ratios

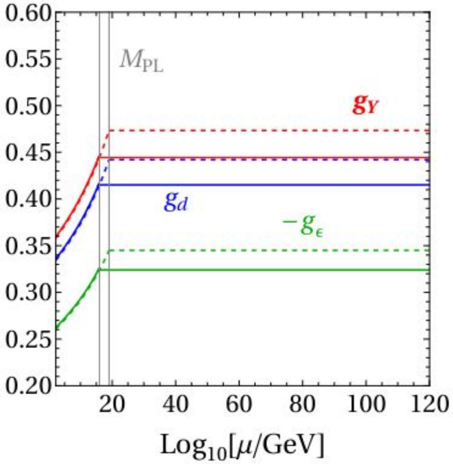
The percent uncertainty

$$\frac{\delta r^*_{g(y),i}}{r^*_{g(y),i}} = \frac{r^*_{g(y),i}(M_{\text{Pl}} \neq 10^{19}\,\text{GeV}) - r^*_{g(y),i}(M_{\text{Pl}} = 10^{19}\,\text{GeV})}{r^*_{g(y),i}(M_{\text{Pl}} = 10^{19}\,\text{GeV})} \qquad r^*_{g,k} = g^*_k/y^*_1,\; r^*_{y,2} = y^*_2/y^*_1$$

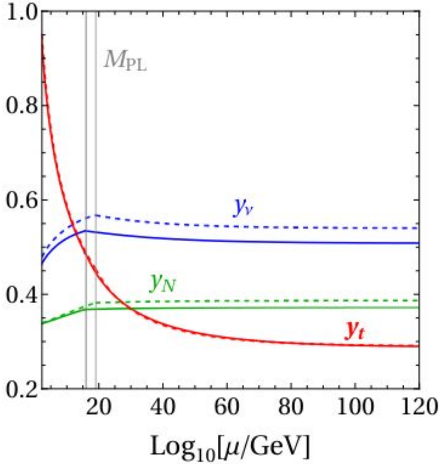
The percent uncertainty on the Yukawa-coupling ratios propagates (neglecting 2-loop contribution)

$$\frac{\delta r^*_{y,2}}{r^*_{y,2}} = \frac{1}{r^{*2}_{y,2}} G_1\left(\sum_{l,k} a'^{(r)}_{lk}\, r^*_{g,l} r^*_{g,k} \cdot \frac{1}{2}\left[\frac{\delta r^*_{g,l}}{r^*_{g,l}} + \frac{\delta r^*_{g,k}}{r^*_{g,k}}\right];\, a^{(r)}_{j\neq 1}\right)$$

$\boldsymbol{B-L}$	f_g	g_Y^*	g_d^*	g_ϵ^*	$\delta g_Y^*/g_Y^*$	$\delta g_d^*/g_d^*$	$\delta g_\epsilon^*/g_\epsilon^*$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
10^{20} GeV	0.0102	0.4843	0.4522	-0.3530	2.3%	2.3%	2.3%	0.0%	0.0%
10^{16} GeV	0.0086	0.4445	0.4151	-0.3240	-6.1%	-6.1%	-6.1%	0.0%	0.0%
	f_y	y_t^*	y_ν^*	y_N^*	$\delta y_t^*/y_t^*$	$\delta y_\nu^*/y_\nu^*$	$\delta y_N^*/y_N^*$	$\delta y_\nu/y_\nu(M_t)$	$\delta y_N/y_N(M_t)$
10^{20} GeV	0.0020	0.2914	0.5523	0.3927	0.4%	2.3%	1.5%	1.3%	0.3%
10^{16} GeV	0.0020	0.2869	0.5069	0.3715	-1.1%	-6.1%	-4.0%	-3.7%	-0.9%
$\boldsymbol{S_3}$ **LQ**	f_y	y_t^*	$y_{\rm LQ}^*$		$\delta y_t^*/y_t^*$	$\delta y_{\rm LQ}^*/y_{\rm LQ}^*$		$\delta y_{\rm LQ}/y_{\rm LQ}(M_t)$	
10^{20} GeV	-0.0006	0.2309	0.1043		-1.3%	-7.8%		-5.1%	
10^{16} GeV	0.00002	0.2422	0.1337		3.5%	18.1%		10.1%	



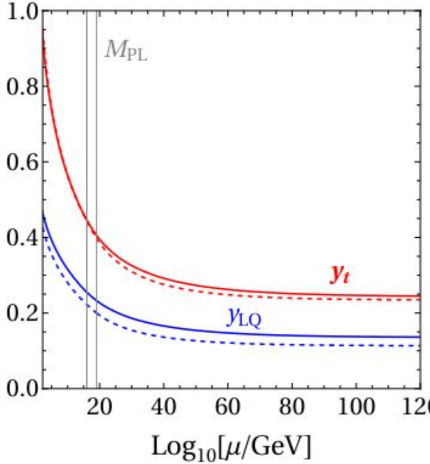
(a)



(b)

10^{16} GeV (dashed)

10^{19} GeV (solid)



(c)

3. Scale-dependence of the gravitational correction

Gauge couplings

$$\frac{dg_i}{dt} = \beta_i^{(\text{matter})} - f_g\, g_i$$

B-L model:
$$\frac{d}{dt}\left(\frac{g_d}{g_Y}\right) = \frac{1}{g_Y}\left(\beta_d^{\text{matter}} - \frac{g_d}{g_Y}\beta_Y^{\text{matter}}\right)[t] \equiv F_d(g_Y(t), g_d(t), g_\epsilon(t), \ldots)$$
$$\frac{d}{dt}\left(\frac{g_\epsilon}{g_Y}\right) = \frac{1}{g_Y}\left(\beta_\epsilon^{\text{matter}} - \frac{g_\epsilon}{g_Y}\beta_Y^{\text{matter}}\right)[t] \equiv F_\epsilon(g_Y(t), g_d(t), g_\epsilon(t), \ldots)$$

exact RG invariants

$$\begin{aligned}
\frac{dg_Y}{dt} &= \frac{1}{16\pi^2}\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right) g_Y^3 - f_g\, g_Y \\
\frac{dg_d}{dt} &= \frac{1}{16\pi^2}\left[\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right) g_d g_\epsilon^2 + \left(b_d + \Pi^{(d)}_{n\geq 2}\right) g_d^3 + \left(b_\epsilon + \Pi^{(\epsilon)}_{n\geq 2}\right) g_d^2 g_\epsilon\right] - f_g\, g_d \\
\frac{dg_\epsilon}{dt} &= \frac{1}{16\pi^2}\left[\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right)\left(g_\epsilon^3 + 2g_Y^2 g_\epsilon\right) + \left(b_d + \Pi^{(d)}_{n\geq 2}\right) g_d^2 g_\epsilon \right. \\
&\quad \left. + \left(b_\epsilon + \Pi^{(\epsilon)}_{n\geq 2}\right)\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.
\end{aligned}$$

Apply total derivative to 1st eq and focus on a sequence of infinitesimal scale intervals . $t_2 < t_1 < t_0$

$$\begin{aligned}
\frac{g_d(t_1)}{g_Y(t_1)} &= r^*_{g,d} + (t_1 - t_0) F_d(g_Y^*, g_d^*, g_\epsilon^*, \ldots) \\
\frac{g_d(t_2)}{g_Y(t_2)} &= \frac{g_d(t_1)}{g_Y(t_1)} + (t_2 - t_1) F_d(g_Y(t_1), g_d(t_1), g_\epsilon(t_1), \ldots) \\
\frac{g_d(t_3)}{g_Y(t_3)} & \quad \ldots
\end{aligned}$$

$$\begin{aligned}
r^*_{g,d}(n \text{ loops}) &\equiv \frac{g_d^*}{g_Y^*}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}\,, \\
r^*_{g,\epsilon}(n \text{ loops}) &\equiv \frac{g_\epsilon^*}{g_Y^*}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}\,,
\end{aligned}$$

$$F_d(g_Y^*, \ldots) = 0$$
$$F_\epsilon(g_Y^*, \ldots) = 0$$
$$\downarrow$$
$$g_\epsilon(t_1)/g_Y(t_1) = r^*_{g,\epsilon}$$
$$g_d(t_1)/g_Y(t_1) = r^*_{g,d}$$

Slope functions:

$$F_d(g_Y(t_1), g_d(t_1), \ldots) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\tilde{b}_Y(t_1) r^{*\,2}_{g,\epsilon} + \tilde{b}_d(t_1) r^{*\,2}_{g,d} + \tilde{b}_\epsilon(t_1) r^*_{g,\epsilon} r^*_{g,d} - \tilde{b}_Y(t_1)\right] r^*_{g,d}$$

$$F_\epsilon(g_Y(t_1), g_d(t_1), \ldots) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\left(\tilde{b}_Y(t_1) r^*_{g,\epsilon} + \tilde{b}_\epsilon(t_1) r^*_{g,d}\right)\left(1 + r^{*\,2}_{g,\epsilon}\right) + \tilde{b}_d(t_1)\, r^{*\,2}_{g,d}\, r^*_{g,\epsilon}\right]$$

$$\tilde{b}_i(t) \equiv b_i + \Pi^{(i)}_{n\geq 2}(t)$$

3. Scale-dependence of the gravitational correction

Gauge couplings

$$\frac{dg_i}{dt} = \beta_i^{(\mathrm{matter})} - f_g\, g_i$$

B-L model:

$$\frac{d}{dt}\left(\frac{g_d}{g_Y}\right) = \frac{1}{g_Y}\left(\beta_d^{\mathrm{matter}} - \frac{g_d}{g_Y}\beta_Y^{\mathrm{matter}}\right)[t] \equiv F_d(g_Y(t), g_d(t), g_\epsilon(t), \ldots)$$

$$\frac{d}{dt}\left(\frac{g_\epsilon}{g_Y}\right) = \frac{1}{g_Y}\left(\beta_\epsilon^{\mathrm{matter}} - \frac{g_\epsilon}{g_Y}\beta_Y^{\mathrm{matter}}\right)[t] \equiv F_\epsilon(g_Y(t), g_d(t), g_\epsilon(t), \ldots)$$

exact RG invariants

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\left(b_Y + \Pi_{n\geq 2}^{(Y)}\right) g_Y^3 - f_g\, g_Y$$

$$\frac{dg_d}{dt} = \frac{1}{16\pi^2}\left[\left(b_Y + \Pi_{n\geq 2}^{(Y)}\right) g_d g_\epsilon^2 + \left(b_d + \Pi_{n\geq 2}^{(d)}\right) g_d^3 + \left(b_\epsilon + \Pi_{n\geq 2}^{(\epsilon)}\right) g_d^2 g_\epsilon\right] - f_g\, g_d$$

$$\frac{dg_\epsilon}{dt} = \frac{1}{16\pi^2}\left[\left(b_Y + \Pi_{n\geq 2}^{(Y)}\right)\left(g_\epsilon^3 + 2 g_Y^2 g_\epsilon\right) + \left(b_d + \Pi_{n\geq 2}^{(d)}\right) g_d^2 g_\epsilon + \left(b_\epsilon + \Pi_{n\geq 2}^{(\epsilon)}\right)\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.$$

Apply total derivative to 1st eq and focus on a sequence of infinitesimal scale intervals . $t_2 < t_1 < t_0$

$$\frac{g_d(t_1)}{g_Y(t_1)} = r^*_{g,d} + (t_1 - t_0) F_d(g_Y^*, g_d^*, g_\epsilon^*, \ldots)$$

$$\frac{g_d(t_2)}{g_Y(t_2)} = \frac{g_d(t_1)}{g_Y(t_1)} + (t_2 - t_1) F_d(g_Y(t_1), g_d(t_1), g_\epsilon(t_1), \ldots)$$

$$\frac{g_d(t_3)}{g_Y(t_3)} \quad \ldots$$

$$r^*_{g,d}(n \text{ loops}) \equiv \frac{g_d^*}{g_Y^*}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}},$$

$$r^*_{g,\epsilon}(n \text{ loops}) \equiv \frac{g_\epsilon^*}{g_Y^*}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}},$$

$$F_d(g_Y^*, \ldots) = 0$$

$$F_\epsilon(g_Y^*, \ldots) = 0$$

$$g_\epsilon(t_1)/g_Y(t_1) = r^*_{g,\epsilon}$$

$$g_d(t_1)/g_Y(t_1) = r^*_{g,d}$$

$$F_d(g_Y(t_1), g_d(t_1), \ldots) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\tilde{b}_Y(t_1) r^{*\,2}_{g,\epsilon} + \tilde{b}_d(t_1) r^{*\,2}_{g,d} + \tilde{b}_\epsilon(t_1) r^*_{g,\epsilon} r^*_{g,d} - \tilde{b}_Y(t_1)\right] r^*_{g,d}$$

$$F_\epsilon(g_Y(t_1), g_d(t_1), \ldots) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\left(\tilde{b}_Y(t_1) r^*_{g,\epsilon} + \tilde{b}_\epsilon(t_1) r^*_{g,d}\right)\left(1 + r^{*\,2}_{g,\epsilon}\right) + \tilde{b}_d(t_1)\, r^{*\,2}_{g,d}\, r^*_{g,\epsilon}\right]$$

$$\tilde{b}_i(t) \equiv b_i + \Pi_{n\geq 2}^{(i)}(t)$$

$$F_d(g_Y(t_1), g_d(t_1), ...) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\tilde{b}_Y(t_1) r_{g,\epsilon}^{*\,2} + \tilde{b}_d(t_1) r_{g,d}^{*\,2} + \tilde{b}_\epsilon(t_1) r_{g,\epsilon}^{*} r_{g,d}^{*} - \tilde{b}_Y(t_1)\right] r_{g,d}^{*}$$

$$F_\epsilon(g_Y(t_1), g_d(t_1), ...) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\left(\tilde{b}_Y(t_1) r_{g,\epsilon}^{*} + \tilde{b}_\epsilon(t_1) r_{g,d}^{*}\right)\left(1 + r_{g,\epsilon}^{*\,2}\right) + \tilde{b}_d(t_1)\, r_{g,d}^{*\,2}\, r_{g,\epsilon}^{*}\right]$$

$$\tilde{b}_i(t) \equiv b_i + \Pi_{n\geq 2}^{(i)}(t)$$

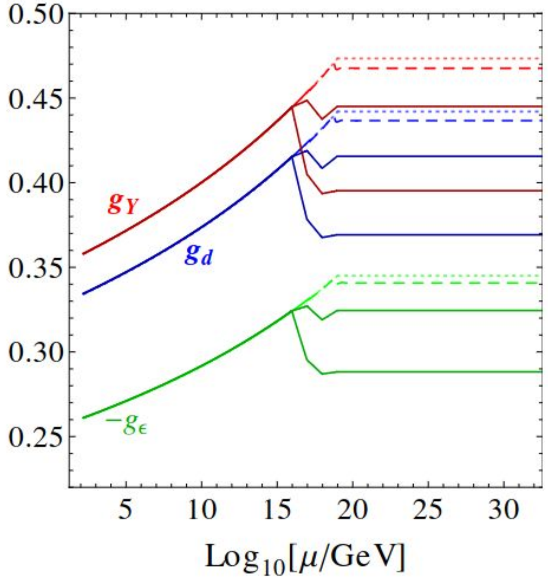
n=1: $F_d^{(n=1)}(t_1) = 0$ and $F_\epsilon^{(n=1)}(t_1) = 0 \longrightarrow$ $g_d(t)/g_Y(t) = r_{g,d}^{*}$, $g_\epsilon(t)/g_Y(t) = r_{g,\epsilon}^{*}$

boundary conditions

$$r_{g,d}^{*}(n \text{ loops}) \equiv \frac{g_d^{*}}{g_Y^{*}}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}},$$

$$r_{g,\epsilon}^{*}(n \text{ loops}) \equiv \frac{g_\epsilon^{*}}{g_Y^{*}}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}},$$

→ the predictions of AS for the gauge couplings do not depend on the particular functional form of the gravitational contribution $f_g(t)$



RG flow of the hypercharge (red), dark gauge (blue), and kinetic mixing (green) couplings of the $B-L$ model at 1 loop. Dotted lines correspond to the benchmark scenario with f_g constant above the Planck scale. Darker solid lines indicate two arbitrary parametrizations of $f_g(t)$. Dashed lines correspond to the FRG results of

Yukawa couplings

$$\frac{dy_j}{dt} = \beta_j^{(\mathrm{matter})} - f_y\, y_j$$

The ratio of two irrelevant Yukawa couplings does not depend on f_y

$$\frac{d}{dt}\left(\frac{y_2}{y_1}\right) = \frac{1}{y_1}\left(\beta_2^{\mathrm{matter}} - \frac{y_2}{y_1}\beta_1^{\mathrm{matter}}\right)[t] \equiv G(y_j(t), g_k(t), \ldots)$$

for the Yukawa couplings the t-dependence of f_g and f_y can impact the running of the y_2/y_1 at 1loop

where $\frac{dy_r}{dt} = \frac{y_r}{16\pi^2}\left(\sum_j a_j^{(r)} y_j^2 - \sum_{l,k} a_{lk}^{\prime(r)} g_l g_k + \sum_{n\geq 2}\widetilde{\Pi}_n^{(r)}\right) + \sum_{m,p,q\neq r}\frac{y_m y_p y_q}{16\pi^2}\left(a''_{mpq} + \sum_{n\geq 2}\tilde{\Delta}_n^{(mpq)}\right) - f_y y_r,$

Repeating the infinitesimal-step analysis

$$\frac{y_2(t_1)}{y_1(t_1)} = \frac{y_2^*}{y_1^*} + (t_1 - t_0)\, G(y_j^*, g_k^*, \ldots)\,,$$

$$\frac{y_2(t_2)}{y_1(t_2)} = \frac{y_2(t_1)}{y_1(t_1)} + (t_2 - t_1)\, G(y_j(t_1), g_k(t_1), \ldots)$$

$$\cdots$$

one finds, that $G(y_j^*, g_k^*, \ldots) = 0$, so that $y_2(t_1)/y_1(t_1) = y_2^*/y_1^*$. However, even at 1 loop, $G(y_j(t_1), g_k(t_1), \ldots) \neq 0.$

$$G(y_j(t_1), g_k(t_1), \ldots) = \frac{y_1^2(t_1)\, r_{y,2}^*}{16\pi^2}\left[\sum_j \left(a_j^{(2)} - a_j^{(1)}\right)\frac{y_j^2(t_1)}{y_1^2(t_1)} - \sum_{l,k}\left(a_{lk}^{\prime(2)} - a_{lk}^{\prime(1)}\right)\frac{g_l(t_1) g_k(t_1)}{y_1^2(t_1)}\right] + \frac{r_{y,2}^*}{16\pi^2}\sum_{n\geq 2}\left[\widetilde{\Pi}_n^{(2)}(t_1) - \widetilde{\Pi}_n^{(1)}(t_1)\right]. \quad (5$$

Yukawa couplings

$$\frac{dy_j}{dt} = \beta_j^{(\mathrm{matter})} - f_y\, y_j$$

The ratio of two irrelevant Yukawa couplings

$$\frac{d}{dt}\left(\frac{y_2}{y_1}\right) = \frac{1}{y_1}\left(\beta_2^{\mathrm{matter}} - \frac{y_2}{y_1}\beta_1^{\mathrm{matter}}\right)[t] \equiv G(\ldots$$

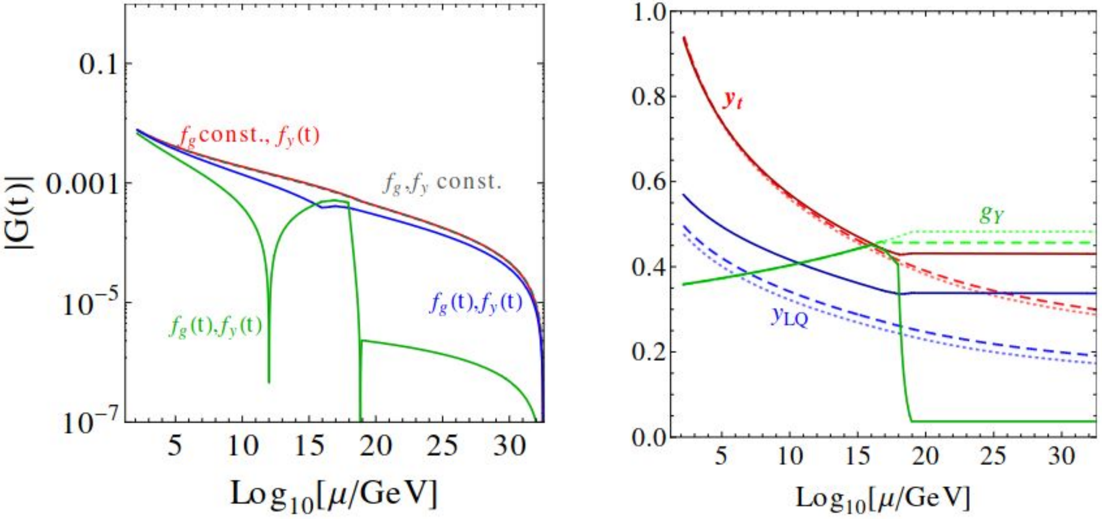
where $\frac{dy_r}{dt} = \frac{y_r}{16\pi^2}\left(\sum_j a_j^{(r)} y_j^2 - \sum_{l,k} a_{lk}'^{(r)} g_l g_k + \sum_{n\geq 2} \widetilde{\Pi}_n^{(r)}\right) + \sum_{m,p,q\neq r} \ldots$

Repeating the infinitesimal-step analysis

$$\frac{y_2(t_1)}{y_1(t_1)} = \frac{y_2^*}{y_1^*} + (t_1 - t_0)\, G(y_j^*, g_k^*, \ldots)\,,$$

$$\frac{y_2(t_2)}{y_1(t_2)} = \frac{y_2(t_1)}{y_1(t_1)} + (t_2 - t_1)\, G(y_j(t_1), g_k(t_1), \ldots$$

$$\ldots$$



(a) (b)

one finds, that $G(y_j^*, g_k^*, \ldots) = 0$, so that $y_2(t_1)/y_1(t_1) = y_2^*/y_1^*$. However, even at 1 loop, $G(y_j(t_1), g_k(t_1), \ldots) \neq 0$.

$$G(y_j(t_1), g_k(t_1), \ldots) = \frac{y_1^2(t_1)\, r_{y,2}^*}{16\pi^2}\left[\sum_j \left(a_j^{(2)} - a_j^{(1)}\right)\frac{y_j^2(t_1)}{y_1^2(t_1)} - \sum_{l,k}\left(a_{lk}'^{(2)} - a_{lk}'^{(1)}\right)\frac{g_l(t_1) g_k(t_1)}{y_1^2(t_1)}\right] + \frac{r_{y,2}^*}{16\pi^2}\sum_{n\geq 2}\left[\widetilde{\Pi}_n^{(2)}(t_1) - \widetilde{\Pi}_n^{(1)}(t_1)\right]. \quad (5$$

$y_2(t)/y_1(t)$ remains fairly stable

Conclusions

- Authors have investigated the issue of the theoretical uncertainties associated with predictions for irrelevant Lagrangian couplings emerging from trans-Planckian AS;

- In the gauge sector the uncertainty induced by relaxing any of the simplifying assumptions never exceeds the 1% level;

- For the Yukawa sector: if the predicted Yukawa couplings are of comparable size to the irrelevant gauge couplings, the uncertainties remain at bay, not exceeding ~ 10% at the fixed point if higher-order corrections are included, or if the Planck scale is moved to, e.g., 10^16 GeV.

- Potentially more dangerous uncertainties could stem from considering the non-trivial scale dependence of the gravitational contributions to the matter beta functions ($f_g(t)$, $f_y(t)$), as in this case we lose the ability of determining the actual value of the Yukawa couplings at the fixed point. However, in the range of variability of the gravitation parameters that can be realistically expected in the framework of the FRG, the resulting uncertainty is moderate.