## Update on Statistical Uncertainty of $D^{0} \rightarrow \pi^{+} K^{-}$SSA

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## Prescription for SSA (and uncertainty) Calculation



Figure 1: Illustrative plot from PHENIX : $\pi^{0}$ (above) and $\eta$ (below) from di-photon invariant mass spectra

- Following the standard practice at STAR, PHENIX and COMPASS:
- From invariant mass spectra in azimuthal $(\phi)$ slices, define signal region (often $2 \sigma$ around the peak), count total, calculate raw asymmetry (and uncertainty)
- Far from signal peak, count pure background, calculate background asymmetry (and uncertainty)
- Correct 'raw' asymmetry with background asymmetry (and relative contribution) to extract 'signal' asymmetry (and uncertainty)


## Now Some Explicit Equations

Transverse Single Spin Asymmetry :

$$
A_{N}(\phi)=\frac{1}{P\langle | \cos (\phi)| \rangle} \frac{N(\phi)-\mathcal{R} \cdot N(\phi+\pi)}{N(\phi)+\mathcal{R} \cdot N(\phi+\pi)}
$$

where $P$ is beam polarization, $\langle | \cos (\phi)\left\rangle=\frac{\int_{\phi_{1}}^{\phi_{2}} \cos (\phi) d \phi}{\phi_{2}-\phi_{1}}\right.$ is the average of the cosine of azimuth in the $\phi$ bin, $\mathcal{R}$ is relative luminosity for opp. pol. dir. of beam, N's are counts in $\phi$ bins. One can use $N(\phi)=N_{L}$ and $N(\phi+\pi)=N_{R}$ for left and right as simplified notation
Statistical Uncertainty of SSA (propagation of error assuming two independent variables $N(\phi)$ and $N(\phi+\pi))$ :

$$
\sigma_{A_{N}}(\phi)=\frac{1}{P\langle | \cos (\phi)| \rangle} \frac{2 \mathcal{R} \cdot N(\phi) \cdot N(\phi+\pi)}{(N(\phi)+\mathcal{R} N(\phi+\pi))^{2}} \sqrt{\left(\frac{\sigma_{N(\phi)}}{N(\phi)}\right)^{2}+\left(\frac{\sigma_{N(\phi+\pi)}}{N(\phi+\pi)}\right)^{2}}
$$

## Simplifications

Assume $\mathcal{R} \sim 1, N(\phi) \sim N(\phi+\pi)=N$ where N is the count of candidates in a $\phi$ bin ( $N=N_{\text {detected }} / n$ if you have n bins in azimuth) and assume Poisson distribution of counts (so that $\sigma_{N}=\sqrt{N}$ )
Simplified version of statistical uncertainty of SSA :

$$
\sigma_{A_{N}}(\phi)=\frac{1}{P\langle | \cos (\phi)| \rangle} \frac{1}{\sqrt{2 N}}
$$

## Finally: The Signal

Corrected signal SSA :

$$
A_{N}^{S i g}(\phi)=\frac{A_{N}^{\text {Raw }}(\phi)-r \cdot A_{N}^{B k g}(\phi)}{1-r}
$$

where $r=\frac{N_{B k g}}{N_{\text {raw }}}$ is background contribution to raw/total count under the signal peak

Corrected signal statistical uncertainty of SSA :

$$
\sigma_{A_{N}^{\text {sig }}}(\phi)=\frac{\sqrt{\sigma_{A_{N}^{\text {Raw }}}^{2}(\phi)+r^{2} \sigma_{A_{N}^{\text {Bkg }}}^{2}(\phi)}}{1-r}
$$

## Procedure

- After background suppression cuts, scale MC counts of signals in $x_{F}$ bins to get counts in 1 year of data
- Using S/B ratio from analysis (1:8), estimate raw/total and background counts ( $N_{t}=9 N_{s}$ and $N_{b}=8 N_{s}$ respectively) - done because we lack enough bkg MC to get bkg count directly
- For each $x_{F}$ bin, distribute $N_{t}$ and $N_{b}$ in $12 \phi$ bins, estimate raw and bakcground uncertainties in each $\phi$ bin
- For each pair of $(\phi, \phi+\pi)$ bins, extract corrected signal uncertainty $\sigma_{A_{N}}(\phi)$
- For $x_{F}$ bin, combine uncertainties for independent measurements in 6 (pairs of left-right) $\phi$ bins

$$
\sigma_{A_{N}}\left(x_{F}\right)=\frac{1}{\sqrt{\sum_{i=1}^{6} \frac{1}{\sigma_{A_{N}}^{2}\left(\phi_{i}\right)}}}
$$

## Calculations

- From 'ideal case' simulation: 4 Million open-charm events $\Rightarrow 2$ Million $D^{0} \rightarrow \pi^{+} K^{-}$(forced decay)
- From cross-sections and 1 year integrated luminosity : 240 Million $D^{0} \rightarrow \pi^{+} K^{-}$produced
- A factor of 120 gain over counts from MC - I applied BR again by mistake in last calculation - underestimated signal
- MC analysis signal counts in $x_{F}$ bins : $2416\left(0.2 \leq x_{F} \leq 0.3\right), 841$ $\left(0.3 \leq x_{F} \leq 0.5\right), 22\left(x_{F}>0.5\right)$
- Use 12 equal azimuthal $(\phi)$ bins and polarization $P=0.7$
- Statistical Uncertainties in $x_{F}$ bins : $0.0156,0.0265,0.1640$


## Projected Asymmetry of $A_{N}^{D^{0}}$




Statistical uncertainties are for $D^{0}$ counts ONLY, whereas calculations are for INCLUSIVE $D / \bar{D}$. So, in a proper comparison, uncertainties will be even shorter (by including $\bar{D}^{0}, D^{+}, D^{-}$).
We get very precise measurement here, but remember this is the 'ideal case' and assumes full statistics of produced counts. Reality will be somewhat different.

## Outlook

- A big 'thank you' to Igor Denisenko - long discussions with him lead to more careful and more detailed calculation
- Igor will probably demonstrate a parallel method of estimating statistical uncerainty in next physics meeting - following the method of least squares in the pdg statistics chapter - calculating the covariance matrix for a linear combination of two independent functions
- Currently looking at $D^{+} \rightarrow \pi^{+} \pi^{+} K^{-}$reconstruction
- Also looking at a multivariate analysis for the ideal $D^{0}$ case with Dimitrije Maletic to see if that can help

