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Neural Network Analysis of S-Star Dynamics: Implications for Modified Gravity

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Aim of the research

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Neural network analysis of S-star dynamics: implications for modified gravity

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- Analyse the dynamics of S-stars, which provide a natural laboratory for testing different gravity theories, using neural networks.
- Show that via neural networks, namely physics informed neural networks (PINN), one can obtain the orbital parameters of the stars and make constrains on different gravity theories, e.g. Λ-gravity.

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Λ-gravity

Newton's shell theorem

a The gravitational field of a sphere acting on external objects can be considered as though all of it mass is concentrated at a point at its center

b Force-free field inside a shell

$$V(r) = -\frac{GM}{r}$$

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Generalized shell theorem (V.G. Gurzadyan (1985))

The most generalized function satisfying shell theorem (a) but not (b), lets us introduce the cosmological constant Λ into the Newtonian gravity

$$V(r) = -\frac{GM}{r} - \frac{\Lambda c^2 r^2}{6}$$

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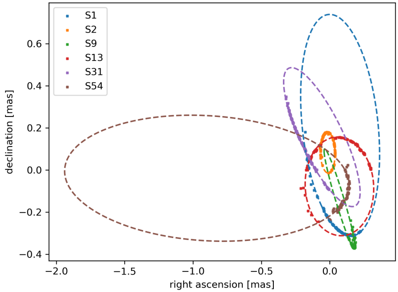
S-stars and S-2 star precession



The coordinates of the considered stars: points indicate the observed data and dashed lines are the ellipses obtained from the orbital parameters of the Keplerian fit by S. Gillessen et al. (2017)

Statistical analysis of S2-star data within the first-order PPN by the GRAVITY Collaboration (R. Abuter et al. (2020)), reports a deviation from Schwarzschild's precession $\delta\varphi_{SP}$ by a magnitude of $f_{SP} = 1.10 \pm 0.19$

$$\delta\varphi_{SP} = 3\frac{r_g}{a(1-e^2)}\pi$$

Taking into account the Λ-term to the precession, we obtain an additional term $\delta\varphi_\Lambda$ to the total observed precession $\delta\varphi_{GRAV}$, which can be used to compensate the observed f_{SP}, i.e. $\delta\varphi_{GRAV} = \delta\varphi_{SP} + \delta\varphi_\Lambda$

$$\delta\varphi_\Lambda = \frac{2a^3(1-e^2)^{\frac{1}{2}}\pi}{r_g}\Lambda$$

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Introduction to PINN

Let the state of a system is governed by the following equation

$$F(x, y, y', \ldots, y^{(n)}) = 0$$

Then the physical loss is given by the following loss function:

$$L_{phys}(f(x), x) = F^2\left(x, f(x), f'(x), f''(x), \ldots, f^{(n)}(x)\right).$$

To calculate the total loss value one should use the actual data $\{(y_i, x_i)\}_{i=1}^{N}$ and sample $\{(\hat{x}_i)\}_{i=1}^{N_p}$ points from a larger data domain. Then the loss function may be calculated:

$$\mathcal{L}(f(\cdot), y, x, \hat{x}) = \frac{1}{N}\sum_{i=1}^{N} L_{reg}(y_i, f(x_i)) + \frac{\alpha}{N_p}\sum_{i=1}^{N_p} L_{phys}(f(\hat{x}_i), \hat{x}_i).$$

where α is a given training step-dependent regularization parameter.

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Physical model

As for physical model to train the PINN we used the Schwarzchild metric, written in the form with relativistic anomaly χ, which allows to explicitly see the GR contribution to the precession

$$u = \frac{\mu}{M}(1 + e\cos\chi)$$

$$\left(\frac{d\chi}{d\varphi}\right)^2 = 1 - 2\mu(3 + e\cos\chi)$$

$$\frac{d^2\chi}{d\varphi^2} = \mu e \sin\chi$$

where (r, φ) are polar coordinates of the star, $u = r^{-1}$, and $\mu = \frac{M}{p}$.

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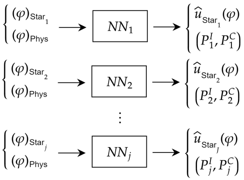
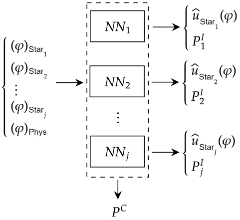
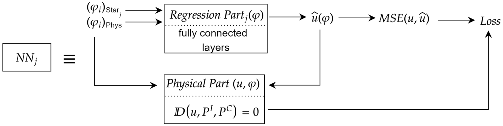
Training schemes



(a) Individual training: Networks are trained separately for each star. They do not have any common parameters.



(b) Parallel training: Networks are trained parallelly, so they have common parameters like the central mass.



(c) NN block: A single network which is used to train on a single star data. These blocks are used in both individual and parallel training cases.

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Metrics and parallel training results 1

$$\mathcal{M}_{\text{model-data}} = \mathbb{E}\left[1 - \frac{|u_{\text{Model}} - u_{\text{Star}}|}{u_{\text{Star}}}\right],$$

$$\mathcal{M}_{\text{data-physics}} = \mathbb{E}\left[1 - \frac{|u_{\text{Phys}} - u_{\text{Star}}|}{u_{\text{Star}}}\right],$$

$$\mathcal{M}_{\text{model-physics}} = \mathbb{E}\left[1 - \frac{|u_{\text{Model}} - u_{\text{Phys}}|}{\frac{1}{2}(u_{\text{Model}} + u_{\text{Phys}})}\right],$$

Star Name	e	$\hat{e}$	p [Au]	$\hat{p}$ [Au]	$\mathcal{M}_{\text{model-data}}$	$\mathcal{M}_{\text{data-physics}}$
S2	0.884	0.884	228	223	0.9755	0.9768
S13	0.425	0.418	1796	1706	0.9844	0.9829
S31	0.550	0.552	2601	2675	0.9803	0.9826
S54	0.893	1.001	2017	4.787	0.9660	< 0

Table: Results for GR case: (e,p) orbital parameters from S. Gillessen et al. (2017), $(\hat{e},\hat{p})$ the network prediction.

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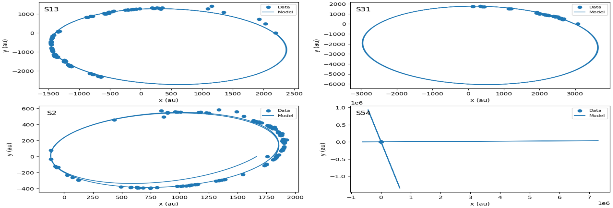
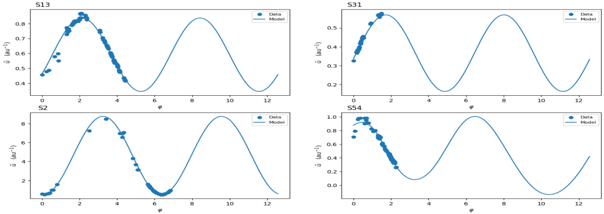
Parallel training results 2





On the above figure, the predicted trajectories are shown in the Cartesian coordinates (x [Au],y [Au]), where $(0, 0)$ is the central mass. On the bottom figure, the models' regression results are shown.

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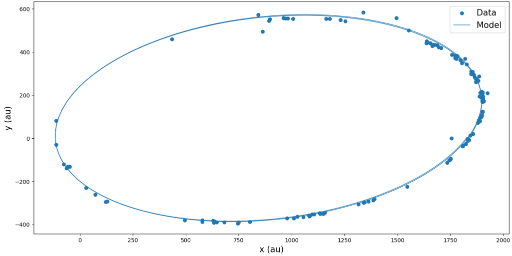
Individual training on S2



The S2 star trajectory in (x [Au],y [Au]) coordinates.

$$\hat{e} = 0.88512 \pm 0.00001, \quad \hat{p} = 219.2 \pm 0.2\,[\text{Au}], \quad \hat{M} = 0.04\,[\text{Au}],$$

$$\mathcal{M}_{\text{model-data}} = 0.9865, \quad \mathcal{M}_{\text{data-phyics}} = 0.9881, \quad \mathcal{M}_{\text{model-physics}} = 0.9977.$$

It is important to note that, after a certain point during training, the value of $\hat{M}$ was fixed to be equal 0.04 [au]. Although the model was able to reach the given value and "understand" the physical meaning of $\hat{M}$, after that point its value was not stable due to the quality of the data.

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Final results

During training, we calculate the two values of the precession rate:

$$\delta\varphi_{\text{Reg}} = \varphi\left(\min_1 \hat{u}\right) - \varphi\left(\min_0 \hat{u}\right) - 2\pi, \quad \text{Precession rate of regression part } \hat{u}(\varphi).$$

$$\delta\varphi_{\text{Phys}} = 3\frac{\hat{r}_g}{\hat{p}}\pi, \quad \text{Precession rate of physical part.}$$

Taking the moving average for every 500 epochs we obtain the following results

$$\delta\varphi_{\text{Reg}} = 11.84'; \quad \sigma_{\text{Reg}} = 0.03'$$
$$\delta\varphi_{\text{Phys}} = 11.82'; \quad \sigma_{\text{Phys}} = 0.02'$$

Using the equations Λ-term precession and the total precession, also taking $\delta\varphi_{\text{Reg}}(+3\sigma_{\text{Reg}})$ as the total precession rate and $\delta\varphi_{\text{Phys}}(-3\sigma_{\text{Phys}})$ as the Schwarzschild precession rate, we obtain for the Λ the following upper constraint

$$\Lambda \leq 5.8 \times 10^{-38}\ [\text{m}]^{-2}.$$

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Conclusion

- The neural network was able to “see” the Schwarzschild precession for S2 star, which made it possible to find the precession rate for both, based on the regression part and the physical part of the network.

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- The neural network was able to “see” the Schwarzschild precession for S2 star, which made it possible to find the precession rate for both, based on the regression part and the physical part of the network.
- The regressed part is more “flexible” and is directly related to the observational data, so the difference in the values of the precession rates can be attributed to an additional precession that occurs due to terms not entered in the physical model, which is Λ-term.

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- Our analysis reveals the efficiency of the neural networks in the study of the S-star dynamics and that stronger constraints on GR and gravity modifications can be expected from forthcoming observational data.

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Thank you!